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Online Appendix:
On the Optimality of Financial Repression

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ABSTRACT

This Appendix contains additional historical material and technical details for the paper.

Contents

1. Early History of Financial Repression

In our discussion of financial repression in the text we focused on recent economic history. Here we discuss the early history of such repression.

Economic historians have argued that, from its beginnings, much of the development of banking is intimately tied to the need to finance government expenditures. Homer and Sylla (1996) and Calomiris and Haber (2014) persuasively argue that the Bank of England was founded specifically to raise funds for the English government to help finance war expenditures. The Bank of England was then governed and regulated to ensure a stable source of funding for the government. Homer and Sylla (1996) also argue that the Banque de France was created to serve a similar need in France.

Bordo, Redish, and Rockoff (2015), Sylla, Legler and Wallis (1987), and Calomiris and Haber (2014) show how the development of state-chartered banks in the United States from 1800 to 1860 was connected to the fiscal needs of state governments. One device used during this period was for individual states to charter banks to issue notes with the proviso that the bank hold an appropriate amount of the debt of the chartering state government. In particular, Calomiris and Haber (2014, p. 167) point out that the Pennsylvania Omnibus Charter bill of 1814 required that all state-chartered "banks had to make loans to the state government at the government's discretion at an interest rate that could not exceed 5%." We also find it notable that when the U.S. federal government needed to raise a large amount of funds during the Civil War, one of its first steps was to set up a system of national banks, which were permitted to issue bank notes but were required to hold government debt to back these notes.

It is worthwhile noting that in many of the examples we cite, dating back to the founding of the Bank of England, there was financial repression in that the government restricted the activities of financial intermediaries and either directly or indirectly prodded banks to hold government debt. Of course, the agreement by banks to hold government debt was often part of a package in which the bank received compensating benefits, such as (local) monopoly rights.

2. Details of Results with Commitment

Proof of Proposition 1. We first show that the aggregate collateral constraint is binding. The reason is that $\beta R_K > 1$, so that if this constraint were slack it would be possible to increase deposits, investment, and hence welfare.

Next, we prove that the Ramsey outcome has no financial repression. To do so, suppose by way of contradiction that in some period t the government is practicing financial repression so that $B_{Bt+1} > 0$. There are three cases: the return on debt equals that on deposits, the return on debt is lower than that on deposits, a case referred to as *interest rate suppression*, the return on debt exceeds that on deposits. In each case under the contradiction hypothesis we construct a variation that improves welfare.

Case 1. $R^e_{t+1} = 1/\beta$.

The welfare-improving variation shifts all debt from banks to households keeping total debt the same. Households keep their total savings the same by reducing their deposits by the amount of increased debt they hold. Formally, consider a variation denoted by carets in which $\hat{R}_{t+1} = 1/\beta$, $\hat{\delta}_{t+1} = 1$, $\hat{B}_{Ht+1} = B_{Ht+1} + B_{Bt+1}$, $\hat{B}_{Bt+1} = 0$, $\hat{D}_{t+1} = D_{t+1} - B_{Bt+1}$, and leaves all allocations in future periods unaffected. This variation clearly satisfies the budget constraints of households, banks, and the government and, because banks borrow less from households, the aggregate collateral constraint of banks becomes slack. Thus it is possible to find a further variation that strictly improves welfare.

Case 2. $R^e_{t+1} < 1/\beta$.

We first show that we can implement the same allocation with an alternative policy that has $\hat{R}^e_{t+1} = 1/\beta$. We can then use the argument in case 1 to prove that it is possible to find a further variation that improves welfare. To see how we can implement the original allocation with $\hat{R}^e_{t+1} = 1/\beta$, define new policies as follows. Consider two subcases. First, suppose $\delta_{t+1} + (1 - \delta_{t+1})\,\iota_{t+1} = 1$ so that either the government is not defaulting or it is defaulting then bailing out the banks so that $R^e_{t+1} = R_{t+1} < 1/\beta$. Let the variation have $\hat{R}_{t+1} = \hat{R}^e_{t+1} = 1/\beta$. Choose $\hat{\tau}_{Kt}$ so that it raises the same amount of revenues as did the sum of the revenues from the repressed interest rate R^e_{t+1} and the original tax on capital, that is,

$\hat{\tau}_{Kt}$ satisfies

$$(1) \quad \hat{\tau}_{Kt} \frac{K_{t+1}}{R_K} = \tau_{Kt} \frac{K_{t+1}}{R_K} + \left(\frac{1}{R_{t+1}} - \beta \right) B_{Bt+1}.$$

It is easy to verify that if at the original allocation, prices, and policies the implementability conditions are satisfied then they also are satisfied under the alternative allocation, prices, and policies.

A similar argument can be made for the subcase in which the government is defaulting on debt and not bailing out, that is with $\delta_{t+1} + (1 - \delta_{t+1}) \iota_{t+1} = 0$. In this case, let the revenues from new debt issuance in the alternative policy be zero, $\hat{B}_{Bt+1}/\hat{R}_{t+1} = 0$ and let $\hat{\tau}_{Kt}$ be defined by

$$(2) \quad \hat{\tau}_{Kt} \frac{K_{t+1}}{R_K} = \tau_{Kt} \frac{K_{t+1}}{R_K} + \frac{1}{R_{t+1}} B_{Bt+1}.$$

Then, using a similar logic to the case above we can establish that the alternative policy supports the same allocation.

Thus, we established that if $R^e_{t+1} < 1/\beta$ then there exists an alternative policy with appropriately modified tax rate on capital that implement the same allocation with $R^e_{t+1} = 1/\beta$. We can then use the argument in case 1 to prove that it is possible to find a further variation that improves welfare.

Case 3. $R^e_{t+1} = R_{t+1} \left[\delta_{t+1} + (1 - \delta_{t+1}) \iota_{t+1} \right] > 1/\beta$.

In order for these prices to be part of an equilibrium the government must be defaulting on debt so that $\delta_{t+1} = 0$. Since the effective return on debt for households is zero, households hold no debt. Moreover, the government must be bailing out the banks so that

$$(3) \quad R^e_{t+1} = R_{t+1} > 1/\beta.$$

Furthermore, in any equilibrium in which investment is interior, the effective rate of return on debt for banks must be weakly smaller than the after tax return on capital, namely

$$(4) \quad R^e_{t+1} \leq \frac{R_K}{1 + \tau_{Kt}}$$

or else banks would not invest in capital.

Now, consider the following variation which in period t moves all debt from banks to households, lowers the interest rate of government debt to the deposit rate, and keeps the government payments for debt and bailouts unaffected in period $t+1$. Let banks increase their investment by the part of net worth that was previously used to buy government debt.

More precisely, the variation is as follows. Set the rate of return on government debt to $1/\beta$, shift all debt to household by setting $\hat{B}_{Ht+1} = B_{Bt+1}$ and $\hat{B}_{Bt+1} = 0$. Also increase investment by ΔK units in period t, reduce current consumption by ΔC with $\Delta C + \Delta K/R_K = 0$ and raise consumption in period $t+1$ by ΔK and leave all allocations for periods $t+2$ on unchanged. Using $\Delta C = -\Delta K/R_K$, the change in the discounted value of utility is

$$(5) \quad -\frac{\Delta K}{R_K} + \beta \Delta K = (\beta R_K - 1)\frac{\Delta K}{R_K}$$

Since $\beta R_K > 1$, this change in utility is positive if ΔK is positive.

We now show that this variation satisfies the government budget constraint in period t, implies that $\Delta K > 0$, and is feasible in that it increases aggregate net worth. Under this variation since $\hat{B}_{Ht+1} = B_{Bt+1}$, (3) implies that the revenues from debt issue in period t rise in that

$$\beta \hat{B}_{Ht+1} > \frac{B_{Bt+1}}{R_{t+1}}.$$

Hence, the government's budget constraint is relaxed in period t even if the government keeps taxes unchanged.

We next show that ΔK is indeed positive. To do so combine the aggregate budget constraint of banks and the aggregate collateral constraint to obtain

$$\Delta K = B_B \frac{\left(\frac{1}{R_{t+1}} - \beta\gamma\right)}{\left(\frac{1+\tau_{Kt}}{R_K} - \beta\gamma\right)} > 0$$

Finally, we check that net worth in period $t+1$ rises. Since the effective rate of return on bonds is less than the after tax return on capital, this variation does increase net worth, that is

$$\Delta N = (1-\gamma)\, B_{Bt+1}\left[\frac{\left(\frac{1}{R_{t+1}}-\beta\gamma\right)}{\left(\frac{1+\tau_{Kt}}{R_K}-\beta\gamma\right)}-1\right] \geq 0.$$

Thus this variation is feasible and it increases welfare, which is a contradiction to the hypothesis that the original allocation is optimal.

Putting all three cases together, we have shown that the Ramsey outcome has no financial repression, $B_{Bt+1} = 0$ for all t and all debt is held by households. *Q.E.D.*

Derivation of the constants A_R and A_N.

Consider the recursive representation of the continuation Ramsey problem given by

$$(6) \quad V_{Rt}(S) = \max_{B',T_K,T} W(T_L) + K - \frac{K'}{R_K} + \beta V_{Rt+1}(S')$$

subject to the government budget constraint

$$(7) \quad G + B = T + T_K + \beta B',$$

$$(8) \quad (1-\beta\gamma R_K)\frac{K'}{R_K} + T_K = \sigma N + (1-\sigma)\bar{n},$$

where $T_K = \tau_K K/R_K$. Here we have substituted in the aggregate bank budget constraint, the binding collateral constraint binds, and use the result that banks hold no debt. Thus, $D' = \gamma K'$ and $N = K + B_B - D$ is the aggregate net worth of continuing banks.

We guess that the value function of this problem has the following form

$$(9) \quad V_{Rt}(S) = A_R + K + A_N N + H_{Rt}(B,G),$$

where the *tax distortion* function H_R is defined as the value of the problem

$$(10) \quad H_{Rt}(B,G) = \max_{B',T,T_K} W(T) - \frac{A_N}{\sigma}T_K + \beta H_{Rt+1}(B',G')$$

subject to the government budget constraint $G + B = T + T_K + \beta B'$.

Substituting the guess for the value function into the right side of (6), we obtain that

$V_{Rt}(S)$ is the maximum over B' and T_K of

$$(11)\quad W\left(T\right)+K-\frac{K'}{R_K}+\beta\left[A_R+K'+A_N\left(K'+B'_B-D'\right)+H_{Rt+1}\left(B',G\right)\right],$$

where $T = G + B - \beta B'$, $D' = \gamma K'$, and

$$(12)\quad \frac{K'}{R_K}=\frac{\sigma N-T_K+(1-\sigma)\bar{n}}{1-\gamma\beta R_K}.$$

Substituting for K' from (12) and $D' = \gamma K'$ into (11) and rearranging terms gives that this value equals

$$\begin{aligned}&W\left(T\right)+\beta H_{Rt+1}\left(B',G'\right)-\frac{\left[\beta A_N R_K(1-\gamma)+\beta R_K-1\right]}{1-\gamma\beta R_K}T_K\\&+K+\frac{\sigma\left[\beta A_N R_K(1-\gamma)+\beta R_K-1\right]}{1-\gamma\beta R_K}N+\beta A_R+(1-\sigma)\bar{n}\frac{\left[\beta\omega_K+\beta A_N R_K(1-\gamma)-1\right]}{1-\gamma\beta R_K}.\end{aligned}$$

The guess is verified when the undetermined coefficients A_N and A_R satisfy

$$A_N=\frac{(\beta R_K-1)\sigma}{1-\beta R_K\left[\sigma+(1-\sigma)\gamma\right]},$$

$$A_R=\frac{(1-\sigma)\bar{n}}{(1-\beta)\sigma}A_N.$$

and the function H_{Rt} solves the functional equation in (10).

Result 1. If v is convex, and v'' is decreasing then $W'(0) = 0$ and $W'' < 0$.

Proof. Since

$$W'\left(T\right)=\frac{T/\ell}{T/\ell-v''\left(\ell\right)\ell}$$

if $T = 0$ then $W'(T) = 0$. Next dividing by T/ℓ gives

$$W'\left(T\right)=\frac{1}{1-\frac{v''\ell^2}{T}}\text{ so }W''\left(T\right)=\left(\frac{1}{1-\frac{v''\ell^2}{T}}\right)^2\frac{1}{T^2}\left[\left(v'''\ell^2+2v''\ell\ell'\right)T-v''\ell^2\right]$$

Since ℓ' is negative, W'' is negative if $v'' > 0$ and $v''' < 0$. *Q.E.D.*

Proof of Proposition 2. As shown in the text, labor taxes are constant. Thus, the marginal cost of raising revenues from labor taxes is constant at the level $-W'(\hat{T})$, and the marginal cost of raising revenues from capital taxes is A_N/σ. Hence, the first order condition for taxing investment implies that the tax on investment is zero in all periods if

$$(13)\quad -W'(\hat{T}) < \frac{A_N}{\sigma},$$

where $\hat{T} = (1-\beta)\left(\sum_{t=0}^{\infty} \beta^t G_t + \delta_{B0} B_{B0} + \delta_{H0} B_{H0}\right)$ is the constant labor tax revenue that needs to be collected if no revenue is raised from investment taxes. The rest of the proposition is proved in the text. *Q.E.D.*

3. Details for Sustainable Equilibrium

First we provide the derivations of the value of the Markov equilibrium.

