Unique Implementation with Market-Based Interventions*

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Abstract

In a large class of economies, competitive equilibria can attain the efficient allocation but coordination failures among private agents can lead to inefficient equilibrium outcomes. This generates a role for policy to uniquely implement the desired outcome. We consider an economy with a firm that needs to raise a fixed amount from a large number of investors in order to finance an investment project. Absent government intervention, there are multiple equilibria. In particular, there are cases when socially efficient projects are not financed due to coordination failures. We study the optimal intervention for a government that evaluates payoffs by using the most adversarial equilibrium selection rule. The government does not know the investment project's return and and can learn about it from market outcomes. We show two main results. First, there does not exist a policy that uniquely implements the efficient outcome. Second, there exist policies which can that uniquely implement allocations that approximate the efficient one with arbitrary precision. These policies require ex-post inefficient interventions by the government. That is, to learn the state of the economy, the government must commit to funding projects that it knows are inefficient.

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Introduction

In a large class of economies, coordination failures prevent the efficient allocation from being uniquely implemented. Examples include the bank run model of Diamond and Dybvig (1983), models with endogenous financial frictions (Alvarez and Jermann (2000), Gu et al. (2013)), models with default and rollover risk (Cole and Kehoe (2000)). See the seminal contribution of Cooper and John (1988) for a general treatment. In some of these cases, private contracts can be suitably amended so that multiplicity is no longer an issue. However, there are also cases in which multiple equilibria emerge despite the presence of unrestricted private contracts. In such economies, there might be a role for government intervention to reduce multiplicity and potentially uniquely implement the efficient outcome.

This paper studies the design of robust policy interventions in economies where coordination failures occur in spite of unrestricted private contracts. In particular, we focus on *robust policy interventions*. That is, we study the optimal intervention for a government that evaluates payoffs according to the most adversarial equilibrium selection rule. To do so, we consider a simple investment model in which coordination failures occur due to non-convexities in the production technology. The government does not know the investment's payoff and relies on market prices to learn the state and make its intervention decision. We show three main results. First, no intervention scheme can uniquely implement the efficient outcome. Second, the optimal intervention scheme approximates the efficient allocation arbitrarily closely and involves subsidies to ex-post inefficient investment projects with strictly positive probability. Third, the intervention scheme does not necessarily worsen the moral hazard problem for the managers. Increasing the probability that good projects are funded increases the incentives to provide ex-ante effort, implying that effort is higher under the optimal intervention scheme as compared with the worst equilibrium.

We consider a simple economy where a firm has access to an investment project that requires a fixed input cost in order to yield returns next period. The manager of the firm designs contracts to raise funds from a continuum of investors. Investors are risk neutral and know the average return of the firm. Absent government intervention, we show that an equilibrium with investment co-exists with one without investment, if the average project returns are larger than the outside returns for the investor. The reason for the latter is a classic coordination failure: if one investor expects other investors not to lend to the firm it is optimal for her not to invest even if she knows the investment project is profitable because she anticipates that the firm will not be able to raise the resources to invest. If average returns are lower than the investors outside option then there is a unique equilibrium with no investment.

We consider the role for government intervention. Governments have the ability to tax investors and raise the required funds if needed. In this sense, the government is a big player and immune from coordination failures. However, the government has an informational disadvantage relative to the private agents: we assume that the government does not know the average return but can learn about it through the market price of private contracts. In the equilibrium with investment, the government can perfectly learn the state. However, in either the no-investment equilibrium when returns are high or the equilibrium when returns are low, the price of private contracts will be zero, implying that the government cannot learn the state.

The first main result is that there does not exist an intervention scheme that uniquely implements the efficient outcome. By the efficient outcome we mean invest if and only if the project returns are greater than the outside return for investors. The reason for this is that efficiency requires the government to not intervene if it observes a zero price. But then it is forced to not intervene in the no-investment equilibrium when project returns are high, implying that some good projects may not get funded.

To solve for the optimal policy, we follow a robust approach and consider the most adversarial selection mechanism from the government's perspective. Our second main result is there exists an optimal intervention scheme that can approximate the efficient allocation with arbitrary precision. As part of this policy, the government commits to funding projects that are ex-post inefficient. Consequently, prices are informative of the project returns which allows the government to ensure that good projects are funded. In the proof, we show that doing so only requires the government to commit to bad projects with a vanishingly small probability. We also show that the restriction to the market-based interventions is without loss of generality and that mechanisms with more general message spaces cannot improve outcomes.

One insight from our work is that commitment is needed to implement the best robust policy for reasons which are in sharp contrast to that with government bailouts. With bailouts, the optimal Ramsey policy has the government committing not to intervene to bail out ex-post efficient projects to provide the correct incentives to private agents ex-ante. Here, the government must commit to fund ex-post inefficient projects to ensure that efficient projects are always implemented. If the government cannot commit to such interventions, either there is no intervention and multiple equilibrium outcomes are possible, or the government directly funds all projects without acquiring any information, essentially shutting down private markets.

In light of the large literature on bailouts and moral hazard one may wonder if such mechanisms may distort the incentives of managers. Suppose the manager can take an action that affects project returns. While the fact that low return projects are occasionally funded has a negative impact on the managers incentives, the ability to always fund good projects increases their incentives to exert effort. As the probability of funding bad projects goes to zero, the negative incentive effects go to zero as well.

Related literature Our paper builds on a literature that studies unique implementation with private contracts. See for instance Winter (2004), Halac et al. (2020), Camboni and Porcellacchia (2021), and Halac et al. (2022). In particular, Halac et al. (2020) study the problem of a firm raising a fixed amount of funds from N investors who are heterogeneous in their endowments. As in our model, this fixed startup cost generates non-convexities which lead to multiple equilibria. They provide conditions under which private contracts can uniquely implement the investment outcome. The main difference with our environment is that we focus on conditions under which private contracts cannot achieve unique implementation. We show that these conditions are equivalent to the existence of sufficient collateral so that lenders can be compensated if investment does not place.

Our paper also relates to the macro literature that studies the role for policy to uniquely implement desired outcomes. See for instance Bassetto (2005), Atkeson et al. (2010), Kirpalani (2015), Roch and Uhlig (2018), Bocola and Dovis (2019), Sturm (2022) and Barthelemy and Mengus (2022). The main difference between our environment and this literature is the assumption that the government/policymaker cannot perfectly observe the state. One of our main results is to show that in this case the optimal mechanism requires both the participation of markets and the government, and that either agent in isolation can only achieve sub-optimal outcomes.

A closely related paper is Valenzuela-Stookey and Poggi (2020) who the study implementable outcomes when a principal (a government) takes an action based on information learned from market outcomes. Like in our paper, this implies that prices in turn can depend on this intervention policy leading to multiple equilibria. The main difference between this and our paper is that we are interested in economies in which there is multiplicity *absent* government intervention and consider the role for policy to help uniquely implement the efficient outcome.

Our paper is closely related to the global games literature that studies how bailouts and government interventions affect the equilibrium set and the incentives to provide effort. See for example Morris and Shin (2006) and Corsetti et al. (2006). We share with this paper the ideas that government intervention can help implement the efficient outcome that interventions can encourageindividual effort and so do not necessarily create moral hazard problems. We differ from these papers because we do not rely on a global games selection to render the equilibrium unique. In our economy, prices are critical because they provide information to the government about the state which in turn affects the optimal intervention probability. It is well known that prices introduce multiplicity even if the common knowledge assumption is relaxed(see Atkeson (2000), Angeletos and Werning

(2006) and Hellwig et al. (2006)).

Our paper provides another perspective on the differences between centralized (governments) and decentralized mechanisms (markets). Acemoglu et al. (2008) argue that centralized mechanisms are worse at collecting information about agents types but do not necessarily satisfy individual rationality constraints. Anonymous markets on the other hand feature the opposite. Similarly, we argue that markets might be better at aggregating information due to price mechanisms. However, the benefit of governments/centralized mechanisms are that they can solve coordination problems and help uniquely implement desired outcomes. This paper emphasizes the complementarity between markets and government interventions: markets aggregate information and the government, a large player, ensures that there are no coordination failures.

The paper proceeds as follows. Section 1 lays out the environment and Section 2 characterizes the set of private equilibria. In Section 3 we consider the problem for a government interested in unique implementation and Section 4 considers the set of implementable outcomes if we have more general mechanisms. Sections 5 considers the incentive effects for managers. Section 6 concludes.

1 Environment

We consider a simple environment in which multiple equilibria can arise because of static coordination problems between non-atomistic lenders.

Let $t = 0, 1$. The economy is populated by a continuum of non-atomistic investors who are risk neutral and require an expected return of $R \geqslant 1$. Their endowment in period 0 is E. There is a firm with access to a project that requires K units of the consumption good in period 0 as a fixed investment cost which if paid generates output $y = \pi(\theta + \nu, \varepsilon)$ in period 1, where $\nu + \theta$ is known in period 0 before investment and ε is mean-zero and realized in period 1. If the investment cost is not paid, the firm generates output ν. Thus, a firm is characterized by (θ, ν). We assume that $\varepsilon \sim F(\varepsilon)$, $\theta \sim G(\theta)$ and $\nu \sim H(\nu)$. In addition, we assume that $supp(y) = [0, \infty)$ and $\mathbb{E}[\pi(\theta + \nu, \varepsilon) \mid \theta + \nu] = \theta + \nu$. For example, we can assume that $\pi(\theta + \nu, \varepsilon) = \exp\left(\theta + \nu - \frac{1}{2}\sigma^2 + \varepsilon\right)$ and $\varepsilon \sim N(0, \sigma^2)$.

