Multiplicity, Information, and Unique Implementation in Credit Markets

Alessandro Dovis
U Penn and NBER

Rishabh Kirpalani
Wisconsin

ASSA, NAWM Econometric Society
January 2023

Motivation

In large class of economies

- Competitive equilibria can attain the efficient outcome (2nd welfare thm)

- But coordination failures can lead to suboptimal outcomes (1st welfare thm does not hold)
  - Static coordination problem: Diamond-Dybvig, Cole-Kehoe
  - Dynamic coordination problem: Alvarez-Jermann, Gu et al

Role of policy to uniquely implement desired outcome

This paper

- Entrepreneur must raise funds to invest from
  - Private investors:
    - Know investment's profitability but static coordination problem
  - Government
    - Big player but does not have information about investment

- Multiple private equilibria
  - Efficient allocation is equilibrium outcome
  - Also equilibria where profitable investment projects not funded

- Study **best robust policy**
  - Maximize value under most adversarial equilibrium selection
  - Focus on market mechanism (wlog)

Results

- Efficient allocation cannot be uniquely implemented
- But it can be approximated arbitrarily closely
- Governments must commit to fund inefficient investment to guarantee that good investment are undertaken for sure
  - Cost to distinguish good and bad investment projects
  - But can make their probability small

Related literature

- Unique implementation with private contracts
  - Halac-Kremer-Winter
  - Finite number of agents

- Market mechanism
  - Valenzuela-Stookey-Poggi
  - No coordination problem without policy

- Unique implementation in Ramsey problem
  - Atkeson-Chari-Kehoe, Bassetto, Sturm, Barthelemy-Mengus
  - Diamond-Dybvig, Roch-Uhlig, Bocola-Dovis
  - Full information

- Governments vs. markets
  - Acemoglu-Golosov-Tsyvinski
  - They consider info vs. IR, we info vs. coordination

Simple economy

Environment

- $t = 0, 1$
- Continuum of non-atomistic investors
  - Risk neutral and outside option return of $R > 1$
  - Endowment of E in period 0
- Entrepreneur has investment opportunity that requires K
  - Investment profitability is θ
- If investment undertaken
  - Output: $y = \pi(\theta, \varepsilon)$
  - $\varepsilon \sim F(\varepsilon)$, support of y is $[0, \infty)$, and $\int \pi(\theta, \varepsilon) dF(\varepsilon) = \theta$
- If no investment
  - Output: 0 (in paper allow for positive payout ν (collateral))
- Investors know θ in period 0, ε is realized in period 1
- Efficient allocation: Invest iff $\theta \geqslant RK$

Private equilibria

- Wlog, the entrepreneur offers debt contracts
  - B : quantity of such contracts
  - q price of such a contract
  - Investors payout conditional on investment is

$$A(\theta, B) \equiv \int \min\{1, \pi(\theta, \varepsilon)/B\}\, dF(\varepsilon)$$

  - Investors payout conditional on no investment is q

- Equilibrium price

$$q(\theta, B) = \frac{1}{R}\left[\mathbb{I} A(\theta, B) + (1 - \mathbb{I})\, q(\theta, B)\right]$$

where $\mathbb{I}$ indicator with value 1 if $qB \geqslant K$ and 0 o/w

Multiple equilibria

With no collateral i.e. no output w/out investment

- If $\theta < RK$: unique equilibrium with no investment
- If $\theta \geqslant RK$: equilibrium with investment coexists with one without

Intuition

- Without investment cannot guarantee return R
- So equilibrium without investment always exists
- Since investors measure zero, cannot design contracts to make investment dominant strategy

Can government uniquely implement efficient outcome?

Government intervention

- Government
  - Can finance investment by itself
  - Lacks knowledge about θ
  - Focus on "market mechanism"
    - Intervention depends on market outcome (B, q)
- Timing
  - Government commits to fund project with probability $\bar{\eta}(B, q)$
  - θ observed by entrepreneur and investors
  - Entrepreneur issues debt B
  - Sunspot ζ realized
  - Price of debt q realized
  - If $qB < K$ manager can ask gov't for assistance
    - Gov't transfers $K - qB$ with prob. $\bar{\eta}(B, q)$

Continuation equilibria

- Given (θ, B)
- The debt price is

$$q = \frac{1}{R}\mathbb{I}A(\theta, B) + \frac{1}{R}(1-\mathbb{I})\left[(1-\bar{\eta}(B,q))\,q + \bar{\eta}(B,q)\,A(\theta,B)\right]$$

- Probability of investment, σ, is
  - $\sigma = 1$ if $qB \geqslant K$
  - $\sigma = \bar{\eta}(B,q)$ if $qB < K$

Continuation equilibria

Can take two forms:

- Investment undertaken without gov't intervention

$$qB \geqslant K$$

$$q = \frac{1}{R} A\left(z, B\right)$$

$$\sigma = 1$$

- Investment undertaken with gov't intervention

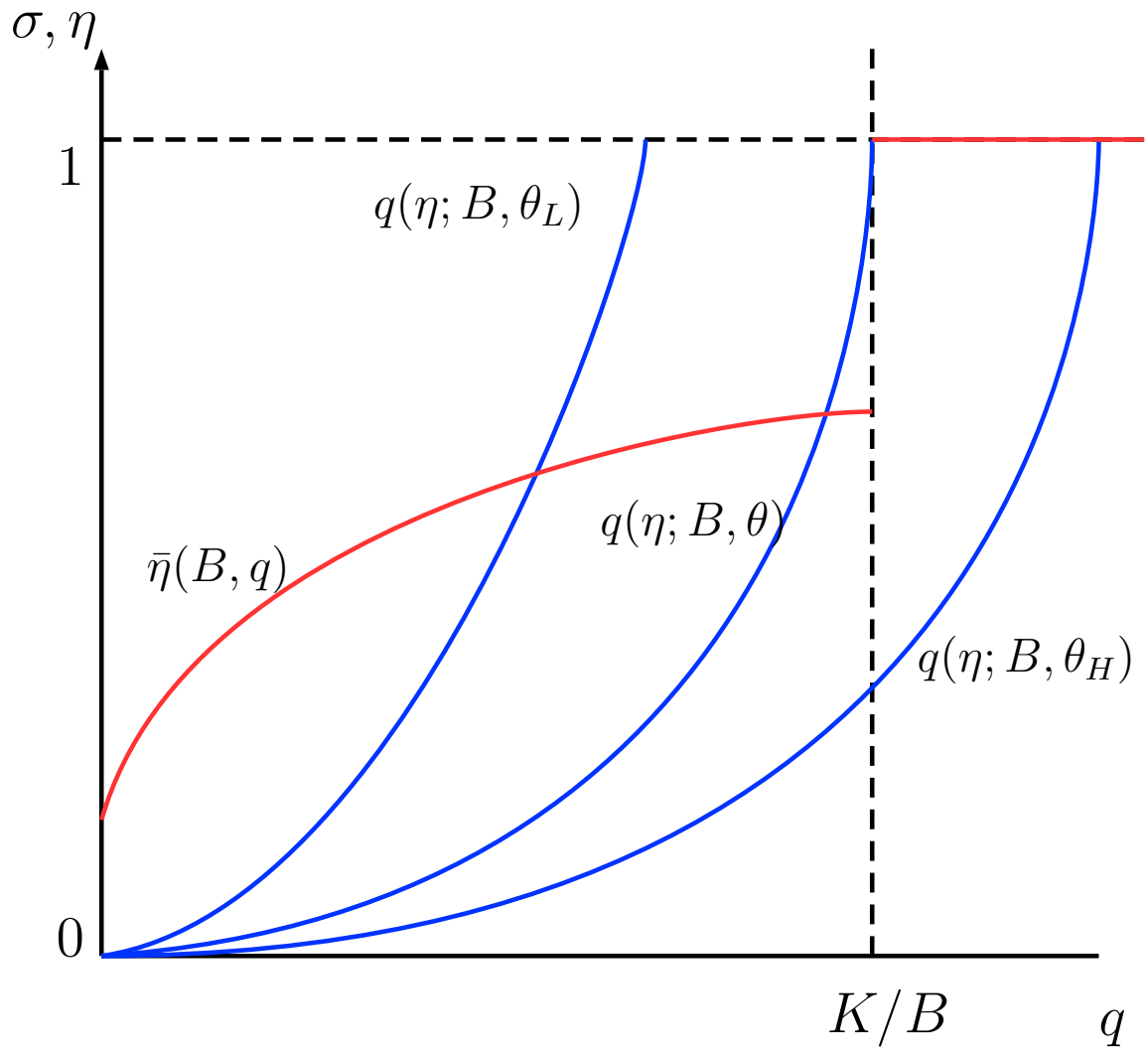
$$qB < K$$

$$q = \frac{\bar{\eta}(B, q)}{R - 1 + \bar{\eta}(B, q)} A\left(z, B\right)$$

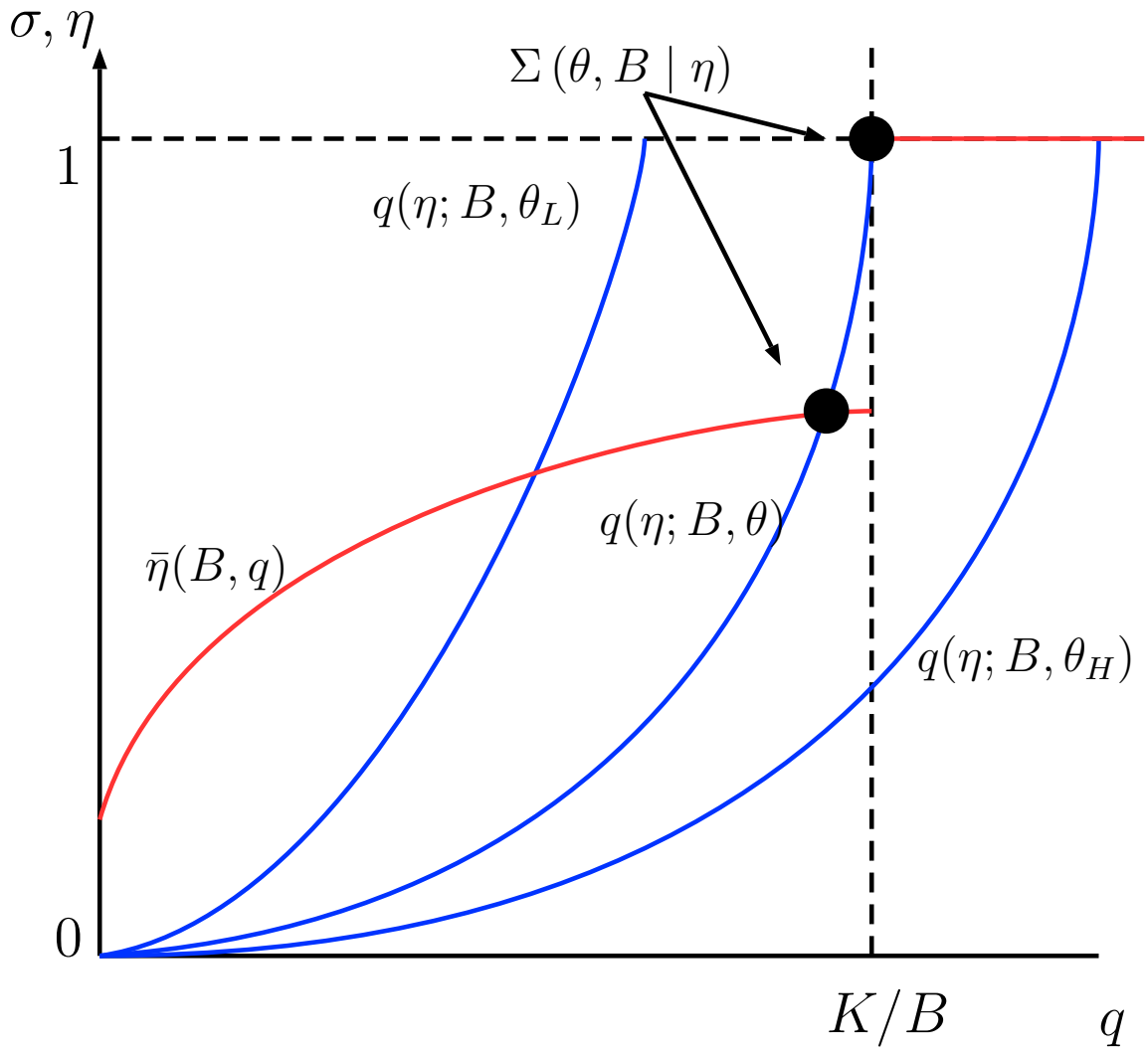
$$\sigma = \bar{\eta}(B, q)$$

$\Sigma\left(\theta, B \mid \bar{\eta}\right)$: investment probabilities consistent w/ equilibrium

