Additional Appendix

A. Sovereign Debt Game: Proof of Lemma 1

Lemma A1: $\mathbf{y}$ is a sustainable equilibrium outcome of the sovereign-debt game if and only if it satisfies the following conditions:

(BC) $c(\theta^t) + y^*(\theta^t) \leq f(m(\theta^{t-1}))$

where

$$y^*(\theta^t) = m(\theta^{t-1}) - \begin{cases} q_S(\theta^t)b_S(\theta^t) + q_L(\theta^t)\left[b_L(\theta^t) - b_L(\theta^{t-1})\right] - b_S(\theta^{t-1}) - b_L(\theta^{t-1}) & \text{if } \delta(\theta^t) = 1 \\ q_S(\theta^t)b_S(\theta^t) + q_L(\theta^t)b_L(\theta^t) - b_L(\theta^{t-1})\frac{r}{1-q} - b_S(\theta^{t-1})r & \text{if } \delta(\theta^t) = r \\ 0 & \text{if } \delta(\theta^t) = 0 \end{cases}$$

(qS) $q_S(\theta^t) = q\sum_{\theta_{t+1}} \mu(\theta_{t+1})\chi_S(\theta^t, \theta_{t+1})$

(qL) $q_L(\theta^t) = q\sum_{\theta_{t+1}} \mu(\theta_{t+1})\chi_L(\theta^t, \theta_{t+1})$

where

$$\chi_S(\theta^t) = \begin{cases} 1 & \text{if } \delta(\theta^t) = 1 \\ r_k & \text{if } \delta(\theta^t) = r_k \\ q\mathbb{E}\left[\chi_S(\delta(\theta^{t+1}))|\theta^t\right] & \text{if } \delta(\theta^t) = 0 \end{cases} \qquad \chi_L(\theta^t) = \begin{cases} 1 + q_L(\theta^t) & \text{if } \delta(\theta^t) = 1 \\ r_k/(1-q) & \text{if } \delta(\theta^t) = r_k \\ q\mathbb{E}\left[\chi_L(\delta(\theta^{t+1}))|\theta^t\right] & \text{if } \delta(\theta^t) = 0 \end{cases}$$

(IC) $\theta_t U(c(\theta^{t-1}, \theta_t)) + \beta v(\theta^{t-1}, \theta_t) \geq \theta_t U(c(\theta^{t-1}, \theta)) + \beta v(\theta^{t-1}, \theta_t)$

(SUST) $\theta_t U(c(\theta^t)) + \beta v(\theta^t) \geq \theta_t U(f(m(\theta^{t-1}))) + \beta v_a$

and the no-Ponzi conditions $b_S(\theta^t), b_L(\theta^t) \leq \bar{B}$.

Proof: First notice that v_a is the value of the worst equilibrium for the perspective of the sovereign borrower, as argued in the paper. Conditions (BC)-(SUST) are necessary for $\mathbf{y}$ to be a sustainable equilibrium outcome. In fact, (BC) is simply the consolidated budget constraint for the stand-in domestic agent, using the no-arbitrage conditions for the stand-in domestic firm and the foreign exporters to substitute for p and τ. Conditions (qS) and (qL) are the no-arbitrage conditions for foreign lenders. Constraints (IC) and (SUST) ensure that the sovereign borrower has no strict incentive to engage in a detectable and undetectable deviation respectively. In (SUST), I use the fact that the equilibrium value for the borrower after a detectable deviation cannot exceed v_a. Conditions (BC)–(SUST) are also sufficient, as $\mathbf{y}$ can be implemented as a sustainable equilibrium outcome using reversion to autarky after any detectable deviation from the equilibrium path of plays. □

Notice that (BC)-(SUST) have a recursive structure in (v, q_S, q_L, b_S, b_L), so I can recover all the equilibrium values (and outcomes) using an operator similar to the one in Abreu, Pearce, and Stacchetti (1990) and Phelan and Stacchetti (2001). In fact, (v, q_S, q_L, b_S, b_L) is part of a sustainable equilibrium iff $(v, q_S, q_L) \in \Phi(b_S, b_L)$ where $\Phi : \mathcal{B} \subset \mathbb{R} \times \mathbb{R}_+ \rightrightarrows \mathbb{R}^3$ is defined as $(v, q_S, q_L) \in \Phi(b_S, b_L)$ iff $\exists\ (v', q'_S, q'_L, b'_S, b'_L, \delta)(\theta)$ such that recursive version of (BC)–(SUST) are satisfied and $(v'(\theta), q'_S(\theta), q'_L(\theta)) \in \Phi(b'_S(\theta), b'_L(\theta))$ for all θ. To characterize the best equilibrium outcome of the sovereign debt game, I use a different strategy. I drop conditions (1)–(3) and substitute them with