A. Preliminaries: Characterization of Markov outcomes

We develop a recursive formulation of the Markov equilibrium. Since it is optimal for the government to default on debt held by household, we impose without loss of generality that $\delta = 0$ and $B'_H = 0$. The *primal Markov* problem given a state S, is to choose current allocations $Y = (C, L, K', D', B'_B)$, current policies, τ_K, T_K, $\iota \in \{0,1\}$, and debt price R, taking as given future policy functions $\iota(S')$ to solve

$$(14)\quad J_{Mt}(S) = \max C - v(L) + \beta J_{Mt+1}(S')$$

subject to implementability constraints, namely the resource constraint and

$$(15)\quad G + \iota B_B \leq (1 - v'(L))\, L + T_K + \frac{1}{R'} B'_B$$

$$(16)\quad \frac{(1+\tau_K)}{R_K} K' + \frac{1}{R'} B'_B - \beta D' = \sigma (K + \iota B_B - D) + (1-\sigma)\bar{n}$$

$$(17)\quad D' \leq \gamma \left[K' + \iota(S')\, B'_B\right]$$

$$(18)\quad R^e(S') = R'\iota(S') \leq \frac{R_K}{1+\tau_K}.$$

$$(19)\quad T_K = \frac{\tau_K K'}{R_K}$$

Next we construct a restricted continuation Markov problem and we use it to characterize the value of the Markov problem. In this restricted problem we do not allow governments to bail out banks in current or future periods and we add a *no temptation to default on banks constraint*. This constraint requires that the continuation value be at least as large as the continuation value if banks held no debt. Formally, let the continuation value of the restricted problem solve

$$(20) \quad V_{Mt}(S) = \max_{y,\tau_K,R} C - v(L) + \beta V_{Mt+1}(S')$$

subject to the implementability constraints with the restriction that $\iota = 1$ and $\iota(S') = 1$ for all S' as well as the no temptation to default on banks constraint, namely,

$$(21) \quad V_{Mt+1}(S') \geq V_{Mt+1}(K', D', 0, 0, G').$$

The idea of this constraint is that the government at t cannot choose policies that lead to a state S' for $t+1$ that would lead to a government to default and not engage in a bailout. Such a policy in $t+1$ is equivalent to simply beginning period $t+1$ with no inherited bank debt and leads to the value on the right side of (21). We show below that a continuation value function $V_{Mt}(S)$ exists.

Given the value of the restricted problem, we claim that the value of the Markov equilibrium is given by

$$(22) \quad J_{Mt}(S) = \max\{V_{Mt}(K, D, 0, 0, G), V_{Mt}(K, D, B_B, 0, G)\}.$$

To prove this claim, suppose that $J_{Mt+1}(S')$ satisfies the analog of (22) at $t+1$ for all S'.

Consider now the problem (14). Suppose first that for some arbitrary state S, the solution to this problem has $\iota(S') = 1$ where $S' = S'(S)$ is the evolution of the state under the solution to this problem. Under this supposition consider the two possibilities for the bailout decision period t. If in (14) the government chooses not to bailout, $\iota = 0$, then $J_{Mt}(S) = V_{Mt}(K, D, 0, 0, G)$. If in (14) the government chooses to bailout, $\iota = 1$, then $J_{Mt}(S) = V_{Mt}(K, D, B_B, 0, G)$. Thus $J_{Mt}(S)$ satisfies (22).

Suppose next that for some arbitrary state S, the solution to (14) has $\iota(S') = 0$ for $S' = S'(S)$ and in it banks are forced to hold a strictly positive level of bank debt B'_B at t. That is, the government has forcibly taken resources B'_B/R' from banks. An alternative and equivalent way of taking these resources is to set B'_B to zero and raising the tax on capital appropriately. Given this observation the rest of the argument that $J_{Mt}(S)$ satisfies (22) is the same as above.

Next, we turn to the proof of the existence of $V_{Mt}(S)$. We guess and verify that $V_{Mt}(S)$ has a form similar to that in the Ramsey equilibrium, namely,

$$(23) \quad V_{Mt}(S) = A_R + K + A_N N + H_{Mt}(B_B, G),$$

where the *tax distortion* function H_{Mt} satisfies the Bellman equation

$$(24) \quad H_{Mt}(B_B, G) = \max_{B'_B, T, R', K', T_K} W(T) - \frac{A_N}{\sigma} T_K - A_B (R') B'_B + \beta H_{Mt+1}(B'_B, G')$$

subject to the government budget constraint $G + B_B \leq T + B'_B/R' + T_K$ where $T_K = \tau_K K'/R_K$, the constraint $R' \leq R_K/(1+\tau_K)$, (16), (17), and a version of the no temptation to default on banks constraint

$$(25) \quad H_{Mt+1}(B'_B, G') + A_N B'_B \geq H_{Mt+1}(0, G'),$$

where we canceled the terms $A_R + K + A_N N$ from both sides.

Lemma M1 (Existence of a Markov Equilibrium) A Markov equilibrium with values given by the conjectured form of the Markov value function and (22) exists.

Proof. First, we show that (24) has a solution. Let $\mathbb{T}$ be the operator defined by the right side of (24). Consider the family X of continuous, bounded, and concave functions h defined over a compact subset of (B, G). Note that this operator maps this space into itself and is continuous. To show that that the induced family of functions $\mathbb{T}(X)$ is equicontinuous, note that for all h in X and for all $B_2 > B_1$, suppressing explicit dependence on G, we have that

$$\begin{aligned} ||(\mathbb{T}h)(B_2) - (\mathbb{T}h)(B_1)|| &\leq ||(\mathbb{T}h)(B_1) - \frac{A_N}{\sigma}(B_2 - B_1) - (\mathbb{T}h)(B_1)|| \\ &= \frac{A_N}{\sigma}||B_2 - B_1|| \end{aligned}$$

where the inequality follows because a feasible solution at B_2 is to repay the additional debt by choosing the policies that are optimal for B_1 and just increase capital taxes by $B_2 - B_1$ which have cost $\frac{A_N}{\sigma}(B_2 - B_1)$. Then every $\mathbb{T}h$ in $\mathbb{T}(X)$ is Lipschitz with Lipschitz constant A_N/σ common to all elements of $\mathbb{T}(X)$. We can then apply a version of the Schauder fixed point theorem, namely, Theorem 17.4 in Stokey, Lucas, and Prescott (1989), to conclude that $\mathbb{T}$ has a fixed point.

Next, substituting this fixed point H_M into the original problem, we can verify the conjecture and calculate the constants in a fashion similar what we did for the Ramsey equilibrium. In particular, it can be verified that the constant $A_B(R)$ is given by

$$\text{(26)} \quad A_B(R) = -\beta\frac{A_N}{\sigma}[\gamma + \sigma(1-\gamma)] + \frac{A_N}{\sigma}\frac{1}{R}$$

and A_R and A_N are the same constants derived in the Ramsey problem. *Q.E.D.*

For notational convenience below, we let the constant A_B with no argument denote $A_B(1/\beta)$.

Next we provide a characterization for the Markov equilibrium that will be useful for characterizing the best sustainable outcome. Replace the no default constraint with

$$\text{(27)} \quad B' \leq \bar{B}(G)$$

where $\bar{B}(G)$ is the maximal level of debt that can be supported and is defined by

$$H_{Mt+1}\left(\bar{B}(G), G\right) + A_N\bar{B}(G) = H_{Mt+1}(0, G).$$

We refer to this problem as the *equivalent* problem.

Consider now the cyclic economy. We make the following *spread* assumption throughout

$$(28)\quad -\frac{W'(G_H)}{R_K} > A_B - \beta W'(G_L).$$

This assumption is satisfied if the spread in spending between high and low states, $G_H - G_L$, is sufficiently large. We strengthen the high marginal value of net worth condition to be

$$(29)\quad -W'(G_H) < \frac{A_N}{\sigma}.$$

Here, for completeness, we characterize the Markov equilibrium in detail, even though we do not need this characterization for the results regarding the best sustainable equilibrium.

Lemma M2. Consider an economy in which at in period 0 the inherited debt is zero and the level of spending is G_H and assumptions (28) and (29) hold. Then in a Markov equilibrium, in all subsequent states G_H, the government issues a strictly positive amount of debt, $B_B' > 0$, the government sets $R' = R_K$. In all states G_L the government issues zero debt.

Proof. Consider a period with current state G_H with zero inherited debt. We first show that $R' = R_K$. To do so, note that the first-order condition for R' is

$$(30)\quad \frac{B_B'}{R_K^2}\left[W'(T) - \frac{A_N}{\sigma}\right] - \mu_{R_K} = 0$$

where μ_{R_K} is the multiplier on $R' \leq R_K$ and we have used that (26) implies that $A_B'(R_K) = A_N/(\sigma R_K^2)$. Since (29) holds, $T \leq G_H$, and W' is decreasing the term in brackets is positive, so if B_B' is positive then (30) implies that $\mu_{R_K} > 0$ so $R' = R_K$.

Next, to show that the government issues a strictly positive amount of debt in all G_H states, rewrite our original problem when inherited debt is zero as

$$H(B_B, G) = \max_{B_B'} W\left(G + B_B - \frac{B_B'}{R_K}\right) - A_B(R_K) B_B' + \beta H(B_B', G')$$

subject to the debt constraint (27). Clearly if the debt constraint (27) is binding a positive amount of debt is issued.

Suppose instead that the debt constraint is slack. The first-order condition with respect to B_B' for the equivalent problem with the debt constraint is

$$(31) \quad -\frac{1}{R_K}W'(T) \leq A_B(R_K) - \beta W'(T')$$

where we have used the envelope condition that $H'(B', G_L) = W'(T')$. Now, suppose by way of contradiction that $B_B' = 0$. Given that we have assumed that inherited debt $B_B = 0$ then $T = G_H$. Since the government cannot sell negative debt in the next period and it has inherited zero debt, $T' \leq G_L$. From the concavity of W it then follows that $-W'(T') \leq -W'(G_L)$. Substituting these results into (31) yields

$$-\frac{1}{R_K}W'(G_H) \leq A_B(R_K) - \beta W'(G_L)$$

which contradicts the spread assumption (28). So it is optimal to issue a strictly positive amount of debt when the current state is G_H and the inherited level of debt is zero.

Next we show that it is optimal to issue zero debt when the current state is G_L. To do so, suppose by way of contradiction that the debt issued at G_L, namely B_B' is strictly positive. Then, using the envelope theorem, the first-order condition can be written as

$$(32) \quad -\frac{1}{R_K}W'(T) = A_B(R_K) - \beta W'(T(B_B', G_H))$$

where $T(B_B', G_H)$ is the tax associated with the optimal policy in the Markov equilibrium when the current state is G_H and the inherited debt is B_B'. Substituting for T from the government budget constraint into (32) and letting $B_B(0, G_H)$ denote the optimal amount of debt issued from the state G_H when the inherited debt in that state is zero gives

$$-\frac{1}{R_K}W'\left(G_L + B_B(0, G_H) - \frac{B_B'}{R_K}\right) = A_B(R_K) - \beta W'(T(B_B', G_H)) > A_B(R_K) - \beta W'(T(0, G_H))$$

where the inequality follows because the tax policy is monotone in inherited debt B_B'. This implies

$$(33)\quad -\beta W'\left(G_L + B_B\left(0, G_H\right) - \frac{B_B'}{R_K}\right) > \beta R_K\left[A_B\left(R_K\right) - \beta W'\left(T\right)\right] > A_B\left(R_K\right) - \beta W'\left(T\right)$$

where in (33), $T = T\left(0, G_H\right)$ and where the second inequality follows from $\beta R_K > 1$.

