On the Design of a Robust Lender of Last Resort *

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Abstract

We study the role that a lender of last resort or a Financial Stability Fund can have to rule out bad equilibrium outcomes in dynamic credit economies when it has imperfect information about the state of the economy. We take a robust approach and study how the Fund policies affect the borrower's worst equilibrium outcome for the borrower. We have three main results. First, to avoid the worst private equilibrium outcome, the Fund must commit to incur losses. The Fund faces a trade-off between the maximal expected losses it is willing to tolerate and the minimal level of utility that the borrower can attain in equilibrium for sure. Second, the optimal timing of the Fund's assistance is back-loaded: the Fund commits to increasing support to the borrower over time. Third, the presence of the Fund does not necessarily generate moral hazard: the borrower's effort is higher than in the worst equilibrium without the Fund.

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1 Introduction

Multiplicity is pervasive in dynamic credit economies. The incentives to repay debts in the current period depend on the ability to borrow in the future. Dynamic coordination failures among lenders can then lead to credit market break-downs and multiple equilibrium outcomes. See Alvarez and Jermann (2000), Gu et al. (2013), Cole and Kehoe (2000), and Passadore and Xandri (2021) for some examples that illustrate this multiplicity. A lender of last resort can help resolve the multiplicity problem. In particular, it can uniquely implement the best equilibrium outcome without having to intervene along the equilibrium path and without incurring any loss on and off path. See for example Kirpalani (2015), Roch and Uhlig (2018), Bocola and Dovis (2019).[1] Such lender-of-last-resort policies require information about the fundamentals of the economy. In particular, the quantity and price of debt that would arise in the best equilibrium outcome. In this paper, we study the role that a lender of last resort or a *Financial Stability Fund* can have if it lacks knowledge about the fundamentals of the economy. Specifically, we ask two questions. First, are there policies that can rule out bad outcomes? Second, can such policies be costlessly implemented by the Fund or do they require incurring losses?

To answer these questions, we take a *robust* approach. That is, we study how the Fund policies affect the *worst* equilibrium outcome for the borrower. We show that the answer to our questions critically depends on whether the value of defaulting is a technological parameter of the economy or it itself an equilibrium outcome.

Our preferred approach is the *endogenous default value* case where the value of defaulting must be a sustainable equilibrium. We show that to avoid the worst private equilibrium outcome, the Fund must incur losses along the equilibrium path. The Fund faces a trade-off between the expected losses it is willing to tolerate in equilibrium and the minimal level of utility that the borrower can attain in equilibrium for sure. The higher the expected looses in the worst case scenario, the higher is the worst equilibrium value. Moreover, we show that more generous schemes are associated with lower variance of outcomes.

Second, we characterize the optimal timing and path of the Fund's interventions. We show that the best intervention is back-loaded: the Fund commits to increasing support to the borrower over time. This implies that the Fund's expected losses are also back-loaded.

Third, we show that the presence of the Fund does not necessarily reduce the borrower's incentives to exert effort: the amount of effort exerted is higher than in the worst equilibrium without the Fund.

We consider a simple pure exchange economy with alternating non-storable endow-

[1]The mechanism for multiplicity is similar to the one in monetary economy where a monetary equilibrium coexists with a non-monetary equilibrium. In these economies, Wallace (1981) studies the role of interventions to guarantee that the monetary equilibrium is the unique outcome. See also Nicolini (1996).

ments for a risk averse borrower that can borrow from risk-neutral lenders. The borrower's endowments can transit to a low-income regime where income is permanently low and no debt can be supported. The state of the economy is the time-varying probability that the income process will transit to this low-income regime. The efficient allocation (without saving) can be implemented as the best sustainable equilibrium outcome – see Chari and Kehoe (1990) – with short-term defaultable debt and defaults along the equilibrium path. Along the equilibrium path, the borrower defaults when he transits to the low-income regime and debt prices reflect the time-varying probability of default. However, this equilibrium is not generally unique. For example, if the value of default is autarky, there is a continuum of sustainable equilibria with values ranging from the efficient allocation to the value in autarky.

We next study the role that a Financial Stability Fund, or *Fund* for short, can play to put a lower bound to the worst equilibrium value. We focus on the worst equilibrium value because private equilibria can attain the efficient allocation without any intervention. If the Fund has full information and knows the state of the economy, it can uniquely implement the efficient allocation by offering to lend to the borrower at the best equilibrium prices any amount of debt up to the one associated with the best equilibrium. In this way, the Fund rules out private lenders' coordination failures and it never has to intervene along the equilibrium path because private lenders are always willing to lend. This result is standard, see for example Kirpalani (2015), Roch and Uhlig (2018), Bocola and Dovis (2019).

Our main contribution is to consider the case in which the Fund has less than perfect information about the borrower's fundamentals. We consider the extreme case in which the Fund has no information about the realization of the borrower's fundamentals but our results extend to less extreme environments where the Fund has imperfect information about the fundamentals. In this case we show that it is not feasible to uniquely implement the efficient allocation in a costless way.

We start by analyzing the effect of a stationary Fund policy that promises to lend to a borrower with no history of past default an amount up to a cap at a price the is the maximum between a *price floor* and the price at which private lenders are willing to lend. The Fund policy can then be summarized by a price floor and a *debt cap*, which effectively places a lower bound on the resources that the borrower can obtain when it wants to borrow.

We show that the Fund must incur losses along the equilibrium path. The Fund faces a trade-off between the maximal expected losses it is willing to tolerate and the minimal level of utility that the borrower can attain in equilibrium for sure.

We study the optimal path for the price floor and the debt. The goal of the Fund is to minimize the net present value of losses to deliver a given value to the borrower in the

worst equilibrium outcome. We show that the debt cap and the losses incurred by the Fund are *backloaded*: both are increasing over time. The logic of this result is reminiscent of the standard motives for backloading in problems with lack of commitment: by promising higher minimal amount of lending to the borrower in the distant future, the Fund is relaxing the entire sequence of sustainability constraints up to that date and so it is also increasing the borrower's value in all previous periods. It is thus cost minimizing for the Fund to commit to large future subsidies.

An implementation for this outcome is that the Fund raises resources in period 0 and commits to an increasing level of support and subsidies. In the the early periods, the Fund's period losses are low and the Fund's balance is growing over time. The Fund can eventually finance the large subsidies with the accumulated assets without the need to raise more funds.

In an extension where we allow for hidden effort, we study how the generosity of the Fund's policy affects the borrower's incentives to make costly investments and reforms to reduce the probability of transiting to the low-income regime, similarly to Atkeson (1991) and Müller et al. (2019). We show that the presence of the Fund increases incentives to exert effort in the worst case scenario because it increases the value of not transiting to the low-income regime relative to the worst private equilibrium. Thus, the presence of the Fund and its subsidies are not necessarily detrimental for reform incentives.

This result is similar to the one in the static economy in Dovis and Kirpalani (2023). It is also related to the catalytic effect of government intervention in the global games literature: Morris and Shin (2006) and Corsetti et al. (2006). We share with these papers the ideas that government intervention can help implement the efficient outcome that interventions can encourage individual effort and so do not necessarily create moral hazard problems. We differ from these papers because we do not rely on a global games selection to render the equilibrium unique.

Finally, we consider the *exogenous default value* case, where the value of defaulting is a model's parameter. If the continuation value of default equals to the autarky value, absent intervention there are multiple equilibria. We show that the Fund by promising a very small uncontingent intervention can guarantee that the efficient allocation is uniquely implemented. The size of the assistance offered by the Fund can be chosen to be small enough (but strictly positive) so that either the Fund's assistance is never asked along the equilibrium path and so losses are zero or such losses are minimal.

Our theory applies to several financial institutions tasked with the role of avoiding crises in debt and in particular sovereign debt markets. A notable example is the International Monetary Fund. Another example is the European Stability Mechanism (ESM) instituted by eurozone countries in the wake of the eurozone debt crises with the stated mission of enabling "*the countries of the euro area to avoid and overcome financial crises and*

to maintain long-term financial stability and prosperity. The ESM carries out this mission by providing loans and other types of financial assistance to member states that are experiencing or are threatened by severe financial distress. In other words, the ESM acts as a "lender of last resort" for euro area countries when they are unable to refinance their government debt in financial markets at sustainable rates." [2]Another example is the Transmission Protection Instrument (TPI) launched by the ECB in Summer 2022 allowing the ECB to buy sovereign bonds in secondary markets "*to counter unwarranted, disorderly market dynamics*"[3]

A common feature of these programs and institutions is the insistence on generating no losses and not costing any resources to tax-payers. An implication of our theory is that, if these claims are credible, such institutions may not be very effective at increasing the value of the worst equilibrium and reducing coordination failures among private lenders if the value of default is endogenous. In this case, to avoid disorderly market dynamics such lenders of last resort must be willing to accept some losses.

