Fiscal Rules, Bailouts, and Reputation in Federal Governments*

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Abstract

Expectations of transfers by central governments incentivize overborrowing by local governments. In this paper, we ask if fiscal rules can reduce overborrowing if central governments cannot commit to enforce penalties when rules are violated. We study a model in which the central government's type is unknown and show that fiscal rules increase overborrowing if the central government's reputation is low. In contrast, fiscal rules are effective in lowering debt if the central government's reputation is high. Even when the central government's reputation is low, binding fiscal rules will arise in the equilibrium of a signaling game.

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1 Introduction

There are numerous examples throughout history in which excessive spending and debt accumulation by subnational governments led to transfers or bailouts by central governments. Examples include provinces in Argentina, states in Brazil, länders in Germany, and most recently countries (Greece, Ireland, and Portugal) in the European Union.[1] One view of such events is that the inability of central governments to commit to not transferring resources to indebted regions leads to profligating fiscal policies ex-ante, which in turn justifies the transfers ex-post. This idea has been formally studied by Chari and Kehoe (2007), Chari and Kehoe (2008), and Cooper et al. (2008) in the economics literature and Rodden (2002) in political science. See also Sargent (2012).

A commonly held view is that *fiscal rules* can correct these incentives to overborrow. In practice, fiscal rules take the form of limits to debt-to-GDP or deficit-to-GDP ratios along with some penalty if these are violated. When thinking about the design of fiscal rules, a natural question that arises is why central governments can commit to enforcing these rules if they cannot commit to not bail out. In this paper, we ask if fiscal rules can be beneficial if central governments cannot commit to enforcing the fiscal rules and if these rules will arise in equilibrium.

We address these questions in a reputation model in the tradition of Kreps and Wilson (1982) and Milgrom and Roberts (1982). The type of the central government is uncertain: it can be either a commitment type or an optimizing type. The commitment type can commit to not transfer resources to indebted regions and enforce fiscal rules. The optimizing type chooses policies sequentially. The *reputation* of a central government is the probability that local governments assign to it being a commitment type.

Our first main result is that if the reputation of the central government is low enough, then fiscal rules lead to more debt accumulation relative to the case with no rules. This is because the punishment associated with the fiscal rule enforcement increases the cost of maintaining a good reputation and thus the optimizing type reveals its type earlier relative to an environment without rules. This early resolution of uncertainty makes overborrowing more attractive for the local governments. In contrast, if the central government's reputation is sufficiently high, fiscal rules are effective in reducing borrowing by local governments. Our second main result is that despite promoting fiscal indiscipline when reputation is low, binding fiscal rules arise in an equilibrium of a signaling game because the commitment type wants to signal its type and it is optimal for the optimizing type to initially mimic and then not enforce the rule once violated.

We show these results in a stylized three-period model populated by local governments and a benevolent central government. The local governments choose the provision

[1] See Rodden et al. (2003), Rodden (2006), and Bordo et al. (2013) for further documentation.

of a local public good and have access to local tax revenues. They can also borrow from the rest of the world at a given interest rate. The central government does not have tax revenues, but it can transfer resources from one local government to another. We consider an institutional setup in which there is a constitution that requires the central government to not impose such transfers (*no-bailout clause*) and the local governments to keep their debt below some level or face a penalty if they violate this rule (*fiscal rule*). The central government can either be a commitment type that enforces the fiscal constitution or an optimizing type that can deviate from the constitution and choose a different policy. This type is initially unknown to the local governments, which learn about it through the actions of the central government.

We first consider the case in which the constitution contains only a no-bailout clause and no fiscal rules. When the central government's initial reputation level is low enough, there is a unique equilibrium in which the optimizing central government does not make transfers to the local governments in the intermediate period. Therefore, there is no revelation of the central government's type until the terminal period. The optimizing central government prefers to delay revealing its type and maintain its reputation. The benefit is that a higher reputation reduces overborrowing by local governments, and the cost is that without transfers public good provision might be unequal across local governments. For low levels of reputation, the benefits of maintaining reputation are first order, while the costs of not equalizing the provision of the local public good in the interim period via transfers are second order. When the local governments are homogeneous, these costs are exactly zero on path. When the local governments are heterogeneous, the distribution of debt inherited in the interim period is non-degenerate; therefore, these costs are positive. However, if the probability of facing the commitment type is close to zero, the provision of the local public good across local governments is almost identical even without transfers in the interim period. This is because the more indebted governments borrow against the transfer they anticipate in the final period.

We next consider a constitution with both a no-bailout clause and a *binding* fiscal rule. Fiscal rules are binding if the debt limits are lower than the equilibrium debt levels without fiscal rules. If the central government's reputation and discount factor are low enough, there exists a unique equilibrium in which fiscal rules are violated in the first period by the local governments and are not enforced ex-post by the optimizing type central government. Therefore, in this equilibrium there is *early resolution of uncertainty* (i.e., the central government reveals its type in the intermediate period). The intuition behind this result is that with fiscal rules, the costs of preserving reputation are higher. This is because the enforcement of the constitution now requires the central government to impose costly penalties on the local governments that violate the rule in addition to enforcing the no-bailout clause.

We then compare the debt levels in the equilibrium outcomes with and without rules. Our main result is that if reputation is low enough, having fiscal rules in the constitution leads to even more debt accumulation relative to the case without rules. The key driver for this result is that the type of the central government is revealed in the intermediate period with rules (early resolution of uncertainty), and only in the terminal period without rules (late resolution of uncertainty). Knowing the type of the central government in the intermediate period allows the local governments to condition their new debt issuances on the government type. This, in turn, lowers the cost of servicing the debt inherited in the intermediate period. Intuitively, if a local government learns that it is facing the commitment type in the interim period, it can spread the losses associated with not receiving a transfer over the intermediate and final periods. In contrast, if the local government learns that it is facing the commitment type only in the final period, all the adjustment must be made in the final period. The former is preferable because local governments like to smooth their public good consumption. Thus, the local governments will issue more debt in the first period when there is early revelation.

In contrast, if the central government's reputation is high enough, there exists a unique equilibrium in which the local governments obey the fiscal rule. This is because they anticipate facing a penalty for violating the rule with sufficiently high probability irrespective of the choice of the optimizing type. Since fiscal rules are respected in equilibrium, debt levels are lower in the equilibrium with fiscal rules.

Our analysis then raises the question of why we would ever see fiscal rules being instituted in practice when governments lack credibility. We study a signaling game in which the rules are chosen at the beginning of time by the central government. We show that for intermediate values of the central government's discount factor, in the equilibrium of this game, the commitment type chooses to announce a fiscal rule, which is mimicked by the optimizing type. However, in this equilibrium the rule is not enforced in the intermediate period by the optimizing type, leading to early resolution of uncertainty and even more debt accumulation.

Our analysis sheds light on historical and contemporary episodes when fiscal rules were instituted but were not enforced ex-post. A leading example is the Stability and Growth Pact (SGP) in the European Union (EU). The SGP calls for all EU member countries to keep budget deficits below 3% of GDP and public debt to below 60% of GDP. EU member countries are liable to financial penalties of up to 0.5% of GDP if they repeatedly fail to respect these limits. The SGP was instituted for the newly formed monetary union, under the pressure of Germany, with the intent of constraining fiscal policy in member countries to insulate the European Central Bank (ECB) from the pressure to inflate or monetize the debt of member countries. However, the enforcement of the SGP has been very lax. For example, in 2003 both Germany and France violated it and sanctions were not imposed. Through the lens of our theory, this corresponds to the case in which the central government reveals its type in the intermediate period. Consistent with our theory, after 2003, the power of the SGP in disciplining fiscal policy was arguably weakened. According to several commentators, this was a major factor in the current European debt crisis in which Greece, Ireland, and Portugal received bailout packages from the European Union and the ECB (the central government), as our theory predicts. Moreover, our theory suggests that even in the period prior to 2003 the debt issued would have been smaller without the SGP.

Arguably, after the bailouts to peripheral member countries, the reputation and credibility of the central European institutions were low. EU member countries and European institutions agreed to impose tough fiscal rules by strengthening the SGP by introducing the so-called "Six-Pack" and "Fiscal Compact", consistent with the prediction of our signaling game. The provisions of the "Six-Pack" were soon violated by Spain and Portugal without any sanction being levied.[2] In 2016 the governor of the Bundesbank, Jens Weidmann, accused the Commission of not enforcing the fiscal rules: "My perception is that the European Commission has basically given up on enforcing the rules of the Stability and Growth Pact."[3]

Another leading example of federal governments with poor fiscal discipline among subnational governments is Brazil. The fiscal behavior of the states and large municipal governments in Brazil was a major source of macroeconomic instability and resulted in subnational debt crises in 1989, 1993, and 1997. "The federal government took a variety of measures to control state borrowing in the 1990s, and at a first glance it would appear to have had access to an impressive array of hierarchical control mechanisms through the constitution, additional federal legislation, and the central bank. Most of these mechanisms have been undermined however, by loopholes or bad incentives that discourage adequate enforcement" (Rodden et al. (2003), page 222). In 1997, the federal government assumed the debts of 25 of the 27 states that were unable to service their debt—an amount equivalent to about 13% of GDP. By September 2001, 84% of state debt was held by the national treasury (see Rodden et al. (2003), page 234). After the bailouts in 1997, the Cardoso administration approved the Fiscal Responsibility Law, which instituted "a rule-based system of decentralized federalism that leaves little room for discretionary policymaking at the subnational level. It has been motivated by the recognition that market control over subnational finances should be replaced, or strengthened, by fiscal rules as well as appropriate legal constraints and sanctions for noncompliance" (Afonso and De Mello (2000)). So, in a manner similar to Europe, the central government in Brazil imposed stringent fiscal rules when its reputation was arguably low.

[2] See https://www.ft.com/content/f66a5c1d-b023-3d0f-ad02-767a9656d4f9

[3] See https://www.ft.com/content/95e7ee7e-ad8e-11e6-ba7d-76378e4fef24.

Related literature Our paper is related to several strands of literature. First, it is related to the literature that studies the free-rider problem in federal governments when the central government cannot commit (e.g., Chari and Kehoe (2007), Chari and Kehoe (2008), Cooper et al. (2008), Aguiar et al. (2015), Chari et al. (2017), and Rodden (2002)). The main result in this literature is that the inability of the central government (or monetary authority) to commit not to transfer ex-post leads to overborrowing ex-ante. In such settings, it is often argued that fiscal rules can improve outcomes by lowering the amount of debt issued (e.g., Beetsma and Uhlig (1999)). Our paper contributes to this literature by analyzing the effects of fiscal rules when the government cannot commit to enforcing them.

Fiscal rules have been studied in several environments as the solution to time inconsistency problems. In the context of delegation, see for instance Athey et al. (2005), Amador et al. (2006), Halac and Yared (2014), and Halac and Yared (2018a). In these papers, fiscal rules are typically thought of as a way to implement the solution to a mechanism design problem. In our paper, we take the set of policy instruments as given and study whether the presence of a fiscal rule allows for better outcomes. Moreover, these papers assume full commitment to the rule, with the exception of Halac and Yared (2018b), who study self-enforcing mechanisms. Under some conditions, the solution to the delegation problem can be implemented with rules that are violated in equilibrium with positive probability. Self-enforcing mechanisms are also the focus of Golosov and Iovino (2016) in the context of an insurance problem.

Hatchondo et al. (2015) and Alfaro and Kanczuk (2016) study fiscal rules in the context of sovereign default. Azzimonti et al. (2016) study the effects of introducing balanced budget rules in a political economy model. All these papers assume full commitment to these rules and do not analyze the enforcement problem, which is the main focus of our paper. Piguillem and Riboni (2018) study the role of fiscal rules as a default option in a legislative bargaining model.

The baseline model uses a reputational setup similar to Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982) with uncertainty about the type of the central government. Cole et al. (1995), Phelan (2006), D'Erasmo (2008), and Amador and Phelan (2018) study environments in which a government with a hidden type interacts with a continuum of private agents. In contrast, in our paper the local governments are strategic and can incentivize the central government to reveal its type via its actions. In addition, we study how varying the costs of maintaining good reputation affects outcomes. In a companion paper, Dovis and Kirpalani (2019a), we study an infinite horizon dynamic game where the local governments cannot commit to repaying their debt, but without fiscal rules, to study the joint dynamics of debt, central government's reputation, and interest rate spreads on local government debt.

Uncertainty about the type of the central government plays a key role in the provision of incentives to local governments. Nosal and Ordoñez (2016) also consider an environment in which uncertainty can mitigate the time inconsistency problem when a central government cannot commit to not bail out banks. The mechanism is very different: here uncertainty about the type of the central government curbs debt issuances by the local governments, while in their paper it is the uncertainty about the state of the economy that restrains the central government from not intervening ex-post.

Layout The rest of the paper is organized as follows. In Section 2 we present some motivating evidence, and in Section 3 we present the model. In Section 4 we analyze the equilibrium without fiscal rules, while in Section 5 we analyze the case with fiscal rules. In Section 6, we show that rules can arise in the equilibrium of a signaling game. Finally, Section 7 concludes the paper.

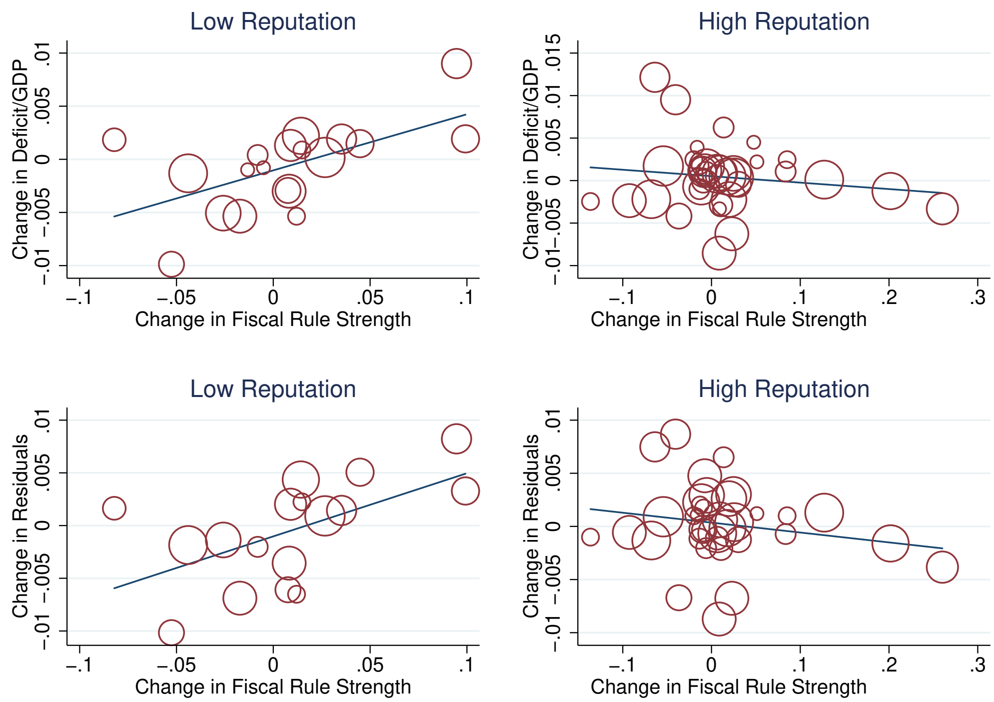
2 Fiscal rules in practice

A large empirical literature studies the ability of fiscal rules to constrain fiscal policy. Heinemann et al. (2018) survey the literature and finds mixed evidence for the efficacy of fiscal rules. Our theory can help rationalize this mixed evidence. Our findings suggest that including a proxy for the central government's reputation is critical when trying to understand the effects of fiscal rules on reducing borrowing. As motivation for our theory, we now provide evidence that accounting for the central government's reputation is important to understanding the effects of fiscal rules from an ex-ante perspective.[4] We find that tighter fiscal rules are associated with more borrowing when the central government's reputation is low, and with less borrowing when its reputation is high.

We consider a sample of European countries and study the changes in subnational primary deficits for European countries as a function of changes in fiscal rule strength for different values of government reputation. While it is challenging to directly measure reputation, we use data on government effectiveness from the World Bank's Worldwide Governance Indicators (WGI) as a proxy. The data on subnational primary deficits and fiscal rule strength are from Kotia and Lledó (2016). They construct an index of subnational fiscal rule strength using data from the European Commission. See the data appendix for more details.