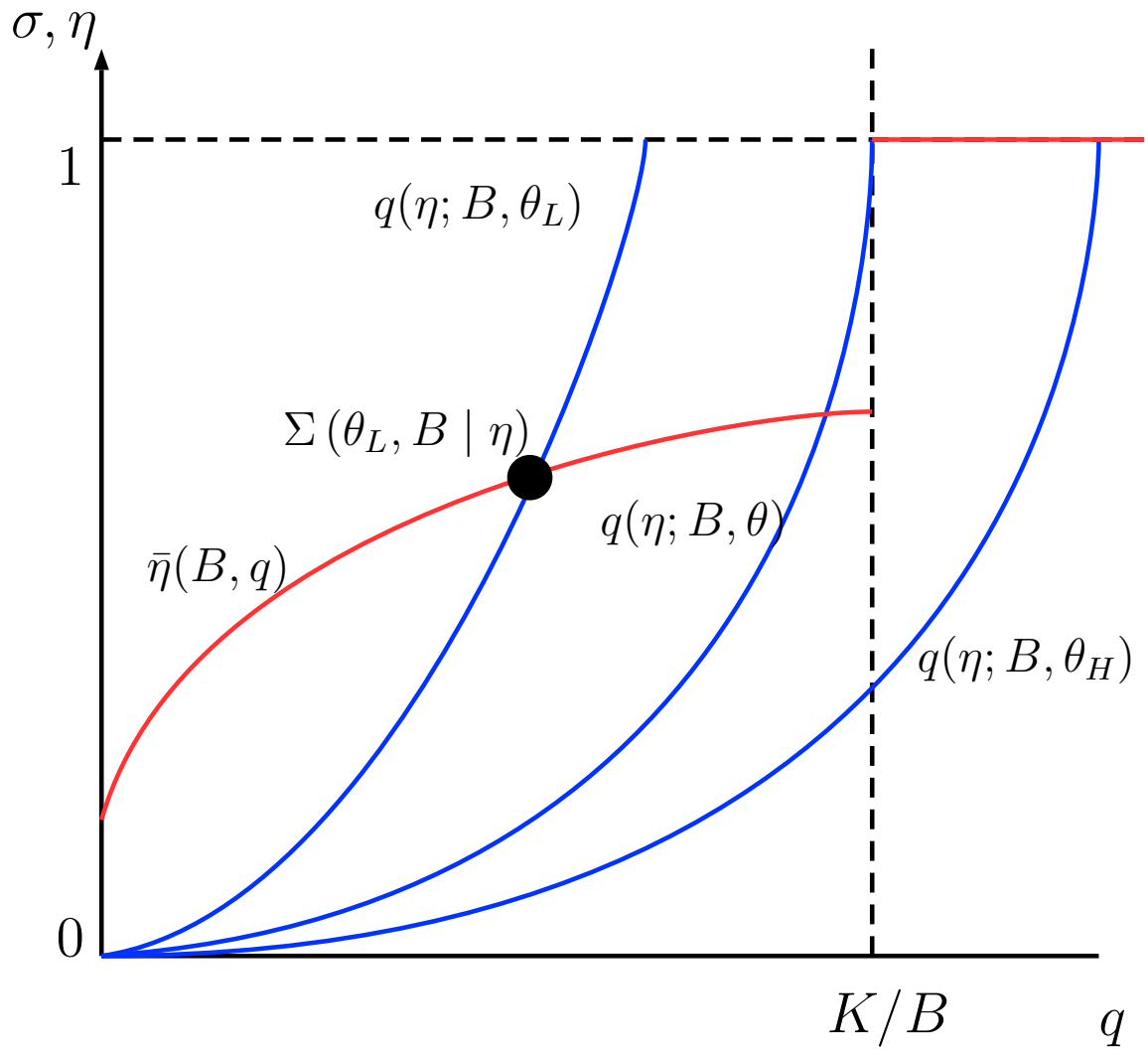
Continuation equilibria



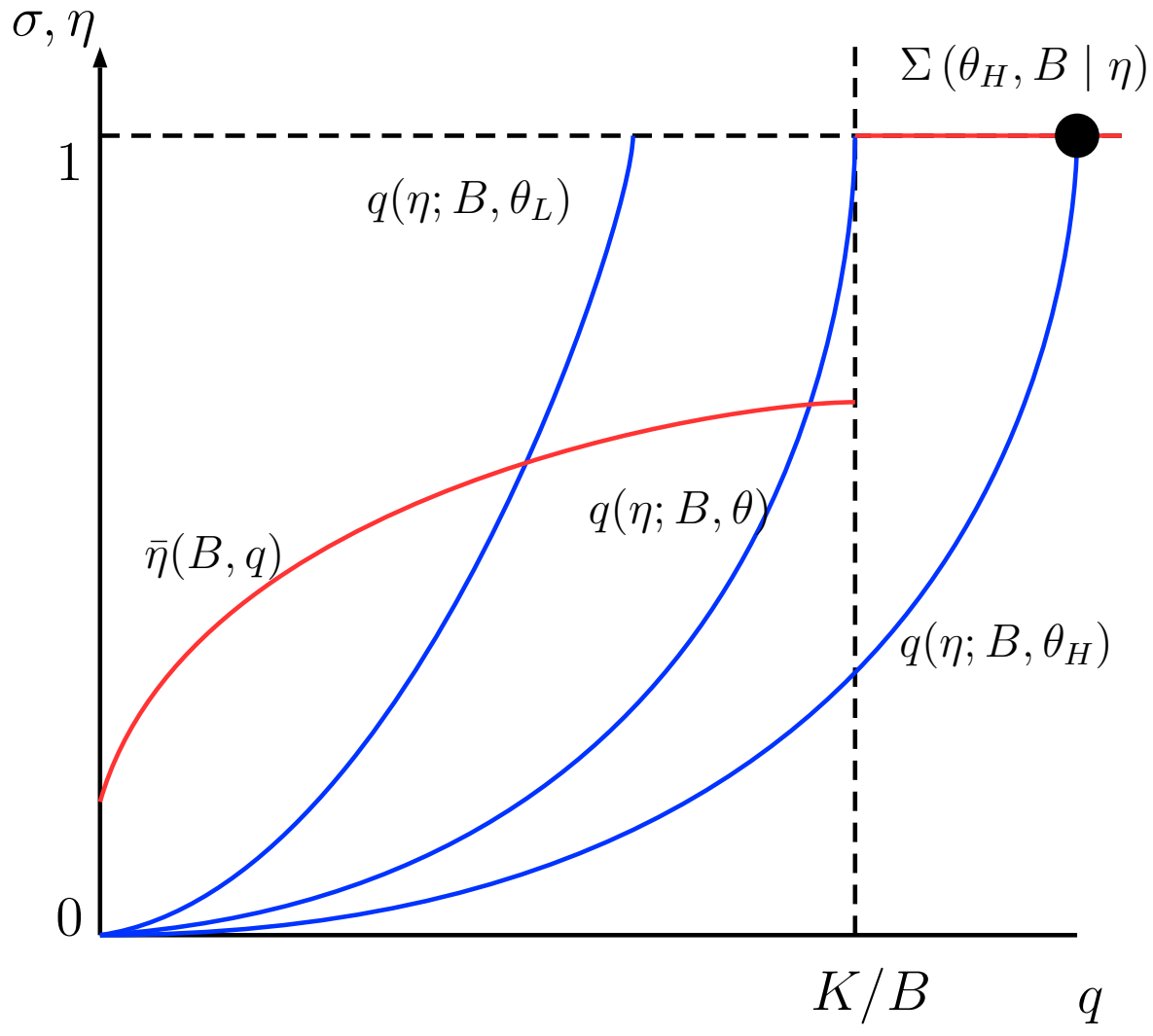
Continuation equilibria



Continuation equilibria



Continuation equilibria



Debt issuance decision

$$B(\theta) = \arg\max_{B} \bar{\eta}\left(q(\theta, B), B\right) \int \max\{\pi(z, \varepsilon) - B\}\, dF(\varepsilon)$$

Best robust policy

Use most adversarial criterion from the gov't perspective

- Highest investment probability if $\theta < \mathsf{RK}$:

$$\sigma = \max \Sigma \left(\theta, \mathrm{B}\left(\theta\right) | \eta\right)$$

- Lowest investment probability if $\theta > \mathsf{RK}$:

$$\sigma = \min \Sigma \left(\theta, \mathrm{B}\left(\theta\right) | \eta\right)$$

Require

$$\Sigma \left(\theta, \mathrm{B} | \eta\right) \neq \emptyset \text{ for all } \left(\theta, \mathrm{B}\right).$$