Investment is undertaken by a manager who is the residual claimant on the firm's cash-flows in period 1. She has no initial endowment and so must raise resources from the lenders. The manager can design any contract in order to raise resources from the lenders but is subject to limited liability.

To simplify the environment, we assume that all investors are symmetrically and perfectly informed about θ. This assumption is without loss of generality since if we assume that investors observe noisy signals about θ, the equilibrium price of the financial contract will perfectly reveal θ to all investors.

Efficient allocation Efficiency dictates that the project should be funded if and only if $\theta \geqslant RK$. Thus, the expected output level associated with the efficient allocation is

$$W^* = \nu + \int_{\theta \geqslant RK} (\theta - RK)\, dG(\theta).$$

2 Private equilibria

We begin by defining a contract. A contract is $\left(R^I(\theta, \nu, \varepsilon), R^N(\nu)\right)$ where $R^I(\theta, \nu, \varepsilon)$ is the return lenders receive conditional on investment and shock realization ε and R^N is the return conditional on no-investment. Let B be the quantity of such contracts issued and q be the price of such a contract. Feasibility requires that

$$R^I(\theta, \nu, \varepsilon) \leqslant \frac{\pi(z, \varepsilon)}{B},$$

where $z \equiv \theta + \nu$ and

$$R^N(\nu) \leqslant \frac{\nu + qB}{B}.$$

Although we let the set of contracts be unrestricted, it is illustrative to start with the assumption that the manager can only issue standard debt contracts. A *debt contract* is one in which

$$R^I(\theta, \nu, \varepsilon) = \min\left\{1, \frac{\pi(\theta + \nu, \varepsilon)}{B}\right\}. \tag{1}$$

That is, the manager pays one unit of the consumption good per unit of debt if there are enough resources available or the output is split equally between all investors. There are two types of debt contracts. The first, which we define as *collateralized* debt, is one in which $R^N > q^{-1}$ so that the manager pledges at least part of ν conditional on no investment. The second, which we define as *non-collateralized* debt is one in which $R^N = q$ so that investors just receive their invested funds back if there is no investment.

The timing is as follows: first the manager issues debt with face value B (either collateralized or non-collateralized); next, after observing B, q, and (z, ν), investors decide whether to lend. The lending decision can depend on the realization of a coordination device ξ that is uniformly distributed over $[0, 1]$. The equilibrium price must satisfy

$$q(B, \theta, \nu, \xi) = \frac{1}{R}\left[\mathbb{I}\int R^I(\varepsilon)\, dF(\varepsilon) + (1 - \mathbb{I})\, R^N\right] \tag{2}$$

where $\mathbb{I}$ is an indicator function which takes a value of 1 if $qB \geqslant K$ and the investment is undertaken.

A *private equilibrium* is a debt contract $\left(R^{I}(\theta,\nu,\varepsilon), R^{N}(\theta,\nu)\right)$, an amount of debt $B(\theta,\nu)$ and debt prices $q(B,\theta,\nu,\xi)$ such that for any (θ,ν), i) the debt contract and quantity are optimal for the manager i.e. $\left(R^{I}(\theta,\nu,\varepsilon), R^{N}(\theta,\nu), B(\theta,\nu)\right)$ solve

$$\max_{R^{I}(\varepsilon),R^{N},B} \int_0^1 \left[\mathbb{I}(\xi)\int \max\left\{\pi(z,\varepsilon)-R^{I}(\varepsilon)B,0\right\}dF(\varepsilon)-(1-\mathbb{I}(\xi))\left(\nu-R^{N}\right)\right]d\xi,$$

ii) equilibrium price satisfies the investors' optimality condition (2), iii) $\mathbb{I}(\xi)=1$ if $q(B,\theta,\nu,\xi)B(\theta,\nu)\geqslant K$ and 0 otherwise.

The following proposition characterizes the set of equilibrium outcomes.

Proposition 1. *i) Suppose that $\nu \geqslant (R-1)K$. For good projects, $\theta \geqslant RK$, the manager issues collateralized debt and there is investment for sure; for bad projects, $\theta < RK$, the manager issues no debt and so no investment takes place. Thus, the unique equilibrium outcome coincides with the efficient allocation. ii) Suppose that $\nu < (R-1)K$. For good projects, an equilibrium with investment always co-exists with one without investment; for bad projects, the manager issues no debt and so no investment takes place. Thus, the efficient outcome is not the unique equilibrium outcome.*

Proof. To prove the proposition, we first define some notation. Assuming there is investment, define

$$q^{I}(B,z)\equiv\frac{1}{R}\int \min\{1,\pi(z,\varepsilon)/B\}\,dF(\varepsilon)$$

This is the debt price in case of investment. Also, for $\theta\geqslant RK$, define $B^*(z)$ and $q^*(z)$ as:

$$\begin{aligned} B^*(z) &\equiv K/q^*(z) \\ q^*(z) &\equiv q^{I}(B^*(z),z). \end{aligned} \tag{3}$$

These are the debt issuance and price that arise in the equilibrium that implements the efficient allocation. If $\mathbb{I}=0$ then define

$$q^{N}(B,\nu)\equiv\frac{1}{R}R^{N}.$$

First, suppose that $\nu \geqslant (R-1)K$. Suppose also that the manager of a good project, $\theta\geqslant RK$, characterized by (ν,z) issues $B^*(z)$ of collateralized debt. First, we show that any equilibrium outcome has investment taking place, i.e., $qB^*(z)\geqslant K$. To see this, suppose by way of contradiction that there exists an equilibrium with $qB^*(z)<K$. In this case, $\mathbb{I}=0$ and

$$q=q^{N}(B,\nu)=\frac{1}{R}\min\left\{1,q+\frac{\nu}{B}\right\}.$$

There are two cases to consider. First, if $q+\nu/B>1$, then $q=1/R$. Since by assumption

$q^*(z) \leqslant 1/R$ then

$$qB^*(z) \geqslant q^*(z)\, B^* = K$$

obtaining a contradiction. Second, if $q + \nu/B < 1$. Then,

$$qB = \frac{\nu}{R-1} \geqslant K$$

where the first equality follows from the definition of price and the second from the assumption on ν. In this case we have a contradiction as well.

Next, we show that it is optimal to issue collateralized debt equal to $B^*(z)$. Since $qB^*(z) \geqslant K$, the manager can always guarantee himself a payoff of

$$V^*(z) = \int \max\left(\pi(z, \varepsilon) - B^*(z), 0\right) dF(\varepsilon) = \nu + \theta - RK.$$

Clearly, it is not optimal for the manager to issue more collateralized debt than $B^*(z)$ as it will simply lower his profits. In addition, issuing less than $B^*(z)$ is not optimal because the manager cannot raise K and implying that his payoff will be less than ν. Finally, issuing non-collateralized debt is not optimal because there is always an equilibrium with no investment and $q = 0$ and so in this case the payoff is

$$\zeta V^*(z) + (1-\zeta)\,\nu \leqslant V^*(z)$$

where ζ is the probability that lenders coordinate on the good equilibrium. Therefore, it is optimal to issue collateralized debt.

Second, suppose that $\nu < (R-1)\,K$. In this case, if the manager issues collateralized debt with face value B and investors believe that no investment will take place,

$$\begin{aligned} qB = q^N(B, \nu)\, B &= \frac{1}{R}\min\left\{1, q + \frac{\nu}{B}\right\} B \\ &\leqslant \left(\frac{q}{R} + \frac{1}{R}\frac{\nu}{B}\right) B \end{aligned}$$

or, rearranging

$$qB \leqslant \frac{1}{R-1}\nu < K$$

where the last inequality follows from the assumption that $\nu < (R-1)\,K$. Of course, this is also true if the manager issues non-collateralized debt. Thus, if lenders believe that there will be no investment, the firm will not be able to raise enough resources thereby validating the beliefs. Therefore, there always exists an equilibrium with no investment even though projects are good.

Finally, it is straightforward to see that a manager with a bad project will always issue zero debt because its value of doing so is less than ν. Q.E.D.

The proposition shows that if the collateral value of the firm is sufficiently large, $\nu \geqslant (R-1)K$, then a manager with a good project, $\theta \geqslant RK$, can always raise K by issuing collateralized debt. This because the collateral value, ν, guarantees the investors a rate of return R even in the case that other investors choose not to invest and the investment project is not funded. This breaks the coordination problem among investors and implies that the unique equilibrium outcome is efficient. This equilibrium is characterized by (q^*, B^*) which solve

$$q^* = \frac{1}{R} \int \min\{1, \pi(z, \varepsilon)/B^*\} dF(\varepsilon)$$

and

$$q^* B^* = K.$$

When the collateral value is small, $\nu < (R-1)K$, it is not possible to guarantee an individual investor a return of R independent of other investors' choices. Thus, there can be an inefficient equilibrium where $\theta > RK$ but $qB < K$ and the investment project is not financed.

Note that if $\nu < (R-1)K$ we cannot characterize the type of debt being issued and the amount of debt. This is because, these choices can in principle affect the equilibrium selection. To see why, let $V^*(z)$ denote the payoff for the manager in the best equilibrium when $\theta > RK$ and he issues B^* units of debt. Then, the manager's expected payoff conditional on issuing fully collateralized debt ($R^N = q + \nu/B$), is

$$\zeta V^*(z)$$

where ζ is the probability that investors coordinate on the best equilibrium. If instead the manager issues non-collateralized debt, his payoff is

$$\zeta' V^*(z) + (1 - \zeta')\nu$$

where ζ' is the probability that lenders coordinate on the investment equilibrium. Note that here the payoff for the manager in the no-investment equilibrium is ν because the debt is not collateralized i.e. it offers a zero payout if $qB < K$. If $\zeta = \zeta'$ then issuing non-collateralized debt is better because it allows the manager to retain ν when investment cannot be financed. However, it may be optimal to issue collateralized debt if the equilibrium selection rule depends on the choice of contract.

