Unique Implementation with Market-Based Interventions

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Motivation

In large class of economies

- Competitive equilibria can attain the efficient outcome (2nd welfare thm)
- But coordination failures can lead to suboptimal outcomes (1st welfare thm does not hold)
  - Static coordination problem: Diamond-Dybvig, Cole-Kehoe
  - Dynamic coordination problem: Alvarez-Jermann, Gu et al

Role of policy to uniquely implement desired outcome

This paper

- Firm's manager must raise fixed amount to invest from
  - Private investors
    - Know investment's profitability but static coordination problem
  - Government
    - Big player but does not have information about investment

- Multiple private equilibria
  - Efficient allocation is equilibrium outcome
  - Also equilibria where profitable investment projects not funded

- Study **best robust policy**
  - Maximize value under most adversarial equilibrium selection
  - Focus on **market mechanism**
    - Government intervention can depend on market outcomes (prices)
    - Show it is wlog

Results

- Efficient allocation cannot be uniquely implemented
- But it can be approximated arbitrarily closely
- Governments must commit to fund inefficient investment to guarantee that good investment are undertaken for sure
  - Cost to distinguish good and bad investment projects
  - But can make their probability small
- Standard moral hazard not present under optimal policy
  - Increasing probability that good projects are funded increases manageer's incentives to exert effort

Related literature

- Unique implementation with private contracts
  - Winter (2004), Halac-Kremer-Winter (2020), Camboni-Porcellachia (2021)
  - Role of collateral
- Market mechanism
  - Valenzuela-Stookey-Poggi (2020)
  - No coordination problem without policy
- Unique implementation in Ramsey problem
  - Atkeson-Chari-Kehoe (2010), Bassetto (2005), Sturm (2022), Barthelemy-Mengus (2022)
  - Diamond-Dybvig (1983), Roch-Uhlig (2018), Bocola-Dovis (2019)
  - Full information
- Governments vs. markets
  - Acemoglu-Golosov-Tsyvinski (2008)
  - They consider info vs. IR, we info vs. coordination and show complementarity
- Companion paper: dynamic coordination problem

Simple economy

Environment

- $t = 0, 1$
- Continuum of non-atomistic investors
  - Risk neutral and outside option return of $R > 1$
  - Endowment E in period 0
- Firm's manager has investment opportunity that requires K
- If investment undertaken
  - Output: $y = \pi(\theta + \nu, \varepsilon)$
  - θ, ν are realized in $t = 0$, ϵ is realized in $t = 1$
  - $\varepsilon \sim F(\varepsilon)$, support of y is $[0, \infty)$, and $\int \pi(\theta + \nu, \varepsilon) dF(\varepsilon) = \theta + \nu$
- If no investment
  - Output: ν (think of ν as collateral)
- Investors know (θ, ν) in period 0, ε is realized in period 1
- Efficient allocation: **Invest iff $\theta \geqslant RK$**

Private equilibria

- Given (θ, ν)
- Manager offers contract: $\left(R^{I}(\varepsilon), R^{N}\right)$.
  - $R^{I}(\varepsilon)$: return for lenders conditional on investment
  - R^{N}:return for lenders conditional on no investment
  - q price of such a contract
  - B : quantity of such contract
- Feasibility

$$R^{I}(\varepsilon) \leqslant \frac{\pi(\theta + \nu, \varepsilon)}{B}$$

$$R^{N} \leqslant \frac{\nu + qB}{B}$$

- Manager is residual claimant

Timing

- (θ, ν) are realized
  - Let $z = \theta + \nu$
- Manager chooses a contract
- Coordination device ξ is realized
- Investors decide whether to lend

Private equilibrium

A *private equilibrium* is a debt contract $\left(R^I(\theta,\nu,\varepsilon), R^N(\theta,\nu)\right)$, an amount of debt $B(\theta,\nu)$ and debt prices $q(B,\theta,\nu,\xi)$ such that:

- $\left(R^I(\theta,\nu,\varepsilon), R^N(\theta,\nu), B(\theta,\nu)\right)$ solve

$$\max_{R^I(\varepsilon),R^N,B} \int_0^1 \left[\mathbb{I}(\xi)\int \max\left\{\pi(z,\varepsilon) - R^I(\varepsilon)B, 0\right\} dF(\varepsilon) + (1-\mathbb{I}(\xi))\left(\nu - R^N\right)\right] d\xi$$

  subject to feasibility

- $q(B,\theta,\nu,\xi)$ satisfies the investors' optimality condition

$$q(B,\theta,\nu,\xi) = \frac{1}{R}\left[\mathbb{I}(\xi)\int R^I(\varepsilon)\, dF(\varepsilon) + (1-\mathbb{I}(\xi))\, R^N\right]$$

