Long-Term Contracts, Commitment, and Optimal Information Disclosure[*]

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Abstract

This paper studies optimal information disclosure in dynamic economies with income risk where an incumbent firm knows more than the market about a consumer's persistent type. When the incumbent can commit to long-term contracts, optimal disclosure prescribes no information revelation. Without commitment, no cross-subsidization is feasible for any disclosure policy due to adverse selection. Partial disclosure is typically optimal, and it implements intertemporal consumption smoothing. Lastly, we show that banning long-term relationships can be beneficial to consumers. We apply these findings to lending markets, offering a new perspective on policies like open banking and consumer data ownership.

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1 Introduction

In many markets, firms engage in long-term relationships with consumers. By observing the history of transactions, incumbent firms acquire information about consumers that competitors do not have access to. For example, health and car insurance companies gradually learn about their customers' health or driving records; credit card companies infer their customers' repayment probability by observing their spending patterns; employers possess information about their employees that is hidden from the public record.[1] This ex-post informational monopoly provides incumbent firms with an advantage relative to the competition.

This paper studies to what extent incumbent firms should be required to share information with competitors. The question is relevant given the prevalence of informational asymmetries across markets and the growing number of policies that regulate firms' disclosure of proprietary data. A prominent example is the European Union's General Data Protection Regulation (GDPR), which grants consumers control over the data collected by firms about them. In the banking sector, this principle underlies *open banking* initiatives, a set of regulations that require banks to make customers' financial histories available to competitors upon the customers' request.[2] We characterize the optimal design of information disclosure, taking into account the equilibrium responses of incumbents and entrants to the available information, and highlighting the role of disclosure policies in shaping insurance and consumption smoothing.

We address these questions in a simple two-period economy in which firms compete to attract consumers, and the incumbent firm learns the consumer's type over time. Consumers are risk averse, value insurance across states and over time, and can always switch to a new firm in the second period. We establish three main results. First, when firms can commit to the terms of the contract, the optimal disclosure policy reveals no information about consumers' histories, to minimize the value of the outside option for high-income consumers ("high types") and maximize the value obtained by low-income consumers ("low types"). Second, when firms cannot commit, cross-subsidization across types is infeasible under any disclosure policy. In this case, partial disclosure can improve welfare by facilitating intertemporal consumption smoothing between the first and second period for high types. Third, when firms lack commitment and the gains from

[1] For example, Kahn (2013) find supporting evidence for the labor market, Cohen (2012) for the auto insurance market, and Ioannidou and Ongena (2010) for the corporate loan market.

[2] In the United States, legislation inspired by the GDPR has been adopted in several states, such as California's Consumer Privacy Act (2018), although its scope in the banking sector remains limited. At the federal level, the Personal Financial Data Rights Rule finalized by the Consumer Financial Protection Bureau (CFPB) in 2024—currently facing legal challenges—seeks to extend open banking principles to U.S. financial institutions. Similar initiatives have been implemented in other jurisdictions, including the United Kingdom, Australia, and Brazil.

cross-subsidization exceed the losses from adverse-selection distortions, long-term relationships become detrimental to consumers. In our main application, we interpret consumers as borrowers and firms as lenders, and relate the model's insights to the ongoing debate on open banking policies and consumer data ownership.

We study a two-period economy where risk-averse consumers trade with risk-neutral firms. Income is uncertain, takes two possible values (high or low), and is Markovian. In the first period, consumers and firms have the same information. Firms compete to attract consumers by offering contracts under perfect competition. At the end of the first period, the consumer and the incumbent firm learn the income realization. Competing firms (outsiders) do not observe the income realization but only a public signal from the disclosure policy. Thus, in the second period, the incumbent has an information advantage relative to its competitors, because the incumbent can condition the contract on the previous income realization, while outsiders' offers must satisfy an incentive constraint. In the second period, firms offer contracts to consumers, who decide whether to switch firms or not. In order to ensure equilibrium existence (in pure strategies), we assume that after all contracts are posted, firms can pay an arbitrarily small cost to withdraw their contracts, as in the game described by Netzer and Scheuer (2014) for a Rothschild and Stiglitz (1976) economy.

