On the Design of a Robust Lender of Last Resort

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Motivation

In dynamic economies where debt sustained by reputational forces

- Private markets can attain efficient allocation
- But coordination failures can lead to suboptimal eq. outcomes

Role of policy to rule out bad equilibrium outcomes

- Are there policies/institutions that can rule out bad outcomes?
- Are such policies costlessly or require incurring losses?

Motivation

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Example: European Stability Mechanism (ESM)

- *The ESM carries out this mission by* ***providing loans and other types of financial assistance to member states that are experiencing or are threatened by severe financial distress****. In other words, the* ***ESM acts as a lender of last resort for euro area countries when they are unable to refinance their government debt in financial markets at sustainable rates.***

This Paper

- Study simple dynamic pure exchange economy where borrower cannot commit to repay
- Multiple equilibria w/out assistance
  - Efficient allocation is a sustainable equilibrium
  - But there are also equilibria with little to no credit
  - Dynamic coordination problem
- Study role of financial stability fund (Fund)
  - Fund commits to lend at some price subject to a debt cap
  - Fund lacks information about borrower's fundamentals
  - Robust approach: maximize value of worst equilibrium

Main Results

- To improve worst eqlbrm, Fund must incur losses on path
  - Trade-off between Fund losses and minimal borrower's utility
  - More generous schemes associated w/ lower variance of outcomes
- Fund's interventions back-loaded
  - Fund commits to increasing support and losses over time
- Fund's assistance does not reduce the borrower's effort
  - Effort is higher than in worst equilibrium without Fund

Related Literature

- Economies w/out commitment
  - Efficient allocation: Kehoe-Levine (1993), Alvarez-Jermann (2000), Kehoe-Perri (2002, 2004), Atkeson (1991), Dovis (2019)
  - Multiplicity: Gu et al (2013) and Passadore-Xandri (2021)
  - Here: maximize worst equilibrium
- Unique implementation in macro
  - Full information: Atkeson-Chari-Kehoe (2010), Bassetto (2005), Sturm (2023), Barthelemy-Mengus (2022), Kirpalani (2015), Roch-Uhlig (2018), Bocola-Dovis (2019)
  - Wallace (1981), Nicolini (1996)
  - Here: lack of info
  - Dovis-Kirpalani (2023): static coordination problem, use price to learn about state
- Unique implementation with private contracts
  - Winter (2004), Halac-Kremer-Winter (2020), Camboni-Porcellachia (2021)
- Role of Fund
  - Abraham et al (2018), Liu et al (2022), Callegari et al (2023)
  - Fund's role: provide state contingent payment
  - Here: Fund's role to reduce uncertainty about eqlbrm selection
  - Global games: Morris-Shin (2006) and Corsetti-Guimaraes-Roubini (2006)

Outline

- Economy
- Private sustainable equilibria
- Sustainable equilibria with Fund assistance
  - Stationary policy
  - Optimal timing of assistance
  - (Moral hazard in extended economy)

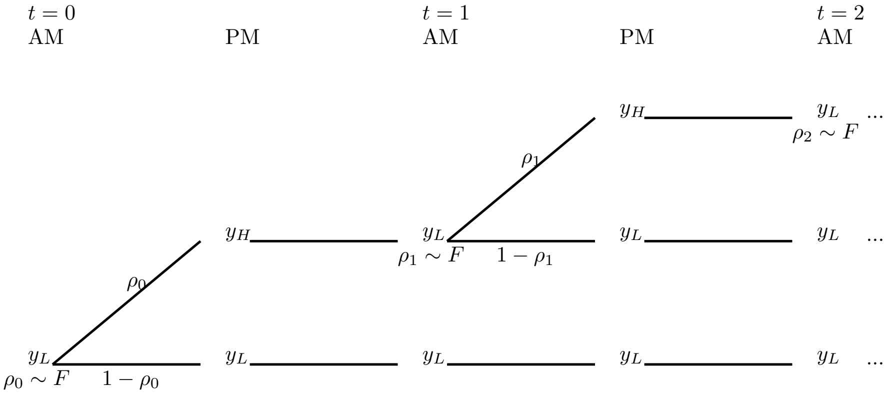
Economy

- $t = 0, 1, ..., \infty$
- Each period is divided into two sub-periods: AM and PM
- Short-lived lenders
  - Require expected gross return of 1 between AM and PM
- Borrower
  - Risk-averse with preferences