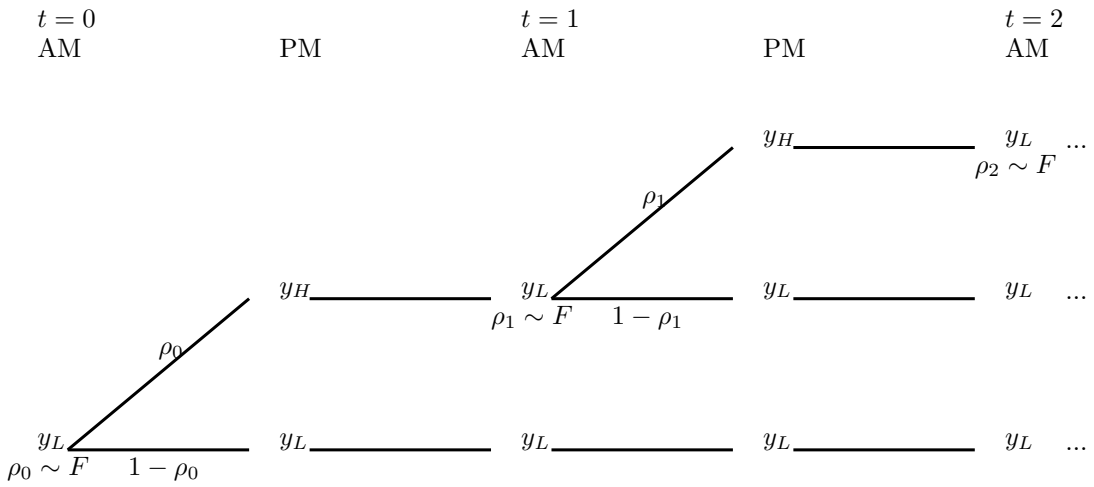
This paper contributes to the literature that uses a optimal dynamic contracting approach to study the efficient borrowing dynamics subject to frictions like lack of commitment. See for instance Atkeson (1991), Kehoe and Levine (1993), Alvarez and Jermann (2000), Kehoe and Perri (2002), Kehoe and Perri (2004), and Dovis (2019). All these papers study the best equilibrium outcome. Two very related papers are Abrahám et al. (2018) and Liu et al. (2022). These two papers also connect the outcome of a dynamic contracting problem with the design of a financial stability fund but they interpret the Fund as the agent tasked with implementing the efficient allocation. As we stressed above, in this class of economies, the best sustainable equilibrium outcome is the efficient allocation. So a version of the Second Welfare Theorem holds. However, differently from Prescott and Townsend (1984), not all equilibria are efficient and a version of the First Welfare Theorem does not hold. Thus, while a Fund is not essential to attain the efficient allocation, it can can be essential to increase the value of the worst sustainable equilibrium from a robust perspective.

We follow this approach in Dovis and Kirpalani (2023) where we study the optimal design of bailout policies in a static model of investment where the multiplicity arises because of strategic complementarities in lenders' investment decisions. The focus of the companion paper is to show how the optimal policy should learn from market prices and the complementarity between markets – that aggregate information – and government policies – that ensure that bad equilibria cannot arise.

[2]https://www.esm.europa.eu/about-us#headline-who_we_are

[3]See https://www.ecb.europa.eu/press/pr/date/2022/html/ecb.pr220721~973e6e7273.en.html

Figure 1: Endowment process



2 Environment

Let $t = 0, 1, 2, ...\infty$. Each period is divided into two sub-periods: AM and PM. There are two types of agents: a borrower and a continuum of short-lived lenders. The borrower is risk-averse with preferences

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_{AMt}\right) + u\left(c_{PMt}\right) \right]$$

where u satisfies standard conditions. There is a single good and the borrower's endowment of this good can take two values $\{y_L, y_H\}$ with $y_L < y_H$. The endowment is always y_L in the AM. In the PM there is a chance that the economy transits to a permanent low-income regime with constant endowment at y_L forever. The probability of transitioning to the low-income regime is $1 - \rho$, where $\rho \sim F(\rho)$, is iid over time and is realized each period at the beginning of the AM. The evolution of the endowment process is illustrated in Figure 1.

The economy also has of a continuum of short-lived lenders who have access to an outside investment option in the AM that yields a net return of zero in the PM. Lenders can transfer resource between the AM and PM but not across periods.

We assume that contracts between borrowers and lenders are subject to limited commitment. In the PM, after observing the endowment's realization, all agents (borrowers and lenders) know if they have transited to the permanent low-income regime or not. At this point, borrowers can choose to walk away from their contracts and live in autarky forever. To simplify the notation, we normalize $u\left(y_L\right) = 0$ so that the value of autarky in

the low-income regime in the PM is

$$V_{aut}(L) \equiv u(y_L) + \frac{\beta}{1-\beta} 2u(y_L) = 0.$$

Similarly, let $V_{aut}(H)$ be the value of autarky for the borrower with the high endowment in the PM,

$$V_{aut}(H) \equiv u(y_H) + \frac{\beta}{1-\beta\mathbb{E}\rho} \int_{\underline{\rho}}^{\bar{\rho}} [u(y_L) + \rho u(y_H)] \, dF(\rho).$$

The efficient allocation without the ability to save across periods solves

$$\max_B \int_{\underline{\rho}}^{\bar{\rho}} [u(y_L + \rho B) + \rho u(y_H - B)] \, dF(\rho)$$

subject to the sustainability constraint

$$u(y_H - B) + \frac{\beta}{1-\beta\mathbb{E}\rho} \int_{\underline{\rho}}^{\bar{\rho}} [u(y_L + \rho B) + \rho u(y_H - B)] \, dF(\rho) \geqslant V_{aut}(H) - \beta\chi$$

where $\chi \geqslant 0$ is a cost of default over and above the inability to being able to borrow in the future.

For simplicity, we assume that β is sufficiently low so that the sustainability constraint is always binding. That is, let b^* be the non-zero solution to

$$u(y_H - b^*) + \frac{\beta}{1-\beta\mathbb{E}\rho} \left[\int_{\underline{\rho}}^{\bar{\rho}} u(y_L + \rho b^*) \, dF(\rho) + \mathbb{E}\rho u(y_H - b^*) \right] = V_{aut}(H) - \beta\chi$$

and assume that

$$u'(y_L + \bar{\rho} b^*) > u'(y_H - b^*). \tag{1}$$

Thus, the efficient allocation has $B = b^*$ for all ρ. Let V^* be the value of the efficient allocation.

Note here that the restriction to no savings across periods implies two constraints: first, the borrower cannot accumulate assets over time to relax the sustainability constraint; second, the borrower cannot insure himself against realizations of ρ. One way to rationalize these constraints is to consider an economy in which lenders are two-period lived and there is no storage technology across periods.

3 Private sustainable equilibria

We show that the efficient allocation can be implemented as a sustainable equilibrium with one period defaultable debt contracts where $b_t = b^*$ and the equilibrium debt price is $q_t(\rho) = \rho$ for all t. However, there can be equilibrium multiplicity. If the exogenous default cost χ is not too high, then there is a continuum of sustainable equilibria.

The timing is as follows:

- In the AM, ρ_t and a coordination device $\xi_{1t} \sim U([0,1])$ are realized;
- The borrower chooses how much debt to issue, b_t;
- The price q is determined in equilibrium;
- In the PM, a second coordination device $\xi_{2t} \sim U([0,1])$ is realized;
- The borrower chooses to repay the debt, $\delta_t = 1$, or not, $\delta_t = 0$.

Let the history of events up to period t be $h^t = \{\rho_s, \xi_{1s}, b_s, q_s, \xi_{2s}, \delta_s\}_{s=0}^t$. Let the history when the borrower acts in the AM in period t be denoted by $h_{AM}^t = (h^{t-1}, \rho_t, \xi_{1t})$; let the history of events in the PM when the borrower chooses whether to repay the debt be denoted by $h_{PM}^t = (h_{AM}^t, b_t, q_t, \xi_{2t})$.

A *sustainable equilibrium* is a debt issuance decision $b(h_{AM}^t)$, a pricing schedule $q(h_{AM}^t, b_t)$, default decision $\delta(h_{PM}^t)$, and borrower's values $V_{AM}(h_{AM}^t)$ and $V_{PM}(h_{PM}^t)$ such that: i) the debt issuance decision is optimal for the borrower in the AM for all h_{AM}^t,

$$V_{AM}(h_{AM}^t) = \max_b u(y_L + q(h_{AM}^t, b)\, b) + \rho_t \int V_{PM}(h_{AM}^t, b, q(h_{AM}^t, b), \xi_{2t})\, d\xi_{2t},$$

ii) the pricing schedule satisfies the break-even constraint for lenders

$$q(h_{AM}^t, b_t) = \rho_t \int \delta(h_{AM}^t, b_t, q_t, \xi_{2t})\, d\xi_{2t},$$

and iii) the optimality of the default decision in the PM for all h_{PM}^t,

$$\begin{aligned} V_{PM}(h_{PM}^t) = \max_{\delta \in [0,1]} \delta \left[u(y_H - b_t) + \beta \int \int V_{AM}(h_{PM}^t, 1, \rho_{t+1}, \xi_{2t+1})\, dF(\rho_{t+1})\, d\xi_{2t+1} \right] \\ + (1-\delta) \left[u(y_H) + \beta \left(\int \int V_{AM}(h_{PM}^t, 0, \rho_{t+1}, \xi_{2t+1})\, dF(\rho_{t+1})\, d\xi_{2t+1} - \chi \right) \right]. \end{aligned}$$

Recursive representation Using standard techniques as in Abreu et al. (1990), the sustainable equilibrium value set, $\mathcal{V}$, is the largest fixed point of the monotone operator

$$P\left[\underline{V}, \bar{V}\right] = \left[\underline{P}\left(\left[\underline{V}, \bar{V}\right]\right), \bar{P}\left(\left[\underline{V}, \bar{V}\right]\right)\right] \tag{2}$$

where

$$\bar{P}\left(\left[\underline{V},\bar{V}\right]\right)=\max_{b(\rho),V'(\rho),V'_d(\rho)}\int\left[u\left(y_L+\rho b\left(\rho\right)\right)+\rho u\left(y_H-b\left(\rho\right)\right)+\beta\rho V'\left(\rho\right)\right]dF\left(\rho\right) \quad (3)$$

subject to

$$u\left(y_H-b\left(\rho\right)\right)+\beta V'\left(\rho\right)\geqslant u\left(y_H\right)+\beta\left(V'_d\left(\rho\right)-\chi\right) \quad (4)$$

$$V'\left(\rho\right),V'_d\left(\rho\right)\in\left[\underline{V},\bar{V}\right] \quad (5)$$

and

$$\underline{P}\left(\left[\underline{V},\bar{V}\right]\right)=\min_{V'(\rho),V'_d(\rho)}\max_{b(\rho)}\int\left[u\left(y_L+\rho b\left(\rho\right)\right)+\rho u\left(y_H-b\left(\rho\right)\right)+\beta\rho V'\left(\rho\right)\right]dF\left(\rho\right) \quad (6)$$

subject to (4) and (5). Starting from an initial guess $\left[\underline{V}_0,\bar{V}_0\right]\supset\mathcal{V}$ and iterating on the P is a way to calculate the equilibrium value set, $\mathcal{V}=\lim_{n\to\infty}P^n\left(\left[\underline{V}_0,\bar{V}_0\right]\right)$.