- $\mathbb{I}(\xi) = 1$ if $q(B,\theta,\nu,\xi)\, B(\theta,\nu) \geqslant K$ and $\mathbb{I}(\xi) = 0$ otherwise

Debt contracts

- Wlog, can consider debt contracts
  - Investors payout conditional on investment is

$$R^I(\varepsilon) = \min\{1, \pi(\theta + \nu, \varepsilon)/B\}$$

  - Investors payout conditional on no investment is either
    - If collateralized

$$R^N = q + \frac{\nu}{B}$$

    - If not collateralized

$$R^N = q$$

- Equilibrium price

$$q(\theta, B) = \frac{1}{R}\left[\mathbb{I}A(\theta, B) + (1 - \mathbb{I})\, q(\theta, B)\right]$$

where

$$A(\theta, B) \equiv \int \min\{1, \pi(\theta, \varepsilon)/B\}\, dF(\varepsilon)$$

Multiple equilibria if collateral is scarce

Suppose that $\nu \geqslant (R-1)\,K$

- If $\theta \geqslant RK$: unique equilibrium with investment
- If $\theta < RK$: unique equilibrium with no investment

Suppose that $\nu < (R-1)\,K$

- If $\theta \geqslant RK$: equilibrium with investment coexists with one without
- If $\theta < RK$: unique equilibrium with no investment

Intuition

If collateral is abundant, $\nu \geqslant (R-1)\,K$:

- Can always raise K w/ collateralized debt
- Guarantee return R even if investment project is not funded
- Unique equilibrium outcome is efficient

If collateral is scarce, $\nu < (R-1)\,K$:

- W/out investment, cannot guarantee return R if $qB = K$, $R^N = 1$

$$R^N \leqslant \frac{\nu + qB}{B} < R$$

- Cannot design contracts to make investment dominant strategy
- Thus, there always exist inefficient equilibrium where $\theta > RK$ but $qB < K$ and the investment project is not funded

Can government uniquely implement efficient outcome?

Government intervention

Assume $\nu = 0$

- Government
  - Can finance investment by itself
  - Lacks knowledge about θ
  - Focus on **market mechanism**
    - Intervention depends on market outcome (B, q)
- Timing
  - Gov't commit to fund project with probability $\bar{\eta}(B, q)$ if $qK < K$
  - θ observed by entrepreneur and investors
  - Entrepreneur issues debt B
  - Sunspot ξ realized
  - Price of debt q realized
  - If $qB < K$ manager can ask gov't for assistance
    - Gov't transfers $K - qB$ with prob. $\bar{\eta}(B, q)$

Continuation equilibria

- Given (θ, B)
- The debt price is

$$q = \frac{1}{R}\mathbb{I}A(\theta, B) + \frac{1}{R}(1-\mathbb{I})\left[(1-\bar{\eta}(B,q))\,q + \bar{\eta}(B,q)\,A(\theta,B)\right]$$

- Probability of investment, σ, is
    - $\sigma = 1$ if $qB \geqslant K$
    - $\sigma = \bar{\eta}(B,q)$ if $qB < K$

Continuation equilibria

Can take two forms:

- Investment undertaken without gov't intervention

$$qB \geqslant K$$

$$q = \frac{1}{R} A\left(z, B\right)$$

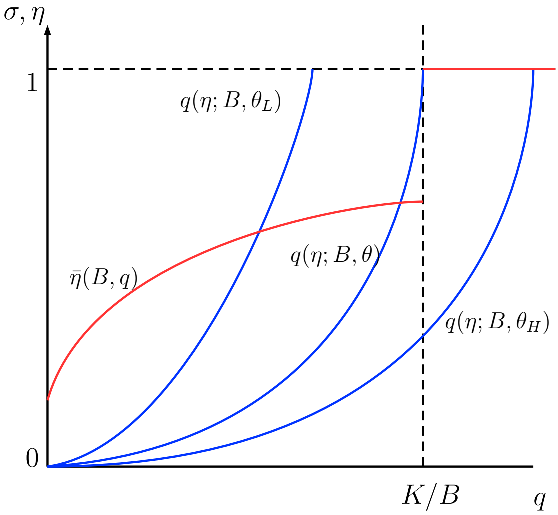
$$\sigma = 1$$

- Investment undertaken with gov't intervention

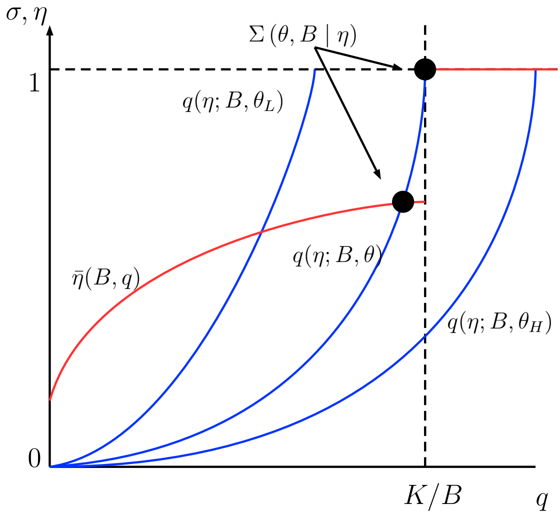
$$\begin{aligned} qB &< K \\ q &= \frac{\bar{\eta}(B, q)}{R - 1 + \bar{\eta}(B, q)} A\left(z, B\right) \\ \sigma &= \bar{\eta}(B, q) \end{aligned}$$

$\Sigma\left(\theta, B \mid \bar{\eta}\right)$: investment probabilities consistent w/ equilibrium

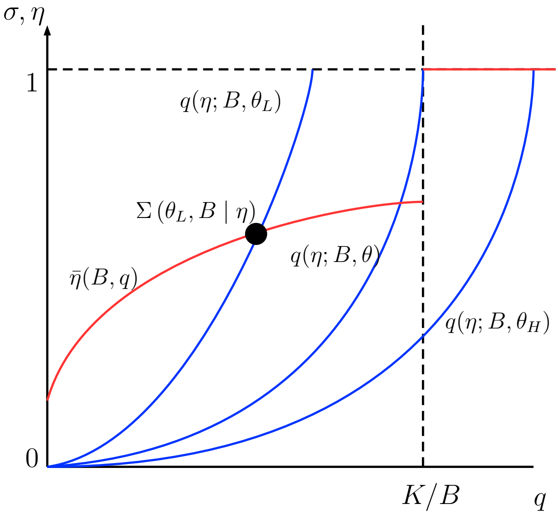
Continuation equilibria



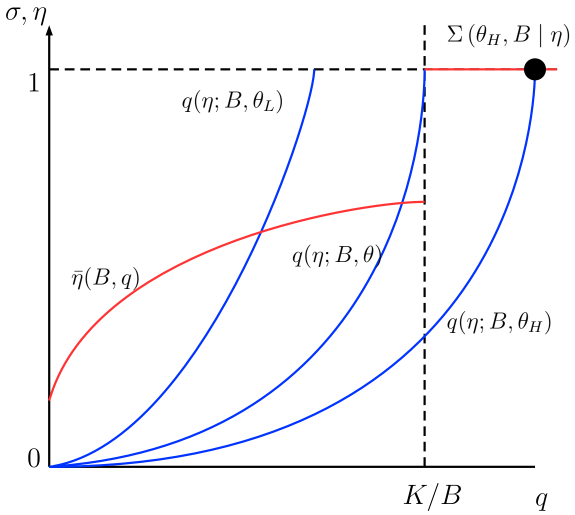
Continuation equilibria



Continuation equilibria



Continuation equilibria



Debt issuance decision

- Debt issuance optimal given belief about equilibrium selection
- Beliefs are drawn from the set $\Sigma(\theta, B \mid \bar{\eta})$

$$B(\theta, \nu) \in \arg\max_{B} \int \zeta(B, \sigma)\, \sigma \Pi(B, \theta)\, d\sigma$$

for some $\zeta(B, \cdot) \in \Delta(\Sigma(\theta, \nu, B(\theta, \nu) \,|\eta))$ where

$$\Pi(B, \theta) \equiv \int \max\{\pi(\theta, \varepsilon) - B, 0\}\, dF(\varepsilon)$$

- If $\Sigma(\theta, B \mid \bar{\eta})$ is singleton, it reduces to

$$B(\theta, \nu) \in \arg\max_{B} \Sigma(\theta, B) \Pi(B, \theta)$$

Best robust policy

Use most adversarial criterion from the gov't perspective

- Highest investment probability if $\theta < RK$:

$$\sigma = \max \Sigma \left(\theta, B\left(\theta\right) | \bar{\eta}\right)$$

- Lowest investment probability if $\theta > RK$:

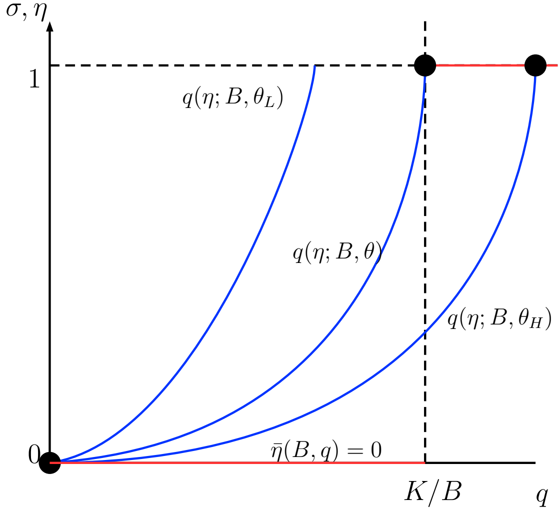
$$\sigma = \min \Sigma \left(\theta, B\left(\theta\right) | \bar{\eta}\right)$$

- Managers choose debt to minimizes the value of the worse equilibrium subject to their IC

Require

$$\Sigma \left(\theta, B | \bar{\eta}\right) \neq \emptyset \text{ for all } \left(\theta, B\right).$$

W/out intervention ⇒ worst eqlbr'm has no investment



Cannot uniquely implement the efficient allocation

- Let efficient outcome be $B^*(\theta), q^*(\theta)$ with investment iff $\theta \geqslant RK$
- Suppose $\exists$ policy that uniquely implements efficient all'n
- If $\theta < RK \Rightarrow$ no investment takes place
    - $q = 0, \bar{\eta}(B, 0) = 0$ for all B
- Now suppose $\theta > RK$
    - Worst private equilibrium has $q = 0$ and no investment
    - Because $\bar{\eta}(B, 0) = 0$, this is also an eq with intervention

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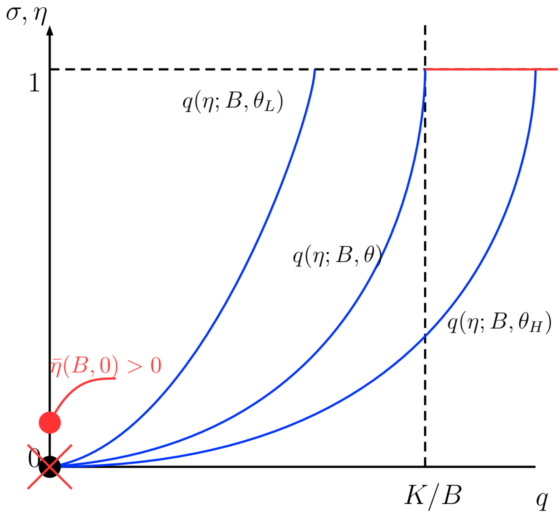
- To avoid no investment when θ high then need $\bar{\eta}\,(B, 0) > 0$
  - **Ex-post inefficient investments are necessary**

Ex-post inefficient investments are necessary



Approximate efficient allocation

- Consider a sequence of $\{\bar{\eta}_n\}$ indexed by parameter $h_n > 0$
- For any θ^* and corresponding $B = B^*(\theta^*)$ let

$$\bar{\eta}_n(B, q) \equiv q \frac{(R-1)}{A(\theta^*, B) - q} + h_n (q^*(\theta^*) - q).$$

- Let $q_n(B, \theta)$ and $B_n(\theta)$ be implicitly defined by

$$\bar{\eta}_n(B, q) = q \frac{(R-1)}{A(\theta, B) - q}.$$

$$B_n(\theta) = \arg\max_B \bar{\eta}_n(B, q(B, \theta)) \int \max\{\pi(\theta, \varepsilon) - B\} \, dF(\varepsilon)$$