We aim to characterize the public disclosure policy that maximizes ex-ante welfare. A public disclosure policy is a map from the individual income realization in the first period—our proxy for a consumer's type—to a signal observed by everyone in the economy. Except under full information, outsiders must either distort the consumption profile they offer to the high types not to attract low types, or they must transfer resources to the latter group. The public disclosure policy effectively controls the composition of the pool of consumers with a particular signal, hence determining the maximal amount outsiders are willing to offer to high types in the second period. The value of the outside option for the high types, in turn, affects the long-term contract that the incumbent firm can offer in the first period before any information is revealed.

We consider two forms of firms' commitment. We find that when firms can commit to long-term contracts, no information disclosure is optimal. Commitment allows incumbent firms to deliver as much intertemporal consumption smoothing and cross-subsidization of low types as permitted by the ex-post participation constraint of the high types. Information disclosure tightens such constraint, hence reducing the amount of insurance that can be sustained, consistent with the idea in Hirshleifer (1971). Notwithstanding the presence of private information, the equilibrium consumption profile features the same insurance pattern of the canonical models by Harris and Holmstrom (1982) and Thomas and Worrall (1988): consumption is downward rigid and *back-loaded*.

We next study the case in which firms cannot commit to the terms of the contract. For an arbitrary disclosure policy in the second period, high types are offered an undistorted allocation at a value that matches the best offer that outsiders could make. Thus, incumbents make profits on the high types, but decreasingly so the more informative the signal. Low-income consumers never receive transfers in equilibrium for *any* public disclosure policy. Incumbents know consumers' history, hence never make negative profits on any given type. The lack of cross-subsidization for the low types is in sharp contrast to the version of the model in Netzer and Scheuer (2014) where all firms are equally uninformed about the consumer's type. The presence of an informed incumbent prevents outsiders from attracting high-income consumers because the incumbent does not need to apply an adverse-selection distortion. Thus, in equilibrium, the outsiders know that if they attract any customers, even if with a good signal, they must be low types. Competition among firms in the first period ensures that these expected ex-post profits are rebated to consumers in the first period.

Next, we characterize the optimal public disclosure policy. Because low types' consumption in the second period is independent of the information structure, the information design is exclusively driven by intertemporal motives between first- and second-period consumption for the high types. We show that the optimal disclosure policy takes the form of a "*bad news*" system in which all high-income consumers receive a good signal but only some low-income ones do. The more low-income consumers are pooled with high-income ones, the lower the outside option for the latter in the second period, and the higher the ex-post profits of the firm which are rebated in the first period. At the optimal policy, the equilibrium contract features an inverse of the Harris and Holmstrom (1982) profile: the consumption is upward rigid and *front-loaded*.

Our analysis sheds light on the value of long-term relationships between firms and consumers. With commitment on the firm's side, having a long-term relationship is always preferable to a sequence of spot contracts. When the firm cannot commit, there is a trade-off: long-term relationships allow for some intertemporal smoothing and dispense with the adverse-selection distortion but prevent cross-subsidization across types in the second period. First, we show that with spot contracts, the amount of information is decreasing in the quality of the pool, contrary to our benchmark case. Second, we show that long-term relationships are preferable when the share of high-income consumers in the economy is low because the benefits of cross-subsidization with spot contracts are limited and the gains from intertemporal smoothing are large. Vice versa, when the share of high-income consumers is high, the lack of insurance in the second period dominates, and spot contracts are preferred.

Along similar lines, we introduce reasons for consumers to leave the incumbent by assuming that the outsider firms deliver utility benefits for exogenous reasons (e.g., better services). This changes the problem in two ways. First, public disclosure has an allocative effect: not only does it affect the degree of potential competition the incumbent faces, but it also determines whether consumers switch in equilibrium. Second, the possibility that the high-income consumers do switch in equilibrium ameliorates the adverse-selection problem and allows for cross-subsidization of low types. Switching induced by better outsiders is a way to implement an equilibrium with spot contracts, although it might require additional information revelation for that to occur in equilibrium.