Next, note that the analog of (32) for state G_H implies

$$(34)\quad -\frac{1}{R_K}W'\left(T\left(0, G_H\right)\right) - A_B\left(R_K\right) = -\beta W'\left(G_L + B_B\left(0, G_H\right) - \frac{B_B'}{R_K}\right).$$

Using the extreme inequalities in (33) to substitute for the right side of (34) to get

$$(35)\quad -\frac{1}{R_K}W'\left(T\left(0, G_H\right)\right) - A_B\left(R_K\right) > A_B\left(R_K\right) - \beta W'\left(T\left(0, G_H\right)\right).$$

Now multiplying by R_K and rearranging gives

$$(36)\quad -W'\left(T\left(0, G_H\right)\right)\left[1 - \beta R_K\right] > 2R_K A_B\left(R_K\right).$$

Since $\beta R_K > 1$ the left side is negative. Since the right side is positive this is a contradiction of the hypothesis that positive debt is sold from G_L. *Q.E.D.*

Consider a Markov equilibrium starting at G_L with zero inherited debt under the spread assumption. Since Lemma M2 implies that it is optimal to issue zero debt in G_L and strictly positive debt in G_H, the payoff in a Markov equilibrium is given by

$$(37)\quad H_M\left(0, G_L\right) = W\left(G_L\right) + \beta\frac{W\left(G_H - \frac{B_M}{R_K}\right) - A_B\left(R_K\right)B_M + \beta W\left(G_L + B_M\right)}{1 - \beta^2}$$

for some positive B_M. Since no debt is issued from a low state G_L then taxes in that state pay for spending and inherited debt if any. Thus, in period 1, $T = G_L$ and in all subsequent low states $T = G_L + B_M$. To determine the level of $B_M > 0$ we use (31) holding with equality, namely

$$(38)\quad -\frac{1}{R_K}W'\left(G_H - \frac{B_M}{R_K}\right) = A_B\left(R_K\right) - \beta W'\left(G_L + B_M\right).$$

For completeness, note that if (28) does not hold then running a balanced budget is optimal and

$$H_M(0, G_L) = W(G_L) + \beta \frac{W(G_H) + \beta W(G_L)}{1-\beta^2}.$$

B. Proofs for the results in the text

Proof of Lemma 4. To show that the continuation value function has the form

$$(39) \quad V_{St}(S) = A_R + K + A_N N + H_{St}(B, G),$$

consider the recursive representation for the continuation of the best sustainable outcome:

$$(40) \quad V_{St}(S) = \max_{B'_B, B'_H, T_K, T, R'} W(T) + K - \frac{K'}{R_K} + \beta V_{St+1}(S')$$

subject to the government budget constraint, the bank budget constraint, the restriction on returns, $R' \leq R_K$, the complementary slackness condition, namely, if $R' > 1/\beta$ then $B'_H = 0$ and the sustainability constraint

$$(41) \quad V_{St+1}(S') \geq A_R + K + A_N(K' - D') + h(B'_B, G')$$

where $h(B'_B, G') \equiv \max\{H_{Mt+1}(0, G'), A_N B'_B + H_{Mt+1}(B'_B, G')\}$ and the value of the right side of (41) is the value of the best deviation, $J_{Mt+1}(S') = A_R + K + A_N(K' - D') + h(B'_B, G')$.

The right side of (40) defines an operator, $\mathbb{T}$. This operator is monotone so using logic similar to that in Abreu, Pearce, and Stacchetti (1990), if we start with an initial value function $V_0(S, G)$ that is pointwise larger than $V(S, G)$, we can construct a sequence of functions $V_n = \mathbb{T}^n V_0$ that converges to V in the sup norm. We will use this montononcity and convergence result to derive the form of the value function. For the initial value function we choose the value of the continuation Ramsey problem. Clearly, since the Ramsey problem solves a less constrained version of the best sustainable problem, V_R is pointwise larger than V and we can use V_R as the initial value function V_0. Recall from our earlier discussion that the Ramsey problem has the form $V_R(B, G) = A_R + K + A_N N + H_R(B, G)$.

We need to show that the sequence of constructed functions V_n have the form $V_n = A_R + K + A_N N + H_n(B, G)$ for the relevant sequence of functions $H_n(B, G)$ that are defined recursively from an initial value $H_0(B, G) = H_R(B, G)$. To do so, substitute our guess for V_n in (40) and inspect the resulting problem to conclude that the optimal B'_B, B'_H, and T_K are independent of K and D and the optimal K' is linear in $\sigma N + (1 - \sigma)\bar{n}$. Using these two properties and some algebra that we detail below, we have that

(42) $\mathbb{T}V_n(S, G) = A_R + K + A_N N + H_{n+1}(B, G),$

where nonnegative B'_B, B'_H, and T_K are chosen to solve

(43) $$H_{n+1}(B, G) = \max_{B'_B, B'_H, T_K} W(T) - \frac{A_N}{\sigma} T_K - A_B(R') B'_B + H_n(B'_B + B'_H, G'),$$

subject to

$$T = G_t + B - T_K - \beta(B'_B + B'_H),$$

$$A_N B'_B + H_n(B, G') \geq H_M(0, G'),$$

$R' \leq R_K$ and if $R' > 1/\beta$ then $B'_H = 0$. The result in (42) implies that in the limit,

$$V(K, N, B, G) = A_R + K + A_N N + H(B, G),$$

where H is defined as the limit of H_n. Moreover, since the sequence H_n converges to H and for all n, the value satisfies (43). Since the operator defined by (43) is monotone, H is the largest fixed point of the functional equation defining the best sustainable equilibrium. *Q.E.D.*

We fill in the algebraic details that justify the claims that the iterations obviously satisfy the independence and linearity properties invoked above in the last section of this Appendix.

In what follows, we assume $-W'(T) \leq A_N/\sigma$ so that it is not optimal to tax capital.

We find it convenient to split the best sustainable problem into two regimes. In the *household debt regime*, denoted by a superscript h, both households and banks hold debt into the next period and, thus, the rate of return on debt must be equal to $1/\beta$. In the *no household debt regime*, denoted by a superscript b, only banks hold debt and the government possibly subsidizes banks. Clearly, the tax distortion function in the best sustainable outcome is the one for which the current regime attains the highest value. Thus, this tax distortion function satisfies

$$(44)\quad H(B,G) = \max\left\{H^h(B,G), H^b(B,G)\right\}$$

where

$$(45)\quad H^h(B,G) = \max_{B'_B, B'_H} W\left(G + B - \beta\left(B'_B + B'_H\right)\right) - A_B B'_B + \beta H\left(B'_B + B'_H, G'\right)$$

subject to $H\left(B'_B + B'_H, G'\right) + A_N B'_B \geq h\left(B'_B, G'\right)$ and

$$(46)\quad H^b(B,G) = \max_{B'_B, R'} W\left(G + B - \frac{B'_B}{R'}\right) - A_B(R)\, B'_B + \beta H\left(B'_B, G'\right)$$

subject to $B'_B \leq \bar{B}_B(G')$ and $R' \leq R_K$ where $\bar{B}_B(G)$ is the maximal debt that can be supported in equilibrium in state G, defined as

$$H\left(\bar{B}_B(G), G\right) + A_N \bar{B}_B(G) = h\left(\bar{B}_B(G), G\right)$$

and $A_B = A_B(1/\beta)$.

We turn now to the proof of Proposition 3. In the proof we assume that it is always optimal for the government to be in the household debt regime and so subsidizing debt in banks is not optimal. In the next section in Proposition S1 we provide sufficient conditions for this to be true.

In the proof we are going to use the following intermediate lemma that establishes that there exists a critical value β^* such that perfect tax smoothing can be supported with trigger strategies alone.

Lemma S1. There is a critical value β^* such that if $\beta > \beta^*$ then the sustainable equilibrium in the cyclic economy can support a strictly positive debt inherited in state G_L and G_H without forcing banks to hold debt. That is, there exists $B_1 > 0$ such that $H(B_1, G_L) \geq H_M(0, G_L)$ and $H(B_1, G_H) \geq H_M(0, G_H)$. Furthermore, if the government inherits zero debt in the low spending state then the perfect tax smoothing outcome can be supported with trigger strategies alone.

Proof. Let T^* be the level of taxes in a Ramsey outcome starting with zero debt inherited in a high spending state. Since the government budget is balanced over every two period cycle we have that

$$(47) \quad \frac{T^*}{1-\beta} = \frac{G_H + \beta G_L}{1-\beta^2}.$$

Since no debt is issued from a low spending state to a high spending state, the associated level of debt issued from a high spending state into a low spending state, B_L^*, satisfies

$$B_L^* = T^* - G_L > 0$$

where the inequality follows from (47). The value associated with this plan is

$$\frac{W(T^*)}{1-\beta} = W(T^*) + \beta \frac{W(T^*) + \beta W(T^*)}{1-\beta}$$

From the discussion following Lemma M1, we know that the value of a best deviations in state G_L and G_H are

$$(48) \quad H_M(0, G_L) = W(G_L) + \beta \frac{W\left(G_H - \frac{B_M}{R_K}\right) - A_B(R_K) B_M + \beta W(G_L + B_M)}{1-\beta^2}$$

$$(49) \quad H_M(0, G_H) = \frac{W\left(G_H - \frac{B_M}{R_K}\right) - A_B(R_K) B_M + \beta W(G_L + B_M)}{1-\beta^2}$$

Next, define the critical value β^* as that value at which the sustainability constraint holds with an equality at G_L, which from (48) is

$$(50)\quad \frac{W\left(T^*\right)}{1-\beta^*}=W\left(G_L\right)+\beta^* \frac{W\left(G_H-\frac{B_M}{R_K}\right)-A_B\left(R_K\right) B_M+\beta W\left(G_L+B_M\right)}{1-\beta^{*2}}.$$

One can show that for all $\beta \geq \beta^*$

$$\frac{W\left(T^*\right)}{1-\beta} \geq H_M\left(0, G_L\right)>H_M\left(0, G_H\right)$$

Thus it is possible to support strictly positive level of debt inherited in state G_L. Moreover, since the sustainability constraint is slack, when zero debt is passed into a high spending state, $W\left(T^*\right) /(1-\beta)>H_M\left(0, G_H\right)$, it is possible to support a strictly positive amount of debt in such a state. *Q.E.D.*

To set up the proof of Proposition 3, for any β, define the maximal debt sold from a low spending state, $B_L(\beta)$, such that the associated Ramsey outcome is sustainable, that is, $B_L(\beta)$ solves

$$(51)\quad \frac{1}{1-\beta} W\left(T^*(\beta)\right)=H_M\left(0, G_L\right)$$

where $B_H(\beta)=B_L(\beta)+\left(G_H-G_L\right) /(1-\beta)$. In the proof of Proposition 3 we will assume that $B_H(\beta)$ and $B_L(\beta)$ are both positive. It is easy to show a sufficient condition for this to be true is that $\beta \geq \beta^*$.

Proof of Proposition 3.

The proof that there exists a critical value G^* such that if $G_0<G^*$ then there is no financial repression in period 0 while if $G_0>G^*$ there is repression in period 0 is provided in the text.

Provisionally assume that $B_t>0$ for all t. We later show that this is true.

From the first-order condition (33), since the multipliers on the sustainability constraint are nonnegative, $\mu_t \geq 0$ for all t, and W is strictly concave, $W^{\prime \prime}<0$, it follows that the tax revenues $\left\{T_t\right\}$ are weakly decreasing. Such a path for tax revenues implies that debt is weakly decreasing over each cycle. To see why, iterate the government budget constraint forward to write that debt is the discounted value of future government surpluses, which, using the cyclical pattern of government spending, gives

$$(52)\quad B_t = \sum_{s=0}^{\infty} \beta^s T_{t+s} - \frac{G_t + \beta G_{t+1}}{1-\beta^2}.$$

Since $\{T_t\}$ is weakly decreasing then $B_{t+2} \leq B_t$ for all t. Furthermore, since the sequence of tax revenues $\{T_t\}$ is weakly decreasing and bounded below by zero, these tax revenues must converge to a positive level so that taxes are perfectly smoothed in the limit.

Next, we show that if $G_0 > G^*$ there is some period $\hat{T}$ such that there is no financial repression for $t \geq \hat{T}$. We do so by showing that eventually the multiplier on the sustainability constraint μ_t satisfies the no repression condition. Since taxes converge to a positive level, it follows from (33) that the multiplier $\{\mu_t\}$ must converge to zero. Thus, there is a finite period $\hat{T} = T(\beta, G_0)$ such that the multiplier μ_t satisfies the no repression condition, that is $\mu_t < A_B/h'(0, G_{t+1})$ for $t \geq T(\beta, G_0)$, so there is no financial repression after $\hat{T}$.

We next prove that our provisional assumption that $B_t > 0$ holds for all t if $\beta > \beta^*$. To show our claim we begin by showing that if $T_t > T^*(\beta)$, then $T_{t+1} \geq T^*(\beta)$. To prove the claim, suppose by way of contradiction that for some t, $T_t \geq T^*(\beta)$ and $T_{t+1}(\beta) < T^*(\beta)$, and suppose for now that period $t+1$ is a low spending state. The concavity of W implies that

$$-W'(T_t) > -W'(T^*(\beta)).$$

The fact that $T_{t+1} < T^*(\beta)$ implies that $B_{t+1} < B_L(\beta)$. To see this, notice that if $T_{t+1} < T^*(\beta)$, we must have perfect tax smoothing in all periods greater than or equal to $t+1$ so that taxes are constant and the amount of debt must be less than $B_L(\beta)$. This result implies that the sustainability constraint between t and $t+1$ is slack and $\mu_t = 0$. Then the necessary first-order condition (33) implies that

$$-W'(T_t) = -W'(T_{t+1}),$$

a contradiction. Then $T_t \geq T^*(\beta)$, and so in high spending periods, $B_t \geq B_H(\beta)$, and in low spending periods, $B_t \geq B_L(\beta)$. Since $B_H(\beta)$ and $B_L(\beta)$ are positive, debt levels are interior. *Q.E.D.*

Result 2. In the cyclic economy in which repression is prohibited, the level of debt from period one onward follows a two-period cycle.