Cannot uniquely implement the efficient allocation

- Let efficient outcome be $B^*(\theta), q^*(\theta)$ with investment iff $\theta \geqslant RK$
- Suppose $\exists$ policy that uniquely implements efficient all'n
- If $\theta < RK \Rightarrow$ no investment takes place
  - $q = 0, \eta(B, 0) = 0$ for all B
- Now suppose $\theta > RK$
  - Worst private equilibrium has $q = 0$ and no investment
  - Because $\eta(B, 0) = 0$, this is also an eq with intervention

Cannot uniquely implement the efficient allocation

- Let efficient outcome be $B^*(\theta), q^*(\theta)$ with investment iff $\theta \geqslant RK$
- Suppose $\exists$ policy that uniquely implements efficient all'n
- If $\theta < RK \Rightarrow$ no investment takes place
  - $q = 0, \eta(B, 0) = 0$ for all B
- Now suppose $\theta > RK$
  - Worst private equilibrium has $q = 0$ and no investment
  - Because $\eta(B, 0) = 0$, this is also an eq with intervention
- To avoid no investment when θ high then need $\eta(B, 0) > 0$
  - **Ex-post inefficient investments are necessary**

Approximate efficient allocation

- Consider a sequence of $\{\bar{\eta}_n\}$ indexed by parameter h_n
- For any θ^* and corresponding $B = B^*(\theta^*)$ let

$$\bar{\eta}_n(B, q) \equiv q\frac{(R-1)}{A(\theta^*, B) - q} + h_n(q^*(\theta^*) - q).$$

- Let $q_n(B, \theta)$ and $B_n(\theta)$ be defined by

$$\bar{\eta}_n(B, q) = q\frac{(R-1)}{A(\theta, B) - q}.$$

$$B_n(\theta) = \arg\max_B \bar{\eta}_n(B, q(B, \theta)) \int \max\{\pi(\theta, \varepsilon) - B\} dF(\varepsilon)$$

Approximate efficient allocation

- Consider a sequence of $\{\bar{\eta}_n\}$ indexed by parameter h_n
- For any θ^* and corresponding $B = B^*(\theta^*)$ let

$$\bar{\eta}_n(B, q) \equiv q\frac{(R-1)}{A(\theta^*, B) - q} + h_n(q^*(\theta^*) - q).$$

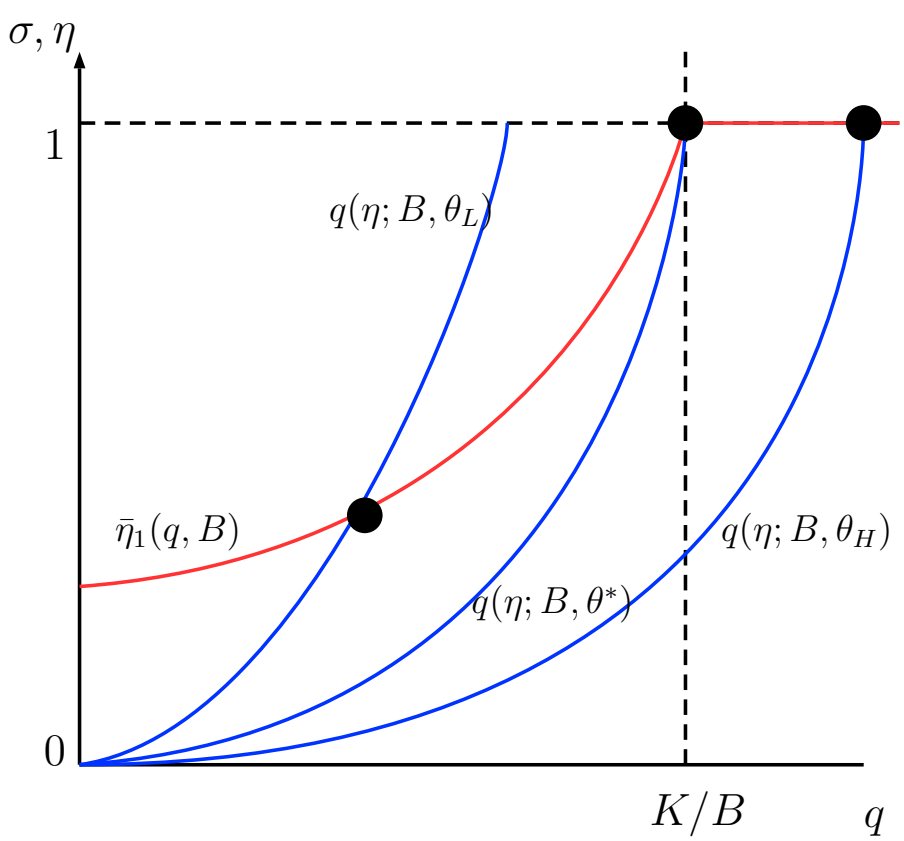
- Let $q_n(B, \theta)$ and $B_n(\theta)$ be defined by