In our main application, we specialize our environment to a bank lending market and derive two main implications. First, making the entire borrowers' history public is never optimal. The information released should be coarsened into a credit rating system that bunches the high types and some low types within the highest rate. A corollary of this result is that data ownership should not be transferred to borrowers, contrary to what the right to data portability of the GDPR and the related open banking reforms dictate. Second, while disclosure mitigates the increase in interest rates within long-term relationships, it might be optimal to outright grant consumers the right to (commit to) data erasure whenever an equilibrium with spot contracts is optimal. Obtaining data erasure, while also possible under the GDPR, might be challenging in the banking sector due to legal and compliance obligations.[3] We show that a similar outcome can be obtained whenever productive entrants (e.g. fintech firms) are able to attract borrowers, hence weakening the adverse selection into switching to a new firm. In the presence of productive entrants, less information should be revealed in order to preserve cross-subsidization.

Related literature This paper contributes to the literature on insurance contracts under lack of commitment, in the spirit of the seminal contributions by Harris and Holmstrom (1982) and Thomas and Worrall (1988). For example, this class of models has been used to study wage-tenure profiles by Burdett and Coles (2003) and, more recently, by Balke and Lamadon (2022) or life-insurance contracts by Hendel and Lizzeri (2003). The agent's outside option plays a critical role in shaping the amount of insurance attainable. We contribute to this literature by assuming that over time the incumbent firm acquires more information about the consumer's persistent type than its competitors do and show how this affects the consumer's outside option.[4] We then study how public disclosure policies affect potential competition and thereby the amount of insurance and consumption

[3] For example, banks must retain records to comply with anti–money laundering and prudential regulations.

[4] Sharpe (1990) studies a model with asymmetric learning between firms and long-term lending relationships. The equilibrium dynamics—for a given disclosure policy—is similar to the one in our economy, but he does not study the optimal information disclosure. More recently, Kaniel and Orlov (2025) study a labor market in which the worker's quality gradually becomes public information and firms can extract rents by affecting the evolution of those beliefs via firing and retention decisions. Another strand of the literature allows for asymmetric information to arise between the agent and all the firms, e.g., Green (1987), or about the outside offer that the agent receives, e.g., Hopenhayn and Werning (2008).

smoothing attainable in equilibrium.

In solving for the optimal disclosure policy, our paper connects to the growing literature on information design: Kamenica and Gentzkow (2011), Bergemann et al. (2015), Bergemann et al. (2017), and Mathevet et al. (2020). Related to our work, Garcia and Tsur (2021) and Immorlica et al. (2022), study optimal information disclosure in adverse-selection economies. Calzolari and Pavan (2006) study the problem of information disclosure in a multiple-principal setting. Farinha Luz et al. (2023) characterize requirements for signals on consumer types to be welfare-improving. Elliott et al. (2024) and Fainmesser et al. (2023) also study how information disclosure affects the market equilibrium. In all these papers, consumers and firms are short-lived, and firms are symmetrically uninformed.

The closest papers to ours are de Garidel-Thoron (2005) and Mukherjee (2008). Both also study the role of information disclosure in dynamic insurance economies with long-term contracts where the incumbent gains an informational advantage. de Garidel-Thoron (2005)'s main analysis is under commitment on the firm's side, and he compares welfare under two polar opposites: full and no information disclosure. His main result is that the value under no information disclosure is higher. The key contribution of our paper is the result showing that some information disclosure is optimal when firms lack commitment.

Mukherjee (2008) studies an environment where outsiders in the second period are more efficient than the incumbent (as in our analysis in Section 8) and finds conditions under which full information disclosure is optimal. Critical to his result are firms' commitment and the ability of the incumbent firm to impose a penalty for agents that choose to leave. If either of those assumptions is violated, information affects not only the amount of resources available to the incumbent but also the way they are distributed across time and state, and that full information is never optimal. We also find conditions under which the mere existence of long-term relationships is harmful to consumers.

Our paper is also related to studies of competition in frictional markets characterized by adverse selection, such as Guerrieri (2008), Guerrieri et al. (2010) and Lester et al. (2019). While in these papers information is symmetric across firms, in our work the level of potential competition ex-post is driven by the incumbent's informational advantage.

Inspired by a growing policy debate, a recent literature studies the consequences of the implementation of open banking policies. This literature considers environments in which information sharing stimulates competition from new fintech firms. See for example Babina et al. (2024) and He et al. (2023). Both papers restrict attention to static contracts, hence neglect intertemporal smoothing and its interaction with cross-subsidization.