It is straightforward to relax the restriction that managers can only offer debt contracts as in (1) and show that our results generalize to any feasible contract. In particular, there is multiplicity if and only if $\nu < (R-1)K$. The idea is that to ensure that investment is undertaken for sure if $\theta > RK$, the manager must guarantee that each investor receives a return R whether the investment is undertaken or not. This is necessary to ensure that an individual investor's decision to lend does not depend on other lenders' decisions. Normalizing the investors endowment to K, the manager needs a measure one of investors to invest in the project. If $v < (R-1)K$ the manager can guarantee a return of at least R if no investment takes place to at most a measure $vK/(R-1) < 1$ of investors. Thus, the remaining investors would participate only if they believe that others will and so multiple equilibria can exist.

The assumption that lenders are non-atomistic is not important for this result but it is useful to think about market mechanisms in which investors act as price takers. Importantly, even if there are a finite number of lenders, private contracts can uniquely implement the efficient outcome only if the collateral value, v, is sufficiently high. For a low value of collateral, private equilibria cannot rule out the no-investment equilibrium for good projects. Consider for example the case in which at least $N \geqslant 2$ investors are needed to fund the project. Let the investors' endowments be E_i such that $\sum_{i=1}^{N} E_i = K$ (ruling out integer issues without loss of generality). Using the insight from Halac et al. (2020), unique implementation of the efficient outcome can be achieved if it is possible to order agents by their endowment size from lowest to highest and make investment a dominant strategy for each investor n given that all previous investors indexed by $i \in \{1, ..., n-1\}$ have invested in the project.[1] This requires $R_i^N \geqslant RE_i$ for $i = 1, ..., N-1$ noting that investor N will always invest since her choice is pivotal. Combining this with the resource constraint

$$\sum_{i=1}^{N-1} R_i^N \leqslant \sum_{i=1}^{N-1} E_i + v$$

implies that the amount of collateral needed is

$$v \geqslant \sum_{i=1}^{N-1} (R-1) E_i.$$

In particular, in the symmetric case in which $E_i = K/N$, the above reduces to

$$v \geqslant \sum_{i=1}^{N-1} (R-1) K \frac{N-1}{N}$$

[1]In our environment, making investment a dominant strategy for investors with small endowments minimizes the amount of collateral needed. In Halac et al. (2020) instead, the cost minimizing way to make investment dominant is to ensure that investors with large endowments invest no matter what investors with small endowments are doing. This is because in their environment investment is probabilistic and depends on the amount invested. Therefore, investors with higher endowments require lower returns in order to participate and so it is cheaper to make investment dominant for high endowment agents.

which converges to the cutoff in Proposition 1 as $N \to \infty$.

3 Market-based interventions

Suppose that $\nu < (R-1)K$ so that private contracts cannot uniquely implement the good outcome. We now introduce a government who lacks knowledge about the state of the firm (θ, ν) but can tax lenders and finance the project.[2] The government can observe the price and the type of the contract offered by firms. We call this type of intervention market-based because the government can condition its actions on the equilibrium objects B and q. There are two main results in this section. First, we show that there exists no market-based intervention that uniquely implements the efficient outcome. Second, we show that that government can design a market-based intervention policy that can approximate the efficient outcome with arbitrary precision. Under this policy, the government learns if the investment project is profitable by observing the equilibrium (B, q) and commits to funding good investment projects (i.e., $\theta > RK$) with probability one but no intervention is necessary along the equilibrium path and bad investment projects (i.e, $\theta < RK$) with a positive (but vanishing) probability. One can interpret such a policy as one in which the government commits to ex-post *inefficient* bailouts.

We focus on cases with $\nu < (R-1)K$ and non-collateralized debt. If $\nu \geqslant (R-1)K$ and $\theta > RK$ then collateralized debt ensures that investment takes place. And so in this case managers will not have an incentive to issue non-collateralized debt.

The timing is as follows:

- The government commits to an intervention probability $\eta(B, q)$;
- The investment project's quality (θ, ν) is realized and observed by the manager and the lenders;
- The manager issues debt B;
- A sunspot ζ is realized and the price of debt q is realized;
- If $qB < K$, the manager can ask the government for assistance. If assistance is requested, the government makes a transfer $T = K - qB$ with probability $\eta(B, q)$;
- Finally, ε is realized and contractual payments are made.

We now characterize the outcomes using backward induction.

[2]We assume that the government knows the investors' opportunity cost R but we could allow that the government does not know θ and/or R as θ/R is a sufficient statistic for the optimality of investment.

Continuation equilibria given (θ, ν, B) The equilibrium price if lenders anticipate that assistance will be requested is

$$q = \frac{1}{R}\mathbb{I}\int \min\{1, \pi(z,\varepsilon)/B\}\,dF(\varepsilon)$$
$$+\frac{1}{R}(1-\mathbb{I})\left[(1-\eta(B,q))\,q + \eta(B,q)\int \min\{1,\pi(z,\varepsilon)/B\}\,dF(\varepsilon)\right]$$

or

$$q = \frac{\frac{1}{R}\left[\mathbb{I} + (1-\mathbb{I})\,\eta(B,q)\right]A(z,B)}{1-\frac{1}{R}(1-\mathbb{I})(1-\eta(B,q))}$$

where

$$A(z,B) \equiv \int \min\{1, \pi(z,\varepsilon)/B\}\,dF(\varepsilon)$$

is the expected payoff on debt conditional on investment. The continuation equilibrium outcome is a debt price q and an investment probability σ; (σ, q) can take on two forms: either the investment is undertaken without government intervention ($\sigma = 1$), and

$$qB \geqslant K \tag{4}$$
$$q = \frac{1}{R}A(z,B),$$

or the investment is undertaken only with government assistance ($\sigma = \eta(B,q)$) if the manager asks for it and

$$qB < K \tag{5}$$
$$q = \frac{\sigma}{R-1+\sigma}A(z,B). \tag{6}$$

The manager will ask for assistance if and only if the payoff associated with assistance,

$$V^{a}(z,\nu,B,q) = \eta\int \max\{\pi(z,\varepsilon)-B\}\,dF(\varepsilon) + (1-\eta)\,\nu$$

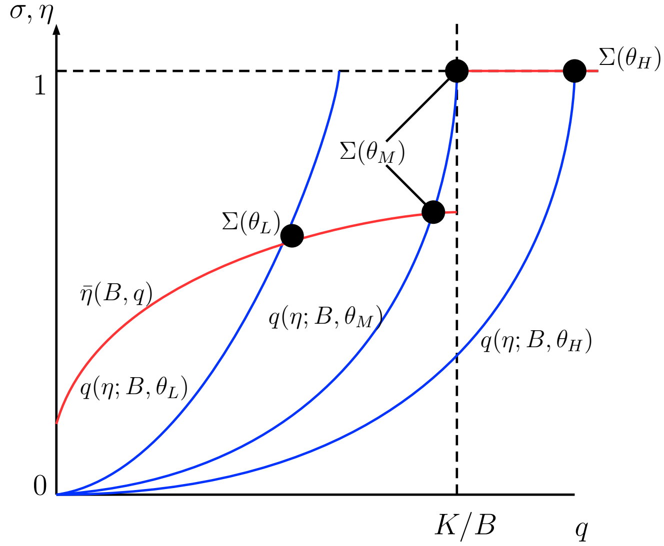
is greater than the value of not requesting assistance, ν. Thus, assistance is requested if and only if

$$\int \max\{\pi(z,\varepsilon)-B\}\,dF(\varepsilon) > \nu. \tag{7}$$

We can then define the set of investment probabilities consistent with a continuation equilibrium given the history (θ, ν, B) as

$$\Sigma(\nu,\theta,B|\eta) = \{\sigma : \exists q \text{ such that either (4) holds or (5)-(7) hold}\}$$

Figure 1: Equilibrium determinacy



Given a policy $\eta(B, q)$, there can be multiple equilibria with different investment probabilities.

Figure 1, illustrates how to identify the set of equilibria for a candidate intervention policy function $\eta(B, q)$. The blue lines are the indifference conditions of investors for a given B and different values of $z = \theta$, $\theta_L < \theta_M < \theta_H$. That is, given a probability of investment σ, q is the maximal price at which they are willing to invest. This is just the inverse of (6), $\sigma = q(R-1)/(A(\theta+\nu, B) - q)$. The probability of investment given q is represented by the red line. If $q < K/B$, then the resources collected from the investors are not sufficient to finance K and the probability that the investment is funded equals to the probability of receiving government assistance. Thus, for $q < K/B$ we have $\sigma = \eta(B, q)$. For $q \geqslant K/B$, the price is sufficiently high so that the investment is implemented with probability 1 with no assistance in equilibrium. Thus, the set of investment probabilities consistent with a continuation equilibrium given (θ, ν, B) is the intersection between the red and the blue lines (for a given θ) in Figure 1. For θ_H and θ_M, there is always an equilibrium with $\sigma = 1$. For θ_H this is the unique equilibrium. For θ_M, there is another equilibrium with assistance. For θ_L, the project output is too low to have an equilibrium without assistance. Thus, there is only one equilibrium with assistance and $\sigma \in (0, 1)$. Finally, note that if $\eta(B, 0) = 0$ and there is no assistance, for any realization of θ, there is always an equilibrium with $\sigma(0) = 0$ and no investment as shown in Proposition 1 for the case with low collateral.