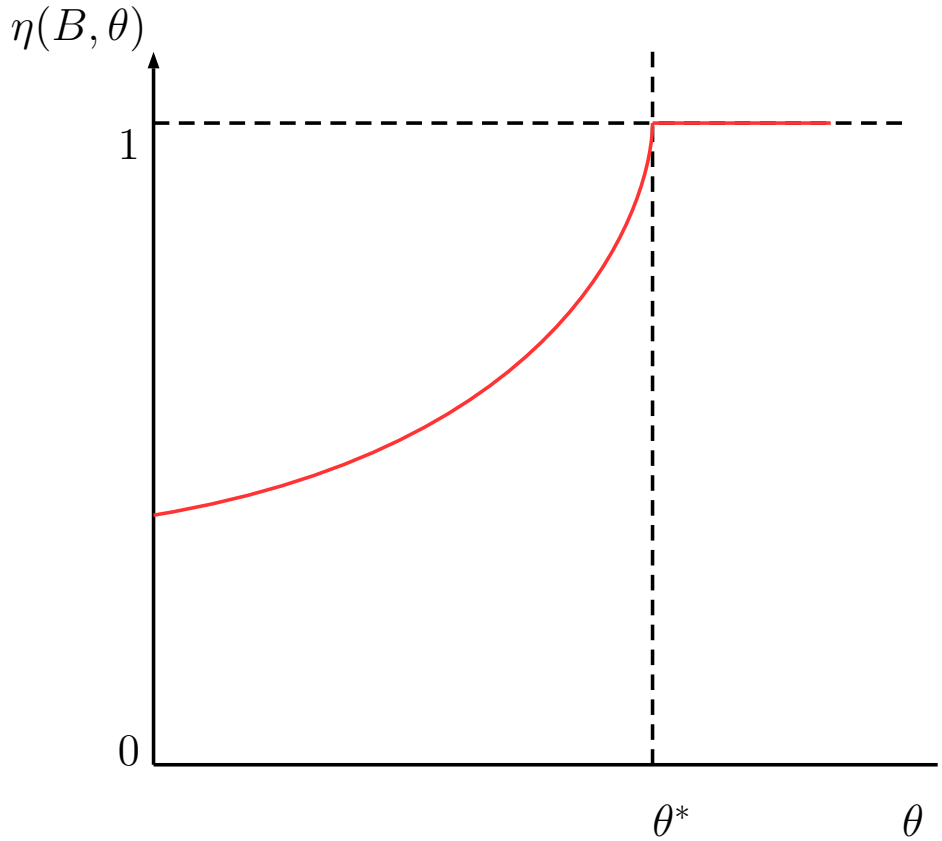
$$\bar{\eta}_n(B, q) = q\frac{(R-1)}{A(\theta, B) - q}.$$

$$B_n(\theta) = \arg\max_B \bar{\eta}_n(B, q(B, \theta)) \int \max\{\pi(\theta, \varepsilon) - B\} dF(\varepsilon)$$

- **If $h_n > 0$, continuation eq is unique for all (B, θ)**
- **As $h_n \to 0$ then converge to efficient allocation**