Our perspective on consumer data ownership differs from that in Jones and Tonetti (2020). They show that assigning data property rights to consumers improves the allocation by allowing multiple firms to use the same data–in line with the *Right to data* *portability* in the GDPR. Ali et al. (2023) also find that consumers' ability to disclose verifiable information about their valuation for a product is typically beneficial, as it increases the volume of trade compared with uniform pricing. In Eeckhout and Veldkamp (2022), an imbalance in the amount of available data between firms competing in a spot product market generates excessive markups that hurt consumers. We find a complementary channel: in the presence of insurance considerations, regardless of the contract duration, assigning full data ownership to consumers is welfare-reducing because doing so kills all insurance within and across periods. At the same time, granting consumers the 'Right to be forgotten', in the spirit of a different prescription of the GDPR, might be beneficial in order to remove the incumbent's informational advantage, but only if consumers can commit to such data erasure upon signing a contract.

Relatedly, Kovrijnykh et al. (2024) study a distinct mechanism through which information sharing can be beneficial. In their model, multiple lenders receive signals about a borrower, and borrowers can maintain multiple lending relationships. Access to a borrower's credit history allows a lender to infer the signals received by its competitors, which can lead to higher credit limits. In their setting, the entire history of accepted contracts is publicly observable, and the optimal design of disclosure is not analyzed.

The rest of the paper is organized as follows. Section 2 presents the environment. Section 3 shows how public information disclosure affects potential competition and the equilibrium in the second period. Section 4 studies the case under firm commitment. Section 5studies the case when also the firm cannot commit. Section 6 discusses the implications of our results and some extensions. Section 7 studies when a sequence of spot contracts is superior to long-term relationships. Section 8 introduces reasons for the consumer to switch firms, and Section 10 concludes the paper. All omitted proofs are in Appendix B.

2 Environment

Consider an economy that lasts for two periods, $t = 1, 2$. There are two types of agents: consumers and firms. Each period is divided into two subperiods. In the first subperiod, consumers receive no income–this is simply a normalization. Consumers are uncertain about their income in the second subperiod, y, which can take a high value, y_H, or a low value, y_L. We assume that $y_1 \sim \pi_1(y_1)$ and $\pi_2(y_2|y_1)$ is the conditional probability of y_2 in the second period after a realization of y_1 in the first period. Thus, period 1 income is useful to forecast period 2 income. We denote consumers with a realization of y_H in period 1 as the high-income consumers or, simply, high types. We define Y_{2H} and Y_{2L} to be the expected income in period 2 conditional on having a high or low income realization in period 1, respectively,

$$Y_{2H} \equiv y_L + \pi_2(y_H|y_H)(y_H - y_L) > Y_{2L} \equiv y_L + \pi_2(y_H|y_L)(y_H - y_L).$$

We assume that $Y_{2H} > Y_{2L}$, so a higher income realization in period 1 forecasts a higher average income in period 2. We also define the (unconditional) expected income in period 1, $Y_1 = y_L + \pi_1(y_H)(y_H - y_L)$, and in period 2, $Y_2 = \sum_{i=L,H} \pi_1(y_i) Y_{2i}$. We assume that $Y_{2L} \leqslant Y_1 \leqslant Y_{2H}$, or, equivalently, that $\pi_2(y_H|y_L) \leqslant \pi_1(y_H) \leqslant \pi_2(y_H|y_H)$. Last, we assume that $\pi_2(y_H|y_1) \in (0, 1)$ for all y_1 so that there is residual uncertainty in period 2.

In the Appendix, we show how this restriction arises in an economy where consumers can be of one of two unobservable types, $\theta \in \{\theta_L, \theta_H\}$, that affect the probability distribution of income $p(y|\theta)$, with $p(y_H|\theta_H) > p(y_H|\theta_L)$, and agents learn about the consumer's type by observing the income realizations. Such an economy can be exactly mapped into our economy with $Y_1 = Y_2$, so the expected income in period 1 and the unconditional expected income in period 2 are the same. We choose to illustrate our results assuming income in the second subperiod follows a Markov chain to minimize the required notation.