(RC) $c(\theta^t) + y^*(\theta^t) \leq f(m(\theta^{t-1}))$

Clearly (BC)–(qL) imply (RC), but the converse in not generally true. Take any $\mathbf{x}$ that satisfies (IC), (SUST), and (RC) and define the implicit market value of debt as the present discounted value of net exports:

$$(1) \quad b(\theta^t) \equiv \left[y^*(\theta^t) - m(\theta^{t-1})\right] + \sum_{j\geq 1}^{\infty} \sum_{\theta^{t+j}} q^j \Pr(\theta^{t+j}|\theta^t) \left[y^*(\theta^{t+j}) - m(\theta^{t+j-1})\right]$$

To implement such an outcome as an equilibrium of the sovereign debt game, it is sufficient (and necessary) to find $\boldsymbol{\pi} = \left\{\delta(\theta^t), b_S(\theta^t), b_L(\theta^t)\right\}_{t=0}^{\infty}$ such that for all $t, \theta^t, \theta_{t+1}$

$$(2) \quad b(\theta^t, \theta_{t+1}) = \begin{cases} b_L(\theta^t)\left[1 + q_L(\theta^t, \theta_{t+1})\right] + b_S(\theta^t) & \text{if } \delta(\theta^t, \theta_{t+1}) = 1 \\ b_L(\theta^t)\frac{r}{1-q} + b_S(\theta^t)r & \text{if } \delta(\theta^t, \theta_{t+1}) = r \\ b_L(\theta^t)q_L(\theta^t, \theta_{t+1}) + b_S(\theta^t)q_S(\theta^t, \theta_{t+1}) & \text{if } \delta(\theta^t, \theta_{t+1}) = 0 \end{cases}$$

where bond prices are defined by (qS) and (qL):

Lemma A2. If an outcome $\mathbf{y}$ is consistent with (qS), (qL), and the no-Ponzi condition, then (BC) is satisfied if and only if (2) is satisfied.

Proof: The sufficiency part is obvious. For the necessary part, iterating forward on the budget constraint (1) at t, θ^t and using (qS) and (qL) we have

$$\chi_S(\theta^t)b_S(\theta^{t-1}) + \chi_L(\theta^t)b_L(\theta^{t-1}) = \left[y^*(\theta^t) - m(\theta^{t-1})\right] + q\sum_{\theta_{t+1}} \mu(\theta_{t+1})\left[\chi_S(\theta^{t+1})b_S(\theta^t) + \chi_L(\theta^{t+1})b_L(\theta^t)\right]$$

$$= ...$$

$$= \lim_{T\to\infty}\sum_{\theta^{t+j}} \Pr(\theta^{t+j}|\theta^t)q^j\left[y^*(\theta^{t+j}) - m(\theta^{t+j-1})\right] + \lim_{T\to\infty} q^T \sum_{\theta^T}\Pr(\theta^T|\theta^t)\left[\chi_S(\theta^{T+1})b_S(\theta^T) + \chi_L(\theta^{T+1})b_L(\theta^T)\right]$$

$$= \left[y^*(\theta^t) - m(\theta^{t-1})\right] + \sum_{j\geq 1}^{\infty}\sum_{\theta^{t+j}} q^r \Pr(\theta^{t+j}|\theta^t)\left[y^*(\theta^{t+j}) - m(\theta^{t+j-1})\right] = b(\theta^{t-1}, \theta_t)$$

where in the last line, I use the no-Ponzi conditions and the fact that χ_L and χ_S are bounded. □

B. Randomization: Relaxing Assumption 2

When Assumption 2 is not satisfied, it may be optimal to use randomization. Here I show how to extend the environment in the paper to allow for randomization. Let ξ_t be the realization of a random variable uniformly distributed over $[0, 1]$ and *i.i.d.* over time. The randomization device ξ is realized before foreign lenders supply the intermediate good to the stand-in domestic firm. The timing within the period is as follows:

1. The public randomization device $\xi_t \in [0, 1]$ is realized;
2. Foreign lenders supply intermediate goods $m_t \geq 0$;
3. θ_t is realized according to μ;
4. Production, consumption, and exporting take place.

Let $s_t = (\xi_t, \theta_t)$ and $s^t = (s_0, s_1, ..., s_t)$. An allocation is a stochastic process $\mathbf{x} \equiv \{m(s^{t-1}, \xi_t), c(s^t), y^*(s^t)\}_{t=0}^{\infty}$.

The programming problem in (J) can be written as

$$\text{(J)} \quad J(v_0) = \max_{\mathbf{x}} \sum_{t=0}^{\infty}\sum_{s^t} q^t \Pr(s^t)\left[y^*(s^t) - m(s^{t-1}, \xi_t)\right]$$

subject to

$$\text{(RC)} \;\; c(s^t) + y^*(s^t) \leq f(m(s^{t-1}, \xi_t))$$

$$\text{(IC)} \;\; \theta_t U(c(s^{t-1}, \xi_t, \theta_t)) + \beta v(s^{t-1}, \xi_t, \theta_t) \geq \theta_t U\left(c(s^{t-1}, \xi_t, \theta')\right) + \beta v(s^{t-1}, \xi_t, \theta')$$

$$\text{(SUST)} \;\; \theta_t U(c(s^t)) + \beta v(s^t) \geq \theta_t U(f(m(s^{t-1}, \xi_t))) + \beta v_a$$