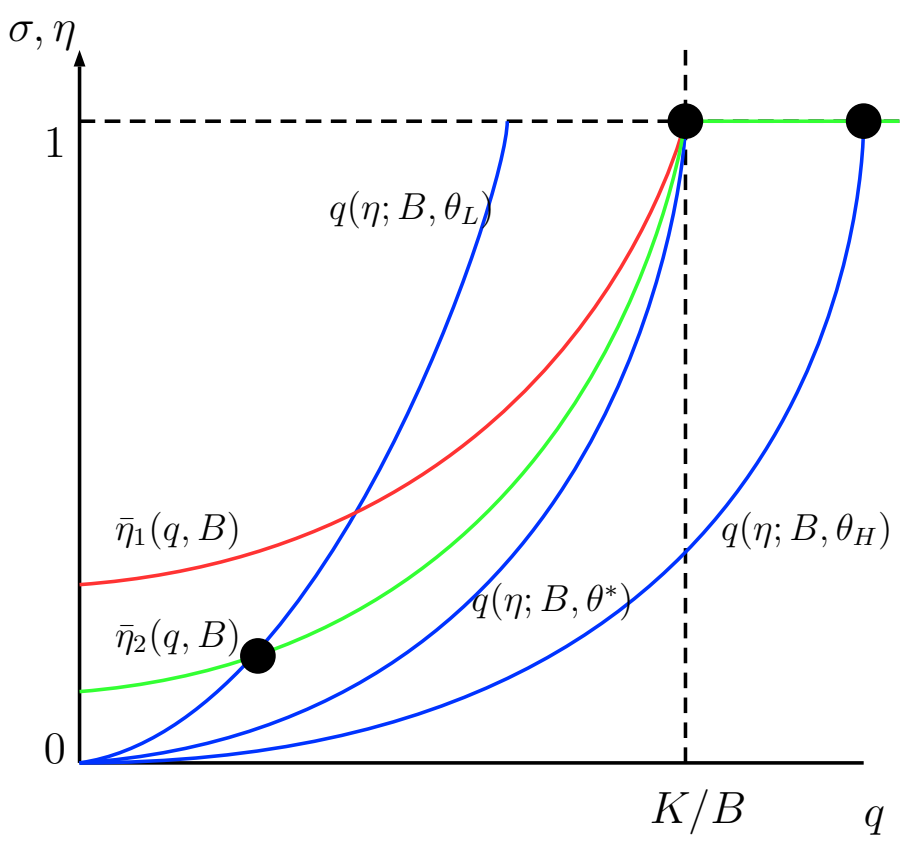
Continuation equilibrium

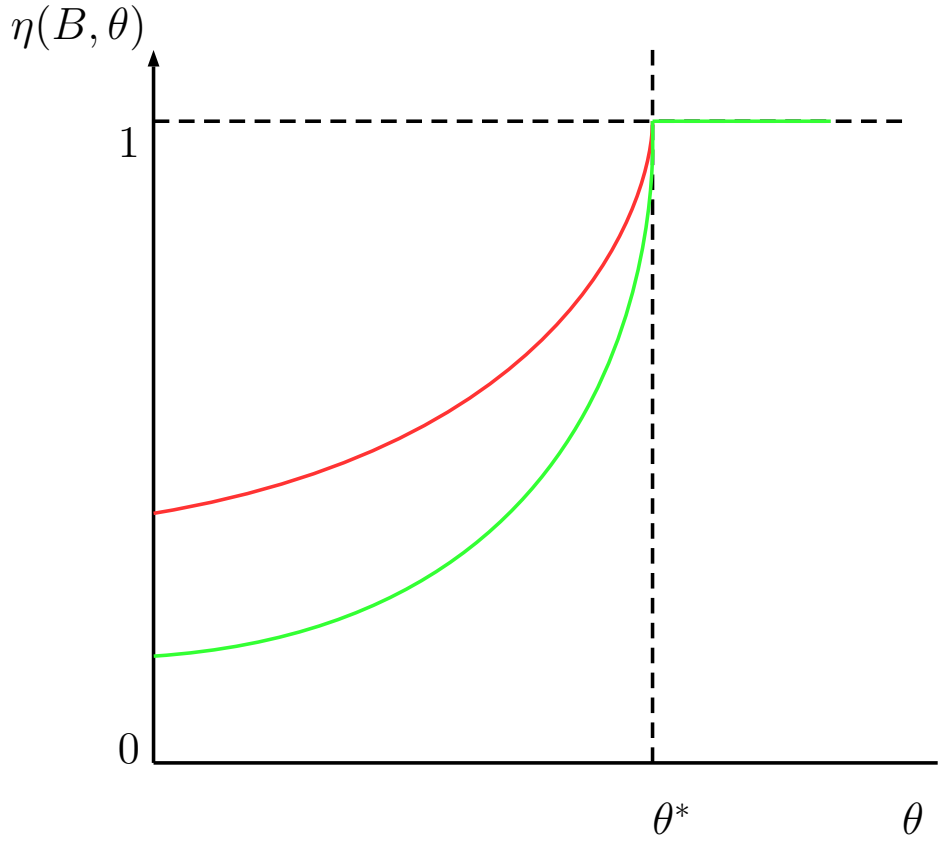




- Unique continuation equilibrium for all θ. Let θ^* s.t. $B = B^*(\theta^*)$
- If $\theta \geqslant \theta^* \geqslant RK$ then invest for sure
- If $\theta < \theta^*$ then invest with probability in $(0, 1)$