[4]Bergman et al. (2016) undertake a similar exercise but for national rules (rules imposed by the central government on itself). They find that that the effects of fiscal rules on primary balance depend on measures of government efficiency. In particular, they find that debt rules are effective only when government efficiency exceeds some threshold.

Figure 1: **Changes in fiscal rule strength and primary deficits**



Note: The size of the circles corresponds to the average length of the regime across the two consecutive regimes considered. The countries are Austria, Belgium, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Luxembourg, the Netherlands, Poland, Portugal, Slovenia, Spain, Sweden, and the United Kingdom. The countries with observations below the 15th percentile of reputation are Czechia, Estonia, Greece, Italy, Latvia, and Poland. The countries with observations above the 50th percentile of reputation are Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Luxembourg, the Netherlands, Spain, Sweden, and the United Kingdom.

We divide the countries into two groups based on the proxy for reputation. In particular, we designate low-reputation countries to be those that are at or below the 15th percentile, and high-reputation countries to be those that are at or above the 50th percentile.[5] To account for the fact that changes in fiscal rule strength can have lagged effects, we construct the average subnational fiscal deficits in a particular fiscal rule regime (defined as periods in which the index for fiscal rule strength stays constant). Figure 1 displays changes in the mean subnational deficit to GDP ratio against the change in the fiscal rule strength between two consecutive regimes, for the two groups of countries defined above. The top panels plot the average changes in deficits, while the bottom panels plot the average change in residuals after controlling for observables such as the cyclical component of GDP and unemployment (including lags), and they also include country fixed effects.[6]

[5]Our results are unchanged if we lower the low-reputation cutoff to below the 15th percentile or increase the high-reputation cutoff to above the 50th percentile.

[6]In particular, in each period we run the following regression: $\mathtt{deficit}_{it} = \beta_0 + \beta X_{it} + \beta_2 \mathtt{deficit}_{it-1} + f_i + \varepsilon_{it}$, where $\mathtt{deficit}_{it}$ is the primary deficit, X_{it} is a vector of control variables (including lags), f_i is a

The graph suggests that tighter fiscal rules are associated with larger deficits when reputation is low and lower deficits when reputation is high. In the data appendix, we also do this exercise by considering only the contemporaneous effect of a change in fiscal rules and obtain similar results.

In a related paper, Grembi et al. (2016) study the effect of a change in law that relaxed fiscal rules for certain Italian municipalities in 2001 and find that deficits increased. While they focus on the particular case of Italy, we look at the effects of fiscal rule changes for subnational governments across a variety of high- and low-reputation countries. While there are certainly cases in our sample in which fiscal rules seem to be effective for low-reputation countries, the data suggest that on average, fiscal rules are less effective for countries with low reputation.

3 Model

Environment The economy lasts for three periods indexed by $t = 0, 1, 2$.[7] Consider a small open economy consisting of N states or regions indexed by $i \in \{1, 2, ..., N\}$. We partition the regions in two groups: the North, $i \in \mathcal{N} = \{1, ..., N_1\}$, and the South, $i \in \mathcal{S} = \{N_1 + 1, N_1 + 2, ..., N\}$. The representative citizen in region i has preferences over the local public good provision $\{G_{it}\}$ given by

$$U_i = \sum_{t=0}^{2} \beta^t u(G_{it}).$$

We make the following assumptions on the utility function throughout the paper:

Assumption 1. *The period utility function u is strictly increasing, strictly concave, $u \in C^1$, $\lim_{c \to 0} u'(c) = \infty$, and $u(0)$ finite.*

The local public good provision is decided by a benevolent *local government* with local tax revenues $\{Y_{it}\}$. For all $n \in \mathcal{N}$ and $s \in \mathcal{S}$ we let[8]

$$Y_{n0} = Y_0 + \Delta \geqslant Y_0 - \Delta = Y_{s0}, \quad Y_{nt} = Y_{st} = Y \quad \text{for } t = 1, 2$$

with $\Delta \geqslant 0$ so the North is (weakly) richer at time 0 relative to the South. Allowing for heterogeneity in local governments' tax revenue is important only when we study the equilibrium constitution in Section 6. The local governments can borrow from the rest of

country fixed effect, and ε_{it} is the residual from the regression. The figure plots the change in the average residual across two consecutive fiscal rule regimes.

[7] In Section 5.2 we discuss how our results extend to any finite horizon model.

[8] Adding heterogeneity in tax revenues for $t > 0$ leaves the results unchanged.

the world at a rate of $1 + r^*$. Let $q = 1/(1 + r^*)$ be the price of a bond that promises to pay one unit of the consumption good next period.

There is also a *central government*. The central government does not have tax revenues, but it can impose transfers from one region to another subject to a budget constraint

$$\sum_{i=1}^{N} T_{it} \leqslant 0, \tag{1}$$

where T_{it} is the transfer to region i in period t.

Efficient allocation As a benchmark, we consider the efficient allocation for utilitarian Pareto weights in this environment. This allocation solves

$$\max_{\{G_{it}\}} \sum_{i=1}^{N} \frac{1}{N} \sum_{t=0}^{2} \beta^t u(G_{it})$$

subject to the consolidated budget constraint

$$\sum_{t=0}^{2} \sum_{i=1}^{N} q^t [G_{it} - Y_{it}] \leqslant 0. \tag{2}$$

This allocation must satisfy

$$qu'(G_{it}) = \beta u'(G_{it+1}), \tag{3}$$

$u'(G_{it}) = u'(G_{jt})$ for all i, j, t, and the consolidated budget constraint (2) with equality. Thus public good provision is equated across regions in every period, and it is efficiently smoothed over time.

Institutional setup and equilibrium Consider an institutional setup in which the central government is subject to a fiscal constitution. The fiscal constitution contains two clauses. The first clause states that the central government should not make any transfers, i.e., $T_{it} = 0$ for all i, t. We call such a provision the *no-bailout clause*. The second clause requires the local governments to keep their debt issued in period 0 below a cap $\bar{b}$. In the case that $b_{i1} > \bar{b}$, the central government must impose a penalty ψ on the region that violated the rule. We assume that the resources collected from penalties are thrown away.[9] We call this constitutional provision a *fiscal rule*. A fiscal rule is then fully described by $(\bar{b}, \psi)$. To simplify notation, we abstract from a cap on debt issued in period 1 and its

[9]This assumption ensures that the cost of imposing the fiscal rule is nonzero for the central government even if it is known that the central government cannot commit.

associated penalty. All our propositions will extend to the case with a cap on debt issued in period 1.

The central government can be one of two types: a *commitment type*, which follows the prescriptions of the constitution, or an *optimizing type,* which is not bound to follow the prescriptions of the constitution, as it chooses policies sequentially to maximize an equally weighted average of the utility of citizens in all regions:[10]

$$W_t = \sum_{j \geqslant t}^{2} \frac{1}{N} \sum_{i=1}^{N} \beta^t u\left(G_{ij}\right)$$

for $t = 1, 2$. An alternative interpretation of these types is that the commitment type suffers a sufficiently large utility cost for violating the constitution, while the optimizing type does not.

The type of the central government is drawn at the beginning of period 0 and is not known to the local governments. They have a common prior π that the central government is the commitment type. The timing is as follows: At $t = 0$, the local governments choose the local public good provision G_{i0} and debt b_{i1} subject to the budget constraints $G_{i0} \leqslant Y_{i0} + qb_{i1}$.

Period 1 can be divided into two subperiods. In the first subperiod, the central government makes transfers $\{T_{i1}\}$ and decides whether to enforce the penalty if the fiscal rule is violated by a local government. The commitment type will always choose zero transfers and enforce the penalty. After observing the central government's actions, the local governments update their prior about the central government's type. In the second subperiod, the local governments decide the provision of the local public good G_{i1} and new debt issuance b_{i2} subject to the budget constraints

$$G_{i1} + b_{i1} \leqslant Y + T_{i1} + qb_{i2} - \psi \mathbb{I}_{\{b_{i1} > \bar{b} \text{ and central government enforces fiscal rule}\}}.$$

At $t = 2$, the central government chooses transfers $\{T_{i2}\}$. As before, the commitment type will always choose zero transfers following the fiscal constitution. Next, the local governments choose G_{i2} subject to the budget constraint $G_{i2} + b_{i2} \leqslant Y + T_{i2}$.

We now comment on some of the assumptions in our model. First, we assume that the local governments can commit to repaying their debt. This can be motivated by the existence of high default costs, which makes repayment always optimal for the local government. We make this choice to focus on the commitment problem of the central gov-

[10]The redistribution motive generates an incentive for the central government to bail out the local government with higher debt. We would obtain similar results if bailouts were motivated by spillovers, as in Tirole (2015).

ernment.[11]

Second, under the fiscal constitution, the commitment type makes no transfers. Transfers may be valuable from an ex-ante utilitarian perspective in the presence of heterogeneity between regions.[12] Thus, in general, the ex-ante welfare associated with the commitment type need not be larger than that of the optimizing type. This is because, on one hand, the commitment type minimizes the intertemporal distortion generated by the anticipation of future transfers, but on the other hand, the intratemporal distortion due to unequal consumption can be large. In this paper we will restrict attention to cases in which heterogeneity is small and so the value of redistribution is minimal. Thus, ex-ante welfare is higher if the local governments are facing the commitment type.

Third, in our model, in period 0, we allow for some heterogeneity in tax revenues but assume that the central government cannot make transfers. We can relax this assumption by allowing both the commitment and optimizing types to make transfers in period 0. In this case our model will be equivalent to one in which $\Delta = 0$ and thus our results are unchanged.

We now define the states, payoffs, and beliefs at each node of the game tree starting from the last period.

Period 2 The state in the last period is the distribution of debt among the local governments, $b_2 = (b_{i2})_{i \in \{1,2,\ldots,N\}}$. If the central government is the optimizing type, it will choose transfers $T_{i2}(b_2)$ such that the consumption of the local public good is equalized between regions:[13] $T_{i2}(b_2) = b_{i2} - \frac{\sum_{j=1}^{N} b_{j2}}{N}$ so that

$$G_{i2} = Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}.$$

We refer to this situation as *debt mutualization*. The value for the central government is

$$W_2(b_2) = \sum_{i=1}^{N} \frac{1}{N} u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right),$$

[11] Dovis and Kirpalani (2019a) study the interaction between the two commitment problems and the behavior of interest rate spreads.

[12] In general, an optimal transfer scheme that takes into account the benefits of redistribution would prescribe positive transfers that depend on (potentially stochastic) observables, such as tax revenues in our model, and not on inherited debt. Allowing for such transfers would not alter our results, as this case is equivalent to the case in which there is no heterogeneity.

[13] Note that there is no benefit to preserving reputation, since the world ends after period 2.

and the value for a local government is

$$V_{i2}(b_2) = u\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right).$$

Note that in this case only average debt matters and not the entire distribution. If instead the central government is the commitment type, transfers are zero and each region will consume $G_{i2} = Y - b_{i2}$. The value for the local government is then

$$V_{i2}^{c}(b_2) = u(Y - b_{i2}).$$

Period 1 The relevant state variables in the second subperiod of period 1 are the updated posterior about the central government's type, π', and the distribution of total obligations owed by the local governments, $a_1 = \{a_{i1}\}_{i\in N}$. The total obligations for the local governments are debt owed to lenders minus transfers received from the central government plus the penalty if the local governments violated the fiscal rule (if enforced):

$$a_{i1} = b_{i1} - T_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b} \text{ and central government enforces fiscal rule}\}}. \tag{4}$$

Facing this state, the local governments choose G_{i1}, b_{i2} to solve

$$V_{i1}(a_1, \pi') = \max_{G_{i1}, b_{i2}} u(G_{i1}) + \beta\pi V_{i2}^{c}(b_{i2}) + \beta(1-\pi) V_{i2}\left(b_{i2}, \left(b_{j2}(a_1, \pi')\right)_{j\neq i}\right) \tag{5}$$

subject to

$$G_{i1} + a_{i1} \leqslant Y + q b_{i2},$$

taking as given the strategy $b_{j2}(a_1, \pi')$ followed by the other local governments.

For later reference, the equilibrium outcome at this node will be given by $\{\mathbf{b}_{i2}(a_1, \pi')\}_{i=1}^{N}$, which solves for all i

$$q u'(Y - a_{i1} + q b_{i2}) = \beta\pi' u'(Y - b_{i2}) + \beta(1-\pi') \frac{u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right)}{N}. \tag{6}$$

Unless the probability of facing the commitment type is one, the optimality condition (6) differs from the Euler equation (3) that characterizes the efficient allocation. In particular, if $\pi' < 1$, there is overborrowing because each local government internalizes only $1/N$ of the marginal cost of repaying its debt if it anticipates a transfer when the central government is the optimizing type.[14]

[14] Depending on the value of β and q, and the inherited debt, it may be optimal for the local government to save, $b_{i2} \leqslant 0$. When we refer to overborrowing, we also include situations in which the local govern-

We now turn to the central government's decision in the first subperiod. The state variables here are distribution of debt among the local governments, $b_1 = (b_{i1})_{i\in\{1,2,\dots,N\}}$, and the prior on the type of the central government, π. We first describe the law of motion for beliefs of the central government's type. Let σ be the probability that the optimizing type mimics the commitment type and follows the constitution in period 1. The law of motion for beliefs follows Bayes' rule and is given by

$$\pi'(\pi, \zeta; \sigma) = \begin{cases} \frac{\pi}{\pi+(1-\pi)\sigma} & \text{if } \zeta = 1 \\ 0 & \text{if } \zeta = 0 \end{cases}, \tag{7}$$

where $\zeta = 1$ denotes the event that the constitution is enforced and $\zeta = 0$ denotes the events in which either transfers are not positive or the penalty is not enforced. Note that we can combine all events in which $T \neq 0$ or the fiscal rule is not enforced, because they signal that the central government is the optimizing type for sure.

The problem of the optimizing type is to choose transfers $\{T_{i1}\}$ and whether to enforce the penalty to maximize

$$\sum_{i=1}^{N} \frac{1}{N}\left[u\left(Y - a_{i1} + q\mathbf{b}_{i2}\left(a_1, \pi'\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(a_1, \pi'\right)\right)\right]$$

subject to the definition of a_1, (4), the central government's budget constraint, (1), and the law of motion for beliefs (7), where $\zeta = 1$ if there are no transfers, $\{T_{i1}\} = \mathbf{0}$, and the penalty is enforced, while $\zeta = 0$ otherwise. We let $W_1(\pi, b_1)$ denote the value of this program.

Next, we show that the problem of the optimizing type can be transformed into one in which it makes a simple binary decision of whether to mimic the commitment type or not. In the latter case, its type is revealed, $\pi' = 0$, and a form of Ricardian equivalence holds in this environment, which implies that the payoffs are independent of the transfers chosen in period 1.[15]

Lemma 1. *If $\pi' = 0$, the continuation values and public good provisions for the local governments are independent of transfers in period 1: for any a_1, a_1' such that $\sum_i \frac{1}{N} a_{i1} = \sum_i \frac{1}{N} a_{i1}'$, we have that $V_{i1}(a_1, 0) = V_{i1}(a_1', 0)$.*

The proof of this lemma is provided in the appendix. The main idea is that if local governments know they are facing the optimizing central government type, then only

ments save less than the efficient level. Clearly, a sufficient condition for the debt levels to be positive is $\beta \leqslant q$ and $Y_{i0} \leqslant Y$.

[15]Of course, the timing of transfers would matter if there were impediments to perfect capital markets, for example, borrowing constraints.

the local government's consolidated budget constraint matters and thus transferring resources across regions in period 1 is irrelevant.

To see why, let us consider two extreme cases when the local governments are certain that they are facing the optimizing type. In the first, the central government makes transfers to equalize the obligations of the local governments in period 1. In the second, it sets $T_{i1} = 0$ for all i. In the first case, it is easy to see that consumption of the local governments will be equalized in both periods 1 and 2. In the second case, absent transfers, the local governments with inherited debt above average will simply borrow more to keep current consumption at the level of other regions, expecting a transfer in the second period. On the other hand, the local governments with inherited debt below average, absent transfers, will reduce new debt issuances because they anticipate a negative transfer in period 2.