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_{AMt}\right) + u\left(c_{PMt}\right)\right]$$

  - Endowment can take two values $\{y_L, y_H\}$ with $y_L < y_H$
  - The endowment is always y_L in the AM
  - In the PM y_H in *high-income regime* and y_L in *low-income regime*
  - In AM draw probability $\rho_t \sim F$ to stay in high-income regime
  - Low-income regime is permanent

Endowment Process



Borrower Cannot Commit

- Can walk away from their debt obligations and live in autarky

- Values of autarky in PM are

$$V_{aut}(L) \equiv u(y_L) + \frac{\beta}{1-\beta} 2u(y_L)$$

$$V_{aut}(H) \equiv u(y_H) + \frac{\beta}{1-\beta\mathbb{E}\rho} \int_{\underline{\rho}}^{\bar{\rho}} [u(y_L) + \rho u(y_H)] \, dF(\rho) \, .$$

  - Normalize $u(y_L) = 0$

- $\chi \geqslant 0$ extra utility cost of defaulting in high-income regime

Efficient Allocation

The efficient allocation solves

$$V^* = \max_b \frac{\int_{\underline{\rho}}^{\bar{\rho}} \left[u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right)\right] dF\left(\rho\right)}{1 - \beta \mathbb{E}\rho}$$

subject to the sustainability constraint in the high-income regime

$$u\left(y_H - b\right) + \frac{\beta \int_{\underline{\rho}}^{\bar{\rho}} \left[u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right)\right] dF\left(\rho\right)}{1 - \beta \mathbb{E}\rho} \geqslant V_{aut}\left(H\right) - \chi$$

Note:

- In low-income regime no repayments can be sustained
- No savings across periods

Assume β low enough so sustainability constraint binding

- Let b^* be largest sol'n

SUSTAINABLE EQUILIBRIUM

Sustainable Equilibrium

- Sustainable equilibrium with one period defaultable debt

- Timing and histories:
  - ρ is realized
  - Coordination device ξ_1
  - Borrower issues b
  - Price of debt q
  - Coordination device ξ_2
  - Borrower chooses to repay, $\delta = 1$, or default $\delta = 0$

Sustainable Equilibrium

- Sustainable equilibrium with one period defaultable debt

- Timing and histories: $h^{t-1} = \{\rho_k, \xi_{1k}, b_k, q_k, \xi_{2k}, \delta_k\}_{k=0}^{t-1}$
  - ρ is realized
  - Coordination device ξ_1
  - Borrower issues b: $h^t_{AM} = \left(h^{t-1}, \rho_t, \xi_{1t}\right)$
  - Price of debt q
  - Coordination device ξ_2
  - Borrower chooses to repay, $\delta = 1$, or default $\delta = 0$: $h^t_{PM} = (h^t_{AM}, b_t, q_t, \xi_{2t})$

Sustainable Equilibrium

A *sustainable equilibrium* is $b\left(h^t_{AM}\right)$, $q\left(h^t_{AM}, b_t\right)$, $\delta\left(h^t_{PM}\right)$, and borrower's values $V_{AM}\left(h^t_{AM}\right)$ and $V_{PM}\left(h^t_{PM}\right)$ such that:
i) $b\left(h^t_{AM}\right)$ is optimal for the borrower in the AM for all h^t_{AM},

$$V_{AM}\left(h^t_{AM}\right) = \max_b u\left(y_L + q\left(h^t_{AM}, b\right) b\right) + \rho_t V_{PM}\left(h^t_{AM}, b\right)$$

ii) $q\left(h^t_{AM}, b_t\right)$ satisfies the lenders' break-even condition

$$q\left(h^t_{AM}, b_t\right) = \rho_t \delta\left(h^t_{PM}\right)$$

iii) $\delta\left(h^t_{PM}\right)$ is optimal for the borrower in the PM for all h^t_{PM},

$$V_{PM}\left(h^t_{PM}\right) = \max_{\delta\in\{0,1\}} \delta\left[u\left(y_H - b_t\right) + \beta\int V_{AM}\left(h^t_{PM}, 1, \rho'\right) dF(\rho')\right] + (1-\delta)\left[u\left(y_H\right) + \beta\left(\int V_{AM}\left(h^t_{PM}, 0, \rho'\right) dF(\rho') - \chi\right)\right]$$