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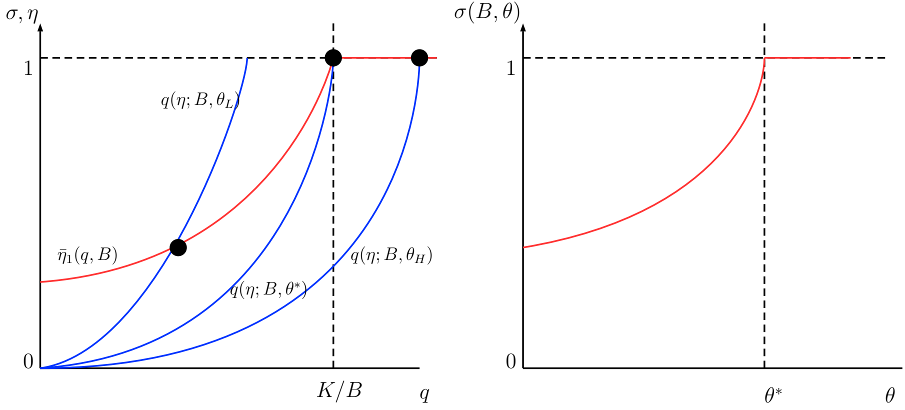
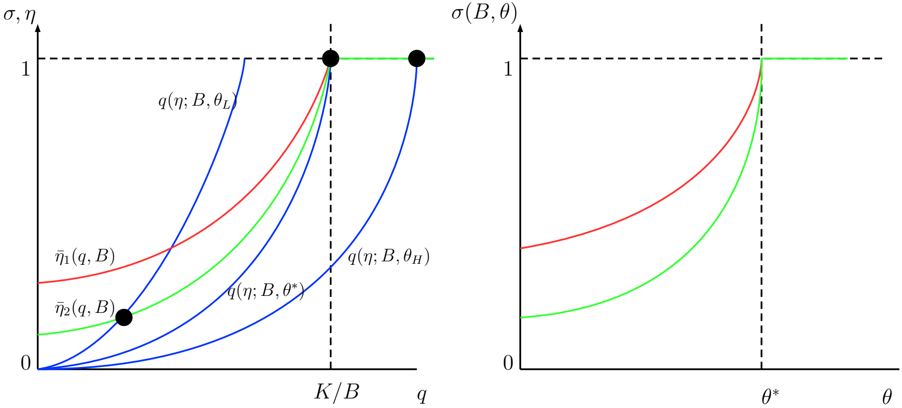
- **If $h_n > 0$, continuation eq is unique for all (B, θ)**
- **As $h_n \to 0$ then converge to efficient allocation**

Continuation equilibrium



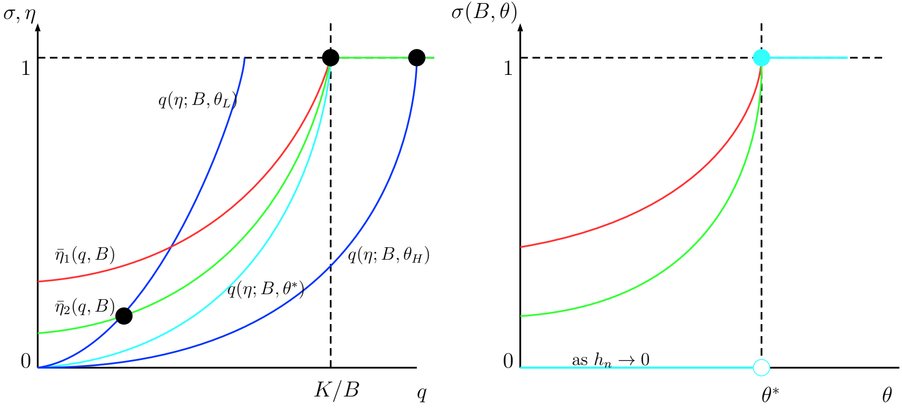
- Unique continuation equilibrium for all θ. Let θ^* s.t. $B = B^*(\theta^*)$
- If $\theta \geqslant \theta^* \geqslant RK$ then invest for sure
- If $\theta < \theta^*$ then invest with probability in $(0, 1)$

Continuation equilibrium



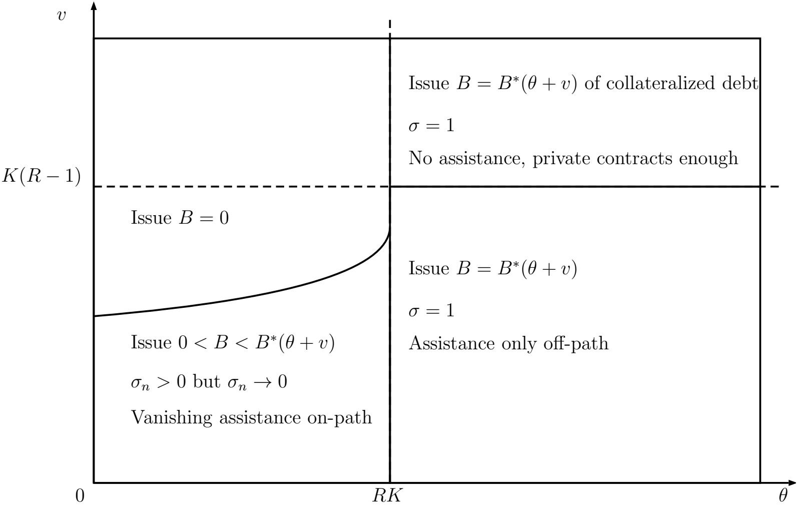
- As h_n decreases:
- If $\theta \geqslant \theta^* \geqslant RK$ then invest for sure
- If $\theta < \theta^*$ invest with smaller probability