Defining $h\left(x\right)$ as the maximal amount of debt that can be supported given that a default induces a drop in continuation value of x:

$$\begin{aligned}&h\left(x\right):u\left(y_H-h\left(x\right)\right)+\beta\left(V_{aut}+x\right)=u\left(y_H\right)+\beta V_{aut}\\&\iff h\left(x\right)\equiv\max\left\{y_H-u\left(u^{-1}\left(y_H\right)-\beta x\right),0\right\},\end{aligned}$$

we can simplify the two programming problems above by noticing that

$$\bar{P}\left(\left[\underline{V},\bar{V}\right]\right)=\int u\left(y_L+\rho h\left(\Delta V+\chi\right)\right)dF\left(\rho\right)+\mathbb{E}\rho u\left(y_H-h\left(\Delta V+\chi\right)\right)+\beta\mathbb{E}\rho\bar{V} \quad (7)$$

$$\underline{P}\left(\left[\underline{V},\bar{V}\right]\right)=\int u\left(y_L+\rho h\left(-\Delta V+\chi\right)\right)dF\left(\rho\right)+\mathbb{E}\rho u\left(y_H-h\left(-\Delta V+\chi\right)\right)+\beta\mathbb{E}\rho\underline{V} \quad (8)$$

where $\Delta V\equiv\bar{V}-\underline{V}$. That is, given a set of possible continuation equilibrium values $\left[\underline{V},\bar{V}\right]$, the best equilibrium value assigns the highest possible continuation value in case of repayment for all ρ because it maximizes the value on-path and maximally relaxes the sustainability constraint (4) subject to (5), and it assigns $V'_d\left(\rho\right)=\underline{V}$ because it maximally relaxes (4) subject to (5). Conversely, for the worst equilibrium value given continuation values $\left[\underline{V},\bar{V}\right]$, to minimize the continuation value and the amount of debt that can be raised in the current period, it is optimal to set $V'\left(\rho\right)=\underline{V}$ and $V'_d\left(\rho\right)=\bar{V}$.

Proposition 1. *The equilibrium value set is the largest fixed point of (2) with*

$$\bar{V} = \frac{u\left(y_L + h\left(\Delta V + \chi\right)\right) + u\left(y_H - h\left(\Delta V + \chi\right)\right)}{1-\beta}$$
$$\underline{V} = \frac{u\left(y_L + h\left(-\Delta V + \chi\right)\right) + u\left(y_H - h\left(-\Delta V + \chi\right)\right)}{1-\beta}$$

If χ is not too large then $\underline{V} < \bar{V}$. If $\chi = 0$ then $\underline{V} = V_{aut}$ and $\bar{V} = V^*$.

This result is standard. First, we argue that the efficient allocation is a sustainable equilibrium outcome. Borrowing b^* at a price ρ is optimal for the borrower because of condition (1) and is feasible because the definition of b^* implies that the borrower has incentives to repay b^* in the PM to retain access to future borrowing. Borrowing more than b^* is not feasible as it will result in a default even if there is no switch to the low-income regime. This implies that the price is $q\left(h^t_{AM}, b\right) = 0$ for all $b > b^*$ and ρ otherwise.

If $\chi = 0$, autarky is also an equilibrium. If lenders expect *future* prices to be zero, then they anticipate that the borrower will default in the PM for sure and so current prices are equal to zero. This is the type of multiplicity present in Alvarez and Jermann (2000), Gu et al. (2013), and Passadore and Xandri (2021).

Intermediate equilibria are also possible. Any equilibrium price between $[0, \rho]$ can be rationalized. Equilibrium prices can be low because, either ρ is low, or agents expect a high probability of coordination to an equilibrium with no future lending and incentives for the borrower to repay, which implies a price $q = \rho\left(1-\xi\right)$. Alternatively, there is an equilibrium with stationary pricing schedules parameterized by $b_{max} \in [0, b^*]$ given $q\left(h^t_{AM}, b\right) = \rho$ if $b \leqslant b_{max}$ and 0 if $b > b_{max}$ that induce the borrower in the AM to borrow $b = b_{max}$ in every period. These pricing schedules are consistent with a sustainable equilibrium because lenders can expect a coordination to the autarkic equilibrium in all future periods if $b > b_{max}$ and rationally anticipate a default. Moreover, there is a continuum of non-stationary equilibria.

4 Positive analysis of Fund's policy

We now take a robust implementation approach to study the role and design of the Financial Stability Fund, or *Fund* for short. That is, we consider the role of the Fund to put a lower bound on the worst equilibrium value by considering policies that maximize the worst possible outcome. Proposition 1 implies that the best private equilibrium is efficient which means that the Fund is not essential to implement the efficient allocation. However, we show that the Fund can be essential from a robust perspective to guarantee that the value of the worst equilibrium outcome is higher than autarky.

If the Fund knew the state of the economy, i.e. the realization of ρ, then it could uniquely implement the efficient allocation by offering to lend at price $\underline{q}(\rho) = \rho$ a maximal amount of debt $\bar{b}(\rho) = b^*$. That is, the borrower can access the Fund's facility if its total debt (private + Fund) is less than $\bar{b}(\rho)$. This implies that pessimistic beliefs on the part of current lenders about the future borrowing ability of the borrower can never be justified, which implies equilibrium uniqueness. Moreover, the Fund only intervenes off-path and never has to use any resources along the equilibrium path.

We now consider the case in which the Fund does not know ρ and cannot condition assistance on the state of the economy. That is, we consider a stationary assistance plan which consists of a maximal amount of debt $\bar{b}$ and a minimal price $\underline{q}$. Under such a plan, the Fund is willing to lend at price $q = \max\left\{\underline{q}, q_m\right\}$ so long as $b \leqslant \bar{b}$ and the borrower has not defaulted before.

We now consider the case in which the default value is not a given value but it is endogenous. We will show that it is not feasible for the Fund to uniquely implement the best equilibrium value in a costless way. If the Fund wants to increase the value of the worst equilibrium outcome, it must incur losses. Moreover, the Fund faces a trade-off between larger losses and higher value for the worst equilibrium outcome.

4.1 Sustainable equilibria with Fund policy

A *sustainable equilibrium* with $\left(\underline{q}, \bar{b}\right)$ is a debt issuance decision $b\left(h_{AM}^t\right)$, a pricing schedule $q\left(h_{AM}^t, b_t\right)$, default decision $\delta\left(h_{PM}^t\right)$, and borrower's values $V_{AM}\left(h_{AM}^t\right)$ and $V_{PM}\left(h_{PM}^t\right)$ such that: i) the debt issuance decision is optimal for the borrower in the AM for all h_{AM}^t

$$V_{AM}\left(h_{AM}^t\right) = \max_b u\left(y_L + q\left(h_{AM}^t, b\right) b\right) + \rho_t \int V_{PM}\left(h_{AM}^t, b, q\left(h_{AM}^t, b\right), \xi_{2t}\right) d\xi_{2t},$$

ii) the pricing schedule is consistent with the Fund's rule and satisfies the break-even constraint for lenders

$$q\left(h_{AM}^t, b_t\right) = \begin{cases} \int \max\left\{\underline{q}, \rho_t \delta\left(h_{AM}^t, b_t, b, q\left(h_{AM}^t, b\right), \xi_{2t}\right)\right\} d\xi_{2t} & \text{if } b_t \leqslant \bar{b}, \delta_{t-k} = 1 \forall k \leqslant t \\ \rho_t \int \delta\left(h_{AM}^t, b_t, b, q\left(h_{AM}^t, b\right), \xi_{2t}\right) d\xi_{2t} & \text{otherwise} \end{cases},$$

and iii) the default decision is optimal in the PM for all h_{PM}^t

$$\begin{aligned} V_{PM}\left(h_{PM}^t\right) = \max_{\delta \in [0,1]} \delta \left[u\left(y_H - b_t\right) + \beta \int \int V_{AM}\left(h_{PM}^t, 1, \rho_{t+1}, \xi_{1t+1}\right) dF\left(\rho_{t+1}\right) d\xi_{1t+1}\right] \\ + (1-\delta)\left[u\left(y_H\right) + \beta \int \int \left(V_{AM}\left(h_{PM}^t, 0, \rho_{t+1}, \xi_{1t+1}\right) dF\left(\rho_{t+1}\right) d\xi_{1t+1} - \chi\right)\right] \end{aligned}$$

Note that $V_{AM}\left(h_{PM}^{t}, 0, \rho', \xi_1'\right)$ is now a generic sustainable equilibrium continuation value without Fund's assistance, i.e. $V_{AM}\left(h_{AM}^{t+1}\right) \in \mathcal{V}$ for all histories h_{AM}^{t+1} with $\delta_{t-k} = 0$ for some $t-k$.