Set of Sustainable Equilibria

Let $\mathcal{V} = [\underline{V}, \bar{V}]$ be the equilibrium value set

Proposition Multiple equilibria: if χ not too large then $\underline{V} < \bar{V}$

- If $\chi = 0$ then $\underline{V} = V_{aut}$ and $\bar{V} = V^*$

Key for multiplicity

- Sustainable debt today depends on future debt availability (continuation value conditional on repayment)
- Same mechanism as Gu et al (2013), Passadore-Xandri (2021), Alvarez-Jermann (2000)

Maximal Sustainable Debt

- Fix $\mathcal{V} = [\underline{V}, \bar{V}]$ and
$$\Delta V \equiv \bar{V} - \underline{V}$$
- Maximal amount of debt that can be supported given that a default induces a drop in continuation value of ΔV is $h(\Delta V)$:

$$u\left(y_H - h\left(\Delta V\right)\right) + \beta \bar{V} = u\left(y_H\right) + \beta\left(\underline{V} - \chi\right)$$

Maximal Sustainable Debt

- Fix $\mathcal{V} = [\underline{V}, \bar{V}]$ and
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- Maximal amount of debt that can be supported given that a default induces a drop in continuation value of ΔV is $h(\Delta V)$:
$$u(y_H) - u(y_H - h(\Delta V)) = \beta(\Delta V - \chi)$$
    - Larger ΔV allows for larger debt

Minimal Sustainable Debt

- Fix $\mathcal{V} = [\underline{V}, \bar{V}]$ and
$$\Delta V \equiv \bar{V} - \underline{V}$$
- Minimal amount of debt that can be supported given that a default induces a drop in continuation value of $-\Delta V$ is $h(-\Delta V)$:
$$u\left(y_H - h\left(-\Delta V\right)\right) + \beta \underline{V} = u\left(y_H\right) + \beta(\bar{V} - \chi)$$

Minimal Sustainable Debt

- Fix $\mathcal{V} = [\underline{V}, \bar{V}]$ and

$$\Delta V \equiv \bar{V} - \underline{V}$$

- Minimal amount of debt that can be supported given that a default induces a drop in continuation value of $-\Delta V$ is $h(-\Delta V)$:

$$u\left(y_H\right) - u\left(y_H - h\left(-\Delta V\right)\right) = \beta\left(-\Delta V + \chi\right)$$

  - If $(-\Delta V + \chi) \leqslant 0$ then $h(-\Delta V) = 0$

Minimal Sustainable Debt

- Fix $\mathcal{V} = [\underline{V}, \bar{V}]$ and
$$\Delta V \equiv \bar{V} - \underline{V}$$
- Minimal amount of debt that can be supported given that a default induces a drop in continuation value of $-\Delta V$ is $h(-\Delta V)$:
$$u(y_H) - u(y_H - h(-\Delta V)) = \beta(-\Delta V + \chi)$$
  - If $(-\Delta V + \chi) \leqslant 0$ then $h(-\Delta V) = 0$

Use APS operator to compute $\mathcal{V} = [\underline{V}, \bar{V}]$

Set Sustainable Equilibria

Sustainable equilibrium value set, $\mathcal{V}$, is the largest fixed point of

$$\mathsf{P}\left[\underline{V}, \bar{V}\right] = \left[\underline{P}\left(\left[\underline{V}, \bar{V}\right]\right), \bar{P}\left(\left[\underline{V}, \bar{V}\right]\right)\right]$$

where

$$\underline{P} = \min_{b, V', V'_d} \int \left[u\left(y_L + \rho b\left(\rho\right)\right) + \rho u\left(y_H - b\left(\rho\right)\right) + \beta\rho V'\left(\rho\right)\right] dF\left(\rho\right)$$

subject to

$$b(\rho) = h(V'(\rho) - V'_d(\rho))$$

$$V'\left(\rho\right), V'_d\left(\rho\right) \in \left[\underline{V}, \bar{V}\right]$$