Formally, we can see this by setting $\pi = 0$ in equation (6):

$$qu'\left(Y - a_{i1} + qb_{i2}\right) = \beta \frac{u'\left(Y - \frac{\sum_{j=1}^{N} b_{j2}}{N}\right)}{N}. \tag{8}$$

Since the right side of the Euler equation is the same for all local governments that anticipate debt mutualization for sure, consumption in period 1 will be equalized even though transfers are zero and the distribution of a_{i1} is non-degenerate.[16]

As a result of Lemma 1, we can then drop $\{T_{i1}\}$ as a choice variable and recast the problem as one in which the optimizing type decides whether to enforce the constitution or not and thus reveal its type. The problem of the central government is

$$\begin{aligned} W_1\left(\pi, b_1\right) = \max_{\tilde{\sigma} \in \{0,1\}} \tilde{\sigma} & \left(\sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - a_{i1} + q\mathbf{b}_{i2}\left(a_1, \pi'\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(a_1, \pi'\right)\right) \right] \right) \\ & + (1 - \tilde{\sigma}) \left(\sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(b_1, 0\right)\right) + \beta V_{i2}\left(\mathbf{b}_2\left(b_1, 0\right)\right) \right] \right), \end{aligned} \tag{9}$$

where $\mathbf{b}_{i2}$ is defined in (6), $a_{i1} = b_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}$, and π' is given by (7). Let $\sigma\left(\pi, b_1\right)$ denote the solution to this problem.

[16]Note that when $N = \infty$ and $\pi = 0$, we have an equilibrium existence issue. In particular, since the cost of issuing debt is zero, local governments will have an incentive to issue an unbounded amount of debt. For an individual government, this is feasible because it is measure 0 and so does not affect the aggregate consolidated budget constraint. We could get existence by adding an exogenous cap on debt. For the later results we will focus on the limit as N goes to infinity. In the appendix we show that this limit is well defined.

Period 0 The state in period 0 is the prior on the type of the central government, π (the realization of Y_{i0} is incorporated by indexing the value functions by t and i). Each local government chooses the local public good provision and debt to solve

$$V_{i0}(\pi) = \max_{G_{i0}, b_{i1}} u(G_{i0}) + \beta[\pi + (1-\pi)\sigma(\pi, b_1)] V_{i1}\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}, \pi'\right) \\ + \beta(1-\pi)[1-\sigma(\pi, b_1)] V_{i1}(b_1, 0) \tag{10}$$

subject to the budget constraint, $G_{i0} \leqslant Y_{i0} + qb_{i1}$, the law of motion for beliefs, (7), taking as given the strategies $b_{-i1}(\pi)$ followed by other local governments, and $\sigma(\pi, b_1)$ followed by the central government.

For later reference, we also define the value for the optimizing type central government in period 0,

$$W_0(\pi) = \sum_{i=1}^{N} \frac{1}{N} u(G_{i0}(\pi)) + \beta W_1(\pi, b_1), \tag{11}$$

where $G_{i0}(\pi)$ and $b_1(\pi)$ are the decision rules in (10). The value for the commitment type is

$$W_0^c(\pi) = \sum_{i=1}^{N} \frac{1}{N} u(G_{i0}(\pi)) + \beta\sigma(\pi, b_1(\pi)) W_1^c\left(\pi, b_1(\pi) + \psi \mathbb{I}_{\{b_{i1}(\pi) > \bar{b}\}}\right) \\ + \beta[1-\sigma(\pi, b_1(\pi))] W_1^c\left(1, b_1(\pi) + \psi \mathbb{I}_{\{b_{i1}(\pi) > \bar{b}\}}\right), \tag{12}$$

where

$$W_1^c(\pi, b_1) = \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, \pi')\right) + \beta V_{i2}^c\left(\mathbf{b}_{i2}(b_1, \pi')\right)\right].$$

Note that to describe the value for the commitment type we need to use the strategy of the optimizing type in period 1 because it affects the posterior after observing enforcement. If $\sigma = 1$, there is no separation and the posterior equals π (first line of (12)), and if $\sigma = 0$, there is separation and the posterior equals 1 (second line of (12)).

Equilibrium definition We can now define a Perfect Bayesian Equilibrium for this policy game.

Definition. A Perfect Bayesian Equilibrium is a set of strategies and beliefs for the local governments, $b_{i1}(\pi)$, $\pi'(\pi, b_1, \zeta)$, $b_{i2}(b_1, \pi)$, a strategy for the optimizing type central government, $\sigma(\pi, b_1)$, and associated value functions, such that i) given $b_{-i1}(\pi)$ and

$\sigma(\pi, b_1)$, $b_{i1}(\pi)$ solves (10); ii) given $b_{-i2}(a_1, \pi)$, $b_{i2}(a_1, \pi)$ solves (5); iii) $\pi'(\pi, b_1, \zeta)$ satisfies (7); and iv) $\sigma(\pi, b_1)$ solves (9).

4 Equilibrium without fiscal rules

We start by characterizing the equilibrium when the fiscal constitution contains only a no-bailout clause and no fiscal rules. We show that if either the initial heterogeneity in tax revenues between regions or reputation is low enough, there exists a unique equilibrium outcome in which the central government's type is not revealed in period 1.

Proposition 1 (No revelation of central government type)**.** *Suppose the constitution has no fiscal rules. Then, for* N *large and either* Δ *or* π *sufficiently small, there exists a unique symmetric pure strategy equilibrium in which the type of the central government is not revealed in period 1. Moreover, the debt issuances* $\{b_1, b_2\}$ *satisfy*

$$qu'(Y_{i0} + qb_{i1}) = \beta u'(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, \pi)) + \beta^2 (1-\pi) u'\left(Y - \mathbf{b}_{j2}(b_1, \pi)\right) \sum_{j \neq i}^{N} \frac{1}{N} \frac{\partial \mathbf{b}_{j2}(b_1, \pi)}{\partial b_{i1}} \tag{13}$$

and $b_2 = \mathbf{b}_{i2}(b_1, \pi)$.

The proof of this and other propositions is provided in the appendix. A key step in the proof of this result is to show that the optimizing type wants to follow the constitution and not implement any transfers along the equilibrium path in period 1.[17] To understand this step, let us consider the *reputation benefits* and *inequality costs* associated with making no transfers in period 1. By making no transfers, the central government preserves its reputation. A higher reputation, in turn, promotes fiscal responsibility, because the local governments expect to repay their debt without a transfer from the central government with higher probability. Hence, the reputation benefits are associated with a reduction in the intertemporal distortions of the local governments' Euler equations (6) relative to the efficient one (3). The inequality costs of making no transfers are associated with intratemporal distortions due to the inequality in the provision of the local public good in period 1. This inequality can be reduced by making transfers (or by the revelation that the central government is the commitment type, as shown in Lemma 1).

We now explain why under our assumptions the reputational benefits are higher than inequality costs. First, if regions are homogeneous and have identical period 0 tax revenues, then along the equilibrium path each local government enters period 1 with the

[17] Note that in this setup the only way for the central government to reveal its type is to make transfers.

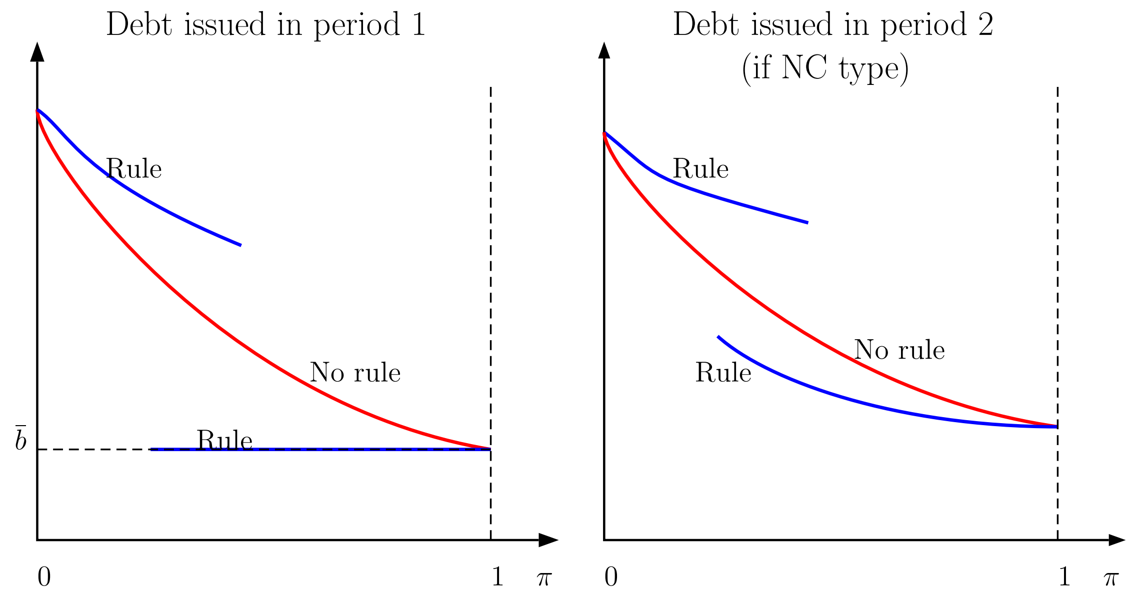
same amount of debt. Thus, the inequality costs are zero, but the reputational benefits are positive. Hence, it is optimal for the optimizing type to make no transfer and maintain its reputation. However, off equilibrium, a local government could potentially increase the debt issued in period 0 to induce the central government to transfer resources to it in period 1. In the proof, we show that such a deviation is not profitable provided that N is sufficiently large.

Second, if all regions are heterogeneous, they will enter period 1 with different levels of debt, which increases the inequality costs of making zero transfers. However, if Δ is sufficiently small, by continuity, it is profitable for the optimizing type to not make any transfers in period 1, since the *inequality* costs are small relative to the reputation benefits for all levels of reputation.

Now suppose that Δ is arbitrary and not necessarily close to zero. We can still guarantee that the optimizing type will not make transfers when its reputation, π, is sufficiently low. To understand this, notice that for π small enough, there is essentially no inequality in the local public good consumption even if the central government makes no transfers in period 1. This is related to Lemma 1. As illustrated by equation (8), since local governments expect debt mutualization with high probability in period 2, as $\pi \to 0$, Southern local governments borrow more to increase their consumption in period 1, while Northern governments borrow less expecting a negative transfer in period 2. This implies that the inequality costs of no transfers in period 1 are second order. However, the reputation benefits from inducing more fiscal discipline are first order, since the Euler equation is distorted relative to the efficient allocation. Hence, it is optimal for the central government to not make any transfers when its reputation is very low, for any $\Delta > 0$. Note that this result relies crucially on the ability of local governments to borrow and lend.

Thus, without fiscal rules, the central government does not reveal its type in period 1. In particular, the local governments' posterior that the central government is the commitment type is equal to its prior. So, when the local governments choose their debt issuance in period 1, they are still uncertain about the type of the central government and about the probability of receiving a transfer in the terminal period. Given these expectations, debt issuances along the equilibrium path are characterized by equation (13) and $b_2 = \mathbf{b}_{i2}(b_1, \pi)$. The first two terms of condition (13) resemble those in a standard intertemporal Euler equation, while the last term on the right hand side captures strategic effects in the debt issuance decision. Each local government understands that its choice of debt issuance in period 0 will affect the debt issuance decisions of the other $N-1$ local governments in period 1, which in turn affects the utility of the local government in period 2 in case of debt mutualization (which happens with probability $1-\pi$). This term vanishes as $N \to \infty$, as shown in Lemma 3 in the appendix.

Figure 2: **Equilibrium outcomes: Debt issued in periods 1 and 2**



5 Equilibrium with fiscal rules

We now consider a constitution with fiscal rules. The main result is that fiscal rules *increase* debt issuances when the central government's reputation is low, but are effective in reducing debt when its reputation is high. Figure 2 offers a preview of our results. The two panels of Figure 2 display the debt issued by a representative local government along the equilibrium path without fiscal rules (blue line) and with fiscal rules (red line) as a function of the prior in period 0 assuming that the central government in power is the optimizing type. For illustration, we set the cap on debt, $\bar{b}$, to be the efficient level of debt which is also the equilibrium level of debt if $\pi = 1$.

When $\pi = 0$, fiscal rules are irrelevant because local governments anticipate that these rules will not be enforced and so the debt issued does not change. When $\pi > 0$ is low enough, fiscal rules are actually detrimental: debt issuances in period 0 are higher with rules. The same is true in period 1 conditional on facing the optimizing type. When π is above a threshold, there exists an equilibrium in which the rules are followed, the central government does not reveal its type in period 1, and total indebtedness is lower than in the case without rules. Hence, fiscal rules may be effective in reducing debt only when the central government's reputation is sufficiently high. But when the central government's reputation is high, the gains from reducing indebtedness are smaller: debt is decreasing in π because the local governments expect that they will not receive a transfer with a high probability. Therefore, fiscal rules are detrimental when the problem of overborrowing is severe, while they are effective only when the gains from enforcement are relatively low. The remainder of this section will establish these results.

5.1 Fiscal rules are irrelevant when type is known

As a useful first step we study the case in which the type of the central government is known. The next proposition shows that if the local governments know that the central government is the optimizing type ($\pi = 0$), then fiscal rules are irrelevant in that the equilibrium outcome is unaffected.

Proposition 2 (Fiscal rules irrelevant if $\pi = 0$)**.** *If $\pi = 0$ the equilibrium outcome is independent of the fiscal rule $(\bar{b}, \psi)$ and it coincides with the one characterized in Proposition 1 for $\pi = 0$.*

If it is known that the central government is the optimizing type, there are no reputational benefits in the interim period of enforcing the fiscal rule because the local governments anticipate a bailout in the last period with probability one. Thus a fiscal rule only imposes costs and so is never enforced. Consequently, the behavior of local governments is unaffected by the presence of a fiscal rule.

If instead the local governments know that the central government is the commitment type ($\pi = 1$), there is no overborrowing. Thus, even though fiscal rules are always enforced, they strictly lower welfare in the case in which they bind and are welfare neutral if they do not bind. Therefore, when the central government's type is known for sure, fiscal rules are either irrelevant (if $\pi = 0$) or weakly welfare reducing (if $\pi = 1$). As a result, in the next subsection we consider the case in which there is uncertainty about the central government's type ($\pi \in (0, 1)$) and show that the presence of fiscal rules alters equilibrium behavior in a meaningful way.

5.2 Fiscal rules promote indiscipline when reputation is low

We now present the first main result of the paper: if the reputation of the central government is low enough, then binding fiscal rules are violated and lead to even more debt accumulation relative to the case with no rules. The key driver for this result is that with binding fiscal rules, the type of the central government is revealed in period 1 because the punishment associated with the fiscal rule enforcement makes it less attractive for the optimizing type to enforce the constitution. This early resolution of uncertainty makes overborrowing more attractive for the local governments.

When the constitution has binding fiscal rules, i.e., $\bar{b}$ is less than the equilibrium outcome without fiscal rules characterized in Proposition 1, we have the following result:

Proposition 3 (Fiscal indiscipline with low reputation)**.** *Suppose the constitution has binding fiscal rules. Then, for N sufficiently large and β and Δ sufficiently small there exists a $\pi_1^* > 0$ such that for all $\pi \in (0, \pi_1^*)$:*

1. *There exists a unique symmetric pure strategy equilibrium in which the fiscal rule is violated in period 0 and not enforced by the optimizing type in period 1 so that the type of the central government is revealed in period 1.*
2. *The debt issued in this equilibrium is larger than if the constitution did not contain fiscal rules.*

We first show that under these assumptions, for π close to zero there exists a unique equilibrium in which fiscal rules are violated. The key step to establish this result is to show that in period 1, if all local governments violate the fiscal rule, the optimizing type central government prefers to not enforce the punishment ψ and reveal its type ($\pi' = 0$ thereafter) than to enforce the punishment and enjoy the reputation gain.[18] The trade-off faced by the government is similar to the one described in the case without rules: by enforcing the rules, the central government enjoys reputation benefits, but it suffers inequality costs and the additional costs associated with imposing the penalty. This extra cost makes the total costs of enforcing the constitution not second order anymore. In the appendix, we show that the government prefers to not enforce the constitution if β is below a threshold $\bar{\beta}$. Intuitively, a lower β implies a lower weight on the dynamic reputational benefits of enforcing the fiscal rule relative to the static costs of imposing the penalty and the inequality costs.