Proof. Consider the following outcomes. The debt level sold into period 1, namely B_1, is set so that the sustainability constraint in period one holds with equality, that is,

$$(53) \quad H_S(B_1, G_L) = H_M(0, G_L).$$

The outcomes in all subsequent periods have debt sold into low spending states equal to B_1 and satisfy perfect tax smoothing in the sense that either taxes are constant in all periods or zero debt is sold into the high spending states. Since the sustainability constraint is relevant only for low spending states, and since the debt level in all low spending states is B_1, this outcome satisfies the sustainability constraint in all periods. To show that this outcome is optimal, note that the maximal amount of debt that can be sold into period 1 is B_1. Selling less debt than B_1 gives less tax smoothing and is not optimal. Selling less debt than B_1 in any subsequent low spending states also leads to less tax smoothing than the candidate allocation and therefore is not optimal. *Q.E.D.*

C. Household debt regime optimal

We turn now to providing conditions under which the household debt regime is optimal in all periods. To do so, let the *fiscal needs* of the government in period t, $F_t = B_t + G_t$, denote the sum of current spending and inherited debt. Clearly, if fiscal needs are sufficiently low the government need to issue relatively small amount of debt supported by reputation and no financial repression is needed. When the fiscal needs are large, the government finds it optimal to practice financial repression. It turns out that a sufficient condition for it to be optimal to choose the household debt is that fiscal needs are below a critical state dependent level denoted by F_L and F_H respectively. The reason that these critical levels are state dependent is not that they depend on the current spending but rather that they depend on the future path of spending. Thus this critical level is F_H is all even periods including period 0 and F_L in all odd periods.

Next we turn to some intuition for why the household debt regime is optimal when fiscal needs are not too large. In determining which regime to choose, the government compares the crowding out costs associated with choosing the household debt regime against those associated with the no household debt regime. In the household debt regime the interest on debt is $1/\beta$ whereas in the no household debt regime, under our assumption that taxing capital is not optimal, the government subsidizes banks to the maximum extent possible, that is it pays R_K on debt.

In comparing regimes, note that a regime is more attractive the lower are the total crowding out costs and the lower is the interest rate paid on debt, because the lower the interest rate the better is the tax smoothing. Since the interest rate on government debt is lower in the household debt regime, this gives an advantage to that regime. If the total crowding out costs in the household debt regime are also lower then that regime is preferred.

Consider then total crowding out costs in the two regimes, namely the product of the marginal crowding out cost per unit of bank debt times the total amount of bank debt. The marginal crowding out costs in the household regime $A_B(1/\beta)$ are higher than those in the no household debt regime, namely $A_B(R_K)$. Let us begin with fiscal needs that are only slightly above those in which no repression is optimal. If the government adopts the household regime, it needs to issue only a small amount of bank debt and so the total crowding out costs are small. In contrast, if the government adopts the no household debt regime, it places all of its debt in banks and doing so leads to a much greater level of total crowding out costs. Hence, in such a region, the household debt regime is optimal.

Now as fiscal needs increase, in the household debt regime the fraction of total debt held by banks also increases and the total crowding out costs rise faster than those in the no household debt regime. As long as the fraction of total debt held by banks stays below some critical value, the household debt regime is optimal. We show that this fraction is below the critical value if current needs are below F_L and F_H in the relevant state.

We begin by considering levels of bank debt B'_B and total debt B' such that at those arbitrary levels, the total crowding out costs in the two regimes are equal, namely

(54) $A_B(1/\beta)B'_B = A_B\left(R_K\right)B'.$

The left side is the crowding out cost of bank debt when banks are paid a rate $1/\beta$ on debt and the right side is the crowding out cost when all the debt is held by banks and banks are subsidized at rate R_K. Let ψ^* equal the ratio of B'_B/B' that satisfies (54), namely

$$(55)\quad \psi^* = \frac{A_B\left(R_K\right)}{A_B(1/\beta)} \in (0,1)\,.$$

We begin by providing sufficient conditions for the household debt regime to be optimal in period 0.

Proposition S1. If $\beta > \beta^*$ then there exists a critical value $F_H > G^*$ such that if $G_0 \in [0, F_H]$ then it is optimal to be in the household debt regime in period 0.

We establish the claim in the proposition using two lemmas. The first lemma shows that if the ratio of bank debt to total debt in the household debt regime is below the cutoff level ψ^*, then the household debt regime is optimal. The second lemma shows that if the initial level of spending is not too large then the ratio of bank debt to total debt in the household debt regime is below the cutoff level. Combining these lemmas gives the result. The bottom panel in Figure A1 may be helpful in understanding this result.

Let B'_{Bh} and B'_h be the solution to the best sustainable problem conditional on the government choosing the household debt regime, namely $H^h\left(B,G\right)$.

Lemma S2. If $B'_{Bh}/B'_h \leq \psi^*$ then $H^h\left(B,G\right) \geq H^b\left(B,G\right)$.

Proof. The idea of the proof is to construct a suboptimal policy in the household debt regime that is better than that in the no household debt regime. The suboptimal policy is the result of restricting the policies of the government to issuing household debt and bank debt in given proportions, that is, $B'_H = (1-\psi^*)B'$ and $B'_B = \psi^* B'$ and to restrict the total debt issue to be less than what was issued in the household debt regime, that is $B' \leq B'_h$.

Since $B'_{Bh}/B'_h \leq \psi^*$ then the sustainability is clearly satisfied for this restricted class of policies. The reason is that for such a class, total debt issued is lower and a higher fraction of that debt is held by banks. Since the sustainability constraint is satisfied we can drop it and write the value of the problem with these restricted policies as

$$(56)\quad \hat{H}\left(B,G\right) = \max_{B' \leq B'_h} W\left(G + B - \beta B'\right) - A_B\left(1/\beta\right)\psi^* B' + \beta H\left(B', G'\right).$$

We claim that in the solution to this problem the constraint $B' \leq B'_h$ is slack. The reason is that with the restricted policies issuing debt is more costly than with unrestricted policies, so less total debt is issued. Hence, $\hat{H}(0, G_0)$ equals the value of the problem in (56) without the constraint $B' \leq B'_h$.

Consider next the utility in the no household debt regime, given by

$$H^b(B, G) = \max_{B'} W\left(G + B - \frac{B'}{R_K}\right) - A_B(R_K)B' + \beta H(B', G')$$

subject to the sustainability constraint. Using the definition of ψ^* in (55), the value of this problem equals

$$(57) \quad H^b(B, G) = \max_{B'} W\left(B + G - \frac{B'}{R_K}\right) - A_B(1/\beta)\psi^* B' + \beta H(B', G')$$

subject to the sustainability constraint. Comparing the objective functions in (56) and (57) and noting that the (57) is more constrained, we have that $\hat{H}(0, G) \geq H^b(0, G)$. Thus, we have established

$$H^h(B, G) \geq \hat{H}(0, G) \geq H^b(B, G).$$

Q.E.D.

Lemma S3. There exists a critical value $F_H > G^*$ such that if $G_0 \leq F_H$ then in the household debt regime

$$\frac{B'_{B1}}{B'_1} \leq \psi^*.$$

Proof. In Proposition 3 we have shown for $G_0 \leq G^*$ there is no financial repression so households hold all the debt. For $G_0 > G^*$ it is optimal to practice financial repression and $B'_B > 0$. By continuity of the policy function, we know that there is some value of $F_H > G^*$ such that for $G_0 \in [G^*, F_H]$, the ratio of government debt held by banks to total government debt issued is less than

$$\frac{B'_B}{B'} < \psi^* = \frac{A_B(R_K)}{A_B(1/\beta)} \in (0,1).$$

Q.E.D.

Then for $G_0 \leq F_H$, we have that the crowding out costs of bank debt in the household branch are less than the crowding out costs in the bank branch. Thus, by Lemma S2 it is optimal to follow the household branch.

We turn now to providing conditions under which the household debt regime is optimal in all periods $t \geq 1$.

Proposition S2. If $\beta > \beta^*$ and

$$(58)\quad A_B(R_K) + \frac{A_N}{\sigma}\left(\left((1-\psi^*)\frac{1+R_K}{1+\beta} - 1\right)\frac{1}{R_K} + \beta\sigma\psi^*\right) > 0$$

then there exists a critical value $F_H > G^*$ such that if $G_0 < F_H$ then it is optimal to be in the household debt regime in all periods $t \geq 1$.

The logic for this result differs for even and odd periods. The logic for even periods is essentially identical to that for period 0 and does not require the sufficient condition (58). Indeed, all we need to show is that in any even period the fiscal needs are smaller than G_0. We establish this result in Lemma S4.

Lemma S4. If $G_0 \in (G^*, F_H)$ the household debt regime is optimal in all even periods.

Proof. From Proposition 3 we have that taxes and debt are decreasing over the cycle. Thus the fiscal needs in an even period are lower than the fiscal needs in period 0. We can now apply the same logic as in Lemma S3 with the sum of inherited debt B_t and G_H playing the role of G_0 to conclude that the household debt regime is optimal in all even periods. *Q.E.D.*

We turn now to establishing that the household debt regime is optimal in all odd periods.

Lemma S5. If (58) holds then the household debt regime is optimal in all odd periods.

Proof. Suppose by way of contradiction that (58) holds but in say, period 1, the no household debt regime is optimal so that

$$(59)\quad H^h(B_1, G_L) < H^b(B_1, G_L) = H(B_1, G_L),$$

$$(60)\quad H(B_1, G_L) = W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - A_B(R_K)\, B_2 + \beta H(B_2, G_H).$$

Note that the sustainability constraint for the programming problem in period 1 must be binding

$$(61)\quad H(B_2, G_H) = \max\{-A_N B_{B2} + H_M(0, G_H), H_M(B_{B2}, G_H)\} = h(B_{B2}, G_H),$$

because if it were not then the household debt regime would be preferred to the no household debt regime.

Next, consider the contrapositive of Lemma S2: if (59) holds then the household branch has $B_{B2} > \psi^* B_2$. Using this, (61) implies

$$(62)\quad H(B_2, G_H) < \max\{-A_N \psi^* B_2 + H_M(0, G_H), H_M(\psi^* B_2, G_H), \}$$

where we have used that $H_M(B_{B2}, G_H)$ is decreasing in B_{B2}. The first term in the maximand is the value of the Markov equilibrium when banks are not bailed out and the second term is the value when they are bailed out.

We establish a contradiction by showing that (62) cannot hold. We do so by considering separately the case in which the first term on the right side is larger and the case in which the second term is larger. In both cases we use the following inequalities

$$(63)\quad H(B_1, G_L) \geq h(B_{B1}, G_L) \geq h(\psi^* B_1, G_L).$$

Here the first inequality follows from the sustainability constraint in period 0 and the second inequality follows because $B_{B1} \leq \psi^* B_1$ from Lemma S3.

Case 1. Second term of (62) larger

Since the second term is larger then

$$(64)\quad H_M(\psi^* B_2, G_H) > -A_N \psi^* B_2 + H_M(0, G_H)$$

and

$$(65)\quad H_M(\psi^* B_2, G_H) > H(B_2, G_H).$$

Here we consider a possibly suboptimal policy in period 1 in a Markov equilibrium of issuing $\psi^* B_2$ units of debt. This policy is feasible because it satisfies the no default constraint since (64) holds. Under this suboptimal Markov policy

$$(66)\quad h\left(\psi^* B_1, G_L\right) \geq W\left(G_L + \psi^* B_1 - \frac{\psi^* B_2}{R_K}\right) - A_B\left(R_K\right)\psi^* B_2 + \beta H_M(\psi^* B_2, G_H).$$

Combining (66) with (63) implies

$$(67)\quad H\left(B_1, G_L\right) \geq W\left(G_L + \psi^* B_1 - \frac{\psi^* B_2}{R_K}\right) - A_B\left(R_K\right)\psi^* B_2 + \beta H_M(\psi^* B_2, G_H).$$

Combining (65) and (67) we obtain

$$(68)\quad H\left(B_1, G_L\right) > W\left(G_L + \psi^* B_1 - \frac{\psi^* B_2}{R_K}\right) - A_B\left(R_K\right)\psi^* B_2 + \beta H\left(B_2, G_H\right).$$

Substituting from (60) in (68) and rearranging gives

$$W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - W\left(G_L + \psi^* B_1 - \frac{\psi^* B_2}{R_K}\right) > A_B\left(R_K\right)\left(1 - \psi^*\right) B_2 > 0.$$

Since W is decreasing, to establish a contradiction it is sufficient to show that

$$\psi^*\left(B_1 - \frac{B_2}{R_K}\right) < B_1 - \frac{B_2}{R_K}$$

which, since $\psi^* \in (0,1)$, is equivalent to

$$(69) \quad B_1 > \frac{B_2}{R_K}$$

Now, the budget constraint in period 1 and 2 are

$$(70) \quad G_L + B_1 = T_1 + \frac{B_2}{R_K}$$

$$(71) \quad G_H + B_2 = T_2 + \beta B_3$$

where in (71) we used that the price of debt issued in period 2 is β because the government is in the household debt regime in even periods. Combining (70) and (71) and using that taxes are decreasing gives

$$\begin{aligned} B_1 &= (G_H - G_L) + (T_1 - T_2) + \frac{1+R_K}{R_K} B_2 - \beta B_3 \\ &\geq (G_H - G_L) + \frac{1+R_K}{R_K} B_2 - \beta B_1 \end{aligned}$$

where the second inequality follows from $T_1 \geq T_2$ and $B_1 \geq B_3$, which implies

$$(72) \quad B_1 \geq \frac{G_H - G_L}{1+\beta} + \frac{1+R_K}{1+\beta} \frac{B_2}{R_K} > \frac{G_H - G_L}{1+\beta} + \frac{B_2}{R_K} > \frac{B_2}{R_K}$$

From (69) we have a contradiction.