and

$$\bar{P} = \max_{b, V', V'_d} \int \left[u\left(y_L + \rho b\left(\rho\right)\right) + \rho u\left(y_H - b\left(\rho\right)\right) + \beta\rho V'\left(\rho\right)\right] dF\left(\rho\right)$$

subject to the two constraints above

Set Sustainable Equilibria

Simplify the two programming problems as:

$$\underline{P} = \int u\left(y_L + \rho h\left(-\Delta V\right)\right) dF(\rho) + u\left(y_H - h\left(-\Delta V\right)\right) + \beta \mathbb{E}\rho \underline{V}$$

$$\bar{P} = \int u\left(y_L + \rho h\left(\Delta V\right)\right) dF\left(\rho\right) + \mathbb{E}\rho u\left(y_H - h\left(\Delta V\right)\right) + \beta \mathbb{E}\rho \bar{V}$$

where $\Delta V \equiv \bar{V} - \underline{V}$

Summing up

- Multiplicity because sustainable debt depends on $\Delta V'$
  - Continuation value upon repayment
  - Minus continuation value upon default
- External observer cannot tell if low q is due to
  - Bad fundamentals: low ρ
  - Bad coordination: high probability of low ΔV
- Next: Can policy help to resolve multiplicity?
  - Fund lacks information about borrower's fundamentals
  - **Robust approach**: most adversarial selection

SUSTAINABLE EQUILIBRIUM WITH FUND ASSISTANCE

Fund Assistance

Let $(\underline{q}, \bar{b})$ be an assistance policy

- Debt cap: $\bar{b}$
- Price floor: $\underline{q}$
- Fund is willing to lend at price $\underline{q}$ if
  - $b \leqslant \bar{b}$
  - Borrower has not defaulted before

Study how $(\underline{q}, \bar{b})$ affects worst equilibrium

Sustainable Equilibrium w/ Assistance $(\underline{q}, \bar{b})$

A *sustainable equilibrium* is $b\left(h^t_{AM}\right)$, $q\left(h^t_{AM}, b_t\right)$, $\delta\left(h^t_{PM}\right)$, and borrower's values $V_{AM}\left(h^t_{AM}\right)$ and $V_{PM}\left(h^t_{PM}\right)$ such that:
i) $b\left(h^t_{AM}\right)$ is optimal for the borrower in the AM for all h^t_{AM},

$$V_{AM}\left(h^t_{AM}\right) = \max_{b} u\left(y_L + q\left(h^t_{AM}, b\right) b\right) + \rho_t V_{PM}\left(h^t_{AM}, b\right)$$

ii) $q\left(h^t_{AM}, b_t\right)$ is consistent with the Fund's rule and satisfies the break-even constraint for lenders

$$q\left(h^t_{AM}, b_t\right) = \begin{cases} \max\left\{\underline{q}, \rho_t \delta\left(h^t_{AM}, b_t\right)\right\} & \text{if } b_t \leqslant \bar{b}, \delta_{t-k} = 1 \forall t-k < t \\ \rho_t \delta\left(h^t_{AM}, b_t\right) & \text{otherwise} \end{cases}$$

iii) $\delta\left(h^t_{PM}\right)$ is optimal for the borrower in the PM for all h^t_{PM},

$$\begin{aligned} V_{PM}\left(h^t_{PM}\right) = \max_{\delta \in \{0,1\}} \delta & \left[u\left(y_H - b_t\right) + \beta \int V_{AM}\left(h^t_{PM}, 1, \rho'\right) dF(\rho')\right] \\ & + (1-\delta)\left[u\left(y_H\right) + \beta \int V_{AM}\left(h^t_{PM}, 0, \rho'\right) dF(\rho') - \chi\right] \end{aligned}$$

Can Uniquely Implement Efficient Allocation?

Yes if can condition $\underline{q}$ on ρ

- In general also need $\bar{b}$ to be contingent on ρ

Proposition Let $\underline{q}(\rho) = \rho$ and $\bar{b} = b^*$. Then the efficient allocation is the unique sustainable equilibrium outcome with assistance. The Fund does no make any loss and it may never be used.