To show that this equilibrium is unique, we show that an equilibrium in which the local governments respect the fiscal rule cannot exist for N large and π small enough. Suppose for contradiction that all local governments respect the fiscal rule. An individual government has an incentive to deviate and violate the fiscal rule. By borrowing more the deviating local government enjoys higher consumption in period 0, while its continuation value is relatively unchanged even if the optimizing type enforces the fiscal rule when N is large and π is small enough. To see the latter, notice that when π is small, continuation values depend only on average obligations for local governments, $(b + \psi)/N + \bar{b}(N-1)/N$. When N is large enough, average obligations are unchanged. Thus, it is not optimal to respect the fiscal rule.

We next show that when the central government's reputation is low, binding fiscal rules promote *more* fiscal indiscipline than a constitution without fiscal rules. That is, the debt levels in this equilibrium are higher than in the equilibrium without fiscal rules. To

[18]The posterior jumps to one as the local governments expect only the commitment type to enforce the fiscal rule.

see this, first note that the debt issuances in period 0 with fiscal rules must satisfy

$$
\begin{aligned}
qu'\left(Y_{i0}+qb_{i1}\right) = {} & \beta\pi u'\left(Y-\left(b_1+\psi\right)+q\mathbf{b}_{i2}\left(b_1+\psi,1\right)\right) \\
& +\beta\left(1-\pi\right)u'\left(Y-b_1+q\mathbf{b}_{i2}\left(b_1,0\right)\right) \\
& +\beta^2\left(1-\pi\right)u'\left(Y-\frac{\sum_{j=1}^{N}\mathbf{b}_{j2}\left(b_1,0\right)}{N}\right)\sum_{j\neq i}^{N}\frac{1}{N}\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}},
\end{aligned}
\tag{14}
$$

where we have used the result that when the local governments choose their new debt levels in period 1, they know with certainty the type of the central government they are facing. This shows up in equation (14), where the right side of the Euler equation is contingent on the type of the central government: with probability π, the local governments observe enforcement of the fiscal rule, learn that they are facing the commitment type, and the new debt issued is $\mathbf{b}_{i2}\left(b_1+\psi,1\right)$; with probability $1-\pi$, they observe no enforcement of the fiscal rule, learn that they are facing the optimizing type, and the new debt issued is $\mathbf{b}_{i2}\left(b_1,0\right)$. This is in contrast to equation (13) that characterizes debt issuances in period 0 without fiscal rules where there is no revelation about the government's type in period 1 and so the local governments cannot make debt issuances contingent on the central government's type. This difference is crucial to the result that there is overborrowing with fiscal rules.

To compare the debt levels in period 0 with and without binding fiscal rules, it is useful to rewrite conditions (13) and (14) to make them more comparable. For simplicity, we consider the limiting case as the number of regions goes to infinity so that there are no strategic interactions among local governments. In the appendix, we show that combining (14) and (13) with the optimality condition for debt issued in period 1, (6), and taking the limit as N goes to infinity, we obtain

$$
u'\left(Y+qb_1\right)q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right) \tag{15}
$$

for the case without fiscal rules and

$$
u'\left(Y+qb_1\right)q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right), \tag{16}
$$

for the case with fiscal rules. These two equations implicitly define the debt issued in period 0 under the two cases. The left side is the marginal benefit of issuing one unit of debt in period 0. The right side is the marginal cost of servicing the debt issued in period 0. Since $N\to\infty$, the local governments only internalize the marginal cost of debt in the state of the world in which the commitment type is in power, which occurs with probability π. If the optimizing type is in power, the continuation value for the local government depends only on average obligations. Thus, as $N \to \infty$ the individual contribution to the average goes to zero and so the marginal cost of servicing the debt internalized by the local government goes to zero if the optimizing type is in power.

We can now compare the right hand side of (15) and (16). Since u is concave, if $\mathbf{b}_{i2}(b_1, \pi) > \mathbf{b}_{i2}(b_1 + \psi Y, 1)$, the expected marginal cost of servicing the debt with rules is smaller than without rules and so local governments optimally choose to issue more debt in period 0 when there are rules. Let's now compare $\mathbf{b}_{i2}(b_1, \pi)$ and $\mathbf{b}_{i2}(b_1 + \psi Y, 1)$. First, since the commitment type imposes a penalty when there are rules, total obligations for the local governments in period 1 are higher with rules than without, $b_1 + \psi Y > b_1$, in the state of the world when the commitment type is in power. This channel increases the debt issued in period 1 with rules when facing the commitment type. There is, however, another force. With rules, the local governments learn the type of the central government in period 1. Thus, conditional on facing the commitment type, the prior jumps to 1. Without rules, instead, there is no learning and the prior stays constant at π. This channel decreases the debt issued in period 1 with rules. For π small enough, we can show that the second channel dominates and $\mathbf{b}_{i2}(b_1, \pi) > \mathbf{b}_{i2}(b_1 + \psi Y, 1)$. This is because as $\pi \to 0$, $\mathbf{b}_{i2}(b_1, \pi) \to Y$ but $\mathbf{b}_{i2}(b_1 + \psi Y, 1)$ is bounded away from Y (see Lemma 2 in the appendix for details). Hence, the local governments will issue more debt in period 0 because of the lower expected marginal cost when there are fiscal rules.

Now consider debt issuances in period 1 if the central government is the optimizing type. In this case, debt issued in period 1 is higher with rules than without for two reasons: first, the inherited debt is larger; second, the local governments face no uncertainty about the type of the central government and therefore internalize only $1/N$ of the cost of issuing debt, while without fiscal rules they internalize the full cost with probability π and $1/N$ of the cost with probability $1 - \pi$.

This result can help rationalize recent events in the eurozone. In particular, our model suggests that debt issued should have been larger after the 2003 SGP violation by France and Germany. Moreover, our model with heterogeneous local governments predicts that debt issued should have increased more for countries with high fiscal needs (the South in our model). This is because, after learning the type of the government, even though the marginal cost of issuing debt decreases for all local governments, Northern regions expect to make transfers to Southern regions in the final period, which induces them to issue less debt than the South. This result is consistent with the standard narrative of the buildup to the eurozone crisis of 2009–2011.

Finally, suppose that the local governments face the commitment type in period 1. Since they do not receive transfers in period 2, they issue less debt than if they faced the optimizing type because they internalize the full cost of servicing the debt with probability 1. However, relative to the case without rules, we cannot sign the change in debt issued in period 1. This is because both reputation and inherited debt are higher, which has opposite effects on debt issuances.

Discussion We now further clarify the features that are critical for fiscal rules to actually incentivize overborrowing if there is uncertainty about the type of the central government and its reputation is low. The main idea is that adding a fiscal rule increases the cost of maintaining reputation in the interim period: not only can the central government not make any transfers but it must also punish the local governments – in particular the ones with higher marginal utility of public consumption good. Hence, with fiscal rules, we have earlier revelation of the type, which – as described above – reduces the cost of servicing the debt and thus more debt issuances in period 0. Therefore, well-intended policies that ask governments to commit to actions period by period in order to alleviate a commitment problem down the road can backfire. There are two features that are critical for this result. First, the punishment associated with enforcing the fiscal rule cannot be delayed. The central government must make a choice between not enforcing the penalty and maintaining its reputation. In contrast, bailout transfers can be delayed. This is because, following the logic of Lemma 1, absent transfers in period 1, agents can use financial markets to endogenously equalize consumption when reputation is sufficiently low. As a result, the central government can achieve the benefits associated with bailout transfers without losing its reputation. Second, the punishment associated with enforcing fiscal rules entails a reduction in aggregate resources for all reputation levels, while enforcing the no-bailout clause has only a distributional cost, which is arbitrarily small for π close to zero.

It is worth noting that there is nothing special about fiscal rules per se. We would obtain similar results if the constitution contained any other action that is costly for the optimizing type to enforce. An example is requiring the optimizing type to throw a fraction of output in the ocean independent of debt issued, or making long speeches. Our modeling choice is motivated by the policy relevance of fiscal rules and the widespread belief that they are a solution to the problem of overborrowing.

Our mechanism relies on the feature that observing enforcement (or not) of the fiscal rule provides information about the probability of obtaining a bailout in the terminal period. This follows from the assumption that the commitment type enforces both the fiscal rule and the no-bailout clause. Thus, observing no enforcement of the fiscal rule in period 1 implies that the local governments are facing the optimizing type and there will be bailout transfers in the terminal period. To see why this is critical, consider the case in which the type of the central government can change over time. As long as the type is sufficiently persistent, our result survives. If instead the type is independent over time, then there are no longer reputational benefits in the interim period, because observing enforcement carries no information about the bailout probability in period 2. Thus, the optimizing type never enforces the constitution in period 1 with or without fiscal rules.[19]

Finally, our characterization can extend to any finite horizon model. To see this, consider a finite period environment in which $\Delta = 0$ (no heterogeneity) and the constitution contains only a no-bailout clause. Clearly, there always exists an equilibrium with enforcement, since on path all local governments will have the same public good provision and debt. Now consider the introduction of the fiscal rule. An almost identical argument to Proposition 3 implies that if β is low enough, there exists an equilibrium with early revelation of the central government's type. Notice that the $\bar{\beta}$ threshold required to get revelation in period 1 in a $T > 3$ period model will be lower than the corresponding threshold in a three-period model since the dynamic gains from enforcement are higher. However, for higher levels of the discount factor, the government will reveal its type in later periods but still before T.

5.3 Fiscal rules promote discipline when reputation is high

We now show that when reputation levels are sufficiently high, fiscal rules are effective, since there exists a unique equilibrium in which these rules are followed. To do this, we require the following assumption:

Assumption 2. $u\left(Y_{i0} + q\bar{b}\right) + \beta V_{i1}\left(\bar{b}, \pi\right) > \max_{b_i > \bar{b}} u\left(Y_{i0} + qb_i\right) + \beta V_{i1}\left(b_i + \psi, \bar{b}_{-i}, \pi\right)$

This assumption requires that the punishment must be large enough or the cap on debt must not be too restrictive so that for all π, if a local government believes that the fiscal rule will be enforced for sure the following period, it will prefer to respect the rule if all other local governments are doing so.

Proposition 4 (Fiscal discipline with high reputation)**.** *Suppose the constitution has binding fiscal rules. Under Assumption 2, for N sufficiently large and Δ sufficiently small there exists a threshold $0 < \pi_2^* < 1$ such that for $\pi \geqslant \pi_2^*$ there exists a unique symmetric pure strategy equilibrium in which the fiscal rule is enforced in period 1. Since fiscal rules are binding, the equilibrium level of debt issued is smaller than if the constitution did not contain fiscal rules. Moreover, if $N = \infty$ an equilibrium with enforcement exists for all π.*

It follows directly from Assumption 2 that if $\pi = 1$, then the local governments will not deviate from the fiscal rule if all other governments are respecting the rule as well.

[19] We reach a similar conclusion if fiscal rules and bailouts are enforced by two different government agencies. Each can be either a commitment or an optimizing type. If these are sufficiently correlated, our result survives. However, if the types of the two agencies are independent, enforcement of the fiscal rule conveys no information about the likelihood of the bailout clause being enforced in the terminal period.

It follows by continuity that when π is sufficiently close but less than 1, the local governments will also continue to respect the rule. Moreover, this is the unique equilibrium. Clearly, since fiscal rules are binding, the equilibrium level of debt issued will be lower than if there were no fiscal rules present.

Proposition 4 also states that if $N = \infty$, an equilibrium in which binding fiscal rules are respected always exists.[20] To see this, notice that when $N = \infty$, since each local government is infinitesimal, there are no costs for the central government to enforce the penalty for a violation of the fiscal rule by an *individual* local government that has measure zero. Hence, if one local government expects that the other local governments will respect the fiscal rule, it is optimal for it to respect the rule as well; so, there always is an equilibrium in which fiscal rules can curb indebtedness and the local governments internalize the free-rider problem.

However, this is not the unique equilibrium in the limiting case. For π low enough, there is always an equilibrium where the rule is ignored by all the local governments and not enforced. In particular, if a local government expects other local governments to violate the rule, it will find it optimal to violate the rule as well because it anticipates no enforcement ex-post. This type of multiplicity is similar to the one in Farhi and Tirole (2012) and Chari and Kehoe (2016).

This result may help rationalize why when two large countries such as Germany and France violated the SGP in 2003, no sanctions were imposed by the European institutions. More generally, Eyraud et al. (2017) provide suggestive evidence that compliance with the SGP rules has been lower among the largest countries. However, it may be possible for institutions such as the IMF to enforce penalties on a small country to preserve their reputation.

Proposition 4 also implies that the existence of an equilibrium with enforcement for low π is fragile: as long as local governments are not measure zero, for π low enough there exists a unique equilibrium where the rule is ignored by all the local governments and not enforced, as shown in Proposition 3. Hence we should expect fiscal rules to be respected – and thus effective in curbing debt – if the initial reputation is high enough. Indeed, for high enough levels of reputation, this is the unique equilibrium. For intermediate levels of reputation, it might be possible that either an equilibrium (in pure strategies) does not exist or multiple equilibria exist.

[20] This implies that the threshold π_1^* defined in Proposition 3 goes to zero as $N \to \infty$. However, for any $N < \infty$, we have that $\pi_1^* > 0$.

6 Equilibrium fiscal constitution

In this section, we ask why fiscal rules might be adopted when reputation is low even though their adoption might lead to higher debt than if there were no rules. We study the *equilibrium fiscal constitution*, that is, the fiscal constitution that arises as the outcome of a signaling game between the two types of government in period 0. We show that if the commitment type is sufficiently patient, it is optimal for it to announce fiscal rules that will promote early resolution of uncertainty in period 1, and the optimizing type will choose to mimic the strategy of the commitment type in period 0 and also announce such rules (and violate them in period 1).

This result rationalizes why we often observe central governments with low reputation setting up tough fiscal rules. Examples include the case of the eurozone after the European debt crisis and the bailouts in Greece, Portugal, Ireland, and Spain with the institution of the "Six-Pack", and the case of Brazil after the bailouts in 1997 and the Fiscal Responsibility Law approved by the Cardoso administration. In both cases, the reputation of the central government was low because of the recent bailouts to local governments.

Formally, we add an additional stage to the policy game described in Section 3. In the initial stage, given the prior π held by local governments, the central government chooses to write a fiscal constitution. A fiscal constitution has a no-bailout clause and a fiscal rule $\alpha = \left(\psi, \bar{b}\right)$, with $\psi \leqslant \bar{\psi}$. After observing the chosen fiscal constitution, the local governments update their prior about the type of the central government, and the subsequent equilibrium outcome is an equilibrium outcome of the policy game described in the previous sections.

In this section, we deviate from the reputation literature following Kreps et al. (1982) by allowing the commitment type to choose an action, the fiscal constitution in the initial stage. In this sense, the commitment type is no longer purely behavioral.[21] We interpret the commitment type as a player that suffers a large utility cost from deviating from past promises. In the game below, the commitment type announces a constitution while internalizing that it will not violate it in the future due to this utility cost. In contrast, the optimizing type suffers no exogenous disutility from deviating from past promises. It does, however, suffer an endogenous cost due to the loss in reputation and trades this off with the static benefits of deviating from the constitution each period.

Definition (Equilibrium fiscal constitution.)**.** An equilibrium fiscal constitution is an equilibrium outcome of the signaling game between the two types of central government. Given a prior π, an equilibrium of the signaling game is a strategy for the commitment

[21] Sanktjohanser (2018) follows a similar approach in a bargaining game.

type central government $\alpha^c = \left(\bar{b}^c, \psi^c\right)$, a strategy for the optimizing type $\alpha^{nc} = \left(\bar{b}^{nc}, \psi^{nc}\right)$, and beliefs π_0' such that i) beliefs evolve according to

$$\pi_0'(\alpha, \pi) = \begin{cases} \pi & \text{if } \alpha = \alpha^{nc} = \alpha^c \\ 0 & \text{if } \alpha = \alpha^{nc} \neq \alpha^c \\ 1 & \text{if } \alpha = \alpha^c \neq \alpha^{nc} \\ 0 & \text{if } \alpha \notin \{\alpha^c, \alpha^{nc}\} \end{cases}, \tag{17}$$

ii) given α^{nc}, the strategy for the commitment type α^c is optimal, in that for all α

$$W_0^c\left(\pi_0'(\alpha^c, \pi); \alpha^c\right) \geqslant W_0^c\left(\pi_0'(\alpha, \pi); \alpha\right),$$

where W_0^c is defined in (12) ; iii) given α^c, the strategy α^{nc} for the optimizing type is optimal, in that for all α

$$W_0\left(\pi_0'(\alpha^{nc}, \pi); \alpha^{nc}\right) \geqslant W_0\left(\pi_0'(\alpha, \pi); \alpha^{nc}\right),$$

where W_0 is defined in (11).