Case 2. First term of (62) larger, that is,

$$(73) \quad H_M(\psi^* B_2, G_H) < -A_N \psi^* B_2 + H_M(0, G_H)$$

Now consider a Markov equilibrium with a possibly suboptimal policy in period 1 of issuing no debt. Under this suboptimal policy, the following inequality must be satisfied

$$h\left(\psi^* B_1, G_L\right) \geq W\left(G_L + \psi^* B_1\right) + \beta H_M\left(0, G_H\right)$$

which, when combined with (63) and (64), implies

(74) $H\left(B_1, G_L\right) > W\left(G_L + \psi^* B_1\right) + \beta\left[H\left(B_2, G_H\right) + A_N \psi^* B_2\right].$

Substituting from (60) in (74) and rearranging

$$W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - W\left(G_L + \psi^* B_1\right) > \beta A_N \psi^* B_2 + A_B\left(R_K\right) B_2 > 0$$

Clearly, if $B_1 - \frac{B_2}{R_K} \geq \psi^* B_1$ we have a contradiction. So consider the case $B_1 - \frac{B_2}{R_K} < \psi^* B_1$. To get a contradiction it is sufficient to show that

(75) $0 > W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - W\left(G_L + \psi^* B_1\right) - \left[\beta A_N \psi^* + A_B\left(R_K\right)\right] B_2.$

To this end, note that

$$\begin{aligned} W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - W\left(G_L + \psi^* B_1\right) &= \int_{\left[B_1 - \frac{B_2}{R_K}, \psi^* B_1\right]} -W'\left(G_L + x\right) dx \\ &< -W'\left(G_L + \psi^* B_1\right)\left[\psi^* B_1 - B_1 + \frac{B_2}{R_K}\right] \end{aligned}$$

where the last inequality follows from the concavity of W. Then, using that it is not optimal to tax capital,

$$-W'\left(G_L + \psi^* B_1\right) < \frac{A_N}{\sigma},$$

the right side of (75) must satisfy the following inequality

$$\begin{aligned} & W\left(G_L + B_1 - \frac{B_2}{R_K}\right) - W\left(G_L + \psi^* B_1\right) - \left[\beta A_N \psi^* + A_B\left(R_K\right)\right] B_2 \\ & < -\frac{A_N}{\sigma}\left(1 - \psi^*\right) B_1 - \left[\beta A_N \psi^* + A_B\left(R_K\right) - \frac{A_N}{\sigma} \frac{1}{R_K}\right] B_2. \end{aligned}$$

Now,

$$
\begin{aligned}
&-\frac{A_N}{\sigma}\left(1-\psi^*\right)B_1-\left[\beta A_N\psi^*+A_B\left(R_K\right)-\frac{A_N}{\sigma}\frac{1}{R_K}\right]B_2 \\
&<-\frac{A_N}{\sigma}\left(1-\psi^*\right)\left(\frac{1+R_K}{1+\beta}\frac{B_2}{R_K}+\frac{G_H-G_L}{1+\beta}\right)-\left[\beta A_N\psi^*+A_B\left(R_K\right)-\frac{A_N}{\sigma}\frac{1}{R_K}\right]B_2 \\
&=-\frac{A_N}{\sigma}\left(1-\psi^*\right)\frac{G_H-G_L}{1+\beta}-\left[A_B\left(R_K\right)+\frac{A_N}{\sigma}\left(\left(\left(1-\psi^*\right)\frac{1+R_K}{1+\beta}-1\right)\frac{1}{R_K}+\beta\sigma\psi^*\right)\right]B_2<0
\end{aligned}
$$

where in the first inequality follows from (72) and the last inequality follows from the sufficient condition (58). We have established a contradiction. *Q.E.D.*

4. Details for Extensions

Here we give that the results of our benchmark model extend to versions with stochastic government spending, liquidity constraints, productivity shocks, nondiscriminatory default, and effective default on deposits.

A. Stochastic Government Spending

Here we extend our results to an environment with stochastic government spending. As will become clear, the result that repression is not optimal under commitment is general and does not depend on the nature of uncertainty. When there is no commitment, we show that in times of abnormally high fiscal needs, repression is desirable. To keep this analysis simple, we let government spending in period 0 be deterministic so that we can vary the level of spending in that period without having any effect on the distribution of government spending from then on. To that end, we let spending in period 0 be given by G_0. From period 1 onward, government spending is stochastic and is given by $G\left(s_t\right)$ where s_t is the state at $t \geq 1$ with N realizations. In particular, in period 1, the state s_1 is drawn from a distribution $\pi_1\left(s_1\right)$, whereas in all subsequent periods the state s_t is drawn from a Markov chain $\pi\left(s_{t+1}|s_t\right)$. This setup captures the idea that the initial period is one with exceptional fiscal needs such as high spending statetime or during a depression. After that initial period, the economy returns to normal times and the fiscal needs during those times are independent of those during the exceptional time at date 0. It will prove convenient to let

$$\hat{G}(s) = G(s) + \beta \sum_{s'} \pi(s'|s) G(s')$$

be the net present discounted value of government expenditure when today's state is s. Without loss of generality, order the states $s_1, ..., s_N$ so that if $s_2 > s_1$, then $\hat{G}(s_2) > \hat{G}(s_1)$.

An individual bank's budget constraint for $t \geq 1$ is now given by

$$\begin{aligned} & x_t(s^t) + (1 + \tau_{Kt}(s^t)) \frac{k_{t+1}(s^t)}{R_K} + \sum_{s^{t+1}} \left[\frac{b_{Bt+1}(s^{t+1})}{R_{t+1}(s^{t+1})} - \frac{d_{t+1}(s^{t+1})}{R_{Dt+1}(s^{t+1})} \right] \\ \leq \; & k_t(s^{t-1}) + \left[\delta_t(s^t) + (1 - \delta_t(s^t)) \iota_t(s^t) \right] b_{Bt}(s^t) - d_t(s^t), \end{aligned}$$

where $s^t = (s_1, \ldots, s_t)$, and $b_{Bt+1}(s^{t+1})$ and $d_{t+1}(s^{t+1})$ are state-contingent government debt and deposits purchased at state s^t promising interest rates $R_{t+1}(s^{t+1})$ and $R_{Dt+1}(s^{t+1})$. For each s^{t+1} the collateral constraint is

$$d_{t+1}(s^{t+1}) \leq \gamma \left[k_{t+1}(s^t) + \left[\delta_{t+1}(s^{t+1}) + (1 - \delta_{t+1}(s^{t+1})) \iota_{t+1}(s^{t+1}) \right] b_{Bt+1}(s^{t+1}) \right]$$

and the regulatory constraint is

$$b_{Bt+1}(s^{t+1}) \geq \phi_t(s^{t+1}) \left[k_{t+1}(s^t) + b_{Bt+1}(s^{t+1}) \right].$$

The bank's problem for period 0 is similar. The household problem is extended in a similar fashion.

The following extension of Proposition 1 is immediate. Indeed, this extension holds for any arbitrary process for the state s_t.

Proposition A1. (Financial Repression Not Optimal with Commitment) The Ramsey outcome has no financial repression, that is, $\phi_t(s^{t+1}) = 0$ for all s^{t+1}. Furthermore, for all t the collateral constraint on banks binds and the effective return on government debt, $R^e_{t+1}(s^{t+1})$, is strictly less than the return on capital so that banks hold no government debt.

The extension of Propositions 3 is also relatively straightforward. To that end, let β^* be the smallest discount factor such that a stationary Ramsey outcome with positive debt in all states is sustainable. To make precise our definition of β^*, consider a vector of debt $\{\underline{B}_R(s_n)\}$ that satisfies $\underline{B}_R(s_N) = 0$, and $\underline{B}_R(s_n) = \hat{G}(s_N) - \hat{G}(s_n)$. If that vector is issued in a given period, say t, then for any subsequent period there will be perfect tax smoothing. The associated tax revenue is constant across all future periods and states s_n and is given by $\underline{T}_R = (1-\beta)\hat{G}(s_N)$. Then β^* is the smallest discount factor that makes the government at least as well of in all states $\{s_n\}$ if it continues with Ramsey plan with the associated inherited debt $\{\underline{B}_R(s_n)\}$ and a plan of defaulting today in s_n and then pursuing the Markov policy from then onward. That is, β^* satisfies

$$\frac{W(\underline{T}_R)}{1-\beta^*} = \max_s \left\{ W(G(s)) + \beta \sum_{s'} \pi(s'|s) H_M(0, G(s')) \right\}.$$

We assume that $\beta \geq \beta^*$ and that it is optimal to be in the household debt regime and note that the sufficient conditions for this to be true are the analogs of those in Proposition S1.

Proposition A2. (Repression in the Best Sustainable Equilibrium) In the best sustainable equilibrium there exists a value G^* of government spending, such that if $G_0 > G^*$, there is financial repression in period 0 and a finite period $T = T(\beta, G_0) \geq 1$ such that there is no financial repression for $t \geq T$ for all histories s^T. Furthermore, government debt is decreasing over each cycle in that $B_t(s^t, s) \geq B_{t+1}(s^{t+r}, s)$ for all $t, s^{t-1}, r > 0$. Moreover, in the limit the economy converges to a limiting cycle with perfect tax smoothing. Instead if G_0 is sufficiently small then there is no financial repression in any period.

Proof. The programming problem for the stochastic economy in period 0 is

$$(76) \quad H_{S0}(0, G_0) = \max_{B'_B, B'} W(T) - A_B \sum_{G'} \pi_1(G') B'_B(G') + \beta \sum_{G'} \pi_1(G') H_S(B', G')$$

subject to

$$\mu_1(G') : H_S(B'(G'), G') \geq h(B'_B(G'), G') = \max\{H_M(B'_B(G'), G'), H_M(0, G') - A_N B'_B(G')\}$$

and the nonnegativity constraints on $B'_H(G')$ and $B'_B(G')$ where $T = G_0 - \beta \sum_{G'} \pi_1(G') B'(G')$.

The first-order conditions for each $B'(G')$ and $B'_B(G')$ are

$$(77)\quad -\beta W'(T) + (\beta + \mu_1(G'))\, H'_S(B'(G'), G') \leq 0$$

$$(78)\quad -A_B + \mu_1(G')\, h'(B_B(G'), G') \leq 0,$$

where (77) and (78) hold with equality if $B(G')$ and $B_{B1}(G')$ are positive, respectively.

We now show that there is a critical value G^* such that for all $G_0 > G^*$, the government does practice financial repression in at least one state. To define the critical value G^*, note that the maximal amount of debt that the government can issue in period 0 without practicing financial repression is the vector $\{B_1^*(G')\}$ with expected value $B_1^* = \sum_{G'} \pi_1(G')\, B_1^*(G')$, where each $B_1^*(G')$ solves

$$(79)\quad H_S(B_1^*(G'), G') = H_M(0, G').$$

Given B_1^* defined from (79), let G^* be defined by

$$-\beta W'(G^* - \beta B_1^*) + \min_{G'} \left[\left(\beta + \frac{A_B}{h'(0, G')} \right) H'_S(B_1^*(G'), G') \right] = 0.$$

Here G^* is the level of government spending such that a government, on the margin, is indifferent to repressing or not in the state G' at which it has the highest incentives to repress when it has issued $\{B_1^*(G')\}$. Clearly, using the same contradiction argument as in the proof of Proposition 3, the equilibrium has financial repression if $G_0 > G^*$.

Next we characterize the transition after period 0 assuming that $G_0 > G^*$. Clearly, from our definition of β^*, if $\beta > \beta^*$ government debt is always positive. The proof is similar to that for the deterministic case.