(PC) $\sum_{t=0}^{\infty}\sum_{s^t}\beta^t \Pr(s^t)\theta_t U(c(s^t)) \geq v_0$

Near Recursive Formulation The problem in (J) admits a nearly recursive formulation using the borrower's promised utility, v, as a state variable. From $t \geq 1$, an efficient allocation solves the following recursive problem for $v \in [v_a, \bar{v}]$:

$$\text{(P)} \quad B(v) = \max_{m(\xi),c(\xi,\theta),v'(\xi,\theta)} \int_0^1 \left\{ f\left(m(\xi)\right) - m(\xi) + \sum_{\theta\in\Theta} \mu(\theta)\left[-c(\xi,\theta) + qB(v'(\xi,\theta))\right] \right\} d\xi$$

subject to

$$\begin{array}{llcl}
(3) & \int_0^1 \left\{ \sum_{\theta\in\Theta} \mu(\theta)\left[\theta U(c(\xi,\theta)) + \beta v'(\xi,\theta)\right] \right\} d\xi & = & v \\
(4) & \theta U(c(\xi,\theta)) + \beta v'(\xi,\theta) & \geq & \theta U(c(\xi,\theta')) + \beta v'(\xi,\theta') \quad \forall\xi,\forall\theta,\theta' \\
(5) & \theta U(c(\xi,\theta)) + \beta v'(\xi,\theta) & \geq & \theta U\left(f(m(\xi))\right) + \beta v_a \quad \forall\xi,\forall\theta \\
(6) & v'(\xi,\theta) & \geq & v_a \quad \forall\xi,\forall\theta
\end{array}$$

At $t = 0$, for all $v_0 \in [v_a, \bar{v}]$ the problem in (J) can be expressed as

$$\text{(7)} \quad J(v_0) = \max_{m(\xi),c(\xi,\theta),v'(\xi,\theta)} \int_0^1 \left\{ f\left(m(\xi)\right) - m(\xi) + \sum_{\theta\in\Theta} \mu(\theta)\left[-c(\xi,\theta) + qB(v'(\xi,\theta))\right] \right\} d\xi$$

subject to (4), (5), (6), and the participation constraint

$$\text{(8)} \quad \int_0^1 \left\{ \sum_{\theta\in\Theta} \mu(\theta)\left[\theta U(c(\xi,\theta)) + \beta v'(\xi,\theta)\right] \right\} d\xi \geq v$$

The constraint set in (P) is not necessarily convex because of the presence of $U \circ f(m)$, a concave function, on the right hand side of the sustainability constraint (5). Thus, randomization may be optimal. The programming problem in (P) can be represented as follows:

$$\text{(P')} \quad B(v) = \max_{\zeta\in[0,1],v_1,v_2\in[v_a,\bar{v}]} \zeta\hat{B}(v_1) + (1-\zeta)\hat{B}(v_2) \quad \text{s.t.} \quad \zeta v_1 + (1-\zeta)v_2 = v$$

where

$$(\hat{\text{P}}) \quad \hat{B}(v) = \max_{m,c(\theta),v'(\theta)} \sum_{\theta\in\Theta} \mu\left(\theta\right)\left[f(m) - m - c(\theta) + qB(v'(\theta))\right]$$

subject to

$$(9) \quad \sum_{\theta\in\Theta} \mu(\theta) \left[\theta U(c(\theta)) + \beta v'(\theta)\right] = v$$

$$(10) \quad \theta U(c(\theta)) + \beta v'(\theta) \geq \theta U\left(c(\theta')\right) + \beta v'(\theta') \quad \forall \theta, \theta'$$

$$(11) \quad \theta U(c(\theta)) + \beta v'(\theta) \geq \theta U(f(m)) + \beta v_a \quad \forall \theta$$

$$(12) \quad v'(\theta) \geq v_a \quad \forall \theta$$

$\hat{B}$ is the maximal value that the lenders can attain *without* using randomization in the current period and using the convexified value for B to evaluate the continuation value. For any $v \in [v_a, \bar{v}]$, the value of $B(v)$ can be obtained from $\hat{B}$ using (P') where, without loss of generality, the randomization is between two values.

When Assumption 2 does not hold, it is not guaranteed that the maximizer of ($\hat{\text{P}}$) is unique. I will assume that this is the case. I further assume that randomization may only occur in the region with *ex-post* inefficiencies:

Assumption A1. If randomization is optimal, there exists $v_r \in (v_a, \tilde{v})$ such that (i) for $v \in (v_a, v_r)$, it is optimal to randomize between v_a and v_r, and (ii) if $v \geq v_r$ then there is no randomization.

This pattern for randomization is what I find in any computed example. Under Assumption A1, the *randomization region* (linear portion of B), is the set $[v_a, v_r]$ with $v_r < \tilde{v}$. For all $v \in [v_a, v_r]$, let

$$(13) \quad \zeta(v) = \frac{v_r - v}{v_r - v_a}$$

be the probability that continuation utility after randomization is equal to v_a. With probability $1 - \zeta(v)$ the post-randomization continuation value is equal to v_r.