Note that in W_0^c and W_0 we highlight the dependence on α of this value function that was left implicit in the definitions (12) and (11). We will do this for all equilibrium objects from now onward.

We can characterize the equilibrium of this game by considering the problem for the commitment type given the prior π. To do so, it is useful to define the value for the optimizing type of enforcement if the inherited debt is b_1 and the posterior after enforcing equals π',

$$\begin{aligned} \omega^e\left(b_1, \pi'; \alpha\right) = \sum_{i=1}^{N} \frac{1}{N} \Big[& u\left(Y - \left(b_{i1} + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}\right) + q\mathbf{b}_{i2}\left(\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}\right), \pi'\right)\right) \\ & + \beta W_2\left(\mathbf{b}_2\left(b_1 + \psi \mathbb{I}_{\{b_1 > \bar{b}\}}\right)\right)\Big], \end{aligned} \tag{18}$$

and the value of non-enforcement,

$$\omega^{ne}(b_1) = \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) + \beta W_2\left(\mathbf{b}_2(b_1, 0)\right)\right]. \tag{19}$$

Clearly, if there is no enforcement, then the posterior jumps to zero.

The problem for the commitment type in period 0 is

$$W_0^c = \max\left\{W_0^{c,sep}, W_0^{c,pool}\right\},$$

where $W_0^{c,sep}$ is the value for the commitment type if it chooses a fiscal rule that ensures separation in period 1, and $W_0^{c,pool}$ is the value for the commitment type if the fiscal constitution it chooses is such that the optimizing type enforces the rule in period 1. The value for $W_0^{c,sep}$ is given by

$$W_0^{c,sep} = \max_{\alpha} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}} - b_{i1}^{er}(\pi,\alpha) + q \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}}, 1\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{er} > \bar{b}}, 1\right)\right) \end{array} \right]$$

subject to

$$\omega^{ne}\left(b_{i1}^{er}(\pi,\alpha)\right) \geqslant \omega^{e}\left(b_{i1}^{er}(\pi,\alpha), 1; \alpha\right),$$

where $b_{i1}^{er}(\pi,\alpha)$ is the debt issued in period 0 when the local governments expect to learn the central government's type in period 1 (early revelation) defined in (14) given $\alpha = (\bar{b}, \psi)$. The last constraint requires that the punishment induces the optimizing type to prefer not to enforce the penalty and lose its reputation rather than enforce and have its reputation jump to 1.

The value for $W_0^{c,pool}$ is given by

$$W_0^{c,pool} = \max_{\alpha} \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i}^{lr}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1}^{lr}(\pi,\alpha) + q \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1}^{lr} > \bar{b}}, \pi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_{i1}^{lr}(\pi,\psi) + \psi \mathbb{I}_{b_{i1}^{lr} > \bar{b}}, \pi\right)\right) \end{array} \right]$$

subject to

$$\omega^{e}\left(b_{i1}^{lr}(\pi,\alpha), \pi; \alpha\right) \geqslant \omega^{ne}\left(b_{i1}^{lr}(\pi,\alpha), 0\right),$$

where $b_{i1}^{lr}(\pi,\alpha)$ is the debt issued in period 0 when the local governments do not expect to learn the central government's type in period 1 (late revelation) defined in (13) given $\alpha = (\bar{b}, \psi)$. The last constraint requires that the optimizing type prefers to mimic the commitment type in period 1.

In setting up these problems we assumed that it was optimal for the optimizing type to mimic the strategy of the commitment type in period 0. In the next proposition we prove that this is the case.

We assume that the commitment type can choose only between two levels of penalties, $\psi \in \{0, \bar{\psi}\}$, and fix the cap on debt to some binding $\bar{b}$.[22] Furthermore, assume that the discount factor for the central government is less than $\bar{\beta}$ in Proposition 3 so that in period 1 it is not optimal for the optimizing type to enforce the penalty if the fiscal constitution has $\psi = \bar{\psi}$. Moreover, we assume that the initial reputation is sufficiently close to zero and N is sufficiently large. Under these assumptions, the next proposition shows that if the commitment type central government is sufficiently patient, then there exists a unique equilibrium fiscal constitution that has fiscal rules. Moreover, the optimizing type central government prefers to mimic the strategy of the commitment type in period 0 and chooses a constitution with fiscal rules despite knowing that it will not enforce the constitution in period 1. If, instead, the commitment type central government is not patient enough, the equilibrium constitution has no fiscal rules:

Proposition 5. *If N is sufficiently large, and Δ and π are sufficiently small, then there exist two cutoffs $\underline{\beta} \leqslant \bar{\beta}$ such that:*

1. *For $\beta \in [\underline{\beta}, \bar{\beta}]$, there exists a unique fiscal constitution with fiscal rules ($\psi = \bar{\psi}$) that are violated by the local governments, and there is early resolution of uncertainty in period 1. If $\Delta > 0$, then $\bar{\beta} > \underline{\beta}$.*

2. *For $\beta < \underline{\beta}$, there exists a unique fiscal constitution with no fiscal rules ($\psi = 0$).*

When the central government's reputation is sufficiently close to zero, for intermediate values of the discount factor β, fiscal rules arise in equilibrium even if they are going to be violated by the local governments. The commitment type chooses to do so to reveal its type in period 1. From its perspective, this has benefits, because in period 1 the reputation of the central government will jump from almost zero to one, promoting fiscal discipline going forward. In particular, the local governments' decision will satisfy the Euler equation, so it is efficient from period 1 onward.[23] But this also has costs. As we have shown in Proposition 3, instituting fiscal rules promotes overborrowing and fiscal indiscipline in period 0. Moreover, the commitment type will suffer the costs of punishment when the local governments violate the rule. When β is above the cutoff $\underline{\beta}$ defined in the Appendix, the benefits outweigh the costs. Conditional on the commitment type announcing a fiscal rule, for π close to zero, the optimizing type always prefers to mimic the strategy of the commitment type in period 0. Intuitively, the reputation cost of not mimicking the strategy of the commitment type is of first order, while the benefit of equalizing consumption is of second order when π is close to zero, using a logic similar to the one in Lemma 1.

[22]Note that it must be that $\overline{\psi} < (1 + r^*) Y - \bar{b}$. Otherwise, it is not resource feasible to impose the punishment when the rule is violated.

[23]Of course, the commitment type central government would like to redistribute resources from the North to the South, but in our setup it has no instruments to do so.

Finally, for the optimizing type to not enforce the rule in period 1, we need to impose an upper bound on the discount factor. In Proposition 3, we define such an upper bound $\bar{\beta}$. In the appendix we show that $\underline{\beta} < \bar{\beta}$ when countries are heterogeneous in period 0, i.e., $\Delta > 0$. This is because there is an additional benefit to the optimizing type of not enforcing the constitution, namely, that it can equalize consumption across regions. This implies that it requires a larger discount factor in order to prefer enforcement. As $\Delta \to 0$, this additional benefit shrinks to zero and so $\overline{\beta} \to \underline{\beta}$. If, instead, β is below $\underline{\beta}$, the commitment type prefers not to institute the rule, and clearly the optimizing type chooses to do the same.

7 Conclusion

Fiscal rules are often thought to be useful in federal states when the central government cannot commit to no-bailout clauses. In this paper, we ask if this is indeed the case when the central government also cannot commit to imposing these rules. We show that in a reputation model in which the local governments are uncertain whether the central government can commit, equilibrium outcomes with rules attain higher debt levels than those without rules when the central government's reputation is low. Our results shed light on the multitude of examples throughout history when fiscal rules were instituted but not enforced. Our analysis of the equilibrium constitution suggests that stringent fiscal rules can arise when the central government's reputation is low even though they increase local governments' debt when such governments are already overborrowing.

In this paper, we assumed that the central government is benevolent and maximizes the utility of the local governments. Another possibility is to study institutional settings where local governments' representatives vote to impose sanctions on the local governments that violate the rule. This is left for future research.

Our results extend to other settings as well. For example, they can be applied to the design of financial regulation. If a government cannot commit not to bailout financial institutions, they may take on more risk than would be optimal from society's perspective, because the price of their liabilities does not respond to the level of riskiness. Kareken and Wallace (1978) and, more recently, Chari and Kehoe (2016) suggest regulating the level of ex-ante risk-taking to solve this problem. However, if it is ex-post costly to fine financial institutions that do not respect the regulation, the forces we emphasize in our model apply to this setup and thus the ex-ante regulation of risk can potentially result in more risk-taking. Another application is to study policies aimed at the prohibition of wage indexation in high-inflation countries.

Finally, in our analysis we take as given the policy instruments available to the central government, such as the form of the fiscal rules. In Dovis and Kirpalani (2019b) we study the optimal design of these rules from an ex-ante perspective taking into account reputation-building incentives.

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Online Appendix

A Omitted Proofs

A.1 Proof of Lemma 1

Let $\pi' = 0$ and consider two vectors a_1 and a_1' that differ only in transfers. We know that debt issuances $\{b_{i2}\}$ are the unique solution[24] to the system

$$qu'\left(Y - a_{i1} + qb_{i2}\left(a_1, 0\right)\right) = \frac{\beta}{N} u'\left(Y - \frac{\sum_{j=1}^{N} b_{i2}\left(a_1, 0\right)}{N}\right) \quad \text{for all } i$$

We can then see that if $\{b_{i2}\left(a_1, 0\right)\}$ solves the system given a_1 then

$$b_{i2}\left(a_1', 0\right) = b_{i2}\left(a_1, 0\right) - \frac{1}{q}\left(T_{i1} + T_{i1}'\right) \quad \text{for all } i$$

solves the system given a_1' and leaves public good provisions in period 1 and 2 unchanged. Hence the value is unaffected by transfers in period 1 when $\pi = 0$. A straightforward extension of these arguments implies that this result holds more generally for any two sequences a_1 and a_1' such that $\sum \frac{1}{N} a_{i1} = \sum \frac{1}{N} a_{i1}'$. Q.E.D.

A.2 Preliminary results for proof of Proposition 1-4

For the following proofs it is useful to define the value of enforcing for the optimizing type if the posterior equals π'

$$\omega^e\left(b_1, \pi'\right) = \sum_{i=1}^{N} \frac{1}{N}\left[u\left(Y - b_{i1} - \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}} + q\mathbf{b}_{i2}\left(b_1, \pi'\right)\right) + \beta W_2\left(\mathbf{b}_2\left(b_1 + \psi \mathbb{I}_{\{b_{i1} > \bar{b}\}}, \pi'\right)\right)\right] \tag{20}$$

[24] To see this note that the solution satisfies $b_{i2} - b_{12} = 1/q\Delta a_i$ where $a_i = [a_{i1} - a_{11}]$ and

$$qu'\left(Y - a_{11} + qb_{12}\right) = \frac{\beta}{N} u'\left(Y - \frac{\sum_{j=1}^{N}\left(b_{12} + \frac{\Delta a_j}{q}\right)}{N}\right)$$

Since u' is strictly increasing there is a unique b_{12} that solves the equation above.

and the the value of non-enforcement

$$\omega^{ne}(b_1) = \sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) + \beta W_2\left(\mathbf{b}_2(b_1, 0)\right) \right] \tag{21}$$

To prove Proposition 3 we use the following two lemmas:

Lemma 2. *As* $N \to \infty$*, the continuation equilibrium in period 1 given inherited debt* b_1 *and posterior* π *is such that:*

1. *If* $\pi > 0$, $\lim_{N\to\infty} \mathbf{b}_{i2}(b_1, \pi) \to b_{i2} < Y$;
2. *If* $\pi = 0$, $\lim_{N\to\infty} \sum_i \frac{\mathbf{b}_{i2}(b_1, 0)}{N} \to Y$ *and*

$$\lim_{N\to\infty} \frac{1}{N} u'\left(Y - \sum_i \frac{\mathbf{b}_{i2}(b_1, 0)}{N}\right) = \frac{q}{\beta} u'\left((1+q)Y - \sum_i \frac{b_{i1}}{N}\right) > 0.$$

Moreover, $\lim_{N\to\infty} V_{i1}(b_1, 0) = u(Y(1+q) - b_1) + \beta u(0)$.

Proof. We know from the text, equation (6), that along a symmetric equilibrium outcome, it must be that

$$qu'\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, \pi)\right) = \beta\pi u'\left(Y - \mathbf{b}_{i2}(b_1, \pi)\right) + \beta(1-\pi)\frac{1}{N} u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b_1, \pi)}{N}\right)$$

whenever $\sum_i \mathbf{b}_{i2}(b_1, \pi)/N < Y$.

Consider part 1 and let $\pi > 0$. Clearly, for each finite N, $\mathbf{b}_{i2} < Y$ due to the Inada condition and so the Euler equation above holds. Suppose by way of contradiction that $\mathbf{b}_{i2}(b_1, \pi) \to Y$ as $N \to \infty$. Then the right side goes to ∞ while the left side goes to $qu'(Y - b_1 + qY)$ which is finite. This is a contradiction.

Consider part 2 and let $\pi = 0$. For all finite N, because of the Inada condition, it must be that $\sum_i \mathbf{b}_{i2}/N < Y$ and so the following Euler equation must hold:

$$qu'\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) = \beta\frac{1}{N} u'\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b_1, 0)}{N}\right) \tag{22}$$

Suppose by way of contradiction that $\frac{\sum \mathbf{b}_{i2}(b_1, 0)}{N} \to B_2 < Y$. Then the left side converges to a positive number, $qu'(Y(1+q) - b_1)$, while the right side converges to zero. This is a contradiction. In particular, since the right side is identical for all i,

$$Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0) \to (1+q)Y - \frac{\sum_i b_{i1}}{N}$$

Therefore, it must be that

$$\lim_{N\to\infty}\frac{1}{N}u'\left(Y-\frac{\sum_i \mathbf{b}_{i2}(b_1,0)}{N}\right)=\frac{q}{\beta}u'(Y-b_{i1}+qY).$$

It follows that, if the posterior equals zero, the value of a continuation equilibrium is

$$u(Y(1+q)-b_1)+\beta u(0).$$

□

Lemma 3. *Suppose* $\pi = 0$. *Then for all* i,

$$\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial \mathbf{b}_{j2}(b_1,0)}{\partial b_{i1}}=-\frac{1}{q}$$

Proof. Step 1:$\lim_{N\to\infty} G_{i1}(\pi=0)=0$.
We know from Lemma 1 that if $\pi = 0$, the equilibrium allocations are identical whether or not there are transfers by the central government in period 1. In the case in which there are transfers $T_{i1} = b_{i1} - \sum_i \frac{1}{N} b_{i1}$, the first order conditions for b_{i1} and b_{i2} respectively are

$$u'(G_{i0})q=\beta\left[\frac{1}{N}u'(G_{i1})+\frac{\beta}{N}u'(G_{i2})\sum_{j\neq i}\frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}}\right] \tag{23}$$

$$u'(G_{i1})q=\frac{\beta}{N}u'(G_{i2}) \tag{24}$$

where the superscript tr denotes outcomes with transfers. Therefore

$$\sum_{j\neq i}\frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}}=\frac{u'(G_{i0})\frac{qN}{\beta}-u'(G_{i1})}{\beta u'(G_{i2})}=\frac{u'(G_{i0})\frac{qN}{\beta}-u'(G_{i1})}{Nu'(G_{i1})q}=\frac{\frac{u'(G_{i0})}{u'(G_{i1})}\frac{q}{\beta}-\frac{1}{N}}{q} \tag{25}$$

We know from Lemma 2 that $\lim_{N\to\infty} G_{i2}(0) = 0$. Now suppose by way of contradiction that $\lim_{N\to\infty} G_{i1}(0) > 0$. Then from (25) we see that

$$\lim_{N\to\infty}\sum_{j\neq i}\frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}}=\frac{u'(G_{i0})}{\beta u'(G_{i1})}>0$$

Next, we can combine (23) and (24) to obtain

$$u'(G_{i0})q=\beta\frac{u'(G_{i1})}{N}\left[1+q\sum_{j\neq i}\frac{\partial b^{tr}_{j2}}{\partial b^{tr}_{i1}}\right] \tag{26}$$

If $G_{i1} > 0$ then the term $\frac{u'(G_{i1})}{N}$ converges to zero as $N \to \infty$, while the argument above establishes that the limit of $q \sum_{j \neq i} \frac{\partial b_{j2}^{tr}}{\partial b_{i1}^{tr}}$ is finite. Therefore, as $N \to \infty$, the right side of (26) converges to zero while the left side is finite. This is a contradiction. Since the equilibrium outcome with transfers in period 1 and the one without are equivalent when $\pi = 0$ then $\lim_{N \to \infty} G_{i1} (\pi = 0) = 0$.