4.2 Worst sustainable equilibrium

We next characterize the *worst* sustainable equilibrium which is associated with the highest payoff the borrower can attain *for sure*. We can characterize the worst equilibrium values with the Fund's assistance by considering a modified version of the operator P without the Fund described in (2). Note that in this case a default must be associated with a value $V_d' \in \mathcal{V} = \left[\underline{V}, \bar{V}\right]$. Let $\left[\underline{V}_F, \bar{V}_F\right]$ be a set of potential continuation equilibrium values if the borrower still has the ability to request the Fund's assistance i.e. he did not default previously. The worst equilibrium value is then

$$\underline{P}_F\left(\left[\underline{V}_F, \bar{V}_F\right]\right) = \int \nu_F\left(\rho, \left[\underline{V}_F, \bar{V}_F\right]\right) dF(\rho) \tag{9}$$

where

$$\nu_F\left(\rho, \left[\underline{V}_F, \bar{V}_F\right]\right) = \min_{b, V', V_d', V_{df}'} \max \left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H\right) + \beta\rho V_{df}' \\ u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \bar{b}\right) + \beta\rho V' \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) + \beta\rho V' \end{array} \right\} \tag{10}$$

subject to

$$u\left(y_H - b\right) + \beta V' \geqslant u\left(y_H\right) + \beta\left(V_d' - \chi\right) \tag{11}$$

$$V' \in \left[\underline{V}_F, \bar{V}_F\right], \quad V_{df}', V_d' \in \left[\underline{V}, \bar{V}\right] \tag{12}$$

Define

$$\begin{aligned} U_F(\rho) &\equiv u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right) \\ U_P\left(\rho, \underline{V}_F - \bar{V} + \chi\right) &\equiv u\left(y_L + \rho h\left(\underline{V}_F - \bar{V} + \chi\right)\right) + \rho u\left(y_H - h\left(\underline{V}_F - \bar{V} + \chi\right)\right) \end{aligned}$$

We can then write (10) as

$$\nu_F\left(\rho, \underline{V}_F\right) = \max \left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H\right) + \beta\rho\left(\underline{V} - \chi\right) \\ U_F(\rho) + \beta\rho\underline{V}_F \\ U_P\left(\rho, \underline{V}_F - \bar{V} + \chi\right) + \beta\rho\underline{V}_F \end{array} \right\}$$

Thus, $\nu_F\left(\rho, \left[\underline{V}_F, \bar{V}_F\right]\right) = \nu_F\left(\rho, \underline{V}_F\right)$ and $\underline{P}_F\left(\left[\underline{V}_F, \bar{V}_F\right]\right) = \underline{P}_F\left(\underline{V}_F\right)$ in that they do not depend on the value of the best equilibrium with assistance. We can then solve for the smallest fixed point of $\underline{P}_F\left(\underline{V}_F\right)$ independently from $\bar{V}_F$, and the smallest fixed point $\underline{V}_F$ is the worst

borrower's value. The function $\underline{P}_F$ is shown in the left column of Figure 2.

The worst equilibrium value for the borrower with Fund's assistance does not coincide with the worst sustainable equilibrium value without assistance if and only if

$$\max \left\{ \begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \mathbb{E}\rho\left[u\left(y_H\right) + \beta\left(\underline{V} - \chi\right)\right] \\ u\left(y_L + \underline{q}\bar{b}\right) + \mathbb{E}\rho\left[u\left(y_H - \bar{b}\right) + \beta\underline{V}\right] \end{array} \right\} > \int U_P\left(\rho, \underline{V} - \bar{V} + \chi\right) dF\left(\rho\right) + \beta\mathbb{E}\rho\underline{V}_F = \underline{V}.$$

If the above condition holds then $\underline{P}_F\left(\underline{V}\right) > \underline{V}$ and the smallest fixed point of $\underline{P}_F$ is greater than $\underline{V}$.

Proposition 2. *Given a stationary assistance policy $\pi = \left(\underline{q}, \bar{b}\right)$, the worst equilibrium value for the borrower, $\underline{V}_F$, is the smallest fixed point $\underline{P}_F\left(\underline{V}_F\right) = \underline{V}_F$. If $\underline{q}\bar{b} > \rho h\left(\underline{V} - \bar{V} + \chi\right)$ then $\underline{V}_F > \underline{V}$. If $\bar{b} \leqslant h\left(\underline{V}_F - \underline{V} + \chi\right)$ the worst equilibrium outcome has no default. If instead $\bar{b} > h\left(\underline{V}_F - \underline{V} + \chi\right)$ then the worst equilibrium has the borrower borrowing from the Fund at maximal capacity and then default on it.*

The first part of the proposition is shown above. The equilibrium outcome in the worst borrower's equilibrium depends on the generosity of the Fund's assistance. Suppose first that $\bar{b} > h\left(\underline{V}_F - \underline{V} + \chi\right)$. In this case the borrower prefers to max out the Fund's assistance and default on the Fund than repaying. In fact, in this case we have that

$$u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u\left(y_H\right) + \beta\left(\underline{V} - \chi\right)\right] > u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u\left(y_H - \bar{b}\right) + \beta V_F\right]. \tag{13}$$

If instead $\bar{b} \leqslant h\left(\underline{V}_F - \underline{V} + \chi\right)$, then there are no default along the equilibrium path in the worst equilibrium because inequality (13) is reversed.

However, if $\bar{b} \leqslant h\left(\underline{V}_F - \underline{V} + \chi\right)$ there can be other equilibria that have a higher value for the borrower in which there is default on the Fund because a default on the Fund is rewarded by a continuation value higher than $\underline{V}$. The condition to have no default on the Fund outside of the low-income regime is

$$\bar{b} \leqslant h\left(\underline{V}_F - \bar{V} + \underline{\chi}\right). \tag{14}$$

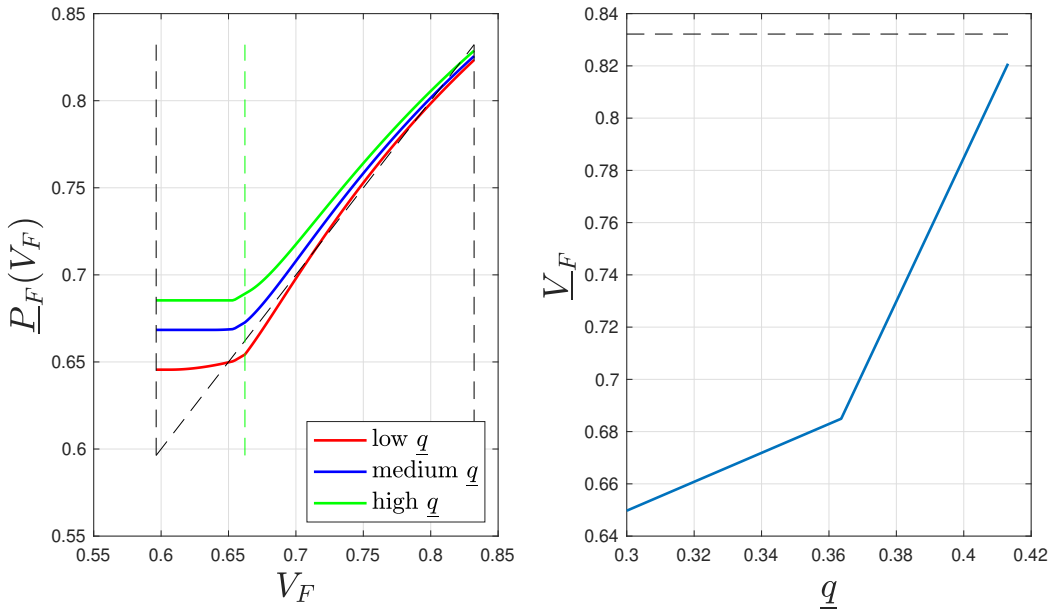
In fact, if $\bar{b} \in \left(h\left(\underline{V}_F - \bar{V} + \underline{\chi}\right), h\left(\underline{V}_F - \underline{V} + \chi\right)\right)$ then there are sustainable equilibria in which it is optimal to default on the Fund. For example, suppose that the continuation value for defaulting on the Fund is $V'_{df} = \bar{V}$ and the continuation value associated to repaying is $V'_r = \underline{V}_F$. Then

$$u\left(y_H\right) + \beta\left(V'_{df} - \chi\right) > u\left(y_H - \bar{b}\right) + \beta V'_r$$

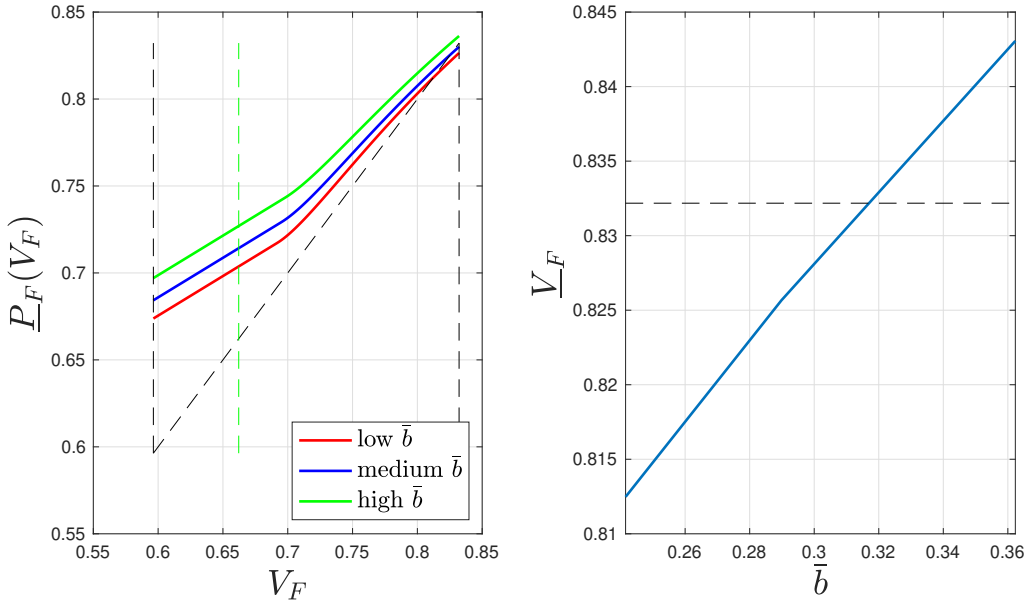
for $\bar{b}$ in such range. However, if $\bar{b} \leqslant h\left(\underline{V}_F - \bar{V} + \underline{\chi}\right)$ then there are no default in any sustainable equilibrium with assistance. This is because in this case for any $V'_{df} \in \left[\underline{V}, \bar{V}\right]$