Debt issuance decision We now characterize the manager's problem at the beginning of the period. Let

$$\Pi(B,z) \equiv \int \max\{\pi(z,\varepsilon) - B, 0\}\, dF(\varepsilon)$$

be the expected manager's payoff conditional on investment. A debt issuance decision is optimal if

$$B(\theta,\nu) \in \arg\max_{B} \int \mu(B,\sigma)\, \sigma \Pi(B, \theta+\nu)\, d\sigma \quad \text{for some } \mu(B,\cdot) \in \Delta(\Sigma(\theta,\nu,B(\theta,\nu)|\eta)).$$

That is, the debt issuance decision must be optimal given some belief about the equilibrium selection, where these these beliefs are drawn from the set of possible equilibrium investment probabilities.

Best robust policy We now solve for the optimal government's policy. To evaluate welfare associated with a given policy, we need to specify a selection mechanism because, in general, multiple equilibria can exist. There are many selection mechanisms we could consider. For example, under the *Ramsey approach* one considers the most optimistic selection in evaluating payoffs, the best equilibrium is always chosen and no intervention is ever necessary. Under the *Robust approach*, we use the most adversarial selection criterion from the government's perspective. In this case, the equilibrium with the highest investment probability is chosen if $\theta < RK$ and the equilibrium with lowest investment probability is chosen if $\theta \geqslant RK$. We denote these two probabilities by $\bar{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)$ and $\underline{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)$:

$$\bar{\sigma}(\theta,\nu,B(\theta,\nu)|\eta) = \max_{\sigma \in \Sigma(\theta,\nu,B(\theta,\nu)|\eta)} \sigma, \quad \underline{\sigma}(\theta,\nu,B(\theta,\nu)|\eta) = \min_{\sigma \in \Sigma(\theta,\nu,B(\theta,\nu)|\eta)} \sigma.$$

Moreover, we consider debt issuance decisions that minimize the government's value under the most adversarial selection. The debt issuances must satisfy the following incentive compatibility constraint:

$$\begin{aligned} B(\cdot) \in \arg\min_{B(\cdot)} & \int_{\underline{\theta}}^{RK} \bar{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)(\theta - RK)\, dG(\theta) \\ & + \int_{RK}^{\bar{\theta}} \underline{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)(\theta - RK)\, dG(\theta) \end{aligned} \tag{8}$$

subject to

$$B(\theta,\nu) \in \arg\max_{B} \sigma(B,\theta,\nu)\, \Pi(B,\theta+\nu) \quad \text{for some } \sigma(B,\theta,\nu) \in \Sigma(\theta,\nu,B(\theta,\nu)|\eta).$$

The government anticipates that the manager will choose the level of debt that minimizes the value of the worse equilibrium for the government subject to the requirement that there exists a set of continuation probabilities consistent with a continuation equilibrium that makes this B a best response. Clearly, if $\Sigma(\theta, \nu, B|\eta)$ is a singleton $\{\sigma(\theta, \nu, B|\eta)\}$ for all (θ, ν, B), then condition (8) reduces to

$$B(\theta, \nu) \in \arg\max_{B} \sigma(B, \theta, \nu) \Pi(B, \theta + \nu). \tag{9}$$

The optimal robust policy solves

$$W = \sup_{\bar{\eta}(B,q), B(\theta,\nu)} \int_{\underline{\theta}}^{RK} \bar{\sigma}(\theta, \nu, B(\theta, \nu)|\eta)(\theta - RK)\, dG(\theta) + \int_{RK}^{\bar{\theta}} \underline{\sigma}(\theta, \nu, B(\theta, \nu)|\eta)(\theta - RK)\, dG(\theta) \tag{10}$$

subject to the incentive constraint for debt issuances (8) and

$$\Sigma(\theta, \nu, B|\eta) \neq \emptyset \text{ for all } (\theta, \nu, B). \tag{11}$$

The last constraint requires that the chosen policy η has at least one equilibrium for all (θ, ν, B). To see why this is needed, consider the following example. Suppose $\eta(0) = a > 0$ and $\eta(q) = 0$ for all $q > 0$. This policy can appear desirable because it ensures that for all $\theta \geqslant RK$ there exists only one equilibrium with $\sigma = 1$ and it rules out the outcome with no investment. However, for $\theta \in (0, RK)$, no equilibrium exists.

The equilibrium prices associated with the best robust policy are

$$q(B, \theta, \nu) = \begin{cases} \frac{\bar{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)}{R-1+\bar{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)} A(z, B) & \text{if } \theta < RK \\ \frac{\underline{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)}{R-1+\underline{\sigma}(\theta,\nu,B(\theta,\nu)|\eta)} A(z, B) & \text{if } \theta \geqslant RK \end{cases}. \tag{12}$$

The following proposition shows that no policy can uniquely implement the efficient outcome- i.e. investment with probability one when $\theta \geqslant RK$ and no investment otherwise.

Proposition 2. *No policy can uniquely implement the efficient allocation.*

Proof. Consider an equilibrium outcome: $(B(\nu, \theta), q(\nu, \theta, B), \eta(B, q))$. If the unique equilibrium outcome coincides with the efficient allocation, then, if $\theta \geqslant RK$ the probability of financing the project is one and zero otherwise. Consider a manager with a project of type $\theta_L < RK$ and $\nu = \nu_L = 0$. Under the efficient allocation, this project is not financed for sure. Thus, $\eta(B, q(\nu_L, \theta_L, B)) = 0$ for all B. The requirement that this must hold for all B follows from the observation that since $\nu_L = 0$ and the manager is subject to limited liability, he will always try to issue some debt if $\eta(B, q(\nu_L, \theta_L, B)) > 0$ for any B. Thus,

$q(\nu_L, \theta_L, B) = 0$ for all B and so

$$\eta(B, 0) = 0 \quad \text{for all } B.$$

Now suppose that $\theta > RK$. Since, $\eta(B, 0) = 0$, for any level of issued debt B, there exists an equilibrium in which $\eta(B, 0) = 0$, $q = 0$ and no investment takes place. In particular, under the adversarial equilibrium selection rule, investment never takes place when $\theta > RK$. Thus, we cannot uniquely implement the efficient allocation. Q.E.D.

The proof follows from the observation that if $\eta(B, 0) = 0$ then the government cannot distinguish between bad projects and bad equilibrium outcomes with good projects based on the price and face value of debt. This implies that any policy which prevents investment when projects are bad (as is consistent with efficient equilibrium) must be also be consistent with an equilibrium outcome where no investment takes place when projects are good.

More generally, if the government wants to avoid ex-post inefficient investments, it must be that $\eta(0, B) = 0$. But then it is not possible to rule out coordination failures among investors for projects with $\theta > RK$. Thus, under the most adversarial selection rule, a necessary condition for investment to take place when $\theta > RK$ is $\eta(0, B) > 0$. If not, then the selection criterion implies that no good project is ever started because $\underline{\sigma}(\theta|\eta) = 0$. This attains a lower value in (10) than a policy that always fund any projects if $\int \theta dG(\theta) > RK$. This implies that under the optimal robust policy, bad projects are occasionally funded. In other words, the government must commit to funding ex-post inefficient investment projects in order to implement the investment outcome when projects are good.

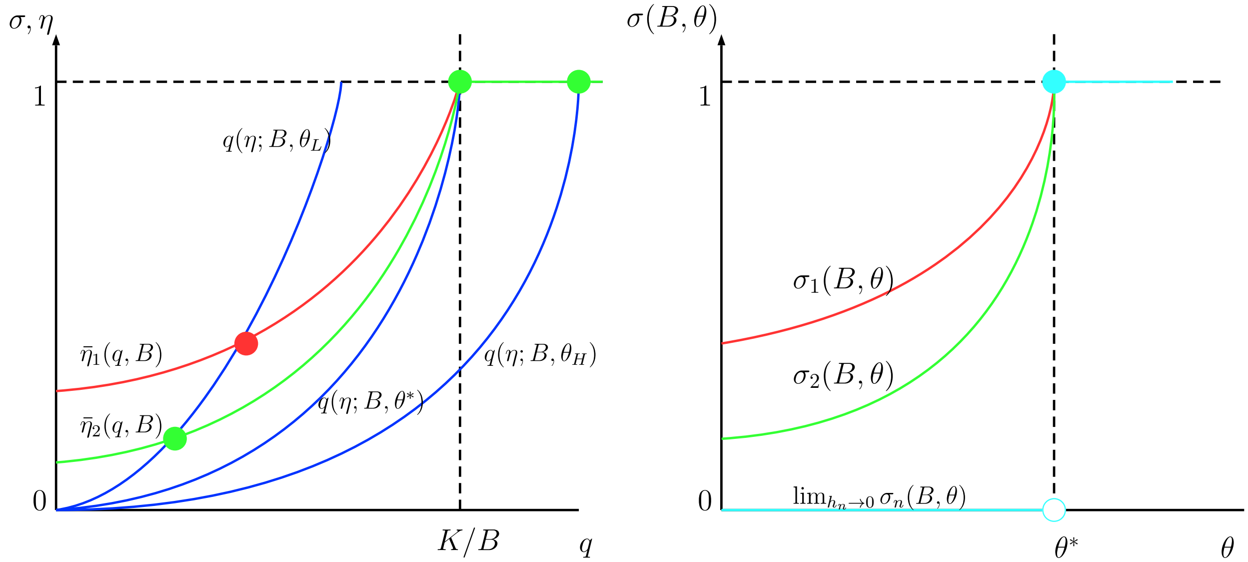
The next proposition shows that we can construct a sequence of policies that uniquely implements an outcome in which efficient investment projects are always financed and the probability of ex-post inefficient investments is vanishingly small. Consequently, the efficient outcome can be approximated with increasing precision. We refer to such policies as the optimal robust policy since there do not exist any others which are feasible in (10) and can strictly dominate them.