Under this assumption I can characterize the solution to (P) using the equivalent representation given by (P')-($\hat{\text{P}}$). Lemma 2–4 and Proposition 1–4 in the paper are still valid. The only thing that I cannot prove is the monotonicity of m for $v < v^*$ (but such result is always true in any numerical simulation).

Implementation with randomization Assume that $\mathbf{x}$ satisfies Assumption A1 and the following properties:

Assumption A2. (i) For all $v > v_a$, $v'(v, \theta_H) < v$. (ii) $v'(\cdot, \theta_L)$ is strictly increasing and $v'(\cdot, \theta_H)$ is strictly increasing for all $v \geq \underline{v}$.

Part (i) of the assumption implies that starting from any v, there is a strictly positive probability of reaching autarky. Part (ii) requires that $v'(\cdot, \theta_L)$ and $v'(\cdot, \theta_H)$ are monotone increasing.

I can then prove the analogue of Proposition 5 in the text:

Proposition 5'. Under Assumptions 1, 3, and $\beta < q$, if $\mathbf{x}$ is an efficient allocation that satisfies Assumptions A1-A2 , then there exist a set of recovery rates $\mathbf{r} = \{1, r_{rL}, r_{rH}, r_{aL}, 0\}$, an initial debt position, $\mathbf{b}_0$, a strategy for the benevolent government, σ, prices, p and $\mathbf{q}$, and an allocation rule, m, such that: (i) $(\sigma, p, m, \mathbf{q})$ is a sustainable equilibrium given $\mathbf{r}$ and $\mathbf{b}_0$, and (ii) $\mathbf{x}$ is the allocation associated with the equilibrium outcome path.

Mapping Between Efficient Allocation and Equilibrium Objects on Path I now construct the candidate equilibrium outcome path that implements an efficient allocation $\mathbf{x}$. Since the efficient allocation can be represented by a time-invariant function of borrower's continuation utility and exogenous shocks, the on-path repayment rule, bond holdings, tariffs, and prices can also be expressed as a function of on-path continuation utility for the borrower and the current realization of the randomization device ξ and the taste shock θ. In particular, the repayment policy, tariff, and intermediate prices are functions of the post-randomization value:

$$(14)\quad \bar{\delta} : [v_a, \bar{v}] \times \Theta \to \mathbf{r} \quad \text{and} \quad \bar{\tau}, \bar{p} : [v_a, \bar{v}] \to \mathbb{R}$$

Bond holdings and prices are functions of the continuation value (for the next period):

$$(15)\quad \bar{q}_S, \bar{q}_L, \bar{b}_S, \bar{b}_L : [v_a, \bar{v}] \to \mathbb{R}$$

An outcome path $\mathbf{y}$ can be recovered in the natural way from (14), (15), and the law of motion for v from the efficient allocation.[1]

The equilibrium repayment policy is such that the borrower defaults only when his pre-randomization continuation value is in $[v_a, v_r]$. For all the other borrower values, there is full

[1] The path for $\mathbf{y}$ can be recovered as follows. Given an exogenous history (s^{t-1}, ξ_t) with associated continuation utility

$$v(s^{t-1}, \xi_t) = \begin{cases} v'\left(v(s^{t-1}), \theta_{t-1}\right) & \text{if } v'\left(v(s^{t-1}), \theta_{t-1}\right) > v_r \\ v_a & \text{if } v'\left(v(s^{t-1}), \theta_{t-1}\right) \in [v_a, v_r] \text{ and } \xi_t \leq \zeta(v'\left(v(s^{t-1}), \theta_{t-1}\right)) \\ v_r & \text{if } v'\left(v(s^{t-1}), \theta_{t-1}\right) \in [v_a, v_r] \text{ and } \xi_t > \zeta(v'\left(v(s^{t-1}), \theta_{t-1}\right)) \end{cases}$$

we have

$$\tau(s^{t-1}, \xi_t) = \bar{\tau}\left(v(s^{t-1}, \xi_t)\right), \ \delta(s^{t-1}, \xi_t) = \bar{\delta}\left(v(s^{t-1}, \xi_t)\right)$$

repayment. I will refer to $[v_a, v_r]$ as the *default region* and to $(v_r, \bar{v}]$ as the *no-default region*. By Assumption 3, for pre-randomization value $v \in [v_a, v_r]$, the post-randomization value is equal to either v_a (with probability $\zeta(v)$) or v_r (with probability $1 - \zeta(v)$). Thus, there are four relevant outcomes for the repayment policy in the default region:

$$(16) \quad \bar{\delta}(v, \theta) = \begin{cases} 0 & \text{if } v = v_a \text{ and } \theta = \theta_H \\ r_{rH} & \text{if } v = v_r \text{ and } \theta = \theta_H \\ r_{aL} & \text{if } v = v_a \text{ and } \theta = \theta_L \\ r_{rL} & \text{if } v = v_r \text{ and } \theta = \theta_L \\ 1 & \text{if } v > v_a \text{ for all } \theta \end{cases}$$

where $\bar{\delta}(v_a, \theta_H) = 0$ because, from Lemma 4, when the borrower's value is autarky, there are no capital flows: $m(v_a) = 0$ and $c(v_a, \theta_H) = f(0)$. When $v = v_r$ and $\theta = \theta_H$, there is a partial repayment today (r_{rH} is generally greater than zero) and there will be again less than full repayment the next period because $v'_H(v_r) < v_r$. I interpret this as a unique protracted default episode.[2] The borrower is out of the default region in the next period only after he draws θ_L ($v'_L(v) \geq \tilde{v} > v_r$ for all v).