To characterize this transition, consider the best sustainable problem

$$(80)\quad H_S(B, s) = \max_{B'_B, B', T} W(T) - A_B \sum_{s'} \pi(s'|s)\, B'_B(s') + \beta \sum_{s'} \pi(s'|s)\, H_S(B'(s'), s')$$

subject to

$$(81)\quad G(s) + B = T + \beta \sum_{s'} \pi(s'|s) B'(s')$$

$$(82)\quad H_S(B'(s'), s') \geq h(B_B(s'), s') = \max\{H_M(B'_B(s'), s'), H_M(0, s') - A_N B'_B(s')\}.$$

Letting $\mu(s')$ denote the multiplier on the sustainability constraint (82) in state s', the first-order condition for the problem (80) implies that if $B(s^{t+1})$ is strictly positive then

$$(83)\quad -\beta W'(T(s^t)) = -[\beta + \mu(s^{t+1})] H'_S(B(s^{t+1}), G(s_{t+1}))$$

$$(84)\quad = -[\beta + \mu(s^{t+1})] W'(T(s^{t+1})),$$

where the second equality follows from the envelope condition on (80), and the first-order condition for $B_B(s^{t+1})$ implies that

$$(85)\quad \mu(s^{t+1}) \leq -A_B/h'(B_B(s^{t+1}), s_{t+1}).$$

Note, there is financial repression in that $B_B(s^{t+1}) > 0$, only if $\mu(s^{t+1}) = -A_B/h'(B_B(s^{t+1}), s_{t+1})$.

Since the multiplier on the sustainability constraint $\mu(s^{t+1}) \geq 0$ for all s^{t+1}, from (83) it follows that the tax revenues $\{T(s^t)\}$ are decreasing. Such a path for tax revenues implies that debt is decreasing over each cycle. To see why, iterate the government budget constraint forward to write that debt is the discounted value of future government surpluses, which, using the cyclical pattern of government spending, gives

$$(86)\quad B(s^t) = \sum_{r=0}^{\infty} \sum_{s^{t+r}} \beta^r \pi(s^{t+r}|s^t) T(s^{t+r}) - \hat{G}(s_t)$$

where $\pi(s^{t+r}|s^r)$ is induced from $\pi(s^{r+1}|s^r)$. Next, since the sequence of tax revenues $\{T(s^t)\}$ is decreasing and bounded below by zero, these tax revenues must converge to a positive level so that taxes are perfectly smoothed in the limit. From (83) it then follows that the sequence of multipliers $\{\mu(s^{t+1})\}$ must converge to zero. Thus, there is a finite $T(\beta, G_0)$ such that for $t+1 \geq T(\beta, G_0)$, the multiplier $\mu(s^{t+1}) < \min_{B_B, s}\{-A_B/h'(B_B, s)\}$, so there is no financial repression. $Q.E.D.$

B. Liquidity Constraints

In the best sustainable equilibrium banks never voluntarily hold government. Here we introduce liquidity considerations in the bank's asset allocations by assuming that bank debt yields liquidity services above those provided by capital. We model these liquidity services as a necessary part of the financial intermediation services offered by banks by assuming that if banks hold at least a fraction κ of their capital stock in the form of government debt then they can intermediate funds efficiently so that the return on intermediated capital is R_K whereas if they hold less than a fraction κ the return on intermediated capital is smaller, namely $\hat{R}_K < R_K$. We assume throughout that κ and $\hat{R}_K$ are sufficiently small so that each bank voluntarily chooses to hold government debt and provide intermediation services efficiently. Under this assumption each bank chooses bank debt so that $b_{Bt+1} \geq \kappa k_{t+1}$.

(An alternative assumption is that government debt held by a bank must exceed some fraction of bank liabilities in order to provide intermediation services efficiently. This alternative assumption is consistent with a variation of our environment along the lines a Diamond-Dybvig model in which some depositors may want to withdraw early and government debt can be sold at better prices than capital in order to meet the demands of depositor. We conjecture that our results will go through under this alternative assumption.)

We can follow the same steps as we did in the body to show that the value function in the best sustainable equilibrium has the form

$$V(S) = A_R + K + A_{N0}N + H(N, B, G)$$

where the tax distortion function with liquidity constraints is given by

$$H(N,B,G) = \max_{B'_B, B'_H, R'} W(T) - A_B(R')B'_B + \beta H(N', B'_B + B'_H, G') \tag{87}$$

subject to

$$T = G_t + B - \frac{B'_B + B'_H}{R'}$$

$$A_R + K' + A_N N' + H\left(N', B'_B + B'_H, G'\right) \geq J_M\left(S', G'\right)$$

$R' \leq R_K$, the complementary slackness condition and the aggregate form of the liquidity constraint

$$B'_B \geq \kappa K'$$

where K' and N' are given by

$$\left[\frac{1}{R_K} + \frac{B'_B}{K'}\frac{1}{R'} - \gamma\beta\left(1 + \frac{R^e}{R'}\frac{B'_B}{K'}\right)\right] K' = \sigma N + (1-\sigma)\bar{n}$$

$$N' = (1-\gamma)\left[1 + \frac{R^e_{t+1}}{R_{t+1}}\frac{B'_B}{K'}\right] K'.$$

Proposition A3. (Dynamics of the Best Sustainable Equilibrium with Liquidity Constraints) For κ sufficiently small, there is a critical value β^* such that for $\beta \geq \beta^*$, there is a finite period $T = T(\beta, G_0)$ such that there is no financial repression for $t \geq T$. Furthermore, if $G_0 > G^*(\beta)$, then $T > 0$ and up until period T, government debt is decreasing over each cycle. Moreover, in the limit, the economy converges to a Ramsey outcome with perfect tax smoothing .

The proof follows immediately from our earlier propositions. In Figure A2 we provide a numerical example which illustrates this proposition.[1]

C. Productivity Shocks

We now consider a version of the model with constant government spending and fluctuating labor productivity and argue that all of results go through.

We assume that output is produced by

$$Y = ZL + K.$$

where labor productivity Z deterministically fluctuates between a high productivity state and a low productivity state. In particular, for all $t \geq 1$ we have $Z_t = Z_L$ if t is odd and

[1] In Figure A1, $G_H = .24$, $G_L = .05, \beta = 99, R_K = 1.02, \gamma = .5, \sigma = .9$, $v(L) = L^{\xi}/\xi$ where $\xi = 2$.

$Z_t = Z_H$ if t is even. In period zero we allow Z_0 to be arbitrary. With productivity Z the function net utility from labor $W(T, Z)$ is given by

$$W(T, Z) = Z\ell(T, Z) - v(\ell(T, Z))$$

where $\ell(T, Z)$ solves $T = Z\ell - v'(\ell)\ell$. Note that the domain of $W(T, Z)$ is $[0, T_{max}(Z)]$ where $T_{max}(Z)$ is the top of the static Laffer curve.

We assume that

$$\text{(88)} \quad W(T, Z) \text{ is concave in } T \text{ and } -W_T(T, Z) \text{ is decreasing in } Z$$

Recall that the key properties that we use in proving our propositions were the analogs of those in (88). Given these properties our proofs go through virtually unchanged. Hence we have the following proposition.

Proposition P1. (Repression in the Best Sustainable Equilibrium) Under (88), in the best sustainable equilibrium there exists a value Z^* of productivity, such that if $Z_0 \geq Z^*$ there is no financial repression in period 0. If $Z_0 < Z^*$, there is a finite period $T = T(\beta, Z_0)$ such that there is no financial repression for $t \geq T$. Furthermore, government debt is decreasing over each cycle.

In the next lemma we provide sufficient conditions for (88).

Lemma P1. If $v(L) = \chi L^{1+\psi}/(1+\psi)$ then (88) holds.

Proof. First, note that

$$\text{(89)} \quad W_T(T, Z) = [Z - v'(\ell)]\,\ell_T < 0$$

since $\ell_T < 0$ on the increasing side of the Laffer curve. Second, note that

$$W_{TZ}(T, Z) = [1 - v''(\ell)\,\ell_Z]\,\ell_T + [Z - v'(\ell)]\,\ell_{TZ}.$$

Under $v(L) = \chi L^{1+\psi}/(1+\psi)$ we have

$$T = [Z - v'(\ell)]\,\ell = Z\ell - (1+\psi)\,v(\ell) = Z\ell - \chi\ell^{1+\psi} \text{ so}$$

$$\ell_T = \frac{1}{Z - (1+\psi)\,\chi\ell^{\psi}} = \frac{1}{Z - (1+\psi)\,v'(\ell)} < 0$$

$$\ell_Z = -\frac{\ell}{Z - (1+\psi)\,\chi\ell^{\psi}} = -\ell\ell_T > 0$$

and then

$$\ell_{TZ} = -\left[\frac{1}{Z - (1+\psi)\,\chi\ell^{\psi}}\right]^2 \left[1 - (1+\psi)\,\psi\chi\ell^{\psi-1}\ell_Z\right]$$
$$= -\ell_T \frac{1 - (1+\psi)\,v''(\ell)\,\ell_Z}{Z - (1+\psi)\,v'(\ell)}.$$

Thus

$$\begin{aligned} W_{TZ}(T,Z) &= [1 - v''(\ell)\,\ell_Z]\,\ell_T + [Z - v'(\ell)]\,\ell_{TZ} \\ &= [1 - v''(\ell)\,\ell_Z]\,\ell_T - [Z - v'(\ell)]\,\ell_T \frac{1 - (1+\psi)\,v''(\ell)\,\ell_Z}{Z - (1+\psi)\,v'(\ell)} \\ &= \left[\frac{-\psi v'(\ell)}{Z - (1+\psi)\,v'(\ell)}\right] + (-\ell_Z\ell_T)\left[\frac{-Z\psi}{Z - (1+\psi)\,v'(\ell)}\right] v''(\ell) > 0 \end{aligned}$$

where the last inequality follows because $Z - (1+\psi)\,v'(\ell) < 0$ since taxes are on the increasing portion of the Laffer curve.

Finally, we check concavity of W in T, namely that $W_{TT} < 0$. From (89) we have

$$W_{TT}(T,Z) = [Z - v'(\ell)]\,\ell_{TT} - v''(\ell)\,\ell_T^2$$

and

$$\ell_{TT} = -\left(\frac{1}{Z - (1+\psi)\,v'(\ell)}\right)^2 \left[-(1+\psi)\,v''(\ell)\,\ell_T\right] = \frac{(1+\psi)\,v''(\ell)\,\ell_T}{\left(Z - (1+\psi)\,v'(\ell)\right)^2} = \frac{(1+\psi)\,v''(\ell)}{Z - (1+\psi)\,v'(\ell)} < 0$$

so

$$W_{TT}(T,Z) = [Z - v'(\ell)]\,\ell_{TT} - v''(\ell)\,\ell_T^2 < 0$$

because both terms are negative since both $Z - v'(\ell) \geq 0$ and $\ell_{TT} < 0$, and $v'' > 0$. *Q.E.D.*

D. Nondiscriminatory Default

Here we state a proposition characterizing the best sustainable equilibrium absent bailouts to banks. This proposition makes clear that the results for nondiscriminatory default are similar to those with discriminatory default.

Proposition A4. (Dynamics of the Best Sustainable Equilibrium with Nondiscrimination) There is a critical value β^* such that for $\beta \geq \beta^*$, there is a finite period $T = T(\beta, G_0)$ such that there is no financial repression for $t \geq T$. Furthermore, if $G_0 > G^*(\beta)$, then $T > 0$ and up until period T, government debt is decreasing over each cycle and the fraction of government debt held by banks decreases over the cycle. Moreover, in the limit the economy converges to a Ramsey outcome with perfect tax smoothing .

The proof builds on the logic in Proposition 3 and is available on request.

E. Effective Default on Deposits

Here we consider an extension in which the government can effectively allow banks to default on deposits. This default can also be interpreted as a policy in which the government taxes away deposits of each bank and returns the proceeds to that bank.

Markov Equilibrium

We start by analyzing the Markov equilibrium. As argued in the text, the government will always default on debt held by the households and so households will hold no government debt along the equilibrium path. Since the banks are collateral constrained, it is always ex post optimal to allow banks to default on its deposits and so households will hold no deposits along the equilibrium path.

Following a similar logic as in the baseline case, the value of a Markov equilibrium for an arbitrary state $S = (K, D, B_B, B_H, G)$ is given by

$$J_M(S) = \max\{V_{Mt}(K,0,0,0,G), V_M(K,0,B_B,0,G)\}$$

The first argument of the max operator is associated with a policy of defaulting on debt, allowing banks to default on deposits, and not bailing out banks. The second argument of the max operator is associated with a policy of defaulting on debt, allowing banks to default on deposits, and bailing out banks.

Analogously to the baseline case, we can prove the following lemma.