Proposition 3. *There exists a policy that can approximate the best outcome arbitrarily closely and attains the supremum in (10). This policy commits to ex-post inefficient investments with some strictly positive probability.*

The formal proof for this proposition is provided in the appendix. Here we describe its logic. We construct a sequence of intervention probabilities, debt issuances and corresponding debt prices $\{\eta_n(B, q), B_n(\theta, \nu), q_n(B, \theta, \nu)\}$ parameterized by a parameter $h_n \in (0, 1)$ as follows: for any z^* and corresponding $B = B^*(z^*)$ define

$$\eta_n(B, q) \equiv q \frac{(R-1)}{A(z^*, B) - q} + h_n(q^*(z^*) - q). \tag{13}$$

Figure 2: Equilibrium for $\{\eta_n\}$



For each (B, θ, ν), the debt prices $q_n(B, \theta, \nu)$ are the unique solution to

$$\eta_n(B, q) = q\frac{(R-1)}{A(z, B) - q}. \tag{14}$$

Finally, debt issuances are defined by

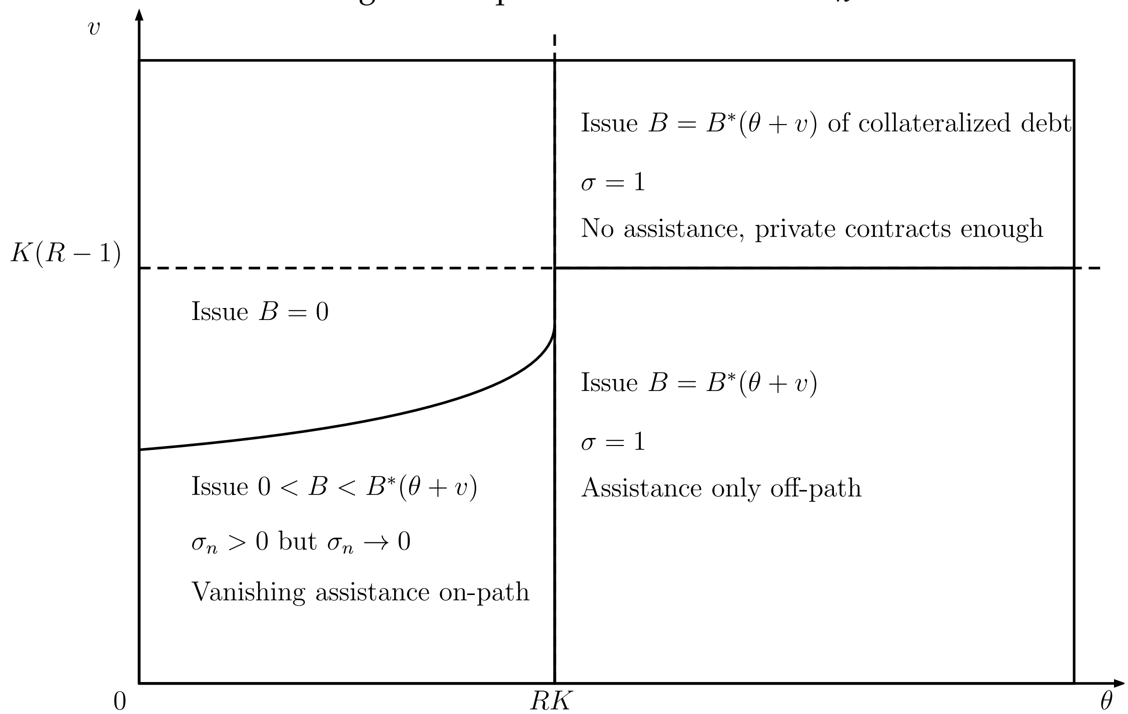
$$B_n(\theta, \nu) = \arg\max_{B \in \mathrm{range} B^*(\cdot)} \eta_n(B, q_n(B, \theta + \nu))[\Pi(B, \theta) - \nu]. \tag{15}$$

The constructed policy is feasible for the problem in (10). In fact, as illustrated in the first panel of Figure 2, for any (B, θ, ν) and $h_n > 0$ there is a unique continuation equilibrium (q, σ) where q is the solution to (14) i.e. the intersection between $\eta_n(B, q)$ and the investors' optimality condition (12) that can be rearranged as the right side of (14). Thus, $\Sigma(\theta, \nu, B|\eta)$ is non-empty and constraint (11) is satisfied. Moreover, since continuation equilibria are unique then $\Sigma(\theta, \nu, B|\eta)$ is a singleton and (8) simplifies to (9) which is satisfied because of (15).

As illustrated in Figure 2, the unique continuation equilibrium has the property that if $z \geqslant z^*$ then there is investment for sure while if $z < z^*$ then the investment probability $\sigma_n(B, z)$ is strictly positive but strictly less than one. As h_n decreases, this probability decreases and it converges to zero as $h_n \to 0$. In the limit, $\sigma_n(B, z)$ converges to a step function that takes value 0 for all $z < z^*$ and 1 for all $z \geqslant z^*$.

The last property ensures that the optimal debt issuance converges to $B^*(\theta + \nu)$ for all θ such that $\theta > RK$. In fact, if $B < B^*(\theta + \nu)$ then the project will be implemented with a probability close to zero while the probability is one for all $B \geqslant B^*(\theta + \nu)$. The fact that $B_n(\theta, \nu) \to B^*(\theta + \nu)$ then follows from the observation that issuing more debt decreases

Figure 3: Equilibrium outcome as $h_n \to 0$



the value of equity as $\Pi(B, z)$ is decreasing in B. For an investment project with $\theta < RK$ there are two possibilities: if v is sufficiently high, then the manager will issue zero debt to avoid losing its collateral value. If instead v is sufficiently small, then the manager will try to issue some debt to receive a subsidy but as h_n goes to zero the probability of receiving this subsidy is arbitrarily small. In this sense, the constructed policy ensures that there is a unique equilibrium that is arbitrarily close to the efficient allocation with minimal subsidy given to inefficient projects on path. Efficient projects instead are implemented without any government intervention. This property of the equilibrium outcome associated with the best robust policy is summarized in Figure 3.

Note that there is a discontinuity at $h_n = 0$. If $h_n = 0$, then the intervention policy coincides with the investors indifference conditions and there is always an equilibrium with no investment when $\theta \geqslant RK$. So for any $h_n > 0$ – no matter how small – we have $\sigma = 1$ for $\theta \geqslant RK$ but at $h_n = 0$ then $\sigma \in [0, 1]$. Thus, the robust policy approximates the efficient allocation but cannot exactly attain it.

In Appendix B we show that the analysis extend to a dynamic model where investment opportunities arise in every period.

Information and commitment Along the proposed sequence of intervention policies, information about the market equilibrium outcome (B, q) allows the government to learn whether the project has expected returns above or below RK. In fact, conditional on B, the unique equilibrium price is a strictly monotone function of $z = \theta + \nu$. Thus, the government can infer z by observing (B, q). Moreover, using information about the chosen B, the government can learn whether $\theta > RK$ or not. As argued above and illustrated in Figure 3, if $\theta > RK$ and $\nu < (R-1)K$ the manager will issue debt $B_n(\theta, \nu) \to B^*(\theta + \nu)$. Thus, if the government observes (B, q) such that it implies $z < B^{*-1}(B)$ then the government knows that this investment has an expected return less than RK because the manager issued a small amount of debt with the prospects of receiving assistance. Thus, the government knows whether the investment it is financing has high or low returns.

Even though the government can ex-post (i.e. after observing the price q) identify managers with low expected returns asking for assistance, it must commit ex-ante to providing them transfers (albeit with a vanishing probability). Not doing so will result in $\eta(B, q) = 0$ and, as argued above, then no-investment is always an equilibrium outcome for high return projects. Thus, funding inefficient investment projects is necessary to guarantee that efficient investment projects are undertaken for sure.

This property of the best robust policy requires commitment on the part of the government. If there is no commitment technology and $\nu = 0$, then there are two possibilities: either there is no intervention at all and there is multiplicity which implies a value for the planner $W = 0$, or the government directly funds all the investment projects without acquiring any information which implies a value for the planner $W = \int (\theta - RK)\, dG(\theta)$.

In particular, if

$$\int (\theta - RK)\, dG(\theta) < 0 \tag{16}$$

then the robust policy without commitment has $\eta(B, 0) = 0$ for all B. If $\eta(B, 0) = 0$ then the worst equilibrium has $q = 0$ for all θ. In this case, the distribution of projects quality is $G(\theta)$ so by (16) it is ex-post optimal to have $\eta(B, 0) = 0$. Thus, there is always an equilibrium with $q(\theta) = 0$ for all θ and $\eta(B(\theta), q(\theta)) = 0$ along the equilibrium path. This is because the government does not have incentives ex-post to finance bad projects.

Consider now the case in which the average project is good, i.e.

$$\int (\theta - RK)\, dG(\theta) > 0 \tag{17}$$

In this case, $q(\theta) = 0$ for all θ cannot be an equilibrium outcome because (17) implies that the government will have incentives to provide assistance with probability one ex-post. Then $q(\theta)$ must be positive for some θ. If the price is positive, the government will learn the underlying θ because there cannot exist $\theta_1 \neq \theta_2$ such that

$$q = \eta(B, q) \frac{A(\theta_1, B)}{R} + (1 - \eta(B, q)) q$$
$$q = \eta(B, q) \frac{A(\theta_2, B)}{R} + (1 - \eta(B, q)) q$$

since A is strictly increasing in θ. Thus, for all $q > 0$, sequential optimality on the part of the government will require that $\eta(B, q) = 0$ for bad projects and $\eta(B, q) = 1$ for good projects. And so, for $\theta < RK$, $q = 0$. But then this implies there can be a coordination failure when $\theta > RK$ which in turn implies that $\eta(B, 0) = 0$ cannot be sequentially optimal. Therefore the set of robust policies is empty because condition (11) cannot be satisfied while imposing that $\eta(B, q)$ is sequentially rational for the government. In this case, what the government can do is to finance all projects directly by providing K to the manager without learning anything from the market. In this case the government can guarantee an expected payoff of $\int (\theta - RK) \, dG(\theta) > 0$.