Given the repayment policy, bond prices are uniquely pinned down by the lenders' optimality conditions. The price for short-term debt is given by:

$$(17) \quad \bar{q}_S(v) = \begin{cases} q & \text{if } v \in (v_r, \bar{v}] \\ q\bar{R}(v) & \text{if } v \in [v_a, v_r] \end{cases}$$

where $\bar{R}(v)$ is the expected recovery rate:

$$(18) \quad \bar{R}(v) = \zeta(v) \frac{\mu(\theta_L) r_{aL}}{1 - q\mu(\theta_H)} + [1 - \zeta(v)] \left[\mu(\theta_L) r_{rL} + \mu(\theta_H) r_{rH}\right]$$

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and

$$\begin{aligned} q_S(s^t) &= \bar{q}_S\left(v'\left(v(s^{t-1}, \xi_t), \theta_t\right)\right), \quad q_L(s^t) = \bar{q}_L\left(v'\left(v(s^{t-1}, \xi_t), \theta_t\right)\right) \\ b_S &= \bar{b}_S\left(v'\left(v(s^{t-1}, \xi_t), \theta_t\right)\right), \quad b_L = \bar{b}_L\left(v'\left(v(s^{t-1}, \xi_t), \theta_t\right)\right) \end{aligned}$$

[2] This is consistent with the fact that there are repeated restructurings, see Cruces and Trebesch (2012).

The price for long-term debt can be written recursively as:

$$(19)\quad \bar{q}_L(v) = \begin{cases} q\sum_{i=L,H}\mu(\theta_i)\left[1+\bar{q}_L(v_i'(v))\right] & \text{if } v\in(v_r,\bar{v}] \\ \frac{q}{1-q}\bar{R}(v) & \text{if } v\in[v_a,v_r] \end{cases}$$

The next lemma shows that $\bar{q}_L$ is increasing in the continuation value for the borrower and it establishes a property of $\bar{q}_L$ that is critical to implement the efficient allocation:

Lemma A3. Under the assumptions in Proposition 5', for a given $\mathbf{r} = \{1, r_{rL}, r_{aL}, r_{rH}, 0\}$ with $r_{rL}, r_{aL}, r_{rH} \in (0,1)$, $\bar{q}_L : [v_a, \bar{v}] \to \mathbb{R}$ is the unique fixed point of the contraction mapping defined by the right hand side of (19) and is strictly increasing and for all v we have that $\bar{q}_L\left(v_L'(v)\right) > \bar{q}_L\left(v_H'(v)\right)$.

Proof. Let Q_L be the space of bounded functions $q_L : [v_a, \bar{v}] \to [0, q/(1-q)]$ and let $T : Q_L \to Q_L$ be defined by the right hand side of (19). That is:

$$\left(Tq_L\right)(v) = \begin{cases} q\sum_{i=L,H}\mu(\theta_i)\left[1+q_L(v_i'(v))\right] & \text{if } v\in(v_r,\bar{v}] \\ \frac{q}{1-q}\bar{R}(v) & \text{if } v\in[v_a,v_r] \end{cases}$$

T satisfies the Blackwell's sufficient condition for a contraction mapping, see Theorem 3.3 in SLP. Then, by the contraction mapping theorem, there exists a unique fixed point of T, $\bar{q}_L$. To see that $\bar{q}_L$ is strictly increasing, first notice that $\bar{q}_L$ must be (weakly) increasing. T maps increasing functions into increasing functions. Then, by a corollary of the contraction mapping theorem (see Corollary 3.1 in SLP), it must be that $\bar{q}_L$ is increasing. Also notice that $\bar{q}_L(v) < q/(1-q)$ for all $v \in [v_a, \bar{v}]$, since there is always a strictly positive probability of the continuation value reaching v_a after a sufficiently long string of high taste shocks. Under the assumptions of Proposition 5', if *randomization* is optimal, then $\bar{q}_L$ is strictly increasing. To see that this is the case, first notice that, by definition, $\bar{q}_L$ is strictly increasing over $[v_a, v_r]$. Second, suppose for contradiction that $\bar{q}_L$ is constant over some interval. Let $[v_1, v_3] \subset [v_r, \bar{v}]$ be the first of such intervals so that for all $v \in [v_a, v_1)$, $\bar{q}_L$ is strictly increasing. For all $v_2 \in (v_1, v_3]$ we have that

$$(20)\quad \bar{q}_L(v_1) = q\sum_{i=L,H}\mu(\theta_i)\left[1+\bar{q}_L(v_i'(v_1))\right] < q\sum_{i=L,H}\mu(\theta_i)\left[1+\bar{q}_L(v_i'(v_2))\right]$$