- Same Kirpalani (2015), Roch-Uhlig (2018), Bocola-Dovis (2019), Callegari et al (2023)

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If $\underline{q}$ not contingent on ρ because Fund lacks info?
Not in general

Worst Sustainable Equilibrium Value

The worst equilibrium value is (smallest) fixed point of

$$\underline{P}_F\left(\left[\underline{V}_F, \bar{V}_F\right]\right) = \int v_F\left(\rho, \left[\underline{V}_F, \bar{V}_F\right]\right) dF(\rho)$$

where

$$v_F\left(\rho, \left[\underline{V}_F, \bar{V}_F\right]\right) = \min_{V', V'_d, V'_{df}} \max \left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H\right) + \beta\rho V'_{df} \\ u\left(y_L + \underline{q}\bar{\bar{b}}\right) + \rho u\left(y_H - \bar{b}\right) + \beta\rho V' \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) + \beta\rho V' \end{array} \right\}$$

subject to

$$V' \in \left[\underline{V}_F, \bar{V}_F\right], \quad V'_{df}, V'_d \in \left[\underline{V}, \bar{V}\right]$$

$$b = h(V' - V'_d)$$

Worst Sustainable Equilibrium Value

The worst equilibrium value is (smallest) fixed point of

$$\underline{P}_F\left(\underline{V}_F\right) = \int v_F\left(\rho, \underline{V}_F\right) dF\left(\rho\right)$$

where

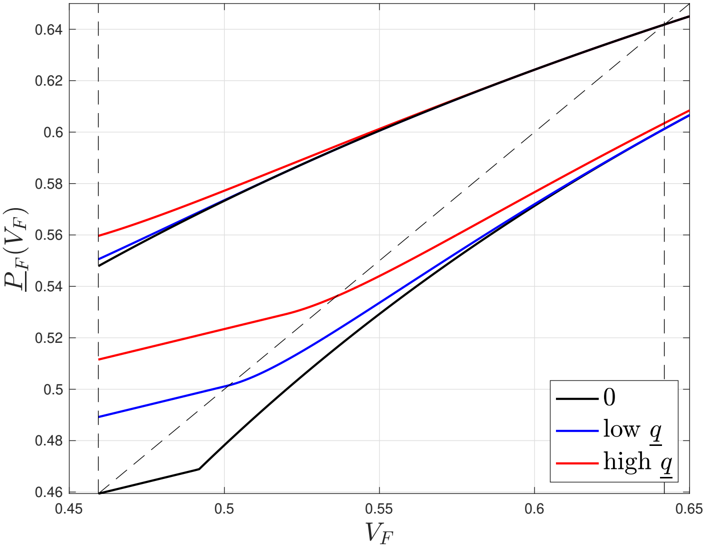
$$v_F\left(\rho, \underline{V}_F\right) = \max\left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H\right) + \beta\rho\underline{V} \\ u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \bar{b}\right) + \beta\rho\underline{V}_F \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) + \beta\rho\underline{V}_F \end{array} \right\}$$

where

$$V'_{df} = \underline{V}, \quad V' = \underline{V}_F, \quad V'_{df} = \bar{V}$$

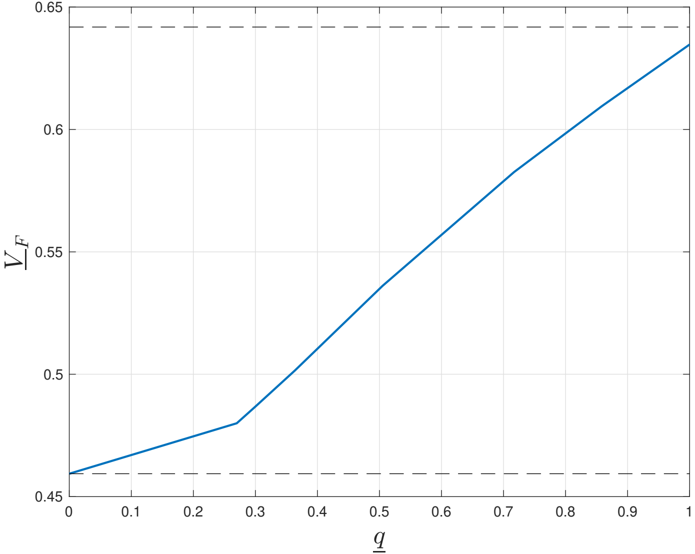
$$b = h(\underline{V}_F - \bar{V})$$

Worst Sustainable Equilibrium Value



- If $\underline{q}\bar{b} > \underline{\rho}h\left(\underline{V} - \bar{V}\right)$ then $\underline{P}_F(\underline{V}) > \underline{V} \Rightarrow \underline{V}_F > \underline{V}$