Continuation equilibrium



- As $h_n \to 0$
  - Investment probability converges to step function
  - Thus, debt levels converge to $B^*(\theta)$ if $\theta \geqslant RK$
  - For $\theta < RK$ try to issue debt but inefficient investment arb. small

Best robust outcome



Commitment to ex-post inefficient investment

- Gov't commit to fund bad projects (even if with small pr)
  - Allows to learn which projects are good
  - Provide enough support for good project so investors coordinate on good outcome

- Commitment technology is necessary
  - On path, gov't knows that projects requesting assistance are bad
  - Want to renege ex-post

- Absent commitment, either
  - No intervention or
  - Gov't directly funds all projects (without collecting any info)

- Cannot rely on reputational forces (Barthelemy-Mengus)

- Opposite result than typical bailout
  - Want to commit to not bailing out

Dynamic version

- Multiplicity only if new external funds needed to fund investment
- Optimal private contract delays investors' payments to minimize need to raise new external funds
- Best robust policy approximates efficient all'n as in static case
- *Bail-in*: Interventions in $t \geqslant 1$ do not provide transfers to $t-1$ investors; such transfers would
  - Reduce information content of debt prices in $t-1$
  - Subsidize investment in the bad project in $t-1$

Moral hazard

- Moral hazard often associated with interventions/bailouts
  - Kareken-Wallace
- How intervention affects managers' incentives to generate investment projects?
- Higher effort than worst case but lower than best case

Higher effort than worst case but lower than best case

- Manager takes costly action a that affects the value for θ
  - $\theta \in \{\theta_L, \theta_H\}$ with and $\theta_L < RK < \theta_H$
  - Let $f(\theta|a) \equiv \Pr(\theta|a)$ and $c(a)$ is effort cost
- Without any intervention, equilibrium a is $[0, a^*]$
  - Efficient effort is $a^* = \arg\max_a f(\theta_H|a)(\theta_H - RK) - c(a)$
  - Effort can be lower because good equilibrium can be selected with probability $\zeta < 1$,

$$\max_a f(\theta_H|a)\,\zeta\,(\theta_H - RK) - c(a)$$

- Under optimal robust policy

$$a_n = \arg\max_a f(\theta_H|a)\,\eta_n(\theta_H)(\theta_H - RK) - c(a) + f(\theta_L|a)\eta_n(\theta_L)\nu(\theta_L)$$

Thus, $0 < a_n < a^*$ and $\{a_n\} \uparrow a^*$

Higher effort than worst case but lower than best case

- Intervention ensures good projects are funded
- This increases rewards for manager's effort relative private equilibria where good projects not implemented for sure
- But also subsidize bad projects
- This reduces incentives so lower effort than efficient equilibrium
- In the limit, as funding of bad projects vanish only positive effect, $\{a_n\} \uparrow a^*$

Can general mechanism improve market mechanism?

- No, if investors observe θ with noise (and market aggregate info)
  - No mechanism uniquely implements the efficient allocation
  - Cannot make dominant strategy for manager with $\theta < RK$ to report something that induces no investment
    - By investing get option value $\int \max\{\pi(\theta, \epsilon) - B, 0\} dF(\epsilon) > 0$
  - For investors, same coordination problem as in debt market

- Yes, if investors observe θ exactly
  - Can uniquely implement the efficient allocation
  - Make one investor pivotal and give them a return $\theta - RK$

Conclusion

- Study investment problem where static coordination problem leads to multiple equilibria
- Study which gov't intervention can uniquely implement desired outcome when gov't lacks info
  - Best robust policy
- Governments must commit to fund inefficient investment to guarantee that good investment are undertaken for sure
- Complementarity between government and market
  - Market aggregates information
  - Government (big player) rules out coordination problems