To see why (20) holds, first notice that $v_L'(v_2) > v_L'(v_1)$ and $\bar{q}_L$ is weakly increasing; it follows that $\bar{q}_L(v_L'(v_2)) \geq \bar{q}_L(v_L'(v_1))$. Second, because $v_H'(v_2) > v_H'(v_1)$ and $v_H'(v_1) < v_1$, it must be that $v_H'(v_1) \in [v_a, v_1)$. Since $\bar{q}_L$ is strictly increasing in that region, $\bar{q}_L(v_H'(v_2)) > \bar{q}_L(v_H'(v_1))$. Hence,

(20) holds. This is a contradiction. Then $\bar{q}_L$ is strictly increasing. Hence, because from Proposition 1 part (ii), we have that $v_L'(v) > v_H'(v)$ for all v. Then it follows that $\bar{q}_L(v_L'(v)) > \bar{q}_L(v_H'(v))$. $\square$

Given the functions for bond prices $\bar{q}_S$ and $\bar{q}_L$, in the no-default region, $\bar{b}_S(v)$ and $\bar{b}_L(v)$ are chosen to match the total value of debt (lenders' value) implied by the efficient allocation after ξ and θ:

$$
\begin{aligned}
(21)\quad b(v,\theta_L) &= \bar{b}_S(v) + \bar{b}_L(v)\left[1 + \bar{q}_L(v_L'(v))\right] \\
(22)\quad b(v,\theta_H) &= \bar{b}_S(v) + \bar{b}_L(v)\left[1 + \bar{q}_L(v_H'(v))\right]
\end{aligned}
$$

A (unique) solution to (21)-(22) is guaranteed by the fact that $\bar{q}_L\left(v_H'(v)\right) < \bar{q}_L\left(v_L'(v)\right)$. Thus, outside of the default region the maturity composition of debt is uniquely pinned down. Simple algebra shows that:

$$
\begin{aligned}
(23)\quad \bar{b}_L(v) &= \frac{b(v,\theta_L) - b(v,\theta_H)}{\bar{q}_L(v_L'(v)) - \bar{q}_L(v_H'(v))} > 0 \\
(24)\quad \bar{b}_S(v) &= b(v,\theta_L) - \bar{b}_L(v)\left[1 + \bar{q}_L(v_L'(v))\right]
\end{aligned}
$$

In the default region, bond holdings are constant. For all $v \in [v_a, v_r]$, $\bar{b}_S(v) = \bar{b}_{Sr}$ and $\bar{b}_L(v) = \bar{b}_{Lr}$, and it must be that:

$$
\begin{aligned}
(25)\quad b(v_r,\theta_L) &= r_{rL}\left[\bar{b}_{Sr} + \frac{\bar{b}_{Lr}}{1-q}\right] \\
(26)\quad b(v_r,\theta_H) &= r_{rH}\left[\bar{b}_{Sr} + \frac{\bar{b}_{Lr}}{1-q}\right] \\
(27)\quad b(v_a,\theta_L) &= r_{aL}\left[\bar{b}_{Sr} + \frac{\bar{b}_{Lr}}{1-q}\right]
\end{aligned}
$$

The other possible outcome follows from (27) and $\delta(v_a,\theta_H) = 0$ because

$$
(28)\quad b(v_a,\theta_H) = q\left[\mu(\theta_L)b(v_a,\theta_L) + \mu(\theta_H)b(v_a,\theta_H)\right] = \frac{q\mu(\theta_L)}{1 - q\mu(\theta_H)}b(v_a,\theta_L)
$$

Thus, fixing any $r_{rL} \in (0,1)$, it must be that

$$
(29)\quad r_{rH} = \frac{b(v_r,\theta_H)}{b(v_r,\theta_L)}r_{rL} \text{ and } r_{aL} = \frac{b(v_a,\theta_L)}{b(v_r,\theta_L)}r_{rL}
$$

If the restrictions in (29) are satisfied, I can choose any $\left(\bar{b}_{Sr}, \bar{b}_{Lr}\right)$ that satisfies (25). Consequently, (26)-(28) will also be satisfied. The split between long and short term debt is indeterminate because the two are perfect substitutes if there is default for sure in the next period. I resolve this indeterminacy by assuming that $\bar{b}_{Sr}/\bar{b}_{Lr} = \lim_{v \to v_r} \bar{b}_S(v)/\bar{b}_L(v)$. The recovery rate $r_{rL} \in (0, 1)$ is a free-parameter. It can be chosen sufficiently low that $\bar{b}_S$ is strictly positive in the default region and for v close to v_r so that a non-full repayment has a natural interpretation.