Worst Sustainable Equilibrium Value



- Trade-off between assistance's generosity ($\underline{q}$) and $\underline{V}_F$

Worst Sustainable Equilibrium Outcome

Fix $\bar{b} > 0$

If $\underline{q}$ low $\Rightarrow \bar{b} > h\left(\underline{V}_F - \underline{V}\right)$

- Borrows from the Fund at maximal capacity and default on it

If $\underline{q}$ high $\Rightarrow \bar{b} \leqslant h\left(\underline{V}_F - \underline{V}\right)$

- There is no default and borrow from Fund iff

$$u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right) > u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right)$$

with $b = h\left(\underline{V}_F - V\right)$.

But for intermediate $\underline{q} \Rightarrow \bar{b} \in \left(h\left(\underline{V}_F - \bar{V}\right), h\left(\underline{V}_F - \underline{V}\right)\right)$

- There are other equilibria (not worst) with default on the Fund
- Borrower is "rewarded" for defaulting on Fund with $V'_{df} > \underline{V}$

Fund's Losses Are Necessary for $\underline{V}_F > V_{aut}$

- If assistance level is low (low $\underline{q}$) then

$$h(\underline{V}_F(\underline{q}, \bar{b}) - \bar{V}) < \bar{b}$$

There are equilibria with default on Fund and so losses

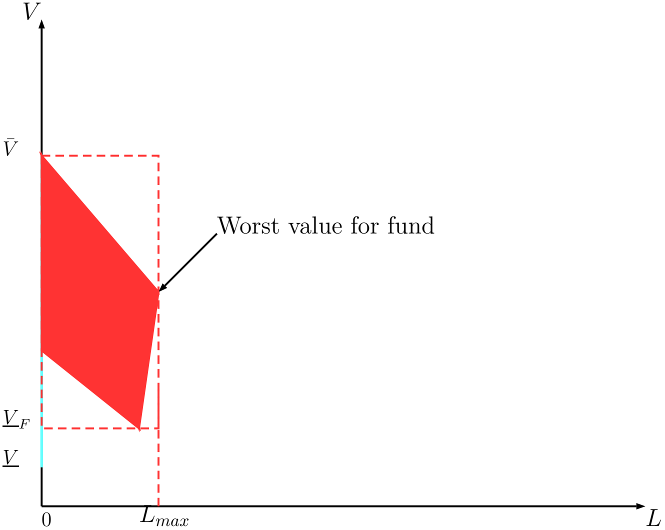
- If assistance level is high (high $\underline{q}$) then

$$h(\underline{V}_F(\underline{q}, \bar{b}) - \bar{V}) > \bar{b}$$

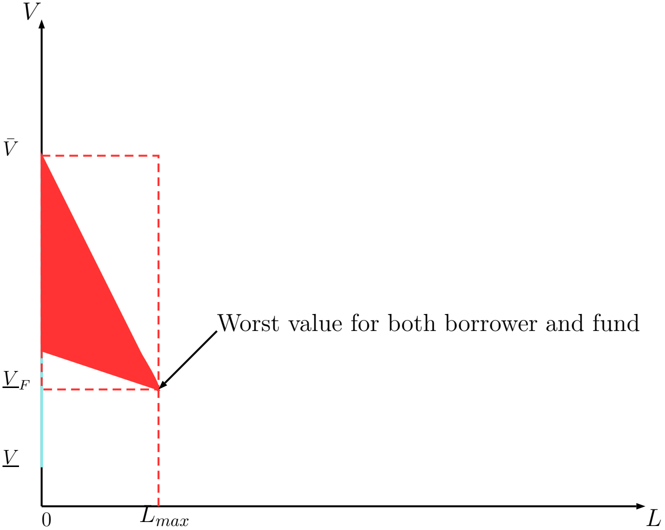
There are no default on the Fund in high-income state but need $\underline{q} > \underline{\rho}$ and so there are price subsidy and losses