Figure 2: $\underline{P}_F(V_F)$

a. Comparative statics with respect to $\underline{q}$ for given $\bar{b}$



b. Comparative statics with respect to $\bar{b}$ for given $\underline{q}$



and $V_r' \in \left[\underline{V}_F, \bar{V}_F\right]$ then:

$$u\left(y_H - \bar{b}\right) + \beta V_r' \geqslant u\left(y_H\right) + \beta\left(V_{df}' - \chi\right).$$

So it is never optimal to default on the Fund outside of the low-income regime. It is also never feasible to default on private lenders outside of the low-income regime because private lenders will simply refuse to lend anticipating a default.

Proposition 2 has an important implication: losses are necessary to increase the worst equilibrium value.

Proposition 3. *If* $\beta\mathbb{E}\rho\left\{\frac{u'(y_L)}{u'(y_H)}\right\} < 1$, *the Fund must incur losses to improve the worst equilibrium value.*

Proof. Clearly, if $\bar{b} > h\left(\underline{V}_F - \bar{V} + \chi\right)$, then there are equilibria with default on the Fund. In this equilibria the Fund is incurring losses in expected value. Suppose next that i) $\bar{b} \leqslant h\left(\underline{V}_F - \bar{V} + \chi\right)$ so that there are no default in the high-income regime on the Fund and ii) the Fund does not subsidize borrowing, $\underline{q} \leqslant \underline{\rho}$. In this case, it must be that $\underline{V}_F = \underline{V}$ so the policy does not improve upon the worst private equilibrium. Suppose by way of contradiction that $\underline{V}_F > \underline{V}$. Note that because of i) and ii) the Fund is never utilized in the equilibrium that attains the minimum value. Thus, it must be that

$$\underline{V}_F = \underline{P}_F\left(\underline{V}_F\right) = \underline{P}_F\left(\underline{V}_F; 0\right)$$

where $\underline{P}_F\left(\underline{V}_F; 0\right)$ is the operator (9) under no assistance i.e. $\underline{q} = 0$ and $\bar{b} = 0$. Clearly, $\underline{V} = \underline{P}_F\left(\underline{V}; 0\right)$. Next, we show that $\underline{P}_F\left(V; 0\right)$ admits a unique fixed point under $\beta\mathbb{E}\rho\left\{\frac{u'(y_L)}{u'(y_H)}\right\} < 1$, implying that $\underline{V}_F = \underline{V}$, a contradiction. To see this, note that

$$\underline{P}_F\left(V_F; 0\right) = \int \left[u\left(y_L + \rho h\left(V_F - \bar{V} + \chi\right)\right) + \rho u\left(y_H - h\left(V_F - \bar{V} + \chi\right)\right) + \beta\rho V_F\right] dF\left(\rho\right)$$

and

$$\begin{aligned}\frac{\partial \underline{P}_F\left(V_F; 0\right)}{\partial V_F} &= \int \left[\left[u'\left(y_L + \rho h\right) - u'\left(y_H - h\right)\right]\rho h' + \beta\rho\right] dF\left(\rho\right) \\ &\leqslant \mathbb{E}\rho\left\{\left[u'\left(y_L\right) - u'\left(y_H\right)\right]h' + \beta\right\}\end{aligned}$$

where

$$h' = \begin{cases} 0 & V_F < \bar{V} - \chi \\ \frac{\beta}{u'(y_H - h)} & V_F > \bar{V} - \chi \end{cases}$$

Thus,

$$\frac{\partial \underline{P}_F(V_F;0)}{\partial V_F} \leqslant \begin{cases} \beta \mathbb{E}\rho & V_F < \bar{V} - \chi \\ \beta \mathbb{E}\rho \left\{ \left[\frac{u'(y_L)-u'(y_H)}{u'(y_H)} \right] + 1 \right\} = \beta \mathbb{E}\rho \left\{ \frac{u'(y_L)}{u'(y_H)} \right\} & V_F > \bar{V} - \chi \end{cases}$$

This imples that $\partial \underline{P}_F(V_F;0)/\partial V_F < 1$ for all V_F and since $\underline{V} = \underline{P}_F(\underline{V};0)$ then $\underline{V}$ is the only fixed point of $\underline{P}_F(V;0)$. Thus, $\underline{V}_F = \underline{V}$, a contradiction. Therefore, if $\bar{b} \leqslant h\left(\underline{V}_F - \bar{V} + \chi\right)$ then it must be that $\underline{q} > \underline{\rho}$ and the Fund subsidizes borrowing in certain states. Q.E.D.

Even if condition (14) is satisfied and there are no default on the Fund in any equilibrium, the Fund must subsidize the bond prices along the equilibrium path to guarantee a value above $\underline{V}$. In fact, if condition (14) holds and $\underline{q} \leqslant \underline{\rho}$ so there are no price subsidy along the equilibrium path then the Fund cannot improve upon the worst equilibrium. The Fund faces a trade-off between larger losses and higher value for the worst equilibrium outcome. Moreover, generous assistance program can lead to a worst equilibrium value that is higher than the maximal value without assistance (as shown in the bottom-right panel of Figure 2) but with subsidies along the equilibrium path.

In what it follows, we consider the case in which the Fund designs its assistance program so it is *never* optimal to default on it and condition (14) is satisfied. In this case, we can show that the worst equilibrium for the borrower is also the one that maximizes expected losses for the Fund:

Proposition 4. *If condition (14) is satisfied, then the worst equilibrium for the borrower's perspective is also the one with the highest losses for the Fund.*

Under condition (14), it is never optimal to default in any sustainable equilibrium outcome andthe period value is

$$\max\left\{U_F(\rho); U_P\left(\rho, \underline{V}_F - \bar{V} + \chi\right)\right\}$$

and, since $\bar{b} \leqslant h\left(\underline{V}_F - \bar{V} + \underline{\chi}\right)$ the amount that the borrower can credibly obtain from the market is higher than what he can obtain from the Fund. Thus, the borrower will ask for assistance only if $\underline{q} > \rho$ and can borrow at subsidized price. Thus, the Fund is losing money in expectations all the time it is active. Next, notice that the worst equilibrium outcome for the borrower maximizes the amount of Fund's intervention along the equilibrium path. In other sustainable equilibrium $V_F \geqslant \underline{V}_F$ and the punishment of defaulting is potentially higher i.e. $V_d' \leqslant \underline{V}$. Thus, accessing private markets may have a larger value than $U_P\left(\rho, \underline{V}_F - \bar{V} + \chi\right)$ and this reduces the states in which it is optimal to ask for intervention and obtain period value $U_F(\rho)$. This is also beneficial for the Fund because the Fund makes negative profits all the times it is active.

5 Optimal dynamic assistance policy

What is the most cost-effective way to deliver V for sure to the borrower? In this section, we show that the optimal path for guarantees $\left\{\underline{q}_t, \bar{b}_t\right\}$ back-loads assistance. This is true given any level of loss that the Fund is willing to sustain. Even if the the NPV of losses, L, is very low, eventually the Fund provides a relatively large subsidy to the borrower. In particular, if L is small, this must happen sufficiently far away in the future to make the NPV small.

We restrict to outcomes where there is no government default in any equilibrium outcome and condition (14) holds. Under this condition, by Proposition 4 we know that the maximal losses for the Fund are the one along the worst equilibrium path. Thus, we can consider the problem of minimizing the losses to attain V is a recursive fashion as

$$L(V) = \min_{\underline{q},\bar{b},b,V'} \int \mathbb{I}\left(\rho|\underline{q},\bar{b},b,V'\right)\left(\underline{q}-\rho\right)\bar{b}\,dF(\rho) + \beta\mathbb{E}\rho L(V') \tag{15}$$

subject to

$$V = \int \max\left\{\begin{array}{c} u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right); \\ u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) \end{array}\right\} dF(\rho) + \beta\mathbb{E}\rho V'$$

$$b, \bar{b} \leqslant h\left(V' - \bar{V} + \chi\right)$$

where

$$\mathbb{I}\left(\rho|\underline{q},\bar{b},b,V'\right) = \begin{cases} 1 & \text{if } u\left(y_L + \underline{q}\bar{b}\right) + \rho u\left(y_H - \underline{q}\bar{b}\right) \geqslant u\left(y_L + \rho b\right) + \rho u\left(y_H - b\right) \\ 0 & \text{otherwise} \end{cases}.$$

Proposition 5. *The optimal assistance program that satisfies the no-default condition (14) is backloaded in that* $\left\{\underline{q}_t \bar{b}_t\right\}$ *is increasing over time. Moreover, the expected present value of subsidies is increasing over time.*