Step 2: $\lim_{N \to \infty} \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}(b_1, 0)}{\partial b_{i1}} = -\frac{1}{q}$.

Now consider the case in which there are no transfers in period 1. In this case the first order conditions imply that

$$\sum_{j \neq i} \frac{\partial \mathbf{b}_{j2} (b_1, 0, N)}{\partial b_{i1}} = \frac{u' (G_{i0}) \frac{qN}{\beta} - u' (G_{i1}) N}{\beta u' (G_{i2})} = N \left(\frac{u' (G_{i0}) \frac{q}{\beta} - u' (G_{i1})}{N u' (G_{i1}) q} \right) = \frac{\frac{u'(G_{i0})}{u'(G_{i1})} \frac{q}{\beta} - 1}{q}$$

Since we just established that $\lim_{N \to \infty} G_{i1} = 0$ and by the Inada condition $\lim_{G \to 0} u' (G) = \infty$, taking limits on both sides of the above equation yields the result since $\lim_{N \to \infty} \frac{u'(G_{i0})}{u'(G_{i1})} \frac{q}{\beta} = 0$. □

Lemma 4. *If $b_1 = \{b_{i1}\}$ is degenerate in that $b_{i1} = b_{j1}$ for all i, j then $\lim_{N \to \infty} \frac{1}{N} \frac{\partial \mathbf{b}_{i2}(b_1, 0)}{\partial \pi} < \infty$.*

Proof. By applying the implicit function theorem to (6) we obtain

$$\frac{\partial \mathbf{b}_{i2} (b_1, 0)}{\partial \pi} = \frac{\beta \frac{N-1}{N} u' (Y - \mathbf{b}_{i2} (b_1, 0))}{\left[q^2 u'' (Y - b_1 + q \mathbf{b}_{i2} (b_1, 0)) + \frac{\beta}{N} u'' (Y - \mathbf{b}_{i2} (b_1, \pi)) \right]}$$

so

$$\frac{1}{N} \frac{\partial \mathbf{b}_{i2} (b_1, 0)}{\partial \pi} = \left(1 - \frac{1}{N} \right) \frac{\beta \frac{1}{N} u' (Y - \mathbf{b}_{i2} (b_1, 0))}{\left[q^2 u'' (Y - b_1 + q \mathbf{b}_{i2} (b_1, 0)) + \frac{\beta}{N} u'' (Y - \mathbf{b}_{i2} (b_1, \pi)) \right]}$$

As $N \to \infty$, the above converges to

$$\frac{\beta \frac{1}{N} \sum_{j \neq i} u' (0)}{\left[q^2 u'' (Y - b_1 + qY) + \beta \frac{u''(0)}{N} \right]}$$

We know from Lemma 2 that the numerator $\beta \frac{1}{N} \sum_{j \neq i} u' (G_{i2})$ converges to a finite number. If $\beta \frac{u''(G_{i2})}{N}$ converges to a finite constant or zero then the denominator converges to a finite number and this the fraction converges to a finite number. If it converges to ∞ then the above converges to zero. In both cases, as $N \to \infty$, $\frac{1}{N} \frac{\partial \mathbf{b}_{i2}(b_1, 0)}{\partial \pi}$ converges to a finite number. □

Lemma 5. *i) For all π, $\omega^e (\cdot, \pi)$ is continuous and differentiable.*

ii) For all b, for π small enough, $\omega^e (b, \cdot)$ is increasing in π.

Proof. For convenience, rewrite (20):

$$\omega^e(b,\pi) = \sum_i \frac{1}{N}\left[u\left(Y - b_i + q\mathbf{b}_{i2}(b,\pi)\right) + \beta u\left(Y - \frac{\sum_i \mathbf{b}_{i2}(b,\pi)}{N}\right)\right]$$

Part i). The fact that ω_1^e is continuous and differentiable in b follows from continuity and differentiability of u and $\mathbf{b_2}$.

Part ii). Consider the derivative with respect to π:

$$\frac{\partial \omega^e(b,\pi)}{\partial \pi} = \sum_i \frac{1}{N}\left[qu'(G_{i1})\frac{\partial \mathbf{b}_{i2}}{\partial \pi} - \beta \frac{u'(G_{i2})}{N}\frac{\partial \sum_i \mathbf{b}_{i2}(b,\pi)}{\partial \pi}\right]$$

While we cannot sign this term in general, at $\pi = 0$, since $qu'(G_{i1}) = \frac{\beta}{N}u'(G_{i2})$, we have

$$\frac{\partial \omega^e(b,\pi)}{\partial \pi} = -\beta \sum_i \frac{u'(G_{i2})}{N^2}\sum_{j\neq i}\frac{\partial \mathbf{b}_{-i2}}{\partial \pi} = -\beta \frac{u'(G_{i2})}{N}\frac{(N-1)}{N}\frac{\partial \mathbf{B}_2}{\partial \pi}$$

where $\mathbf{B}_2 \equiv \sum_i \mathbf{b}_{i2}$ and so if $\frac{\partial \mathbf{B}_2}{\partial \pi} < 0$, then $\frac{\partial \omega^e(b,\pi)}{\partial \pi} > 0$.

We now show that $\mathbf{B}_2(b_1,\pi)$ is decreasing in π for π small enough. Recall the first order condition in period 1, (6), rewritten here for convenience:

$$qu'(Y - b_{i1} + q\mathbf{b}_{i2}) = \beta\pi u'(Y - \mathbf{b}_{i2}) + \beta(1-\pi)\frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{i2}}{N}\right)}{N} \tag{27}$$

First define

$$\Delta MU_i \equiv \beta\left[u'(Y - b_{i2}) - \frac{u'\left(Y - \frac{\sum_j \mathbf{b}_{j2}}{N}\right)}{N}\right]$$

$$A_i \equiv \left[-\beta\pi u''(G_{i2}^c) - \frac{\beta(1-\pi)}{2N}u''(G_{i2}) - qu''(G_{i1})\right] > 0$$
$$a_i \equiv \frac{2N}{\beta(1-\pi)}A_i > 0$$

where $G_{i2}^c = Y - \mathbf{b}_{i2}$. Using the implicit function theorem we have

$$A_i d\mathbf{b}_{i2} = \frac{\beta(1-\pi)}{2N}u''(G_{i2})\,d\mathbf{b}_{-i2} - \Delta MU_i d\pi$$

and so

$$\frac{\partial \mathbf{b}_{i2}}{\partial \pi} = \frac{1}{1 - \frac{u''(G_{i2})}{a_i}\frac{u''(G_{-i2})}{a_{-i}}} \frac{-\Delta MU_i}{A_i} + \frac{u''(G_{i2})}{a_i}\frac{-\Delta MU_{-i}}{A_i}.$$

Next, we have

$$\begin{aligned} \frac{\partial \mathbf{B}_2}{\partial \pi} &= \frac{1}{1 - \frac{u''(G_{s2})}{a_s}\frac{u''(G_{n2})}{a_n}} \frac{-\Delta MU_s}{A_s} + \frac{u''(G_{s2})}{a_s}\frac{-\Delta MU_n}{A_s} \\ &+ \frac{1}{1 - \frac{u''(G_{s2})}{a_s}\frac{u''(G_{n2})}{a_n}} \frac{-\Delta MU_n}{A_n} + \frac{u''(G_{n2})}{a_n}\frac{-\Delta MU_s}{A_n} \end{aligned}$$

At $\pi = 0$,

$$\begin{aligned} A_i &= \left[-\frac{\beta}{4} u''(G_{i2}) - q u''(G_{i1}) \right] = A > 0 \\ a_i &= \frac{4}{\beta} A_i = a > 0 \end{aligned}$$

Therefore evaluating $\frac{\partial \mathbf{B}_2}{\partial \pi}$ at $\pi = 0$, we obtain

$$\frac{d\mathbf{B}_2}{d\pi} = \left[-\frac{1}{1 - \frac{u''(G_{s2})}{a}\frac{u''(G_{s2})}{a}} - \frac{u''(G_{n2})}{a} \right] \frac{1}{A} [\Delta MU_s + \Delta MU_n] \tag{28}$$

We know that

$$\frac{1}{1 - \frac{u''(G_{s2})}{a}\frac{u''(G_{n2})}{a}} > 1$$

and

$$\frac{u''(G_{n2})}{a} = \frac{u''(G_{n2})}{\left[-u''(G_{n2}) - q\frac{4}{\beta}u''(G_{n1}) \right]} > -1$$

Therefore

$$-\frac{1}{1 - \frac{u''(G_{s2})}{a}\frac{u''(G_{n2})}{a}} - \frac{u''(G_{n2})}{a} < -1 + 1 = 0$$

Next, notice that

$$\Delta MU_s + \Delta MU_n = \beta \left[u'(Y - \mathbf{b}_{s2}) + u'(Y - \mathbf{b}_{n2}) - u'\left(Y - \frac{\mathbf{b}_{s2} + \mathbf{b}_{n2}}{2} \right) \right]$$

Clearly, if $\Delta = 0$ then $\Delta MU_s + \Delta MU_n = \beta u'(Y - \mathbf{b}_{s2}) > 0$. Thus, by continuity, $\Delta MU_s + \Delta MU_n > 0$ if Δ is small enough.[25] Therefore, for π close to zero, $\frac{\partial \mathbf{B}_2}{\partial \pi} \leqslant 0$ because all three

[25] One can prove the same result for arbitrary Δ if $u''' > 0$.

terms in (28) are positive. □

A.3 Proof of Proposition 1

Assume first that the local governments expect that the central government will not make any transfers in period 1 and will mutualize debt in period 2 with probability $1-\pi$. We will denote the proposed equilibrium outcome with a superscript "no-rules." The optimality condition of problem (10) and the envelope condition from problem (5) imply that debt issuance in period 0 satisfies (13) and the debt issuance in period 1 is $b_2^{\text{no-rules}} = \mathbf{b}_{i2}\left(b_1^{\text{no-rules}}, \pi\right)$.

We now study the incentives for the central government to implement positive transfers in period 1 on-path. First we show that it is optimal not to make transfers if Δ small enough. Fix some $\pi > 0$. Clearly, for $\Delta = 0$, the central government strictly prefers to not transfer due the reputational benefits because the inherited debt distribution is degenerate. By continuity, for Δ small but positive, it will also strictly prefer to implement zero transfers and enforce the constitution.

Next we show that it is optimal not make transfers if π is small enough. Fix some $\Delta > 0$. We now show that even though the central government faces a non-degenerate distribution of debt $\left\{b_{i1}^{\text{no-rules}}\right\}$ in period 1, it does not have incentives to implement positive transfers if π is small enough. Define the difference between the value of enforcement if $\pi' = \pi$ and not for a central government that inherits debts $b_1^{\text{no-rules}}(\pi) = \left\{b_{i1}^{\text{no-rules}}\right\}$ as

$$\mathcal{W}(\pi) \equiv \omega^e\left(b_1^{\text{no-rules}}(\pi), \pi\right) - \omega^{ne}\left(b_1^{\text{no-rules}}(\pi)\right)$$

where since there are no fiscal rules we set $\psi = 0$ in the definition of ω^e in (20). Note that for an equilibrium with enforcement to exist, it must be that $\mathcal{W}(\pi) \geqslant 0$. Since the utility and policy functions are continuous in π, $\mathcal{W}$ is continuous in π. Moreover $\mathcal{W}(0) = 0$ so it is enough to show that $\mathcal{W}'(0) > 0$. Differentiating $\mathcal{W}$ we obtain:

$$\mathcal{W}'(\pi) = \sum_i \left(\left[\frac{\partial\omega^e\left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial b_{i1}} - \frac{\partial\omega^{ne}\left(b_1^{\text{no-rules}}(\pi)\right)}{\partial b_{i1}}\right]\frac{\partial b_{i1}^{\text{no-rules}}(\pi)}{\partial \pi}\right) + \frac{\partial\omega^e\left(b_1^{\text{no-rules}}(\pi), \pi\right)}{\partial \pi}$$

Evaluating the expression above at $\pi = 0$, using that $\omega^{ne}(\cdot) = \omega^e(\cdot, \pi = 0)$ when $\psi = 0$ and so $\partial\omega^e\left(b_1^{\text{no-rules}}(0), 0\right)/\partial b_{1i} = \partial\omega^{ne}\left(b_1^{\text{no-rules}}(0)\right)/\partial b_{1i}$, we obtain

$$\mathcal{W}'(0) = \frac{\partial\omega^e\left(b_1^{\text{no-rules}}(0), 0\right)}{\partial \pi} > 0$$

as desired. That ω^e is increasing in π for π close to zero is established in Lemma 5 part ii).

We are left to show that an individual government has no incentives to increase its debt and force the central government to make a transfer. Suppose local government i chooses $b_{i1} > b_1^{\text{no-rules}}$ to induce the central government to make a transfer to region i in period 1 with some positive probability. The value for the best deviation for such local government is:

$$V_i^{dev} = \max_{b_{i1}} u\left(Y_{i0} + qb_{i1}\right) + \beta\left[\pi + (1-\pi)\,\sigma\left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right] V_{i1}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi'\right)$$
$$+ \beta(1-\pi)\left[1 - \sigma\left(\pi, b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right] V_{i1}\left(b_{i1}, b_{-i1}^{\text{no-rules}}, 0\right)$$

subject to

$$\omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) \leqslant \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right) \tag{29}$$

Let V_i be the value along the conjectured equilibrium and $\Delta V_i = V_i - V_i^{dev}$. At $\pi = 0$ Note that by construction, $b_1^{\text{no-rules}}$ solves (10) or

$$V_i = \max_{b_{i1}} u\left(Y + qb_{i1}\right) +$$
$$+ \beta\pi\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right)\right]$$
$$+ \beta(1-\pi)\left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right) + \beta u\left(Y - \frac{\sum_j \mathbf{b}_{j2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)}{N}\right)\right]$$

Note that for $\pi = 0$, $\Delta V_i = 0$. Now suppose that $\pi > 0$. Notice that as N gets large, b_{i1} needs to increase in order to induce the central government to make a transfer. In particular, for any finite b_{i1}, as $N \to \infty$ then, eventually, $\omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) > \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right)$. This is because

$$\lim_{N\to\infty}\left(\omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right) - \omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right)\right)$$
$$= u\left(Y - b_1^{\text{no-rules}} + q\mathbf{b}_{-i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), \pi\right)\right)$$
$$- \left[u\left(Y - b_1^{\text{no-rules}} + q\mathbf{b}_{i2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), 0\right)\right) + \beta u\left(Y - \mathbf{b}_{j2}\left(\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right), 0\right)\right)\right]$$
$$> 0$$

As a result, a necessary condition for $\omega^{ne}\left(b_{i1}, b_{-i1}^{\text{no-rules}}\right) \geqslant \omega^e\left(b_{i1}, b_{-i1}^{\text{no-rules}}, \pi\right)$ as $N \to \infty$ is that $b_{i1} \to \infty$ which violates feasibility when facing the commitment type. For each π there exists $N(\pi)$ such that for $N > N(\pi)$, the deviation is infeasible. And so for $N > \max_\pi N(\pi)$, the constructed outcome is an equilibrium outcome.

We are left to show that such an equilibrium is unique (among symmetric pure strategy equilibria). First, fix some $\Delta > 0$. Suppose there exists an interval $(0, \pi_1)$ such that for all $\pi \in (0, \pi_1)$, there exists an equilibrium in which the optimizing type implements positive transfers with strictly positive probability. Then, it must be that $\mathcal{W}(\pi) \leqslant 0$. However, this contradicts our earlier argument that $\mathcal{W}(\pi) > 0$ for π sufficiently close to zero. As a result, an equilibrium in which $\sigma > 0$ cannot exist for π sufficiently small.