Finally, on-path tariff rates and prices for the intermediate good, $\bar{\tau}, \bar{p}$, are given by:

$$(30) \quad f'(m(v)) = \frac{1}{1 - \bar{\tau}(v)} = \bar{p}(v)$$

Then the outcome path $\mathbf{y}$ constructed from the efficient allocation $\mathbf{x}$ using (16), (17), (19), (23), (24), (25), and (30) satisfies the optimality conditions for exporters and firms – conditions (17) and (18) in the paper – and the equilibrium bond pricing equations – conditions (19) and (21) in the paper. Moreover, it supports the level of consumption implied by the efficient allocation.

C. Data and Facts

In this appendix, I discuss an extensive literature which documents the behavior of GDP, consumption, and imports of intermediate goods around sovereign default episodes and other key aspects of the data and I replicate some of the findings.

Data Description I consider the same 23 default events as Mendoza and Yue (2012): Argentina (1982, 2002), Chile (1983), Croatia (1992), Dominican Republic (1993), Ecuador (1999), Indonesia (1998), Mexico (1982), Moldova (2002), Nigeria (1983, 1986), Pakistan (1998), Peru (1983), Philippines (1983), Russia (1998), South Africa (1985, 1993), Thailand (1998), Ukraine (1998), Uruguay (1990), Uruguay (2003), Venezuela (1995), Venezuela (1998).

Annual data for GDP and consumption are gathered from the World Development Indicators (WDI). These are measured in real US dollars. As in Mendoza and Yue (2012), imported intermediates are the sum of categories for intermediate goods based on the Broad Economic Category (BEC) classification. The categories for intermediate goods are: (111) Food and beverages, primary, mainly for industry, (121) Food and beverages, processed, mainly for industry, (21) Industrial supplies not elsewhere specified, primary, (22) Industrial and lubricants, processed, (other than motor spirit), (42) Parts and accessories of capital goods (except transport equipment), (53) Part and accessories of transport equipment. For years 1962 through 2000, data is available from Feenstra et al. (2005) but is classified using the Standard International Trade Classification, revision 4 (SITC4). I use UN concordances to map SITC4 into BEC codes. For years 1976 through 2010, data is available through the World Bank's World Integrated Trade Solution (WITS) database, which has information from the UN's Comtrade database. This database provides the series for the above BEC codes when available. When I have data from both sources, I use the WITS data, which does not rely on the concordances. For years in which both sources provide data, I have cross referenced the values. Although the levels are not exactly the same, deviations from the trend (my variable of interest) are very similar across the two sources. I deflate the intermediate import data using the US producer price index (PPI) from the Bureau of Economic Analysis (BEA). Each annual series is logged and HP-filtered with a smoothing parameter of 100.

Sovereign debt crises are associated with severe output and consumption losses for the debtor country The first two panels of Figure 1 illustrate the dynamics for GDP and consumption around a sovereign default episode. On average, in the 23 default episodes considered, output is 4.5% and 5.2% below trend in the year of a default and the year after, respectively. On average, consumption is 3.1% and 3.6% below trend in the year of a default and the year after, respectively. The same is true if I instead consider the median. This confirms the findings in Mendoza and Yue (2012). This pattern has been documented in several studies. See the references in the survey by Panizza, Sturzenegger, and Zettelmeyer (2009). Tomz and Wright (2007) is a notable exception. They find only a weak association between default episodes and output being below trend.

Sovereign debt crises are associated with trade disruptions A large literature documents that sovereign default episodes are accompanied by large drops in trade. For instance, see Rose (2005) and the references in the survey by Panizza, Sturzenegger, and Zettelmeyer (2009). Also, Borensztein and Panizza (2008) document that a default has a negative impact on trade credit. Arteta and Hale (2008) show that foreign credit to the private sector collapses in the aftermath of a default. Fuentes and Saravia (2006) show that defaults lead to a fall in FDI flows into the country.

As noted in Mendoza and Yue (2012), the drop in imports of intermediate goods is very large: it drops on average from 4.4% above trend the year before a default to about 15.5% below trend the year of a default and the year after that. See the third panel in Figure 1. This drop is larger than in recessions of similar magnitudes. To document this fact, I regress imported intermediates at time t on a constant, GDP at time t, and dummy variables that take value of one if there is a default in the country at time t, $t-1$, $t-2$ and $t-3$ from 1962 to 2010 for the 18 countries in the sample for which I have data on intermediate imports. The result for this simple regression are reported in the table below. The drop in intermediate inputs in the year of and the year following a sovereign default is more than 10 percent larger than what one would expect from a drop in output of the same magnitude, absent default. This drop can have a non-trivial impact on the economy. Gopinath and Neiman (2012) present a model calibrated to replicate the crisis in Argentina in 2002 and show that the decline in imports of intermediate goods can account for up to a 5 percentage point decline in the welfare relevant measure of productivity.

OLS Regression: Intermediate Imports at time t

Variable	Coefficient Estimate	Standard Error
Constant	0.007	0.960
GDP at t	1.810	0.145
Default at t	-0.119	0.044
Default at $t-1$	-0.108	0.044
Default at $t-2$	-0.040	0.044
Default at $t-3$	-0.005	0.043

R^2=0.225; Number of observations = 714

Intermediate imports and GDP are logged and HP-filtered.