Equilibrium Values w/out Assistance

Equilibrium Values w/ Low Assistance



Equilibrium Values w/ High Assistance



Exogenous Default Value (autarky)

- What if default is followed by autarky?
- Multiple private equilibria only if $\chi = 0$
- Fund can uniquely implement the efficient allocation w/ no costs
  - If $\underline{q}\bar{b} > 0$ there exists a unique equilibrium with value $V_F \geqslant V^*$
  - If $\bar{b}$ and $\underline{q}$ are sufficiently small and $\underline{\rho} > 0$
    $\Rightarrow$ assistance never used on-path (no cost) and $V_F = V^*$
- Similar to Wallace (1981), Nicolini (1996)

Details

OPTIMAL TIMING OF ASSISTANCE

Non-Stationary Assistance

- Allow for $\left\{\underline{q}_t, \bar{b}_t\right\}$

- What is the most cost-effective way to deliver V for sure to the borrower?

- Restrict to no default in any equilibrium outcome so

$$\bar{b}_t \leqslant h(\underline{V}_{Ft} - \bar{V})$$

- Show optimal path $\left\{\underline{q}_t, \bar{b}_t\right\}$ is back-loaded

- Eventually large losses even if Fund agrees to small losses in NPV

Problem

$$L(V) = \min_{\underline{q},\bar{b},b,V'} \int \mathbb{I}\left(\rho|\underline{q},\bar{b},b,V'\right)\left(\underline{q}-\rho\right)\bar{b}\,dF(\rho) + \beta\mathbb{E}\rho L\left(V'\right)$$

subject to

$$V = \int \max\left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right); \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) \end{array} \right\} dF(\rho) + \beta\mathbb{E}\rho V'$$

$$b = h\left(V' - \bar{V}\right), \quad \bar{b} \leqslant h\left(V' - \bar{V}\right)$$

where

$$\mathbb{I} = \begin{cases} 1 & \text{if } u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right) \geqslant u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) \\ 0 & \text{otherwise} \end{cases}.$$

Problem

$$\mathsf{L}(V) = \min_{v, V'} \mathsf{K}(v, V') + \beta \mathbb{E} \rho \mathsf{L}(V')$$

subject to

$$v + \beta \mathbb{E} \rho V' = V$$

where

$$\mathsf{K}(v, V) \equiv \min_{\underline{q}, \bar{b}, b} \int \mathbb{I}\left(\rho | \underline{q}, \bar{b}, b, V'\right) \left(\underline{q} - \rho\right) \bar{b} \, d\mathsf{F}(\rho)$$

subject to

$$\int \max \left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right); \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) \end{array} \right\} d\mathsf{F}(\rho) = v$$

$$b = h\left(V' - \bar{V}\right), \quad \bar{b} \leqslant h\left(V' - \bar{V}\right)$$

Backloading is Optimal

$$L(V) = \min_{v, V'} K(v, V') + \beta \mathbb{E} \rho L(V')$$

subject to

$$v + \beta \mathbb{E} \rho V' = V$$

FOC + envelope imply

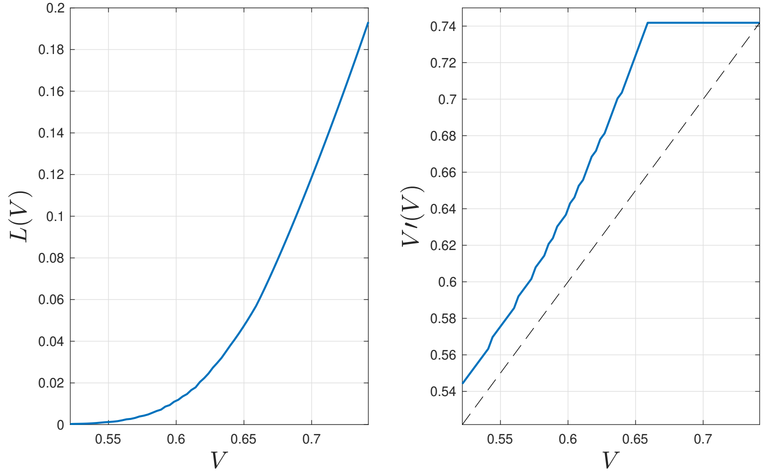
$$K_V + \beta \mathbb{E} \rho [L'(V') - L'(V)] = 0$$

with $K_V \leqslant 0$ so if K is convex (so L is convex)

$$L'(V') > L'(V) \Rightarrow V' > V$$

- Expected present value of subsidies is increasing
- Higher future assistance $V' \uparrow \Rightarrow$ support more private debt today
- Minimize cost of providing V