Interestingly, longer horizons and repetition of the policy game do not help provide the government with incentives to finance inefficient investments ex-post. This is because if the good equilibrium can be uniquely implemented in the best sustainable equilibrium, then this is also true in the worst. Therefore, there are no dynamic gains that can provide incentives. See Barthelemy and Mengus (2022) for a general version of this argument.

4 General mechanism

So far, we have relied on a market-based mechanism where dispersed information is aggregated through prices which serve as the only way through which the government can learn about the state. Government interventions are therefore functions of market outcomes, namely B and q. An alternative way for the government to learn about the state is to use a general mechanism where the manager and the investors send a message to the government. In this section, we show that the market-based intervention described above cannot be improved upon by a general mechanism.

To ease notation, in this section we assume that $\nu = 0$. A mechanism $\mathcal{M} = (M, \mathbf{y})$ is a message space $M = \prod_{i \in [0,1]} M^i \times \bar{M}$, where M^i denotes the message space of investor i and M^i the message space of the manager, and an outcome function $\mathbf{y} = (\iota, k, r_I, r_{NI}, x_I, x_{NI}) : M \to \mathbb{R}^6$ where $\iota(m, \bar{m}) \in [0, 1]$ denotes the investment decision and $k^i(m, \bar{m})$, the contribution of investor i, $r_I^i(m, \bar{m}, y)$ and $x_I(m, \bar{m}, y)$ denote the payoffs to the investor i and the manager if investment takes place, and $r_N^i(m, \bar{m})$ and $x_N(m, \bar{m})$ denote the payoffs if no investment takes place.

A mechanism is feasible if

$$\text{if } \iota(m, \bar{m}) = 1 \text{ then} \int k^i(m, \bar{m})\, di \geqslant K, \tag{18}$$

$$\int r_I^i(m, \bar{m}, y)\, di + x_I(m, \bar{m}, y) \leqslant y, \tag{19}$$

and

$$\int r_{NI}^i(m, \bar{m})\, di + x_{NI}(m, \bar{m}) \leqslant 0 \tag{20}$$

where $y = \pi(\theta, \varepsilon)$. The first constraint says that investment only takes place if at least K units of resources and raised while the second two require that the payoffs to the manager and investors are resource feasible in each state. Finally, we require that a mechanism satisfies the manger's limited liability constraints,

$$x_I(m, \bar{m}, y) \geqslant 0, \quad x_{NI}(m, \bar{m}, y) \geqslant 0. \tag{21}$$

A mechanism $\mathcal{M}$ induces a reporting game with the following timing: First, each investor receives a noisy signal of θ: $\theta^i = \theta + u^i$ with $u^i \sim N(0, \sigma_u)$ while the manager observes θ. Next, the managers and investors report $\left(\bar{m}, m^i\right)$. Given the message profile, investment and payoffs are determined according to $\mathbf{y}$. Thus, a reporting strategy profile $(\bar{m}, m)$ is an equilibrium of the reporting game induced by $\mathcal{M}$ if and only if

$$\begin{aligned} \bar{m}(\theta) \in \arg\max_{\bar{m}} \int \iota(\bar{m}, m(\theta)) \int x_I(\bar{m}, m(\theta), y)\, dP(y|\theta + \nu)\, dG(\theta, \nu) \\ + \int (1 - \iota(\bar{m}, m(\theta)))\, x_{NI}(\bar{m}, m(\theta), \nu)\, dG(\theta, \nu) \end{aligned} \tag{22}$$

and

$$\begin{aligned} m_i(\theta + u_i) \in \arg\max_{m^i} \int \iota(m_i, m_{-i}(\theta)) [K - k_i(m_i, m_{-i}(\theta)) \\ + \frac{1}{R} \int r_{Ii}(m_i, m_{-i}(\theta), \theta + \varepsilon)\, dF(\varepsilon) \Big]\, dG(\theta|\theta + u_i) \\ + \int (1 - \iota(m_i, m_{-i}(\theta)))\, K dG(\theta|\theta + u_i) \end{aligned} \tag{23}$$

where the first condition is the incentive compatibility constraint for the manager and the second the analogous constraint for the investors.

We now consider the outcomes that can be implemented by a mechanism. We focus on investment outcomes. As in the previous section, we let $\sigma(\theta)$ be the probability that investment project θ is undertaken. A mechanism $\mathcal{M}$ implements the outcome σ if there is an equilibrium reporting strategy profile $(\bar{m}, m)$ of the reporting game induced by $\mathcal{M}$ such that $\iota(\bar{m}(\theta), m(\theta)) = \sigma(\theta)$ for all θ. We say that the mechanism uniquely (or strongly) implements the outcome function $\sigma(\theta)$ if such reporting strategy profile is the unique equilibrium of the reporting game induced by $\mathcal{M}$.

Proposition 4. *There exists no mechanism that uniquely implements the efficient allocation if investors have noisy information about fundamentals* θ*.*

Suppose by way of contradiction that there is a mechanism $\mathcal{M}$ that uniquely implements the efficient allocation with $\sigma(\theta) = 1$if $\theta > RK$ and $\sigma(\theta) = 0$ if $\theta < RK$. First, we show that the investment decision cannot depend only on the manager's report. To see why, suppose we have an investment rule $\iota(m, \bar{m})$. Consider a manager with $\theta < RK$. The manager's value with no investment is then $x_{NI} = 0$. Thus, such manager will always have incentives to report a message such that investment take place with positive probability even if $\theta < RK$. This is because to induce investment when $\theta > RK$, it must be that $x_I > 0$ for some realization of y.

Thus, the mechanism must elicit some information from the continuum of investors. Consider θ_H for which it is optimal to invest and $\theta_L = \theta_H - a$ for which it is not optimal to invest. By the argument above, we can abstract from the manager's report since it will always make a report that leads to investment. Let $m_H = \left\{m^i\left(\theta_H + u^i\right)\right\}$ and $m_L = \left\{m^i\left(\theta_L + u^i\right)\right\}$ be the equilibrium report profile sent by the investors in these two states. Under the assumption that the efficient allocation is the unique outcome then

$$\iota(m_H) = 1, \quad \iota(m_L) = 0.$$

But now, fixing the mechanism, consider these alternative reporting strategies

$$\hat{m}^i\left(\theta^i\right) = m^i\left(\theta^i - a\right)$$

If the mechanism is anonymous i.e. it depends only on the distribution of reports, then $\hat{m}^i$ is an equilibrium reporting strategy because no agent i can affect the equilibrium outcome. This implies that the mechanism implements another outcome other than the efficient allocation because $\iota\left(\left\{\hat{m}^i\left(\theta_H + u^i\right)\right\}\right) = \iota(m_L) = 0$.

Suppose now that the mechanism is not anonymous. In particular, consider the extreme case of a dictatorial mechanism. That is, the investment decision is determined by the report of one agent only. Since signals are noisy, the designer will not learn the true θ and there will be cases with inefficient investments or efficient investments not undertaken.

More generally, consider any mechanism that only depends on the reports of a subset of investors $\mathcal{J} \subset I$. A necessary condition to implement the efficient allocation is that the mechanism designer needs to know θ with certainty. But then, this implies that $\mathcal{J}$ must be a continuum. In this case, using an identical argument to that above we can show that since no agent is pivotal, multiple coordination equilibria exist.

If the signals received by the investors are perfectly informative, $\sigma_u = 0$, one can design a mechanism that uniquely implements the efficient outcome by making any investor pivotal. For instance, let $M^i = \Theta$ and make agent i^* be the dictator so

$$\iota(m) = \begin{cases} 1 & m^{\iota^*} \geqslant RK \\ 0 & o/w \end{cases}$$

and let

$$r_I^{i^*}(\theta) = y, \quad r_{NI}^{i^*}(\theta) = 0, \quad k^i = \begin{cases} K & m^{\iota^*} \geqslant RK \\ 0 & o/w \end{cases}$$

Investor's i^*'s reporting strategy then solves

$$\max_m \iota(m, m_{-i}) \left(\frac{1}{R}\theta\right) + (1 - \iota(m, m_{-i})) K = \max \left\{\frac{1}{R}\theta, K\right\}$$

Thus, investor's i^* will report $m \geqslant RK$ if and only if $\theta \geqslant RK$. This is the unique reporting strategy as it does not depend on the reporting strategies of other agents. Thus, the efficient allocation is uniquely implemented.

Note that the market mechanism is not a special case of the general mechanism we just described. When investors receive a noisy signal $\theta^i = \theta + u^i$ and there are no noise traders, the equilibrium price is fully revealing about the state and individual investment decisions do not depend on the signal θ^i. Thus, we cannot recast the competitive equilibrium as a reporting game where investors report $a(\theta^i)$ and the outcome functions depend on equilibrium prices as a sufficient statistic of the profile of reports. We conjecture that there is a mapping between the market mechanism and the general mechanism in the presence of noise traders as in Grossman and Stiglitz (1980) or the more recent analysis in Albagli et al. (2021).

5 Moral hazard

We now discuss how the intervention affects managers' incentive to generate investment projects. A large literature emphasizes the potential problem of interventions and bailout policies for incentives as in the seminal paper by Kareken and Wallace (1978). These concerns are only partially operating in our environment. The best robust policy can in fact increase incentives to exert effort ex-ante because it ensures that good projects are implemented and therefore rewards the manager's effort.