Recoveries are accompanied by trade surpluses Periods following a sovereign default are associated with sustained trade surpluses as the economy recovers, see Mendoza and Yue (2012), among others. Moreover, typically as the economy recovers from the recession associated with the default episode, there is a partial repayment of the defaulted debt, after which the country regains access to international credit markets. Benjamin and Wright (2009) document that settlements tend to occur when output has returned to trend.

Maturity of debt shortens when a default is more likely, as measured by interest rate spreads Broner, Lorenzoni, and Schmukler (2010) use data from 11 emerging economies from 1990 to 2009 to document that during emerging market debt crises (when spreads are high), the maturity of debt issued shortens. Moreover, when the spreads are low, long term spreads are generally higher than short term spreads. During debt crises, the gap between long and short-term spreads tends to narrow and the term spread curve flattens or even inverts. Arellano and Ramanarayanan (2012) confirm these findings.

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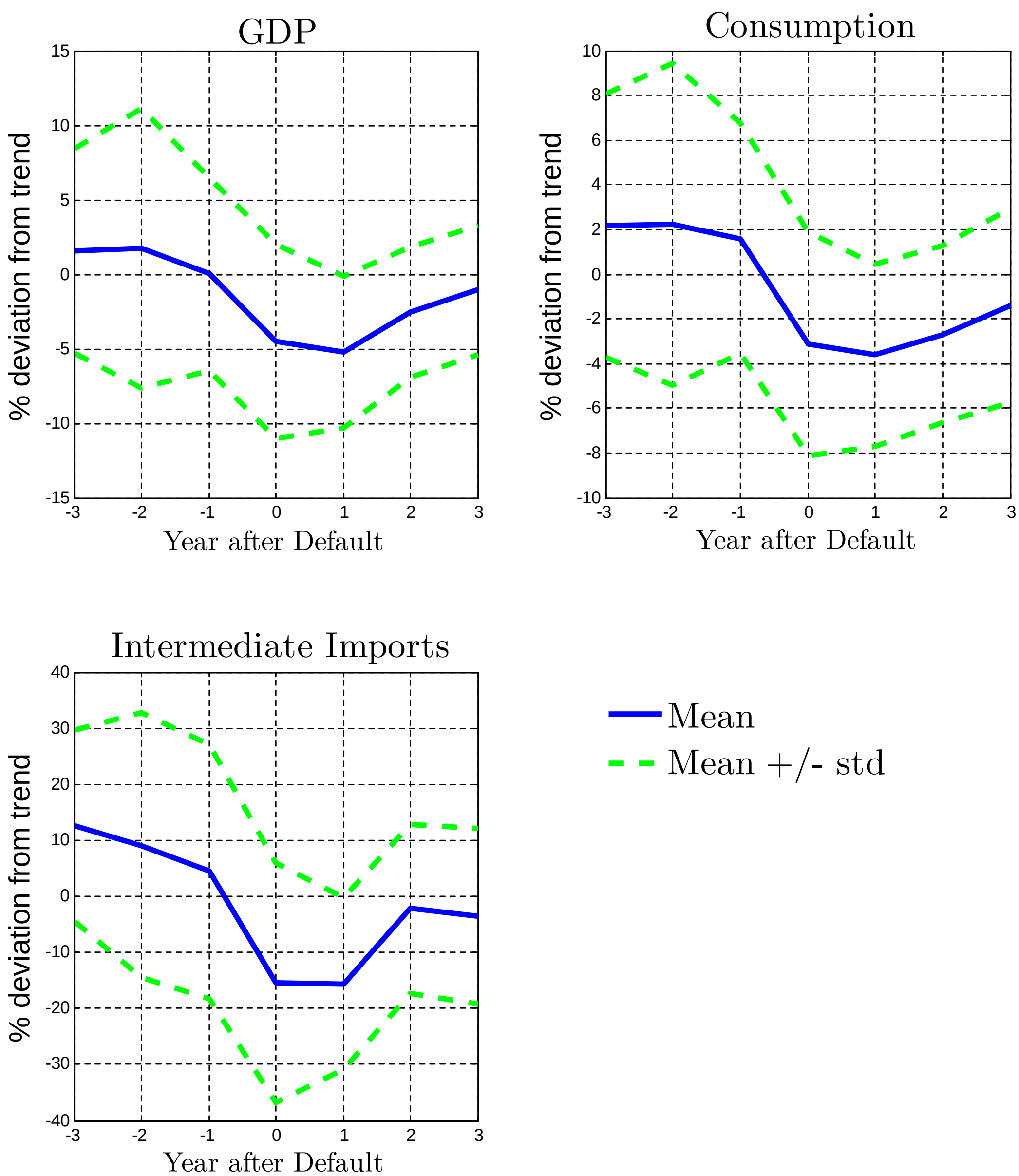
Real Variables Around Sovereign Default Episodes



Data for 23 default events over the 1977- 2009 period. Same sample as in Mendoza and Yue (2012). See Data Appendix for a description of each variable.