To see this, rewrite the problem in (15) as

$$L(V) = \min_{\nu,V'} K(\nu, V') + \beta\mathbb{E}\rho L(V')$$

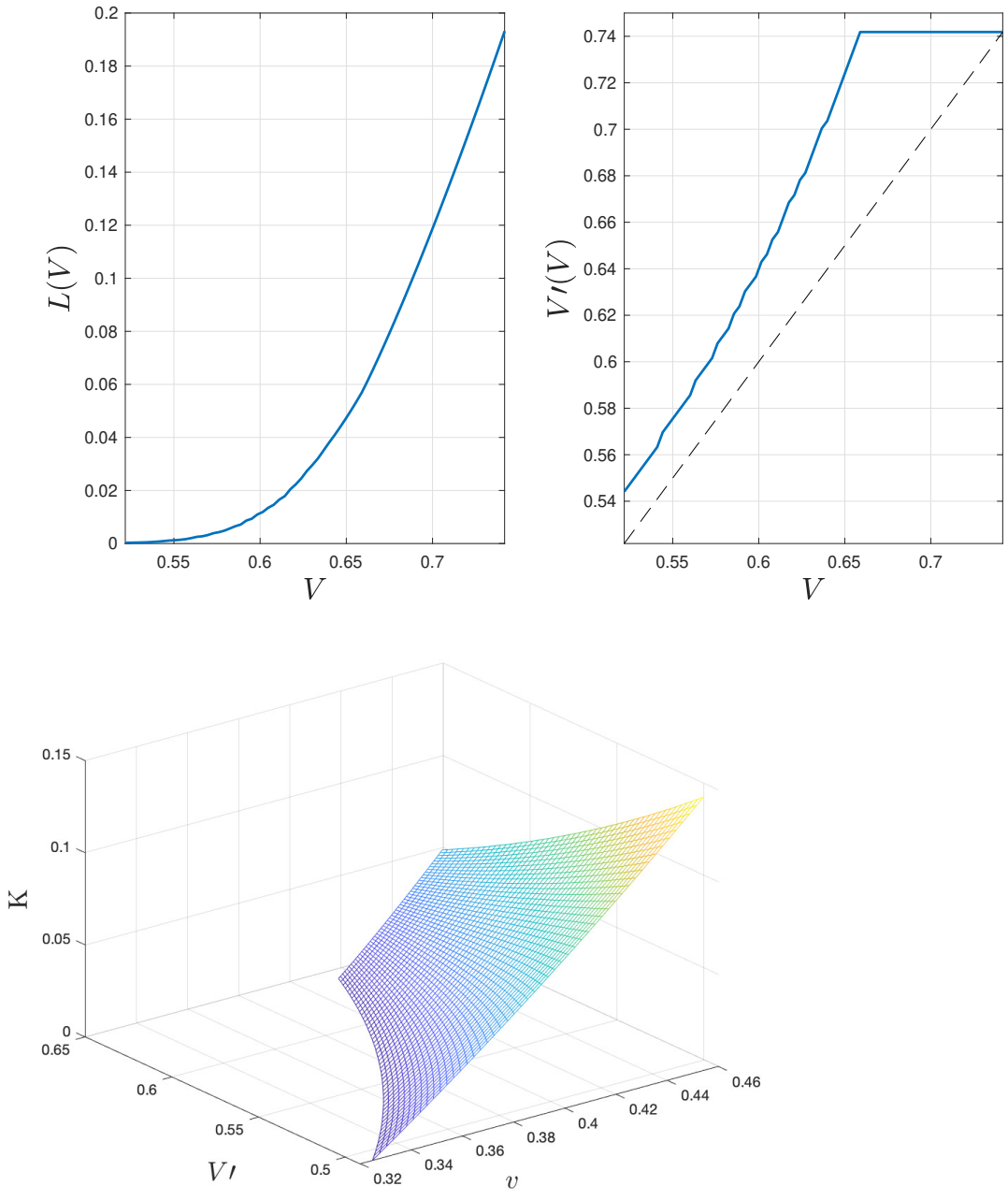
subject to

$$\nu + \beta\mathbb{E}\rho V' = V$$

where

$$K(\nu, V') \equiv \min_{\bar{b},T} \int \mathbb{I}(\rho)\left(T - \rho\bar{b}\right) dF(\rho)$$

Figure 3: L (V) and optimal dynamic



subject to

$$\int \max \left\{ \begin{array}{c} u\left(y_L + T\right) + \rho u\left(y_H - \bar{b}\right); \\ u\left(y_L + \rho h\left(V' - \bar{V} + \chi\right)\right) + \rho u\left(y_H - h\left(V' - \bar{V} + \chi\right)\right) \end{array} \right\} dF\left(\rho\right) = v$$

$$\bar{b} \leqslant h\left(V' - \bar{V} + \chi\right)$$

Note that the focs

$$K_1 + \lambda = 0$$
$$K_2 + \beta \mathbb{E}\rho\left[L'\left(V'\right) + \lambda\right] = 0$$

and envelope condition

$$L'\left(V\right) = -\lambda$$

require that

$$K_2 + \beta \mathbb{E}\rho\left[L'\left(V'\right) - L'\left(V\right)\right] = 0.$$

Since $K_2 \leqslant 0$ i.e. higher V' implies lower period losses, then it must be that

$$L'\left(V'\right) - L'\left(V\right) \geqslant 0.$$

If $K\left(v, V'\right)$ is convex then standard arguments imply that $J\left(V\right)$ is also convex. Thus, $L'\left(V\right)$ is increasing and $V' \geqslant V$.

We cannot prove that K is convex but we can verify it in numerical examples. As shown in Figure 3, K and J are convex functions and $V'\left(V\right) \geqslant V$. That is, the optimal assistance program is backloaded.

The last result implies that the Fund must commit to large future outlays. An implementation for this outcome is that the Fund raises L in period 0 and commits to an increasing level of support and subsidies. In the early periods, the Fund's period losses are lower than $rL = \left(1/\beta - 1\right) L$ and the Fund's balance – or its endowment – is increasing over time. Thus, the Fund can eventually finance the large subsidies with the accumulated assets without the need to raise more funds. In the worst equilibrium that delivers a borrower value of V_0 the Fund exhausts all of its resources but a better equilibrium can be selected and the Fund can potentially repay some of the initial contribution back to its shareholders/donors.

6 Exogenous default value

So far, we assumed that the continuation value of a borrower that defaults on his debt is any continutation sustainable equilibrium value. We refer to this case as the *endogenous default value* case. In this section, we consider the case in which a borrower's default is followed by a continuation value $V_{aut}(H) - \beta\chi$. We will refer to this case as the *exogenous default value* case. This is the case typically considered in the quantitative sovereign default literature. In this case, the extent to which there is multiplicity is limited. First, we show that there are multiple private sustainable equilibria only if $\chi = 0$. If $\chi > 0$ then the efficient allocation is the unique equilibrium outcome. Then we show that even if $\chi = 0$, the Fund can uniquely implement an allocation arbitray close to the efficient allocation simply by committing to a small intervention that if $\underline{\rho} > 0$ it is only outside the equilibrium path.

6.1 Private sustainable equilibria

We first show that, in the exogenous default value case, multiplicity of equilibrium arises if and only if $\chi = 0$. When $\chi > 0$ there is a unique sustainable equilibrium.

A *sustainable equilibrium* is a debt issuance decision $b\left(h_{AM}^t\right)$, a pricing schedule $q\left(h_{AM}^t, b_t\right)$, default decision $\delta\left(h_{PM}^t\right)$, and borrower's values $V_{AM}\left(h_{AM}^t\right)$ and $V_{PM}\left(h_{PM}^t\right)$ such that: i) the debt issuance decision is optimal for the borrower in the AM for all h_{AM}^t,

$$V_{AM}\left(h_{AM}^t\right) = \max_b u\left(y_L + q\left(h_{AM}^t, b\right) b\right) + \rho_t \int V_{PM}\left(h_{AM}^t, b, q\left(h_{AM}^t, b\right), \xi_{2t}\right) d\xi_{2t},$$

ii) the pricing schedule satisfies the break-even constraint for lenders

$$q\left(h_{AM}^t, b_t\right) = \begin{cases} \rho_t \int \delta\left(h_{AM}^t, b_t, q_t, \xi_{2t}\right) d\xi_{2t}, & \text{if } \delta_{t-k} = 1 \quad \forall k \leqslant t \\ 0 & \text{if } \delta_{t-k} = 0 \text{ for some } t-k < t \end{cases},$$

and iii) the optimality of the default decision in the PM for all h_{PM}^t,

$$\begin{aligned} V_{PM}\left(h_{PM}^t\right) = \max_{\delta \in [0,1]} \delta & \left[u\left(y_H - b_t\right) + \beta \int \int V_{AM}\left(h_{PM}^t, 1, \rho_{t+1}, \xi_{2t+1}\right) dF\left(\rho_{t+1}\right) d\xi_{2t+1}\right] \\ & + (1-\delta)\left[V_{aut}(H) - \beta\chi\right] \end{aligned}$$

Let V_{aut} be the value of autarky for the borrower in the AM, where

$$V_{aut} \equiv u\left(y_L\right) + \rho V_{aut}(H) + (1-\rho) V_{aut}(L) = \rho V_{aut}(H).$$

Proposition 6. *When the value of default is exogenous:*

- *If* $\chi = 0$ *then the equilibrium value set is* $[V_{aut}, V^*]$.
- *If* $\chi > 0$ *then the efficient allocation is the unique equilibrium.*

This result is standard.

Consider first the case with $\chi = 0$. As in the endogenous default case, the efficient allocation is a sustainable equilibrium outcome. Autarky is also an equilibrium because if lenders expect *future* prices to be zero, then they anticipate that the borrower will default in the PM for sure and so current prices are equal to zero.