Next, fix some $\pi > 0$. We know that for $\Delta = 0$, in any symmetric equilibrium, $\mathcal{W}(\pi) > 0$. Therefore, by continuity this inequality will continue to hold for Δ sufficiently small by positive. As a result, an equilibrium in which $\sigma > 0$ cannot exist for Δ sufficiently small. Q.E.D.

A.4 Proof of Proposition 3

We first show that under our assumptions, there exists a unique equilibrium with no enforcement if π is sufficiently small.

To this end consider first the problem a local government i that expects that i) other local governments are going to violate the fiscal rule, ii) the optimizing type central government is not going to enforce the fiscal rule punishment in period 1. Consequently, local government i expects to learn the type of the central government in period 1. We will denote the proposed equilibrium outcome with a superscript "rules." The problem for the local government at time 0 is then:

$$\Omega(\pi) = \max_{b_{i1}} u(Y_{i0} + qb_{i1}) + \beta\pi V_{i1}\left(\left(b_{i1} + \psi, b_{-i1}^{\text{rules}} + \psi\right), 1\right) + \beta(1-\pi) V_{i1}\left(\left(b_{i1}, b_{-i1}^{\text{rules}}\right), 0\right)$$

where $b_{-i1}^{\text{rules}} > \bar{b}$ is the debt chosen by the other local governments and b_{i1}^{rules} is the solution to the problem above and $b_1^{\text{rules}} = \left(b_{i1}^{\text{rules}}, b_{-i1}^{\text{rules}}\right)$. The optimality condition is:

$$qu'\left(Y_{i0} - qb_{i1}^{\text{rules}}\right) = \beta\pi \frac{\partial V_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)}{\partial b_{i1}} - \beta(1-\pi)\frac{\partial V_{i1}\left(b_1^{\text{rules}}, 0\right)}{\partial b_{i1}}$$

and using the envelope conditions for $V_{i1}\left(b_1^{\text{rules}} + \psi, 1\right)$ and $V_{i1}\left(b_1^{\text{rules}}, 0\right)$ we obtain

$$\begin{aligned} qu'\left(Y_{i0} + qb_{i1}^{\text{rules}}\right) = {} & \beta\pi u'\left(Y - \left(b_{i1}^{\text{rules}} + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{\text{rules}} + \psi, 1\right)\right) \\ & + \beta(1-\pi) u'\left(Y - b_{i1}^{\text{rules}} + q\mathbf{b}_{i2}\left(b_1^{\text{rules}}, 0\right)\right) \\ & + \beta^2(1-\pi) u'\left(Y - \frac{\sum_{j=1}^N \mathbf{b}_{j2}\left(b_1^{\text{rules}}, 0\right)}{N}\right) \sum_{j=1, j\neq i}^{N} \frac{1}{N} \frac{\partial \mathbf{b}_{j2}\left(b_1^{\text{rules}}, 0\right)}{\partial b_{i1}}, \end{aligned} \tag{30}$$

which is equation (14) in the text. Note that for Δ small enough, $b_{i1}^{\text{rules}} > \bar{b}$ for all i.

We now show that for N large enough and π small enough no individual local government has an incentive to deviate from b_{i1}^{rules} and choose $b_{i1} = \bar{b}$ to attain value

$$\bar{\Omega}(\pi) = u\left(Y_{i0} + q\bar{b}\right) + \beta\left[\pi + (1-\pi)\,\sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right)\right] V_{i1}\left(\bar{b}, b_{-i1}^{rules}, \pi'\right)$$
$$+ \beta(1-\pi)\left[1 - \sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right)\right] V_{i1}\left(\bar{b}, b_{-i1}^{rules}, 0\right)$$

First notice that as $N \to \infty$,

$$\omega^{e}\left(\bar{b}, b_{-i1}^{rules}(\pi), 1\right) - \omega^{ne}\left(\bar{b}, b_{-i1}^{rules}(\pi)\right) \to \omega^{e}\left(b_{1}^{rules}(\pi), 1\right) - \omega^{ne}\left(b_{1}^{rules}(\pi)\right)$$

This is because as $N \to \infty$, the value for the central government is independent of the debt issued by an individual local government. Further

$$\omega^{e}\left(b_{1}^{rules}(\pi), 1\right) - \omega^{ne}\left(b_{1}^{rules}(\pi)\right) < 0$$

since we are constructing an equilibrium in which the central government finds it optimal not to enforce. Therefore there exists $\tilde{N}_1$ such that for $N \geqslant \tilde{N}_1$, $\sigma\left(\pi, \bar{b}, b_{-i1}^{rules}\right) = 0$. Next, we have that

$$\Omega(\pi) - \bar{\Omega}(\pi) = \left[u\left(Y_{i0} + qb_{i1}^{rules}(\pi)\right) - u\left(Y_{i0} + q\bar{b}\right)\right]$$
$$+ \beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi) + \psi, b_{-i1}^{rules}(\pi) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi) + \psi\right), 1\right)\right]$$
$$+ \beta(1-\pi)\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi), b_{-i1}^{rules}(\pi)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi)\right), 0\right)\right]$$

Clearly, since $b_{i1}^{rules}(\pi) > \bar{b}$ we know that

$$\left[u\left(Y_{i0} + qb_{i1}^{rules}(\pi)\right) - u\left(Y_{i0} + q\bar{b}\right)\right] > 0,$$
$$\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi) + \psi, b_{-i1}^{rules}(\pi) + \psi\right), 1\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi) + \psi\right), 1\right)\right] < 0.$$

Notice that as $N \to \infty$, $\left[V_{i1}\left(\left(b_{i1}^{rules}, b_{-i1}^{rules}\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}\right), 0\right)\right] \to 0$. Let $\tilde{N}_2^{*}$ be the threshold, such that for $N \geqslant \tilde{N}_2^{*}$,

$$\left[u\left(Y_{i0} + qb_1\right) - u\left(Y_{i0} + q\bar{b}\right)\right] + \beta\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi), b_{-i1}^{rules}(\pi)\right), 0\right) - V_{i1}\left(\left(\bar{b}, b_{-i1}^{rules}(\pi)\right), 0\right)\right] > 0$$

for all π. Notice that

$$\begin{aligned}
\Omega(\pi)-\bar{\Omega}(\pi) &= \left[u\left(Y_{i0}+qb_{i1}^{rules}(\pi)\right)-u\left(Y_{i0}+q\bar{b}\right)\right] \\
&\quad +\beta(1-\pi)\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi),b_{-i1}^{rules}(\pi)\right),0\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{rules}(\pi)\right),0\right)\right] \\
&\quad +\beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi)+\psi,b_{-i1}^{rules}(\pi)+\psi\right),1\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{rules}(\pi)+\psi\right),1\right)\right] \\
&\quad \geqslant \left[u\left(Y_{i0}+qb_{i1}^{rules}(\pi)\right)-u\left(Y_{i0}+q\bar{b}\right)\right] \\
&\quad +\beta\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi),b_{-i1}^{rules}(\pi)\right),0\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{rules}(\pi)\right),0\right)\right] \\
&\quad +\beta\pi\left[V_{i1}\left(\left(b_{i1}^{rules}(\pi)+\psi,b_{-i1}^{rules}(\pi)+\psi\right),1\right)-V_{i1}\left(\left(\bar{b},b_{-i1}^{rules}(\pi)+\psi\right),1\right)\right]
\end{aligned}$$

Since for $N \geqslant \tilde{N}_2$ the first two terms are positive, there exists a $\tilde{\pi}_1$ such that for $\pi \leqslant \tilde{\pi}_1$, $\Omega(\pi)-\bar{\Omega}(\pi)>0$, and thus a local government has no incentives to satisfy the rule in the conjectured equilibrium.

The next step in establishing that the conjectured equilibrium exists is to show that the optimizing type central government when faced with debt $b_1 = b_1^{rules}$ for all i prefers to not enforce the punishment ψ and reveal its type ($\pi' = 0$ thereafter) than enforce the punishment and have the posterior jump to one (as the local governments expect only the commitment type to enforce the fiscal rule). That is, it must be that

$$\omega^e\left(b_1^{rules}(\pi)+\psi,1\right) \leqslant \omega^{ne}\left(b_1^{rules}(\pi)\right)$$

which is true if π and β is sufficiently small. In particular, this is true for $\beta \leqslant \bar{\beta}(\pi,N)$ where $\bar{\beta}(\pi,N) \equiv$

$$\frac{\sum_{i=1}^{N}\frac{1}{N}\left[u\left(Y-b_{i1}^{rules}(\pi)+q\mathbf{b}_{i2}\left(b_1^{rules}(\pi),0\right)\right)-u\left(Y-\left(b_{i1}^{rules}(\pi)+\psi\right)+q\mathbf{b}_{i2}\left(b_1^{rules}(\pi)+\psi,1\right)\right)\right]}{u\left(Y-\frac{\sum\mathbf{b}_{i2}\left(b_1^{rules}(\pi)+\psi,1\right)}{N}\right)-u\left(Y-\frac{\sum\mathbf{b}_{i2}\left(b_{i1}^{rules}(\pi),0\right)}{N}\right)}.$$

The right side of the expression above implicitly defines the maximal discount factor under which it is optimal not to enforce. Therefore, if $\beta < \bar{\beta}(\pi,N)$, $\omega^e\left(b_1^{rules}(\pi)+\psi,1\right) \leqslant \omega^{ne}\left(b_1^{rules}(\pi)\right)$. Therefore, we have shown that under our assumptions an equilibrium in which fiscal rules are violated and not enforced exists.

Next, we show that an equilibrium with enforcement cannot exist for π small. For this to be an equilibrium, it must be that if all other regions are following the rule, no single region has an incentive to deviate and violate it. The value of such a deviation is given by

$$V_i^{dev}(\pi) = \max_{b_{i1} > \bar{b}} u(Y_{i0} + q b_{i1}) + \beta\left[\pi + (1-\pi)\sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right)\right] V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, \pi'\right)$$
$$+ \beta(1-\pi)\left[1 - \sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right)\right] V_{i1}\left(b_{i1}, \bar{b}_{-i}, 0\right)$$

First, notice that because the reputational benefit shrinks to zero as π goes to zero,

$$\lim_{\pi \to 0}\left(\omega^e\left(b_{i1}^{rules}, \bar{b}, \pi\right) - \omega^{ne}\left(b_{i1}^{rules}, \bar{b}\right)\right) < 0$$

so that $\lim_{\pi\to 0} \sigma\left(\pi, b_{i1}, \bar{b}_{-i}\right) = \sigma_0 < 1$. But then

$$\lim_{\pi\to 0} V_i^{dev}(\pi) = u(Y_{i0} + q b_{i1}) + \beta\sigma_0 V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right) + \beta\left[1 - \sigma_0\right] V_{i1}\left(b_{i1}, \bar{b}_{-i}, 0\right)$$

where we used that $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, \pi'\right) = V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right)$ since

$$\lim_{\pi\to 0} \pi' = \lim_{\pi \to 0} \frac{\pi}{\pi + (1-\pi)\sigma} = 0.$$

Next, recall from Lemma 1, that the value $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right)$ depends on the average level of debt $\frac{1}{N}(b_{i1} + \psi) + \frac{(N-1)}{N}\bar{b}$. Therefore, as $N \to \infty$, $V_{i1}\left(b_{i1} + \psi, \bar{b}_{-i}, 0\right) \to V_{i1}\left(\bar{b}, 0\right)$ which implies that value of punishment for the deviating local government shrinks to zero. Therefore, this deviation is strictly profitable. And so there exists some $\tilde{N}_3$ such that for $N \geqslant \tilde{N}_3$ there exists $\tilde{\pi}_2$ such that for $\pi \leqslant \tilde{\pi}_2$, this deviation is strictly profitable.

We can then conclude that if $N \geqslant \max\left\{\tilde{N}_1, \tilde{N}_2, \tilde{N}_3\right\}$ and $\pi \leqslant \min\{\tilde{\pi}_1, \tilde{\pi}_2\}$ there exists a unique equilibrium with non-enforcement.

To compare the debt levels in period 0 with and without binding fiscal rules, it is useful to rewrite conditions (13) and (14) to make them more comparable. For the case without fiscal rules, we can combine (13) with (6) to obtain a condition that characterizes the debt issuance in period 0:

$$u'(Y + q b_1) q = \frac{\beta^2 \pi}{q} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) + \frac{\beta^2(1-\pi)}{qN} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) \tag{31}$$
$$+ \frac{\beta^2(1-\pi)}{N} u'(Y - \mathbf{b}_{i2}(b_1, \pi)) \sum_{j \neq i} \frac{\partial \mathbf{b}_{j2}(b_1, \pi)}{\partial b_{i1}}.$$

For the case with fiscal rules, we can combine (14) with (6) to obtain

$$u'\left(Y+qb_1\right)q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi,1\right)\right)+\frac{\beta^2\left(1-\pi\right)}{qN}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right) \quad (32)$$
$$\frac{\beta^2\left(1-\pi\right)}{N}u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}}.$$

Taking the limit as N goes to infinity for $\pi>0$ but small, since $\lim_{N\to\infty}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right)<\infty$, as shown in Lemma 2, condition (31) reduces to

$$u'\left(Y+qb_1\right)q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right) \quad (33)$$

as the sum of the second and third terms on the right side converge to zero. Condition (32) instead reduces to

$$u'\left(Y+qb_1\right)q=\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right), \quad (34)$$

because, as shown in Lemma 2 and 3,

$$\lim_{N\to\infty}\frac{\beta u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{N}\frac{1}{q}=-\lim_{N\to\infty}\frac{u'\left(Y-\mathbf{b}_{i2}\left(b_1,0\right)\right)}{N}\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}\left(b_1,0\right)}{\partial b_{i1}}.$$

We can then compare the right hand side of (33) and (34). We know that for π small enough, $\mathbf{b}_{i2}\left(b_1,\pi\right)>\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)$, because as $\pi\to 0$, $\mathbf{b}_{i2}\left(b_1,\pi\right)\to Y$ but $\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)$ is bounded away from Y (see Lemma 2 for details). This observation along with the concavity of u implies that

$$\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1+\psi Y,1\right)\right)<\frac{\beta^2\pi}{q}u'\left(Y-\mathbf{b}_{i2}\left(b_1,\pi\right)\right).$$

Therefore, from (33) and (34) we see that the expected marginal cost of issuing debt in period 0 is lower when there is early revelation of the central government's type. Hence, local governments will issue more debt in period 0 because of the lower expected marginal cost. Q.E.D.

A.5 Proof of Proposition 4

We first show that for π close to 1, there exists an equilibrium with enforcement. At $\pi=1$, the value for a local government of respecting the fiscal rule is $u\left(Y_{i0}+q\bar{b}\right)+\beta V_{i1}\left(\bar{b},\pi\right)$ while the value of violating is $\max_{b_i>\bar{b}}u\left(Y_{i0}+qb_i\right)+\beta V_{i1}\left(b_i+\psi,\bar{b}_{-i},\pi\right)$. That the latter is larger than the former follows directly from Assumption 2. By continuity, there exists some $\tilde{\pi}_1 < 1$ such that for $\pi \geqslant \tilde{\pi}_1$, the inequality continues to hold.

Next, we want show that there is an interval around $\pi = 1$ for which the enforcement equilibrium is unique. For an equilibrium with non-enforcement $\left(b_1 = b_1^{rules}\right)$ to exist, it must be that it is optimal for a local government to violate the fiscal rule rather than obeying the rule when all other local governments are violating the rule. That is, $\Omega(\pi) \geqslant \bar{\Omega}(\pi)$ where these objects were defined in the proof of Proposition 3. Note that

$$\begin{aligned}
\bar{\Omega}(1) &= u\left(Y_{i0} + q\bar{b}\right) + \beta V_{i1}\left(\bar{b}, 1\right) \\
&> \max_{b_i > \bar{b}} u\left(Y_{i0} + q b_i\right) + \beta V_{i1}\left(b_i + \psi, \bar{b}_{-i}, 1\right) \\
&= \max_{b_i > \bar{b}} u\left(Y_{i0} + q b_i\right) + \beta V_{i1}\left(b_i + \psi, b_{-1}^{rules} + \psi, 1\right) \\
&= \Omega(1)
\end{aligned}$$

where the first line is the definition of $\bar{\Omega}(1)$, the second line follows from Assumption 2, the third line follows from the fact that the debt holdings of other regions are irrelevant if the central government is the commitment type for sure ($\pi = 1$), and the last line is the definition of $\Omega(1)$. Hence, by continuity, if π is sufficiently close to 1, $\bar{\Omega}(\pi) > \Omega(\pi)$, and the local government i will prefer to deviate from b_{i1}^{rules} and not violate the fiscal rule. Therefore there exists some $\tilde{\pi}_2$ such that $\pi \geqslant \tilde{\pi}_2$, an equilibrium with non-enforcement cannot exist. Thus, for $\pi \geqslant \max\{\tilde{\pi}_1, \tilde{\pi}_2\}$ there exists a unique equilibrium with enforcement. Q.E.D.