To see this, suppose that the manager can take an action that affects the value for θ. Say $\theta \in \{\theta_L, \theta_H\}$ and $\theta_L < RK < \theta_H$. Without any intervention, the manager's problem is

$$\begin{aligned}
&\max_a \sum_\theta f(\theta|a)\,\zeta \int \max\{\pi(\theta,\varepsilon) - B^*(\theta), 0\}\, dF(\varepsilon) - c(a) \\
&= \max_a \sum_\theta f(\theta|a)\,\zeta\,(\theta - RK) - c(a)
\end{aligned}$$

where ζ is the probability that investors coordinate on the good equilibrium. As we showed in Section 2, $\zeta \in [0,1]$. Thus, the private equilibrium effort a can be anything from 0 to the efficient level a^*, $a^* = \arg\max_a \sum_\theta f(\theta|a)(\theta - RK) - c(a)$.

Under the optimal robust policy in Section 3, if $\theta = \theta_H$, the manager's value converges to $(\theta - RK)$; if $\theta = \theta_L$ the manager's value is greater than 0 but it converges to 0 as $h_n \to 0$. Thus, under the sequence of intervention policies that approximates the efficient allocation, effort a is going to be lower than a^* along the sequence but $a \to a^*$ as $h_n \to 0$ and the probability of supporting inefficient projects goes to zero. Thus, the intervention increases effort relative to the worst case scenario in the private economy without intervention. Conversely, it reduces incentives to exert effort relative to the best case scenario for the market economy but this effect vanishes as the probability of financing inefficient projects vanishes.

The next proposition summarizes this argument:

Proposition 5. *Along the sequence of intervention policies that approximate the efficient allocation described in (13), $0 < a_n < a^*$ but $a_n \to a^*$ as $h_n \to 0$.*

Suppose manager can take an action that affects the value for ν. Then, absent government intervention, there could be over-production in ν because the manager values the benefits of restricting the set of states where multiplicity can take place. In this case, $a < a^{eq}$ but $a^{eq} > a^* = 0$. Thus, the optimal robust intervention is welfare improving in this dimension by reducing the excessive amount of effort in private equilibria. For example, for fear of bad equilibrium, private agents may produce too much collateralizable assets or – thinking more broadly – borrowers may issue too much long-term debt in economies with rollover risk as in Cole and Kehoe (2000). The optimal intervention reduces these incentives.

6 Conclusion

We study an economy where coordination failures lead to multiple equilibria. We show how a government that lacks information about the state of the economy can use information contained in prices to uniquely implement an outcome that approximates the efficient one arbitrarily closely. The government does so by committing to fund projects that are ex-post inefficient with a small probability. This paper highlights the complementarity between markets and government interventions: markets aggregate information and the government, a large player, ensures that there are no coordination failures.

We expect that the arguments in this paper to extend to other economies where non-convexities and increasing returns lead to multiple equilibria, for example, industrial policies in the presence of external effects as in Sturm (2022). One area that deserves further study are economies in which multiplicity arises due to dynamic coordination problems. We study this problem in a companion paper, Dovis and Kirpalani (2023).

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Appendix

A Proof of Proposition 3

We construct a sequence of intervention policies and show that the sequence of induced equilibrium outcomes converges to the efficient allocation. Thus, this class of intervention policies can uniquely implement an outcome which approximates the efficient one arbitrarily closely.

Fix any z^* and corresponding $B = B^*(z^*)$ where $B^*(z^*)$ is defined in (3). For some $h_n \in (0,1)$ define

$$\eta_n(B,q) \equiv q\frac{(R-1)}{A(z^*,B)-q} + h_n(q^*(z^*)-q).$$

Recall that in order for an outcome to constitute an equilibrium, from the investors optimality condition (12), it must be that

$$\eta_n(B,q) = q\frac{(R-1)}{A(z,B)-q}.$$

Thus, if $z = z^*$ then, given the definition of η_n, we have an equilibrium iff $q^*(z^*) - q = 0$. Thus, there exists a unique equilibrium outcome: $q = q^*(z^*) = A(z^*,B)/R$ and $\sigma = 1$.

Consider now $z \neq z^*$. The relevant case is if $z < z^*$. Suppose that a firm with this z issues $B = B^*(z^*) \neq B^*(z)$, where $B^*(z) > 0$ if $\theta \geqslant RK$ and $B^*(z) = 0$ if $\theta < RK$. In this case, the equilibrium price solves

$$l(q) = r(q) \tag{24}$$

where

$$l(q) \equiv q\frac{(R-1)}{A(z^*, B) - q} + h_n(q^*(z^*) - q), \quad r(q) \equiv q\frac{(R-1)}{A(z, B) - q}$$

Note that $l(q)$ and $r(q)$ are increasing, continuous, and convex functions of q. Moreover, $l(0) > r(0)$ and $l(1) < r(1)$. Thus, there exists q such that $l(q) = r(q)$. The solution is also unique because of the convexity of $l(q)$ and $r(q)$. Given $\eta_n(B, q)$, define $q_n(z, z^*)$ to be the corresponding solution to (24) and $\bar{\eta}_n(z, z^*)$ to be the implied intervention probability.

Next, we show that $\bar{\eta}_n(z, z^*)$ and $q_n(z, z^*)$ converge to zero as $h_n \to 0$. To this end, rearrange (24) as

$$h_n = (R-1)\frac{f(q_n(z, z^*))}{(q^*(z^*) - q_n(z, z^*))} \tag{25}$$

where

$$f(q) \equiv \frac{q}{A(z, B) - q} - \frac{q}{A(z^*, B) - q}$$

and, because $A(z, B) < A(z^*, B)$,

$$f'(q) = \frac{A(z, B)}{(A(z, B) - q)^2} - \frac{A(z^*, B)}{(A(z^*, B) - q)^2} > 0.$$

From (25), as h_n decreases, it must be that q_n decreases since $f(q)/(q^*(z^*) - q)$ is increasing in q. Thus, $\{q_n(z, z^*)\}$ is a decreasing sequence. Suppose by way of contradiction that it converges to something strictly positive. Then the right side, $f(q)/(q^*(z^*) - q)$, is strictly positive and we have a contradiction. Thus, $q_n(z, z^*) \to 0$ and so does $\bar{\eta}_n(z, z^*)$.

Summing up, we have shown that

$$\lim_{h_n \to 0} \bar{\eta}_n(z, z^*) = \begin{cases} 1 & \text{if } z \geqslant z^* \\ 0 & \text{if } z < z^* \end{cases} \tag{26}$$

We now turn to construct the series for debt issuances. For any debt level in range of $B^*(z)$, we can define $\sigma_n(z, B) = \eta_n(z, z^*)$ where z^* is such that $B = B^*(z^*)$. We now have to check the incentives to issue debt for a manager with (θ, ν). The optimal debt solves

$$B_n(\theta, \nu) = \arg\max_{B \in rangeB^*(\cdot)} \sigma_n(\theta + \nu, B)\left[\int \max\{\pi(\theta + \nu, \varepsilon) - B\} dF(\varepsilon) - \nu\right] + \nu. \tag{27}$$

By construction, this level of debt satisfies the incentive compatibility constraint (9).

Thus, the constructed sequence $\{\sigma_n(B,q), q_n(B,\theta,\nu), B_n(\theta,\nu)\}$ is feasible for the problem in (10) for any $h_n > 0$. Next, we show that the equilibrium outcome must converge to the efficient allocation. Note that if a manager with $\theta \geqslant RK$ issues $B = B^*(z^*)$ then he will find it optimal to request assistance as (7) holds. Therefore, if the manager chooses the debt level associated with the good equilibrium, $B^*(z^*)$, the efficient outcome is the unique equilibrium outcome.

We now show that $\{B_n(\theta,\nu)\}$ converges (point-wise) to $B^*(\theta+\nu)$ for all $\theta > RK$. To see this, fix (θ,ν) such that $\theta > RK$. Consider any $\Delta > 0$. If the manager chooses $B = B^*(\theta+\nu)$ then the manager's value is $V^*(\theta,\nu) = \theta - RK + \nu$. Clearly, it will never be optimal to choose $B > B^*(\theta+\nu)$ as this only increases repayments. If the manager chooses $B < B^*(\theta+\nu) - \Delta$, the manager's value is

$$\begin{aligned} V_n &= \sigma_n(\theta+\nu, B)\left[\int \max\{\pi(\theta+\nu,\varepsilon) - B\}\, dF(\varepsilon) - \nu\right] + \nu \\ &< \sigma_n(\theta+\nu, B^* - \varepsilon)\,\theta + \nu \end{aligned}$$

because $\sigma_n(\theta+\nu, B^*-\Delta)$ is strictly increasing in B, so $\sigma_n(\theta+\nu,B) \leqslant \sigma_n(\theta+\nu,B^*-\Delta)$ for all $B < B^* - \Delta$. Since $\{\sigma_n(\theta+\nu,B^*-\Delta)\}$ converges to zero, there exists N_Δ such that $\sigma_n(\theta+\nu,B^*-\Delta) \leqslant \delta$ for all $n \geqslant N_\Delta$ where δ is defined as $\delta = (\theta - RK)/\theta$. Thus, for all $n \geqslant N_\Delta$, $V_n < V^*$ and the optimal $B_n(\theta,\nu)$ in (27) cannot be less than $B^*(\theta,\nu) - \Delta$. Therefore, for any $\Delta > 0$, there exists N_Δ such that for all $n \geqslant N_\Delta$ we have $|B^*(\theta,\nu) - B_n(\theta,\nu)| < \Delta$. Thus, $B_n(\theta,\nu)$ converges to $B^*(\theta,\nu)$ for any $\theta > RK$.