Intermediate equilibria are also possible. Any equilibrium price between $[0, \rho]$ can be rationalized. Equilibrium prices can be low because, either ρ is low, or agents expect a high probability of coordination to an equilibrium with no future lending and incentives for the borrower to repay, which implies a price $q = \rho(1-\xi)$. Alternatively, there is an equilibrium with stationary pricing schedules parameterized by $b_{max} \in [0, b^*]$ given $q(h^t_{AM}, b) = \rho$ if $b \leqslant b_{max}$ and 0 if $b > b_{max}$ that induce the borrower in the AM to borrow $b = b_{max}$ in every period. These pricing schedules are consistent with a sustainable equilibrium because lenders can expect a coordination to the autarkic equilibrium in all future periods if $b > b_{max}$ and rationally anticipate a default. Moreover, there is a continuum of non-stationary equilibria indexed by $V_0 \in (V_{aut}, V^*)$ that converge to autarky as $t \to \infty$.

Consider now the case with $\chi > 0$. We already established that the efficient allocation is a sustainable equilibrium outcome. This is the unique sustainable equilibrium outcome when the value associated to a default is exogenous and $\chi > 0$. To see this, note that the value for the worst and best equilibrium outcome are the smaller and the largest fixed point of the following operator:

$$\begin{aligned} P_0(V') &= \int \left[u(y_L + \rho h(V' - V_{aut} + \chi)) + \rho\left[u(y_H - h(V' - V_{aut} + \chi)) + \beta V'\right]\right] dF(\rho) \\ &= \int u(y_L + \rho h(V' - V_{aut} + \chi))\, dF(\rho) + \mathbb{E}\rho u(y_H - h(V' - V_{aut} + \chi)) + \mathbb{E}\rho\beta V' \end{aligned}$$

The operator P_0 is shown in Figure 4 for the case with $\chi = 0$ and $\chi > 0$. Note that P_0 is concave in V' as

$$\begin{aligned} P_0'(V') &= \left[\int u'(y_L + \rho h)\, dF - \mathbb{E}\rho u'(y_H - h)\right] h' + \beta\mathbb{E}\rho \\ P_0''(V') &= \left[\int u'(y_L + \rho h)\, dF - \mathbb{E}\rho u'(y_H - h)\right] h'' + \left[\int u''(y_L + \rho h)\, dF + \mathbb{E}\rho u''(y_H - h)\right](h')^2 \end{aligned}$$

The second derivative is negative because $[u'(y_L + \rho h) - \rho u'(y_H - h)] \geqslant 0$, $u'' < 0$ and

$h'' < 0$ since

$$h' = \frac{\beta}{u'(y_H - h)} > 0, \quad h'' = \left(\frac{\beta}{u'(y_H - h)}\right)^2 u''(y_H - h)\, h' < 0.$$

When $\chi = 0$, autarky is clearly a fixed point, $V_{aut} = P_0(V_{aut})$, as well as $V^* = P_0(V^*)$. When $\chi > 0$ instead $P_0(V_{aut}) > V_{aut}$ and the smallest equilibrium value is V^*. Thus, there is a unique sustainable equilibrium with value V^*.

6.2 Positive analysis of Fund's policy

Our main result in this section is that if the exogenous default value case, then promising a very small intervention can guarantee that the efficient allocation is uniquely implemented. The size of the assistance offered by the Fund can be chosen to be small enough (but strictly positive) so that either the Fund's assistance is never asked along the equilibrium path and so losses are zero or such losses are minimal.

6.2.1 Sustainable equilibria with Fund policy

From Proposition 6, we know that assistance is needed only if $\chi = 0$ because when $\chi > 0$ the efficient allocation is the unique sustainable equilibrium outcome. Thus, we will concentrate on the case where exclusion from borrowing is the only cost of default.

A *sustainable equilibrium* with assistance $\pi = (\underline{q}, \bar{b})$ is a debt issuance decision $b(h^t_{AM})$, a pricing schedule $q(h^t_{AM}, b_t)$, default decision $\delta(h^t_{PM})$, and borrower's values $V_{AM}(h^t_{AM})$ and $V_{PM}(h^t_{PM})$ such that: i) the debt issuance decision is optimal for the borrower in the AM for all h^t_{AM}

$$V_{AM}(h^t_{AM}) = \max_b u(y_L + q(h^t_{AM}, b)\, b) + \rho_t V_{PM}(h^t_{AM}, b),$$

ii) the pricing schedule is consistent with the Fund's rule and satisfies the break-even constraint for lenders

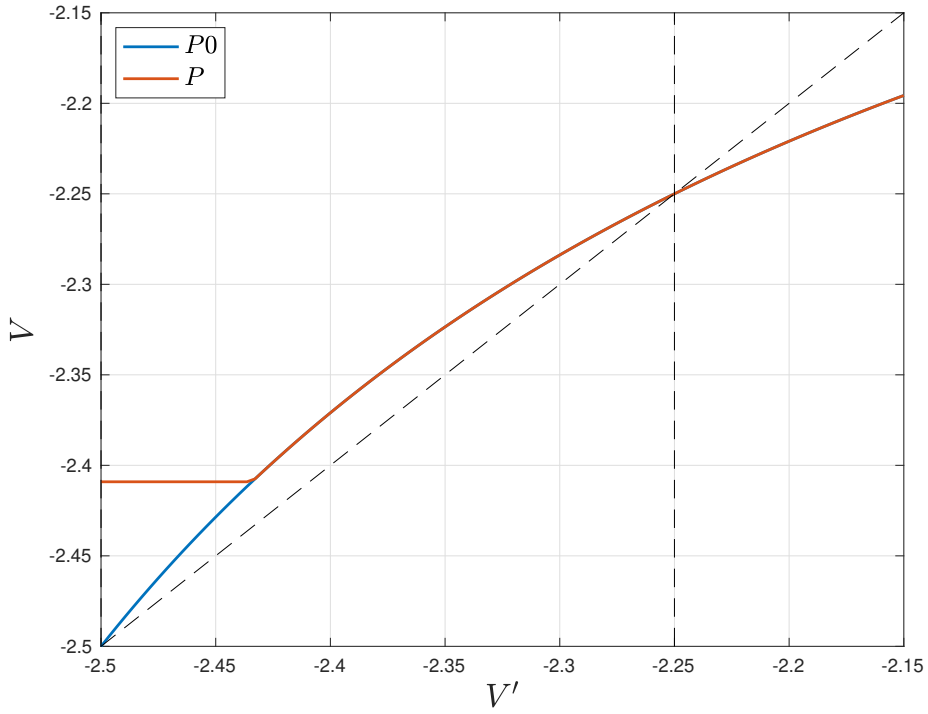
$$q(h^t_{AM}, b_t) = \begin{cases} \max\{\underline{q}, \rho_t \delta(h^t_{PM})\} & \text{if } b_t \leqslant \bar{b} \text{ and } \delta_{t-k} = 1 \quad \forall k \leqslant t \\ \rho_t \delta(h^t_{PM}) & \text{if } b_t > \bar{b} \text{ and } \delta_{t-k} = 1 \quad \forall k \leqslant t \\ 0 & \text{if } \delta_{t-k} = 0 \text{ for some } t - k < t \end{cases},$$

and iii) the optimality of the default decision in the PM for all h^t_{PM}

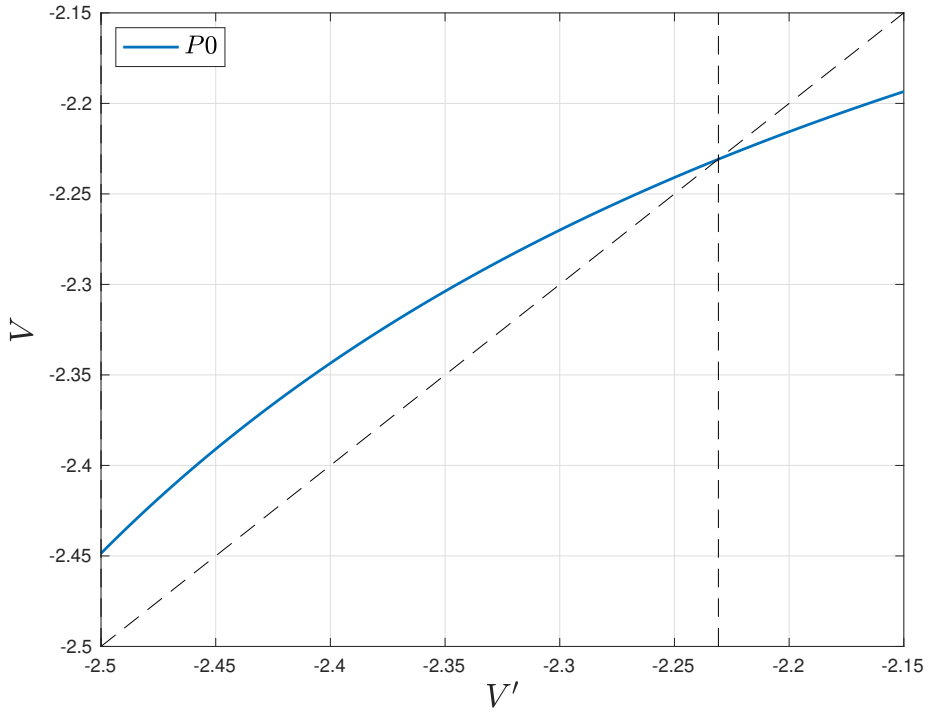
$$V_{PM}(h^t_{PM}) = \max_{\delta \in [0,1]} \delta \left[u(y_H) + \beta \int V_{AM}(h^t_{PM}, 1, \rho')\, dF(\rho') \right] + (1 - \delta) V_{aut}(H).$$