Backloading is Optimal



Implementation

- Fund raises L in period 0 and commits to increasing support
- In early periods, Fund's period losses are small and the Fund's balance – its endowment – is increasing over time
- Fund can eventually finance large subsidies with the accumulated assets w/out raising more funds
- In worst equilibrium Fund exhausts all its resources
- But there are better equilibria where can repay some of the initial contribution back to its shareholders/donors

Conclusion

Study Fund's role to rule out bad equilibrium outcomes

- To increase worst value, Fund must incur losses
  - Trade-off between maximal losses and lowest borrower's value
- Optimal timing of Fund's interventions is backloaded
  - Fund commits to increasing support over time
- Assistance does not reduce borrower's incentives to exert effort
  - Effort is higher than in worst equilibrium without Fund

Policy lesson for ESM, TPI program of ECB, IMF ...

- If limited information about borrower's fundamentals
- And Fund insist on making no losses
- Then its role can be limited

ADDITIONAL SLIDES

What if Autarky after default?

Proposition Suppose $\chi = 0$.

- Without assistance, the equilibrium value set is $[V_{aut}, V^*]$
- With assistance, if $\underline{q}\bar{b} > 0$ there exists a unique equilibrium with value $V_F \geqslant V^*$
- If $\bar{b}$ and $\underline{q}$ are sufficiently small and $\underline{\rho} > 0$ assistance is never used in equilibrium (no cost) and $V_F = V^*$

Worst Equilibrium with Assistance

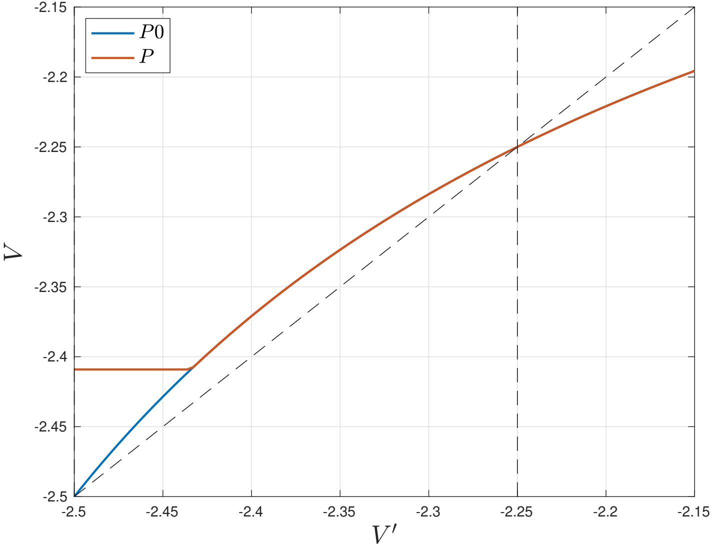
Worst equilibrium value is the smallest fixed point of

$$P\left(V';\pi\right) = \int \max\left\{P_1\left(\rho;\pi\right), P_2\left(\rho;\pi\right), P_3\left(V',\rho;\pi\right)\right\} dF\left(\rho\right).$$

where

$$\begin{aligned}
P_1\left(\rho;\pi\right) &= u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u\left(y_H\right) + \beta\left(V_{aut} - \chi\right)\right] \\
P_2\left(V',\rho;\pi\right) &= u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u\left(y_H - \bar{b}\right) + \beta V'\right] \\
P_3\left(V',\rho;\pi\right) &= u\left(y_L + \rho h\left(V' - V_{aut}\right)\right) + \rho u\left(y_H - h\left(V' - V_{aut}\right)\right) \\
&\quad + \beta\rho V'
\end{aligned}$$

Sustainable Equilibria if $\chi = 0$ and Autarky after Default



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