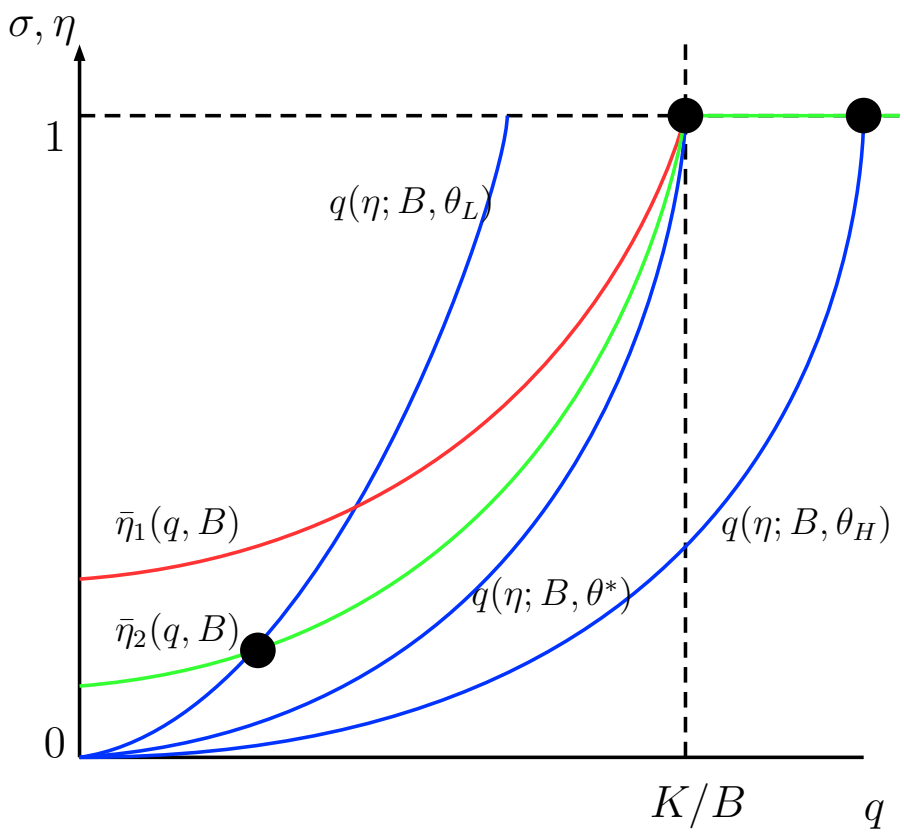
Continuation equilibrium

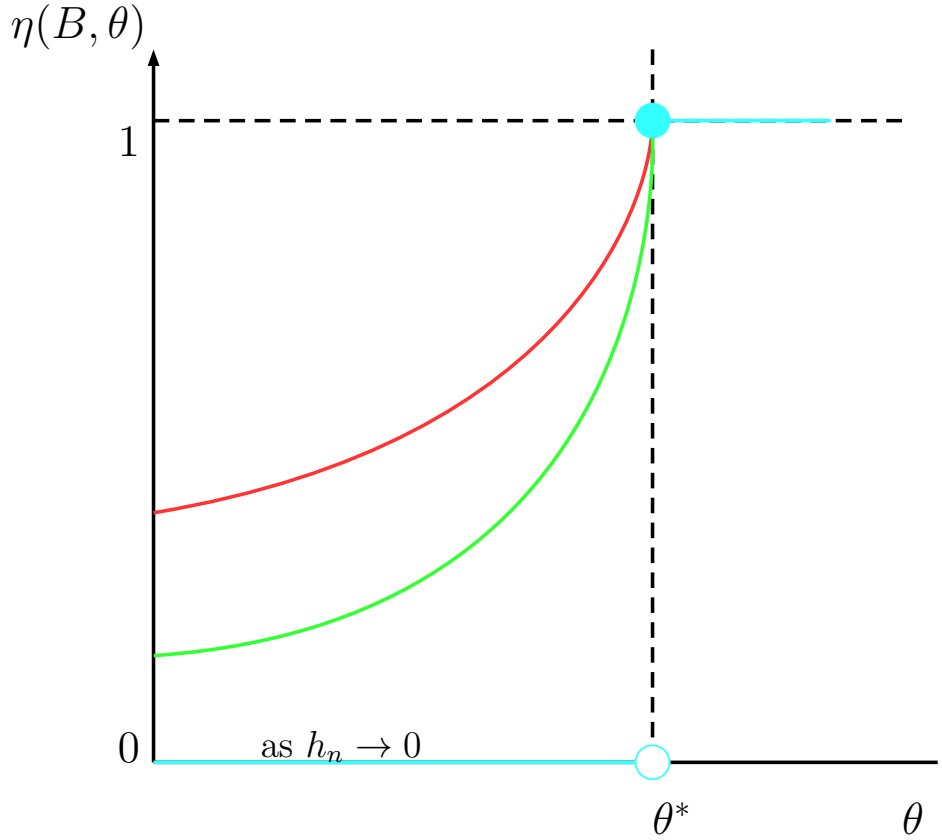




- As h_n decreases:
- If $\theta \geqslant \theta^* \geqslant RK$ then invest for sure
- If $\theta < \theta^*$ invest with smaller probability

Continuation equilibrium





- As $h_n \to 0$
  - Investment probability converges to step function
  - Thus, debt levels converge to $B^*(\theta)$ if $\theta \geqslant RK$
  - For $\theta < RK$ try to issue debt but inefficient investment arb. small

Commitment to ex-post inefficient investment

- Need to commit to fund bad projects (even if with small pr)
  - Allows to learn which projects are good
  - Provide enough support so investors coordinate on good outcome
- Opposite logic of typical bailout argument
  - Typically want to commit to not bailing out
- Commitment technology is necessary
- Absent commitment, either
  - No intervention or
  - Gov't directly funds all projects (without collecting any info)
- Cannot rely on reputational forces (Barthelemy-Mengus)

Moral hazard

- Moral hazard often associated with interventions/bailouts
  - Kareken-Wallace
- How intervention affects entrepreneurs' incentives to generate investment projects?
- Higher effort than worst case but lower than best case
  - Ensures good projects are implemented
  - This increases rewards for manager's effort relative private equilibria where good projects not implemented for sure
  - But also subsidize bad projects
  - This reduces incentives so lower effort than efficient equilibrium

Moral hazard, cont.

- Entrepreneur takes costly action a that affects the value for θ
  - $\theta \in \{\theta_L, \theta_H\}$ with and $\theta_L < RK < \theta_H$
  - Let $f(\theta|a) \equiv Pr(\theta|a)$ and $c(a)$ is effort cost
- Without any intervention, equilibrium a is $[0, a^*]$
  - Efficient effort is $a^* = \arg\max_a f(\theta_H|a)(\theta_H - RK) - c(a)$
  - Effort can be lower because good equilibrium can be selected with probability $\zeta < 1$,

$$\max_a f(\theta_H|a)\,\zeta(\theta_H - RK) - c(a)$$

- Under optimal robust policy

$$a_n = \arg\max_a f(\theta_H|a)\,\eta_n(\theta_H)(\theta_H - RK) - c(a) + f(\theta_L|a)\eta_n(\theta_L)\nu(\theta_L)$$

Thus, $0 < a_n < a^*$ and $\{a_n\} \uparrow a^*$

Can general mechanism improve market mechanism?

- No, if investors observe θ with noise (and market aggregate info)
  - No mechanism uniquely implements the efficient allocation
  - Cannot make dominant strategy for entrepreneur with $\theta < RK$ to report something that induces no investment
  - For investors, same coordination problem as in debt market

- Yes, if investors observe θ exactly
  - Can uniquely implement the efficient allocation
  - Make one investor pivotal and give them a return $\theta - RK$

Conclusion

- Study investment problem where static coordination problem leads to multiple equilibria
- Study which gov't intervention can uniquely implement desired outcome when gov't lacks info
  - Best robust policy
- Complementarity between government and market
  - Market aggregates information
  - Government (big player) rules out coordination problems
- Governments must commit to fund inefficient investment to guarantee that good investment are undertaken for sure