A.6 Proof of Proposition 5

The proof proceeds as follows. We first show that there exists $\underline{\beta}$ such that for $\beta \geqslant \underline{\beta}$, the commitment type chooses $\psi = \bar{\psi}$ to separate in period 1. In our construction we assume (and later verify) that the optimizing type chooses the same fiscal constitution as the commitment type in period 0 and does not enforce the fiscal rule if $\psi = \bar{\psi}$ in period 1. We showed in Proposition 3 that the latter is true if $\beta \leqslant \bar{\beta}$. Next, we show that if $\Delta > 0$ then $\underline{\beta} < \bar{\beta}$.

Recall that $b_{i1}^{er}(\pi, \alpha)$ denotes the debt issued in period 0 when the local governments expect to learn the central government type in period 1 defined in (14) given $\alpha = \left(\bar{b}, \psi\right)$; $b_{i1}^{lr}(\pi, \alpha)$ denotes the debt issued in period 0 when the local governments do not expect to learn the central government type in period 1 defined in (13) given $\alpha = \left(\bar{b}, \psi\right)$.

If the commitment type chooses $\psi = \bar{\psi}$ and $\beta \leqslant \bar{\beta}$ where $\bar{\beta}$ is defined in the proof of Proposition 3, since $\bar{b}$ is binding, we know that for π small enough there exists a unique equilibrium with separation in period 1 and early resolution of uncertainty. Thus we can write $W_0^{c,sep}$ as

$$W_0^{c,sep} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_{i1}^{er}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - \left(b_{i1}^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}\right) + q\mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{er}(\pi,\alpha) + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) \end{array} \right]$$

If instead the commitment type chooses $\psi = 0$, for π close to zero, there is no separation in period 1 and so its value $W_0^{c,pool}$ is

$$W_0^{c,pool} = \sum_i \frac{1}{N} u\left(Y_{i0} + q b_i^{lr}(\pi,\alpha)\right) + \\ + \beta \sum_i \frac{1}{N} \left[\begin{array}{c} u\left(Y - b_{i1}^{lr}(\pi,\alpha) + q\mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \\ + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1^{lr}(\pi,\alpha), \pi\right)\right) \end{array} \right].$$

The commitment type will then impose a binding rule if and only if $W_0^{c,sep} \geqslant W_0^{c,pool}$. Let $\Gamma(\pi) = W_0^{c,sep} - W_0^{c,pool}$. As $\pi \to 0$, $\Gamma(\pi) \to$

$$\beta \sum_i \frac{1}{N} \left[u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) + \beta u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right] \\ -\beta \sum_i \frac{1}{N} \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) + \beta u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]$$

since $b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha) = b_{i1}$. (From now on we use $b_{i1} = b_{i1}^{er}(0,\alpha) = b_{i1}^{lr}(0,\alpha)$.) Rearranging the expression above we obtain

$$\frac{\beta^2}{N} \sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right] \\ -\frac{\beta}{N} \sum_i \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right].$$

Note that both terms in square brackets are positive, thus we can define the cutoff β such that the expression above equals zero:

$$\underline{\beta}(\pi, N) \equiv \frac{\sum_i \left[u\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) - u\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right]}{\sum_i \left[u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) - u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right]}$$

Then for $\beta < \underline{\beta}(\pi, N)$, $\Gamma(0) < 0$. Thus, for π small, $W_0^{c,sep} < W_0^{c,pool}$ and the unique constitution will feature no fiscal rules. Conversely, for $\beta > \underline{\beta}(\pi, N)$, $\Gamma(0) > 0$. Thus, for

π small, $W_0^{c,sep} > W_0^{c,pool}$ and the unique constitution will feature fiscal rules.

To show that this is an equilibrium for $\beta > \underline{\beta}(\pi, N)$, we need to show that the optimizing type does indeed not want to enforce the constitution in period 1 (and induce separation). We know from the proof of Proposition 3 that if $\beta < \bar{\beta}(\pi, N)$, where $\bar{\beta}(\pi, N) \equiv$

$$\frac{\sum_{i=1}^{N} \frac{1}{N} \left[u\left(Y - b_{i1}^{rules}(\pi) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi), 0\right)\right) - u\left(Y - \left(b_{i1}^{rules}(\pi) + \psi\right) + q\mathbf{b}_{i2}\left(b_1^{rules}(\pi) + \psi, 1\right)\right)\right]}{u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_1^{rules}(\pi)+\psi,1\right)}{N}\right) - u\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b_{i1}^{rules}(\pi),0\right)}{N}\right)},$$

then for π close to zero, the optimizing will strictly prefer to not enforce the rule at $t = 1$. Thus we have our desired result for $\beta \in \left[\underline{\beta}(\pi, N), \bar{\beta}(\pi, N)\right]$. To show that this a well defined interval, we need to show that $\bar{\beta}(0, N) > \underline{\beta}(0, N)$. This is true if

$$\begin{aligned} 0 > &\left[Nu\left(Y - \frac{\sum \mathbf{b}_{i2}\left(b + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)\right] \\ &- \left[Nu\left(Y - \frac{\sum \mathbf{b}_{i2}(b_1, 0)}{N}\right) - \sum_i u\left(Y - \mathbf{b}_{i2}(b_1, 0)\right)\right] \end{aligned}$$

Given the concavity of u, this is true if $\mathbf{b}_{s2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) < \mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0)$. From the first order conditions for $\mathbf{b}_{i2}(b_1, 0)$ we have

$$u'\left(Y - b_{i1} + q\mathbf{b}_{i2}(b_1, 0)\right) q = \frac{\beta}{N} u'\left(Y - \frac{\sum \mathbf{b}_{i2}(b_1, 0)}{N}\right)$$

This implies that

$$\mathbf{b}_{s2}(b_1, 0) - \mathbf{b}_{n2}(b_1, 0) = \frac{b_{s1} - b_{n1}}{q} \tag{35}$$

Next from the first order conditions for $\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)$ we have

$$u'\left(Y - \psi \mathbb{I}_{b_{i1} > \bar{b}} - b_{i1} + q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right) q = \beta u'\left(Y - q\mathbf{b}_{i2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right)\right)$$

Then, if the rule is not binding for the North:

$$\begin{aligned} &u'\left(Y - \psi - b_{s1} + qb_{s2}\right) - u'\left(Y - b_{n1} + qb_{n2}\right) \\ &= \beta u'\left(Y - qb_{s2}\right) - \beta u'\left(Y - qb_{n2}\right) > 0 \end{aligned}$$

and so

$$\mathbf{b}_{s2}\left(b_1 + \psi \mathbb{I}_{b_{i1} > \bar{b}}, 1\right) - \mathbf{b}_{n2}(b_1, 1) < \frac{\psi + b_{s1} - b_{n1}}{q} \tag{36}$$

If instead the rule is binding for the North as well we have

$$\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1,1\right)<\frac{b_{s1}-b_{n1}}{q} \tag{37}$$

So from (35) and (36)-(37) it follows that for ψ small enough, $\mathbf{b}_{s2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)-\mathbf{b}_{n2}\left(b_1+\psi\mathbb{I}_{b_{i1}>\bar{b}},1\right)<\mathbf{b}_{s2}\left(b_1,0\right)-\mathbf{b}_{n2}\left(b_1,0\right)$ and so $\bar{\beta}\left(0,N\right)>\underline{\beta}\left(0,N\right)$. Therefore, for β in this range and π small enough, we have an equilibrium in which $\psi=\bar{\psi}$ and the rules are not enforced in period 1 by the optimizing type.

Finally, we need to show that the optimizing type will mimic the commitment type in period 0 and announce the same rule anticipating it will not enforce it in period 1. The value of choosing the same constitution as the commitment type in period 0 is given by

$$\begin{aligned}W_0^m\left(\pi,\alpha\right)&=\sum_i u\left(Y_{i0}+qb_{i1}^{er}\left(\pi,\alpha\right)\right)+\beta W_1^{er}\left(b_1^{er}\left(\pi,\alpha\right)\right)\\&=\sum_i \left[u\left(Y_{i0}+qb_{i1}^{er}\left(\pi,\alpha\right)\right)+\beta u\left(Y-\psi-b_{i1}^{er}\left(\pi\right)+q\mathbf{b}_{i2}\left(b_{i1}^{er}\left(\pi,\alpha\right),0\right)\right)\right.\\&\quad\left.+\beta^2u\left(Y-\frac{\sum_j\mathbf{b}_{j2}\left(b_{i1}^{er}\left(\pi,\alpha\right),0\right)}{N}\right)\right]\end{aligned}$$

while the value of choosing a different constitution is $W_0^m\left(0,\alpha\right)$ because the local governments learn that they are facing the optimizing type. We will establish that $\frac{\partial}{\partial\pi}W_0^m\left(\pi,\alpha\right)>0$, at $\pi=0$ which in turn implies that if π is close to 0, the optimizing type will always find it optimal to mimic. Differentiating $W_0^m\left(\pi,\alpha\right)$ with respect to π and evaluating at $\pi=0$ yields

$$\begin{aligned}\frac{\partial}{\partial\pi}W_0^m\left(0,\alpha\right)=\sum_i&\left[u'\left(G_{i0}\right)q\frac{\partial b_{i1}^{er}\left(0\right)}{\partial\pi}-\beta u\left(G_{i1}\right)\frac{\partial b_{i1}^{er}\left(0\right)}{\partial\pi}+\right.\\&\left.+u'\left(G_{i1}\right)q\frac{\partial\mathbf{b}_{i2}}{\partial b_{j1}}\frac{\partial b_{j1}^{er}\left(0\right)}{\partial\pi}-\frac{\beta^2}{N}u'\left(G_{i2}\right)\frac{\partial\mathbf{B}_2}{\partial b_{j1}}\frac{\partial b_{j1}^{er}\left(0\right)}{\partial\pi}\right]\end{aligned}$$

Recall the first order conditions for the local government in periods 1 and 2

$$\begin{aligned}u'\left(G_{i0}\right)q&=\beta u'\left(G_{i1}\right)+\frac{\beta^2}{N}u'\left(G_{i2}\right)\sum_{j\neq i}\frac{\partial\mathbf{b}_{j2}}{\partial b_{i1}}\\u'\left(G_{i1}\right)q&=\frac{\beta}{N}u'\left(G_{i2}\right)\end{aligned}$$

Substituting these into the previous equation yields

$$\frac{\partial}{\partial \pi} W_0^m (0, \alpha) = \sum_i u' (G_{i1}) \, q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial b^{er}_{-i1} (0)}{\partial \pi}$$
$$= u (G_{i1}) \, q \frac{\partial \mathbf{b}_{i2}}{\partial b_{j1}} \frac{\partial B^{er}_1 (0)}{\partial \pi} > 0$$

since at $\pi = 0$, $\frac{\partial}{\partial b_{N1}} \mathbf{b}_{S2} (b_1, 0) = \frac{\partial}{\partial b_{S1}} \mathbf{b}_{N2} (b_1, 0) < 0$ and $\partial B^{er}_1 (0) / \partial \pi < 0$. Q.E.D.

B Data underlying Figure 1

We use two datasets:

1. Dataset used in Kotia and Lledó (2016). They construct an index for the strength of subnational fiscal rules using a database from the European Commission (EC), measuring the strength of all the fiscal rules present in each EU country. The EC dataset includes all types of numerical fiscal rules—budget balance rules, debt rules, expenditure rules, and revenue rules—covering different levels of government—central, regional, and local—in force since 1990 across EU countries. They then weight the scores for the components applicable at the subnational level: regional and local. See Appendix B in Kotia and Lledó (2016) for details about the construction of the index.

   The dataset also contains information on

   (a) subnational primary balances—based on authors' own consolidation of total revenue and expenditures across local and (when applicable) state or regional governments using non-consolidated fiscal data from Eurostat;

   (b) output gap from the World Economic Outlook;

   (c) population above 65 years of age from the World Development Indicators;

   (d) unemployment from the World Economic Outlook;

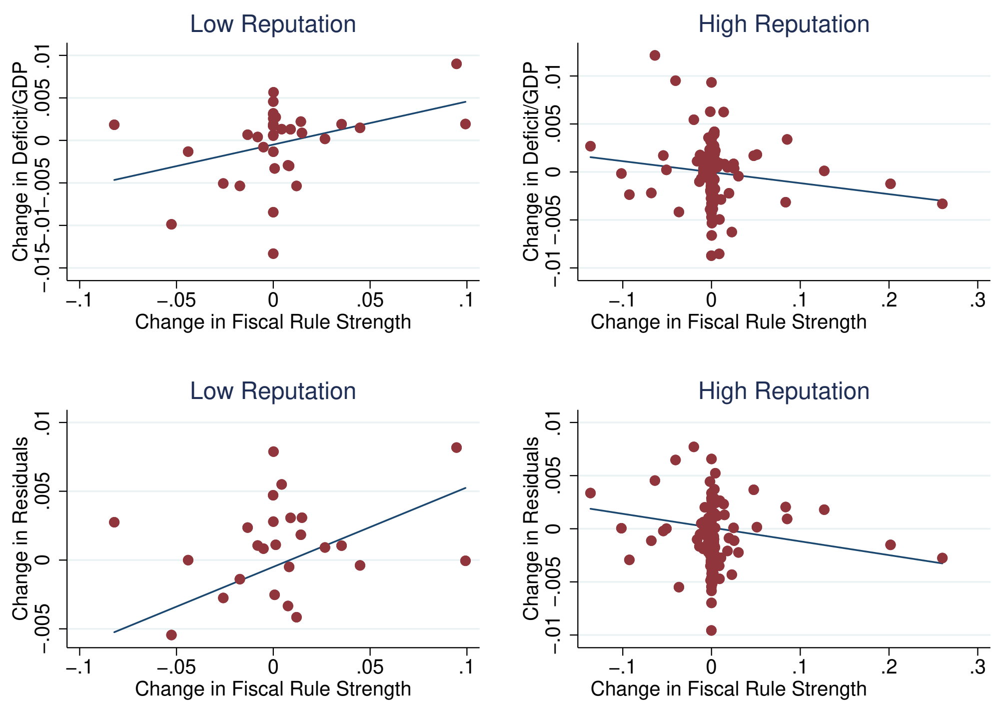
   (e) legislative election dummy taking the value of 1 if a national legislative election was held in that year, and zero otherwise, from the Database for Political Institutions (DPI).

2. World Bank's Worldwide Governance Indicators (WGI) data. This dataset consists of data on the quality of governance provided by a large number of enterprise, citizen, and expert survey respondents in industrial and developing countries. The WGI consists of aggregate indicators of six broad dimensions of governance: (i)

Voice and Accountability, (ii) Political Stability and Absence of Violence/Terrorism, (iii) Government Effectiveness, (iv) Regulatory Quality, (v) Rule of Law, and (vi) Control of Corruption. The governance indicator ranges from around -2.5 to 2.5, with higher values implying better outcomes. The data on government efficiency are biannual from 1996 until 2002 and then annual. We use linear interpolation to add observations in 1997, 1999, and 2001. Our preferred measure of reputation, π, is Government Effectiveness.

In figure 3 we plot the raw data and look at the changes in deficits for contemporaneous changes in fiscal rules.

Figure 3: Scatter plot of changes in primary deficits to changes in fiscal rule strength



In the bottom panels of Figure 1 and 3, we report the change in residuals after controlling for an estimated fiscal reaction function. In particular, we run the following regression

$$\mathtt{deficit}_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 \mathtt{deficit}_{it-1} + f_i + \varepsilon_{it},$$

where $\mathtt{deficit}_{it}$ is the primary deficit; X_{it} is a vector of control variables (including lags) consisting of output gap, population above 65 years of age, unemployment, legislative election dummy, and inflation; f_i is a country fixed effect; and ε_{it} is the residual from the regression. The figures plot the change in the average residual across two consecutive fiscal rule regimes.