Lemma D1. The value for the Markov equilibrium has the following form:

$$V_M(K,0,B_B,0,G) = \hat{A}_R + K + \hat{A}_N(K + B_B) + H_M(B_B, G)$$

where the tax distortion function solves the functional equation:

$$H_M(B_B, G) = \max_{B'_B, T, R', T_K} W(T) - \hat{A}_B(R') B'_B + \beta H_M(B'_B, G')$$

subject to the government budget constraint $G + B_B \leq T + B'_B/R'$, the constraint $R' \leq R_K$ and the no temptation to default on banks constraint

$$H_M(B'_B, G') + \hat{A}_N B'_B \geq H_M(0, G')$$

where the constants are

$$\text{(90)} \qquad \begin{aligned} \hat{A}_N &= \frac{(\beta R_K - 1)\sigma}{1 - \beta R_K \sigma} < A_N = \frac{(\beta R_K - 1)\sigma}{1 - \beta R_K[\sigma + (1-\sigma)\gamma]} \\ \hat{A}_R &= \frac{(1-\sigma)\bar{n}}{(1-\beta)\sigma}\hat{A}_N < A_R = \frac{(1-\sigma)\bar{n}}{(1-\beta)\sigma}A_N \\ \hat{A}_B(R) &= \frac{\hat{A}_N}{\sigma}\frac{1}{R} - \beta\hat{A}_N \end{aligned}$$

Proof. Since banks cannot raise deposits the Markov equilibrium is equivalent to one in which $\gamma = 0$. Hence, $\hat{A}_N$, $\hat{A}_R$, and $\hat{A}_B(R)$ are simply our earlier expressions for A_N, A_R, and $A_B(R)$ with γ set to zero.

Best Sustainable Equilibrium

Consider now the best sustainable equilibrium. One possibility is that in the best sustainable equilibrium the government allows banks to default on deposits. In this case, the analysis of our baseline case goes through unchanged since this case is equivalent to setting $\gamma = 0$ in the baseline model.

The other, more interesting possibility, is that on the equilibrium path, from period 1 onward, it is optimal not to allow banks to default on deposits so banks issue deposits at interest rate $R_D = 1/\beta$. Following the same logic as in the baseline case, the value of the best sustainable outcome in period 0 is

$$J_S(S) = \max\{V_S(K,0,0,0,G), V_S(K,0,B_B,0,G)\}$$

Clearly in period 0 is always optimal to default on debt and allow banks to default on deposits as well. The first argument is associated with a policy of no bailouts and the second argument is associated with a policy of bailouts. The continuation value $V_S(S)$ can be written as

$$V_S(S) = \max_{T,K',B'_B,B'_H,R',N'} W(T) + K - \frac{K'}{R_K} + \beta V_S(S')$$

subject to

$$T = G + B - \frac{B'_B + B'_H}{R'}$$

$$\frac{K'}{R_K} = \frac{\sigma N + (1-\sigma)\bar{n}}{(1-\gamma\beta R_K)} - \frac{\left(\frac{1}{R'} - \gamma\beta\right)}{(1-\gamma\beta R_K)} B'_B$$
$$N' = (1-\gamma)(K' + B'_B)$$

and

(91) $V_S(S') \geq J_M(S')$

Here, unlike in the baseline case, the tax distortion function needs to include net worth and we have the following lemma.

Lemma D2. The continuation value for the best sustainable equilibrium has the form

(92) $V_S(K, B_H, B_B, 0, G) = A_R + K + A_N N + H_S(N, B_B + B_H, G)$

where $N = (1-\gamma)(K + B_B)$ and $H_S(N, B, G)$ is the largest fixed point of

(93) $$H_S(N, B, G) = \max_{T, K', B'_B, B'_H, R', N'} W(T) - A_B(R') B'_B + \beta H_S(N', B'_B + B'_H, G')$$

subject to the budget constraint,

$$G + B \leq T + \frac{B'_B + B'_H}{R},$$

the constraint $R' \leq R_K$, the complementary slackness and

(94) $$\frac{K'}{R_K} = \frac{\sigma N + (1-\sigma)\bar{n}}{1 - \gamma\beta R_K} - \frac{1/R' - \gamma\beta}{1 - \gamma\beta R_K} B'_B$$

(95) $N' = (1-\gamma)(K' + B'_B)$

and the sustainability constraint

(96) $$\begin{aligned} &A_R + A_N(1-\gamma)(K' + B'_B) + H_S(N', B'_B + B'_H, G') \\ &\geq J_M(K', B'_H, 0, G') = \max\{V_{Mt}(K', 0, 0, 0, G'), V_M(K', 0, B'_B, 0, G')\} \\ &= \hat{A}_R + \hat{A}_N K' + \max\left\{\hat{A}_N B'_B + H_M(B'_B, G'), H_M(0, G')\right\} \end{aligned}$$

The proof of this lemma is nearly identical to that in the baseline case.

Next, we want to show that moving one unit of debt from households to banks relaxes the sustainability constraint. To do so we use the following sufficient condition

(97) $$A < A_N - \left(A_N(1-\gamma) - \hat{A}_N\right) R_K \frac{\beta(1-\gamma)}{1 - \gamma\beta R_K}$$

where A is defined below. Throughout we focus on the case in which the government does not bail out banks in the deviation to the Markov equilibrium. The argument for the case in which the government bails out banks in this deviation is similar.

Lemma D3. If the sufficient condition (97) is met then moving one unit of debt from households to banks relaxes the sustainability constraint.

Proof. First note that we can rewrite (96)

$$(98)\quad \left(A_R - \hat{A}_R\right) + \left(A_N\left(1-\gamma\right) - \hat{A}_N\right) K' + A_N\left(1-\gamma\right) B'_B + H_S\left(N', B'_B + B'_H, G'\right) \geq H_M\left(0, G'\right),$$

Next, since household choices are interior $R = 1/\beta$ so that using (94) in (98) we obtain

$$(99)\quad \left(A_R - \hat{A}_R\right) + \left(A_N\left(1-\gamma\right) - \hat{A}_N\right) R_K \left[\frac{\sigma N + (1-\sigma)\bar{n}}{1-\gamma\beta R_K} - \frac{\beta\left(1-\gamma\right)}{1-\gamma\beta R_K} B'_B\right] + A_N\left(1-\gamma\right) B'_B + H_S\left(N', B'_B + B'_H, G'\right) \geq H_M\left(0, G'\right),$$

or collecting terms:

$$(100)\quad \alpha\left(N\right) + \left[A_N - \left(A_N\left(1-\gamma\right) - \hat{A}_N\right) R_K \frac{\beta\left(1-\gamma\right)}{1-\gamma\beta R_K}\right] B'_B + H_S\left(N', B'_B + B'_H, G'\right) \geq H_M\left(0, G'\right)$$

where

$$\alpha\left(N\right) = \left(A_R - \hat{A}_R\right) + \left(A_N\left(1-\gamma\right) - \hat{A}_N\right) R_K \frac{\sigma N + (1-\sigma)\bar{n}}{1-\gamma\beta R_K}$$

Thus, to prove that (100) is relaxed by moving one unit of debt from households to banks it is sufficient to show that

$$(101)\quad \left[A_N - \left(A_N\left(1-\gamma\right) - \hat{A}_N\right) R_K \frac{\beta\left(1-\gamma\right)}{1-\gamma\beta R_K}\right] + \frac{\partial H_S}{\partial N'}\frac{\partial N'}{\partial B'_B} > 0$$

We know that $\frac{\partial N'}{\partial B'_B} < 0$ and

$$\frac{\partial H_S}{\partial N'} = \mu' \alpha' \left(N'\right) = \mu' \frac{\left(A_N \left(1-\gamma\right) - \hat{A}_N\right) R_K \sigma}{1 - \gamma \beta R_K}$$

where μ' is the multiplier on the sustainability constraint. Using the definition of A_N and $\hat{A}_N$ in (90) we have

$$\frac{\partial H_S}{\partial N'} = \mu' \frac{\sigma}{1-\gamma\beta R_K} \left(\beta R_K - 1\right) R_K \sigma \left(\frac{\left(1-\gamma\right)}{1 - \beta R_K \left[\sigma + \left(1-\sigma\right)\gamma\right]} - \frac{1}{1-\beta R_K \sigma} \right)$$

Thus

$$\begin{aligned} \operatorname{sign}\left(\frac{\partial H_S}{\partial N'}\right) &= \operatorname{sign}\left(\left(1-\gamma\right)\left(1-\beta R_K \sigma\right) - 1 + \beta R_K \left[\sigma + \left(1-\sigma\right)\gamma\right]\right) \\ &= \operatorname{sign}\left(\gamma \left[\beta R_K - 1\right]\right) = + \end{aligned}$$

Intuitively: the marginal value of net worth N is higher in the best sustainable equilibrium than in the Markov equilibrium and so high net worth relaxes the sustainability constraint. So the term $\frac{\partial H_S}{\partial N'} \frac{\partial N'}{\partial B'_B}$ goes in the wrong direction. However, we can bound this term because

$$(102) \quad \mu' \leq \frac{A_B}{h'\left(B'_B, G'\right)}$$

and

$$\begin{aligned} N' &= \left(1-\gamma\right)\left(R_K \left(\frac{\sigma N + \left(1-\sigma\right)\bar{n}}{1-\gamma\beta R_K} - \frac{1/R' - \gamma\beta}{1-\gamma\beta R_K} B'_B\right) + B'_B\right) \\ &= -\left(1-\gamma\right)\left(R_K \frac{1/R' - \gamma\beta}{1-\gamma\beta R_K} - 1\right) B'_B + \text{terms indep. of } B'_B \end{aligned}$$

so using $R' = 1/\beta$ we have

$$\frac{\partial N'}{\partial B'_B} = -\left(1-\gamma\right)\left(\frac{R_K\left(1-\gamma\right)\beta}{1-\gamma\beta R_K} - 1\right).$$

Thus,

$$
\begin{aligned}
\left|\frac{\partial H_S}{\partial N'}\frac{\partial N'}{\partial B_B'}\right| &= \mu' \frac{\sigma}{1-\gamma\beta R_K}\left(\beta R_K - 1\right)\sigma R_K\left(\frac{(1-\gamma)}{1-\beta R_K\left[\sigma+(1-\sigma)\gamma\right]} - \frac{1}{1-\beta R_K\sigma}\right) \\
&\times (1-\gamma)\left(\frac{R_K(1-\gamma)\beta}{1-\gamma\beta R_K} - 1\right) \\
&\leq \frac{A_B}{-h'\left(B_B', G'\right)}\frac{\sigma}{1-\gamma\beta R_K}\left(\beta R_K - 1\right)\sigma R_K\left(\frac{(1-\gamma)}{1-\beta R_K\left[\sigma+(1-\sigma)\gamma\right]} - \frac{1}{1-\beta R_K\sigma}\right) \\
&\times (1-\gamma)\left(\frac{R_K(1-\gamma)\beta}{1-\gamma\beta R_K} - 1\right) \\
&\equiv A
\end{aligned}
$$

where the inequality follows from the bound on μ' from (102) and

$$
-h'\left(B_B', G'\right) = \hat{A}_N
$$

So the sufficient condition is

$$
(103)\quad A < A_N - \left(A_N(1-\gamma) - \hat{A}_N\right) R_K \frac{\beta(1-\gamma)}{1-\gamma\beta R_K}
$$

Q.E.D.

In the two panels of Figure A3 we plot the total debt and the share of debt held by banks in an economy that begins with exceptional fiscal needs and the returns to normal times. Clearly, allowing for effective default on deposits does not change the qualitative pattern of the main results from our benchmark model.

5. Detailed Algebra for Best Sustainable Equilibirum

Here we fill in additional details for the proof of Lemma 3. Consider the operator defined by the right side of (40), namely

$$
(104)\quad \mathbb{T}V(S) = \max_{B_B', B_H', T, R', K'} W(T) + K - \frac{K'}{R_K} + \beta V_{St}\left(S'\right)
$$

subject to the government budget constraint, the bank budget constraint, the restriction on return, $R' \leq R_K$, the complementary slackness condition, namely, if $R' > 1/\beta$ then $B_H' = 0$

and the sustainability constraint

(105) $V_{St+1}(S') \geq A_R + K + A_N N' + h(B'_B, G')$

where $h(B'_B, G') \equiv \max\{-A_N B'_B + H_{Mt+1}(0, G'), H_{Mt+1}(B'_B, G')\}$ and the value of the right side of (105) is the value of the best deviation, $J_{Mt+1}(S') = A_R + K + A_N N' + h(B'_B, G')$.