Figure 4: Sustainable equilibrium values when default value is exogenous

a. $\chi = 0$



b. $\chi > 0$



6.2.2 Worst sustainable equilibrium

Next, we characterize the *worst* sustainable equilibrium which is associated with the highest payoff the borrower can attain *for sure.* With the Fund's assistance scheme, the borrower can unilaterally borrow from the Fund and then default, obtaining value

$$P_1(\rho;\pi) = u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u(y_H) + \beta\left(V_{aut} - \chi\right)\right] \tag{16}$$

Moreover, the borrower could borrow from the Fund and repay and being assigned the lowest continuation value V', obtaining value

$$P_2(V', \rho;\pi) = u\left(y_L + \underline{q}\bar{b}\right) + \rho\left[u\left(y_H - \bar{b}\right) + \beta V'\right]. \tag{17}$$

Thus, no equilibrium can have a value lower than $\int \max\{P_1(\pi,\rho), P_2(\pi,\rho,V')\}\, dF(\rho)$. Note that we do not need to worry about incentives to repay the Fund because if $P_2(V',\rho;\pi) \geqslant P_1(\rho;\pi)$ then it must be that $u\left(y_H - \bar{b}\right) + \beta V' \geqslant u(y_H) + \beta\left(V_{aut} - \chi\right)$ and the Fund has incentive to repay the Fund. If instead $P_1(\rho;\pi) > P_2(V',\rho;\pi)$ then the borrower prefers the option to borrow at to capacity from the Fund and default and we do not need to check repayment incentives.

Finally, again given the lowest continuation value V', the borrower can borrow from private lenders and obtain at least a value

$$P_3(V',\rho;\pi) = u\left(y_L + \rho h\left(V' - V_{aut}\right)\right) + \rho u\left(y_H - h\left(V' - V_{aut}\right)\right) + \beta V'.$$

Thus, the worst equilibrium value is the smallest fixed point of

$$P(V';\pi) = \int \max\{P_1(\rho;\pi), P_2(\rho;\pi), P_3(V',\rho;\pi)\}\, dF(\rho).$$

Proposition 7. *Suppose $\chi = 0$. If $\underline{q}\bar{b} > 0$ then there exists a unique equilibrium with value $V_\pi \geqslant V^*$. Assistance is never used in equilibrium if $\bar{b}$ and $\underline{q}$ are sufficiently small and $\underline{\rho} > 0$.*

Proof. First, notice that for all V'

$$P(V';\pi) \geqslant P_0(V')$$

because $\int P_3(V'\rho,\pi)\,dF(\rho) = P_0(V')$ and P_3 is one feasible option. If $\underline{q}\bar{b} > 0$ then

$$\begin{aligned}\int P_1(\rho;\pi)\,dF(\rho) &= u\left(y_L + \underline{q}\bar{b}\right) + \mathbb{E}\rho\left[u(y_H) + \beta V_{aut}\right] \\ &> u(y_L + h(0)) + \mathbb{E}\rho\left[u(y_H) + \beta V_{aut}\right] \\ &= V_{aut}.\end{aligned}$$

Thus, the concavity of P_0 and the observation that that $P(V';\pi) \geqslant P_0(V')$ with strict inequality for V_{aut} implies that P has a unique fixed point that is greater than V^*. If $\underline{q}$ and $\bar{b}$ are sufficiently small and $\underline{\rho} > 0$, then for $V' = V^*$ we have that $P(V^*;\pi) = P_0(V^*) = V^*$ and the borrower chooses to borrow only from the private markets for all possible ρ. In this case $V_F = V^*$. If instead $\underline{q}$ and $\bar{b}$ are relatively large and/or $\underline{\rho} > 0$ then it may be optimal for the borrower to use the Fund when ρ is sufficiently small (or equal to 0) and $P(V^*;\pi) > P_0(V^*) = V^*$ and so $V_F > V^*$. Q.E.D.

The content of this proposition is illustrated in panel a of Figure 4 that illustrates $P(V)$ and shows that it has a unique fixed point.

Thus, when the default value is exogenous, a small Fund intervention can avoid the worst equilibrium and support the efficient allocation as the unique sustainable equilibrium outcome. In particular, if $\underline{\rho} > 0$ then this can be achieved without the Fund incurring any losses in equilibrium. In this case, the Fund is just an off-path threat and its assistance is never utilized. Even when $\underline{\rho} = 0$, it is possible to minimize the cost of intervention by setting $\bar{q}$ to be sufficiently close to zero (but strictly positive). Thus, when the value of default is exogenous, promising a very small intervention can guarantee that the efficient allocation is uniquely implemented with essentially no costs for the Fund.

The result in Proposition 7 is related to the analysis of the possibilities of hyperinflation episodes in model of fiat-money. Wallace (1981) and Nicolini (1996) show that a small convertibility program that exchange fiat currency with consumption good is sufficient to ensure that only the efficient monetary equilibrium is an equilibrium outcome.

7 Moral hazard

We next study how the generosity of the Fund's policy affects the borrower's incentives to make costly investments and reforms to reduce the probability of transiting to the low-income regime. We show that the presence of the Fund increases incentives to exert effort in the worst case scenario because it increases the value of not transiting to the low-income regime relative to the worst private equilibrium. This result is similar to what we show in a companion paper, Dovis and Kirpalani (2023), for in the case of a static coordination problem.

We make this point within the case with exogenous default value with $\chi = 0$ so there is multiplicity by Proposition 6. Suppose the borrower at the beginning of each period can make an investment x to affect the probability of drawing ρ so that $\rho \sim F(\cdot|x)$ and $F(\cdot|x_H) \succeq^{fosd} F(\cdot|x_L)$ if $x_H > x_L$. The cost of x is $k(x)$ and x is not observable by either the Fund or short-term lenders.

Consider now the private equilibria without the Fund. Recall that $(1 - \mathbb{E}(\rho|x))\, V_{LL}$ is normalized to zero so, if the borrower is excluded from borrowing, the optimal investment effort is

$$x_{aut} = \arg\max_x u(y_L) + \mathbb{E}(\rho|x)\,[u(y_H) + \beta V_{aut}] - k(x)$$

where $V_{aut} = \max_x u(y_L) + \mathbb{E}(\rho|x)\,[u(y_H) + \beta V_{aut}] - k(x)$. The optimal effort level then satisfies

$$k'(x) = \frac{\partial \mathbb{E}(\rho|x)}{\partial x}\,[u(y_H) + \beta V_{aut}] \tag{18}$$

In the best equilibrium, if the sustainability constraint is binding – as we are assuming throughout – the borrower's value and the debt issued satisfy

$$V^* = \max_x \int [u(y_L + \rho b^*) - u(y_H - b^*)]\, dF(\rho|x) + \mathbb{E}(\rho|x)\,\beta V^* - k(x)$$

and

$$u(y_H - b^*) + \beta V^* = u(y_H) + \beta V_{aut}.$$

Thus, the optimal effort level solves

$$x^* = \arg\max_x \int u(y_L + \rho b^*)\, dF(\rho|x) + \mathbb{E}(\rho|x)\,[u(y_H) + \beta V_{aut}] - k(x)$$

or

$$k'(x) = \int u(y_L + \rho b^*)\, dF_x(\rho|x) + \frac{\partial \mathbb{E}(\rho|x)}{\partial x}\,[u(y_H) + \beta V_{aut}] \tag{19}$$

Comparing (18) and (19), it follows that $x^* > x_{aut}$. Thus, any $x \in [x_{aut}, x^*]$ is an equilibrium level of investment effort.

Consider now the problem with a stationary intervention $(\underline{q}, \bar{b})$.

Proposition 8. *If $\underline{q} < 1$ and $\bar{b} > 0$ then the equilibrium level of effort in the worst equilibrium with assistance is higher than the effort in the worst equilibrium without assistance.*

By Proposition 7, we know that there is a unique equilibrium outcome with maximal debt. If the assistance program is small enough then the efficient outcome is the unique equilibrium (and so the worst as well) and so $x = x^*$. If the assistance program is large, the value of not transiting to the low-income regime is higher and so there is even more effort than in the efficient allocation.

[... to be completed ...]

8 Parting takes

In this paper, we study how a Financial Stability Fund can increase the value that a borrower can attain for sure by acting as a lender of last resort and avoid the coordination failures in private credit markets which prevent the borrower from accessing credit. We show that if the Fund has imperfect information about the borrower's fundamentals and cannot contingent its policies on them then if the value of default is autarky then the Fund can uniquely implement the efficient allocation without intervening along the equilibrium path by choosing a small enough intervention. If instead the value of default is a sustainable equilibrium outcome then then the Fund cannot uniquely implement the efficient allocation without cost and it must commit to losses in order to guarantee a minimal value to the borrower that is above the autarkic level.

Moreover, we showed that the cost minimizing intervention prescribes increasing losses for the Fund. This last feature may be problematic for the Fund as it may be politically not feasible to sustain such large losses in the future and which implies that current promises may not be credible. To account for this issue, one may want to add a participation constraint for the Fund problem in (15) requiring that either the net present value of future losses or the per-period losses cannot exceed some value. Our results would be largely unaffected by this change. Proposition 5 suggests that such additional constraints are likely to be binding.

A related issue concerns the uncertainty about the political support for the Fund to sustain losses in the future. Our focus on the worst case scenario makes uncertainty about future support very costly in terms of the worst equilibrium outcome. In this sense, opacity about future assistance and willingness to sustain losses should be detrimental. Thus, the Fund should be very transparent about its modus operandi.

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