For $\theta < RK$, if ν is small enough, it may be optimal for the manager to issue $B > 0$ in order to induce intervention on the part of the government. However, by (26), when h_n is close enough to zero, the probability of such an intervention is close to zero.

Summing up: we show that as $h_n \to 0$: i) if $\theta > RK$ then $B_n(\theta,\nu) \to B^*(\theta+\nu)$ and $\sigma_n(\theta+\nu, B_n(\theta,\nu)) \to 1$; ii) if $\theta < RK$ then $\sigma_n(\theta+\nu, B_n(\theta,\nu)) \to 0$. Thus, in the limit the welfare converges to W^* and we can uniquely approximate the efficient allocation. Q.E.D.

B Dynamic model

We now consider a dynamic version of the economy to study the interaction between private contracts and government interventions. We start with a simple 3-period version of the model with $t = 0, 1, 2$. It is straightforward to show that our characterization holds for any arbitrary horizon. As before, in $t = 0$ a manager with no funds attempts to raise funds to finance an investment project that yields $y_1 = \pi(\theta_0, \varepsilon_1)$ in period 1. In addition, in period 1, the manager has available another project which also requires an investment K in period 1 and yields output $y_2 = \pi(\theta_1, \varepsilon_2)$ in period 2. We assume θ_0 and θ_1 are independent. Private investors and the manager observe θ_0 in period 0 and θ_1 in period 1. Production efficiency dictates that a project should be funded if and only if $\theta_t \geqslant RK$.

Private equilibria In period 0, a manager with a project θ_0 offers an amount B_0 of contracts that promise a state-contingent return $\{R_{01}(y_1, \theta_1), R_{02}(y_1, \theta_1, y_2)\}$. Let q_0 be the price of such contracts. In period 1, after observing (y_1, θ_1), the manager can issue B_1 new contracts to new investors promising a repayment $R_{12}(y_2)$, which has price q_1. If investment is undertaken in period 0, feasibility requires that

$$q_0 B_0 \geqslant K.$$

If investment is undertaken in period 1, feasibility requires

$$\begin{aligned} y_1 + q_1 B_1 &\geqslant K + B_0 R_{01}(y_1, \theta_1), \\ y_2 &\geqslant B_0 R_{02}(y_1, \theta_1, y_2) + B_1 R_{12}(y_2) \end{aligned}$$

and if there is no investment $y_1 \geqslant B_0 R_{01}(y_1, \theta_1)$. Critically, we assume that $R_{01}(y_1, \theta_1) \geqslant 0$ so that period 0 investors cannot be forced to make payments in period 1. Alternatively, we could assume that they have zero endowment in period 1.

Proposition 6. *In period 0, if* $\theta_0 > RK$ *then an equilibrium with investment in period 0 always co-exists with one without investment; if* $\theta_0 < RK$ *depending on equilibrium selection in period 1, there may be an equilibrium with investment. In period 1, conditional on investment in period 0, if* $\theta_1 > RK$*, an equilibrium with investment co-exists with one without investment if and only if* $y_1 < K$.

In period 1, there can be multiplicity if and only if $y_1 < K$ and the manager needs to rely on new external funds to finance the investment. In fact, suppose that there is a state (y_1, θ_1) with $y_1 \geqslant K$ and $\theta_1 > RK$ where the manager relies on period 1 investors to fund the investment and with some positive probability there is coordination on the equilibrium outcome without investment. If this is the case, then it must be that $R_{01}(y_1, \theta_1) = \bar{R}$ and available funds are less than K, $y_1 - \bar{R} < K$. This cannot be part of an equilibrium because the coordination failure can be avoided by setting $R_{01}(y_1, \theta_1) = y_1 - K \geqslant 0$ and delaying payments until period 2 by setting $R_{02}(y_1, \theta_1)$ such that $\int R_2(y_1, \theta_1, \pi(\theta_1, \varepsilon_2))\, dF(\varepsilon_2) = R\bar{R}$. Finding such delayed payments is feasible because $\theta_1 > RK$ and $\bar{R} \leqslant K$. Clearly, the manager is better off delaying payments and investing since he is the residual claimant of output. If $y_1 < K$, the constraint that $R_{01}(y_1, \theta_1) \geqslant 0$ requires that the manager must raise at least some funds from period 1 investors. Thus he is exposed to coordination failures as in Section 2.

The above argument implies that private equilibrium contracts must involve the delay of payments in periods in which period 1 investment project is profitable. This is possible also if contracts cannot depend on θ_1 because it is not verifiable. In this case, the manager can issue state contingent debt that promises to repay in period 1 contingent on y_1 only, $R_{01}(y_1)$ but with the option to delay or rollover the payments until period 2 upon investing in the new project and repay $R_{02}^{roll}(y_1)$ implicitly defined as

$$R_{01}(y_1) = \int \min\left\{\pi(RK, \varepsilon), R^{roll}(y_1)\right\} dF(\varepsilon).$$

We call this the *rollover option*. The manager will only exercise the rollover option if $\theta_1 > RK$. In fact, the manager's payoff for exercising the option is

$$V^{roll}(\theta_1) = y_1 - K + \frac{1}{R}\int \max\left\{\pi(\theta_1, \varepsilon) - R^{roll}(y_1), 0\right\} dF(\varepsilon).$$

Thus, if $\theta_1 < RK$

$$\begin{aligned}
V^{roll}(\theta_1) &< y_1 - K + \frac{1}{R}\int \max\left\{\pi(RK, \varepsilon) - R^{roll}(y_1), 0\right\} dF(\varepsilon) \\
&= y_1 - K + \frac{1}{R}\left[\int \pi(RK, \varepsilon)\, dF(\varepsilon) - R_{01}(y_1)\right] \\
&= y_1 - R_{01}(y_1) = V^{no-roll}.
\end{aligned}$$

If instead $\theta_1 > RK$, a symmetric argument implies that exercising the rollover option is better than no investment in period 1:

$$V^{roll}(\theta) > y_1 - R_{01}(y_1) = V^{no-roll}.$$

Note that the rollover option does not have to be necessarily exercised in equilibrium for ensuring that all projects with $\theta_1 > RK$ are undertaken. The mere existence of this option rules out coordination failures in period 1. In fact, an individual investor is assured that even if other new investors do not invest, the manager will exercise the option. Thus, the equilibrium is unique. In the case proposed above, the manager does indeed prefer to issue new debt from period 1 investors because it can do so at a lower interest rate than $R^{roll}(y_1)$ if $\theta_1 > RK$. This because $R^{roll}(y_1)$ must be sufficiently high to prevent manager with $\theta_1 < RK$ to roll over the debt repayments and gamble for a high realization of y_2 when θ_1 is not contractible.

Multiplicity in period 0 is similar to the two-period version, but in the multi-period economy there can be equilibria with over investment while in the two-period economy, the bad equilibrium always has under investment. In particular, in the multi-period economy there are equilibria with $\theta_0 < RK$ and investment being undertaken in period 0 to reduce set of states where multiplicity can arise in period 1. To see this, assume that in period 1 investors coordinate on the bad equilibrium for sure. Thus, the value for a manager with $\theta_0 < RK$ is 0 if there is no investment. If instead he invests in period 0 his value is

$$-\left(K - \frac{1}{R}\theta_0\right) + \frac{1}{R}\int_{\pi(\theta_0,\varepsilon_1)\geqslant K}\int_{\theta_2\geqslant RK}\left(-K + \frac{1}{R}\theta_2\right) dF(\varepsilon_1)\, dG(\theta_2)$$

This value can be positive if the option value of investing in period 1 is sufficiently high. This is an example of when inefficient over-investment in private equilibria can arise to increase the collateral value, as discussed in the last paragraph of Section 5.

Robust intervention We now show how the government can uniquely implement an allocation that is arbitrary close to the efficient allocation using a scheme similar to the one described for the two-period economy.

Proposition 7. *There exists a policy that can approximate the best outcome arbitrarily closely. This policy commits to ex-post inefficient investments with some strictly positive probability in period 0 and in period 1 if $y_1 < K$. The optimal scheme does not make any transfers to investors.*

To minimize the amount of intervention, the government commits to provide assistance only to managers that issue debt contract with the rollover option. In particular, $\eta_0(B_0, q_0)$ is the same as the two-period case, while $\eta_1(B_0, q_0, y_1, B_1, q_1)$ equals to zero if $y_1 \geqslant K$ and it is equal to the one in the two-period case whenever $y_1 < K$. The government intervention does not make any transfers to debt holders.

To understand how this intervention scheme approximates the efficient allocation, consider first period 1. As shown above, if $y_1 \geqslant K$ there is a unique continuation private equilibrium that corresponds with the efficient one. Thus, there is no need for government intervention, The government will then commit not to intervene whenever $y_1 \geqslant K$.

Suppose now that $y_1 < K$. In this case, the environment is identical to the two-period one described in previous section except that the amount of resources to be raised is $K + R_{01}(y_1) - y_1 > 0$. Thus, absent government intervention there are two continuation equilibria. The government can use an identical policy to the previous section to uniquely implement an allocation arbitrary close to the efficient outcome with some vanishing assistance for inefficient project.

It is important that the intervention in period 1 does not insure period 0 investors against the realizations of y_1. This is because of two reasons. First, insurance will affect the information content of debt prices in period 0. In particular, full insurance will make period 0 price insensitive to θ_0 and therefore the government cannot use prices to learn the investment profitability. [3] Second, the government does not want to subsidize investment in the bad project in period 0. If the government provides any transfers to period 0 investors, it incentivizes investment in investment projects with $\theta_0 < RK$. And so not providing transfers avoids the usual over-investment issue inherent in models with moral hazard and bailouts.

[3]See Dovis and Kirpalani (2022) and Bond and Goldstein (2015) for economies where interventions affect the information content of prices.