We need to show that the sequence of functions $\{V_n\}$ defined recursively as $V_{n+1} = \mathbb{T}V_n$ from an initial value $H_0(B, G)$ given by that of the Ramsey equilibrium $H_R(B, G)$ have the form

(106) $V_n = A_R + K + A_N N + H_n(B, G)$

Consider the first iteration:

$$V_1(S) = \max_{B'_B, B'_H, T, R', K', N'} W(T) + K - \frac{K'}{R_K} + \beta\left[A_R + K' + A_N N' + H_0(B', G')\right]$$

subject to

$$T = G + B - \frac{B'_B + B'_H}{R'},$$

(107) $A_R + K' + A_N N' + H_0(B', G') \geq A_R + K' + A_N N' + h(B'_B, G')$,

$R' \leq R_K$, and the complementary slackness condition, where K' and N' are given by

$$\frac{K'}{R_K} = \frac{\sigma N + (1-\sigma)\bar{n}}{[1 - \gamma\beta R_K]} - \frac{\left[\frac{1}{R'} - \gamma\beta\right]}{[1 - \gamma\beta R_K]} B'_B$$

(108) $N' = (1-\gamma)\left[1 + \frac{B'_B}{K'}\right] K'$.

Note that (107) simplifies to

(109) $H_0(B', G') \geq h(B'_B, G')$,

Rearranging and using (108) to substitute out for N' gives

$$V_1(S) = \max_{B'_B, B'_H, T, R', K'} K + (\beta R_K - 1)\frac{K'}{R_K} + \beta A_R + \beta A_N(1-\gamma)K' + \beta A_N(1-\gamma)B'_B$$

$$+ W(T) + \beta H_0(B', G')$$

$$= \max_{B'_B, B'_H, T, R', K'} \beta A_R + K + [(\beta R_K - 1) + \beta R_K A_N(1-\gamma)]\frac{K'}{R_K}$$

$$+ W(T) + \beta H_0(B', G') + \beta A_N(1-\gamma)B'_B$$

subject $R' \leq R_K$,

$$T = G + B - \frac{B'_B + B'_H}{R'}$$

and

$$H_0(B', G') \geq h(B'_B, G')$$

which follows from (109), together with the transition

$$\frac{K'}{R_K} = \frac{\sigma N + (1-\sigma)\bar{n}}{[1-\gamma\beta R_K]} - \frac{\left[\frac{1}{R'} - \gamma\beta\right]}{[1-\gamma\beta R_K]}B'_B. \quad (110)$$

Rearranging again using (110) to substitute out for K' gives

$$V_1(S) = \max_{B'_B, B'_H, T, R'} \beta A_R + K + [(\beta R_K - 1) + \beta R_K A_N(1-\gamma)]\frac{\sigma N + (1-\sigma)\bar{n}}{[1-\gamma\beta R_K]}$$

$$+ W(T) + \beta H_0(B', G') + \beta A_N(1-\gamma)B'_B$$

$$- [(\beta R_K - 1) + \beta R_K A_N(1-\gamma)]\frac{\left[\frac{1}{R'} - \gamma\beta\right]}{[1-\gamma\beta R_K]}B'_B$$

subject to $R' \leq R_K$,

$$T = G + B - \frac{B'_B + B'_H}{R'}$$

$$H_0(B', G') \geq h(B'_B, G')$$

Thus, collecting terms, we obtain:

$$
\begin{aligned}
V_1(S) = &\max_{B'_B, B'_H, T_K, T, R'} \left[\beta A_R + \left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right] \frac{(1-\sigma)\bar{n}}{[1-\gamma\beta R_K]}\right] + K \\
&+ \frac{\left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right]\sigma}{[1-\gamma\beta R_K]} N \\
&+ W(T) + \beta H_0(B', G') - A_B(R') B'_B
\end{aligned}
$$

subject $R' \leq R_K$,

$$T = G_t + B - \frac{B'_B + B'_H}{R'}$$

$$H_0(B', G') \geq h(B'_B, G')$$

where the coefficient of B'_B in $V_1(S)$ *is* given by

$$A_B(R') = \left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right] \frac{\left[\frac{1}{R'} - \gamma\beta\right]}{[1-\gamma\beta R_K]} - \beta A_N(1-\gamma)$$

Next note that

$$\frac{\left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right]\sigma}{[1-\gamma\beta R_K]} = A_N$$

$$\left[\beta A_R + \left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right] \frac{(1-\sigma)\bar{n}}{[1-\gamma\beta R_K]}\right] = A_R$$

To check this algebra we note that

$$\left[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)\right]\sigma = A_N\left[1-\gamma\beta R_K\right]$$

implies

$$\frac{(\beta R_K - 1)\sigma}{[1-\beta R_K(\gamma + \sigma(1-\gamma))]} = A_N$$

and that

$$\left[\beta A_R + \frac{[(\beta R_K - 1) + \beta R_K A_N(1-\gamma)]}{[1-\gamma\beta R_K]}(1-\sigma)\bar{n}\right] = \beta A_R + \frac{A_N}{\sigma}(1-\sigma)\bar{n}$$
$$= \frac{(1-\sigma)\bar{n}A_N}{(1-\beta)\,\sigma} = A_R$$

Finally, note that

$$\begin{aligned} A_B(R') &= [(\beta R_K - 1) + \beta R_K A_N(1-\gamma)]\,\frac{\left[\frac{1}{R'} - \gamma\beta\right]}{[1-\gamma\beta R_K]} - \beta A_N(1-\gamma) \\ &= \frac{A_N}{\sigma}\left[\frac{1}{R'} - \gamma\beta\right] - \beta A_N(1-\gamma) \\ &= \frac{A_N}{\sigma}\frac{1}{R'} - A_N\left(\frac{\gamma\beta}{\sigma} + \beta(1-\gamma)\right) \\ &= \frac{A_N}{\sigma}\frac{1}{R'} - \beta\frac{A_N}{\sigma}(\gamma + \sigma(1-\gamma)) \end{aligned}$$

Thus

$$V_1(S) = A_R + K + A_N N + H_1(B,G)$$

with

$$H_1(B,G) = \max_{B'_B, B'_H, T, R'} W(T) - A_B(R')\,B'_B + \beta H_0(B',G') \tag{111}$$

subject to $R' \leq R_K$,

$$T = G + B - \frac{B'_B + B'_H}{R'}$$

$$H_0(B',G') \geq h(B'_B, G')$$

Thus, we proven that the first iteration preserves the form (106). Iteratively applying the same steps above shows that V_n remains of the same form, namely (106) where in each iteration we continue to have

$$A_N = \frac{(\beta R_K - 1)\,\sigma}{[1 - \beta R_K\,(\gamma + \sigma(1-\gamma))]}$$

$$A_R = \frac{(1-\sigma)\bar{n}A_N}{(1-\beta)\,\sigma}$$

$$A_B\,(R') = \frac{A_N}{\sigma}\frac{1}{R'} - \beta\frac{A_N}{\sigma}\,(\gamma + \sigma\,(1-\gamma))\,.$$

and H_{n+1} satisfies the analog of (111) with H_n replacing H_0.

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Figure A1: Policy functions

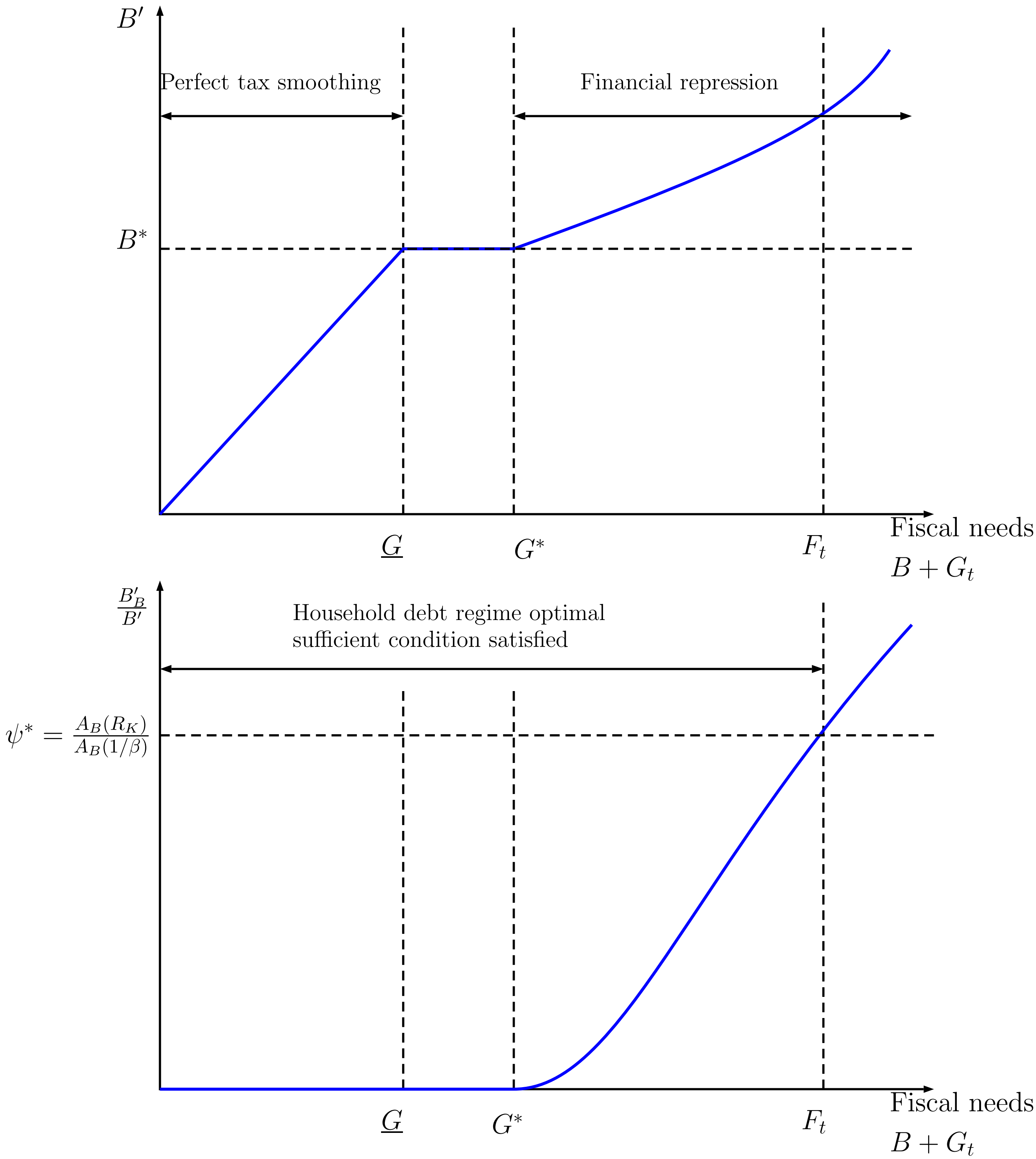

Figure A2: Equilibrium outcome with liquidity needs

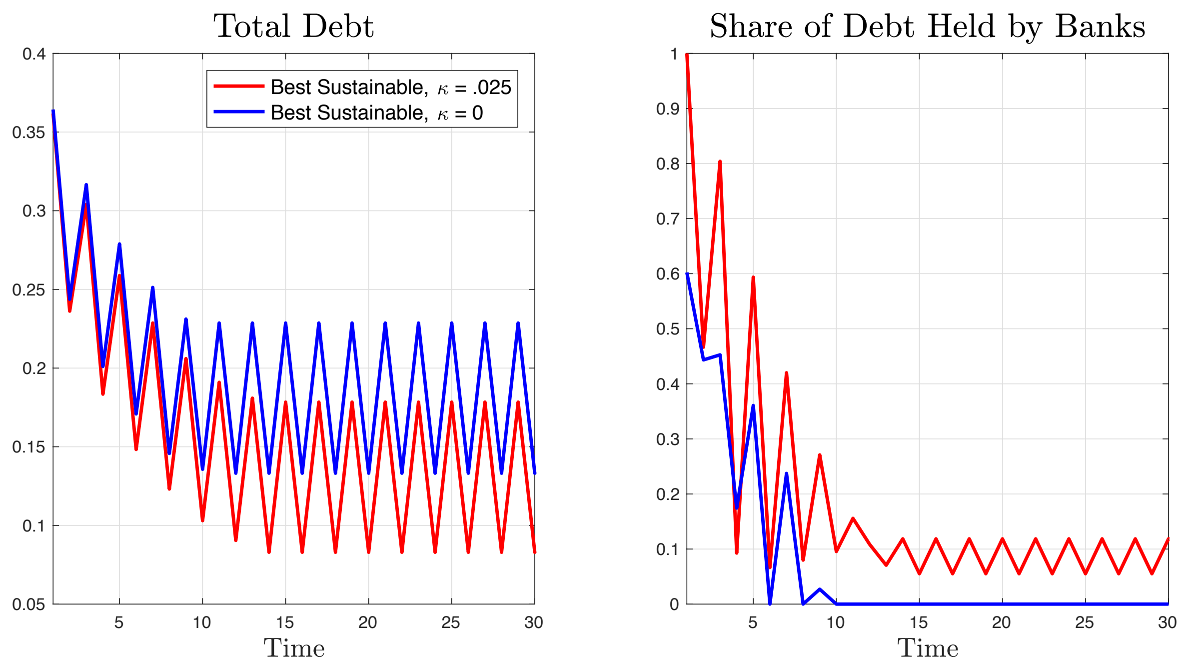

Figure A3: Equilibrium outcome with default on deposits