Consumers are risk-averse and have common preferences over expected per-period consumption, C, given by $u(C)$, with u strictly increasing and strictly concave. We let $h = u^{-1}$. Consumption within the period is aggregated linearly: consumers value consumption in the first subperiod, c_{am}, at factor 1 and in the second subperiod at factor $\beta < 1$. We assume that consumers have limited liability so that their consumption must be non-negative in each subperiod. As we show below, it is never optimal to offer strictly positive consumption in the second subperiod after the realization of y_L. We can then denote consumption after y_H simply as c_{pm} and express, $C_1 = c_{am,1} + \beta\pi_1(y_H) c_{pm,1}$ in period 1 and $C_2 = c_{am,2} + \beta\pi_2(y_H|y_1) c_{pm,2}$ in period 2.

Firms are risk-neutral. Firms value income in both subperiods at factor 1. To simplify notation, without loss of generality, we assume no discounting between period 1 and period 2 for neither firms nor consumers.

We briefly comment on the main features of our preferences. The differential discounting within each period generates gains from trade: within each period, firms aim at front-loading their transfers to consumers, subject to the uncertainty about their income realization. If within-period discounting was the same for consumers and firms, perfect separation would be costless, effectively eliminating the information advantage of the incumbent. Risk-aversion generates a utility loss whenever expected consumption is unequally distributed across time and state, period 1 and period 2 after each realization of y_1. Absent risk-aversion, consumers only care about the present value of consumption which always equal to the present value of income due to competition ex-ante. Thus, both differential discounting and risk-aversion are needed for information design to affect consumer welfare.

Our environment captures the same forces as a standard Rothschild-Stiglitz economy within each period: the usual trade-off between separating and pooling consumer types emerges. Instead of assuming insurance motives *within a period*, we adopt a linear aggregation, augmented with differential discounting between consumers and firms, in order to obtain closed-form expressions for the allocation and a sharper characterization of the separating and pooling regions.[5] However, concavity of utility *across periods* still generates standard insurance motives. (A previous version of the paper, Dovis and Martellini (2024), considers a more canonical two-period insurance economy that delivers the same results under an assumption on endogenous objects, satisfied in numerical examples and for which we provide a sufficient condition.)

Information and market structure Firms compete to attract consumers by offering contracts that specify consumption levels conditional on all the available information at the time. At the beginning of period 1, all agents share the same information. Firms simultaneously offer contracts $(c_{1,am}, c_{1,pm})$ to consumers, who choose one firm to sign with. We will call the firm chosen by the consumer in period 1 the *incumbent*. The firm pays consumption $c_{1,am}$. At the end of period 1, income y_1 is realized and it is observed by the consumer and the incumbent. The payment $c_{1,pm}$ is made. The *outsiders* (i.e., the other firms) do not observe y_1 directly but observe a public signal m in some signal space M. Public signals are distributed according to some distribution $\mu \in \Delta(M)$, where $\mu(m|y_1)$ denotes the share of consumers with income y_1 that receive signal m. We will refer to (M, μ) as the *public disclosure policy*.

At the beginning of period 2, the incumbent acts as a Stackelberg leader and offers a contract $(c_{2,am}, c_{2,pm})$ conditional on the consumer's history (y_1, m). Outsiders observe the distribution of contracts offered by the incumbent and can poach consumers by offering a menu of contracts $(c_{2,am}, c_{2,pm})$ to all consumers with a given public signal m. Critically, outsiders do not observe the realization of y_1 and the contract offered by the incumbent to a particular consumer. All firms observe the contracts offered and decide whether to withdraw their contracts at a cost $\varepsilon \geqslant 0$. Finally, consumers choose the contract that maximizes their utility among those that are left after the withdrawal stage. A consumer with history (y_1, m) can either accept the single contract offered by the incumbent or select one of the contracts in the menu offered by the outsiders to consumers with history m. If there are contracts that offer the same value, we consider the following tie-breaking rule: if the incumbent is among those offering the contract with the highest

[5] In doing so, we connect with the literature on redistribution in linear environments (e.g., Dworczak et al. (2021)).

value, then the consumer stays with the incumbent; if not, the consumers are evenly split among the outsiders with the same offer.[6]

We always assume that consumers can walk away from the incumbent and sign with another firm.[7] On the firm side, we consider two cases. First, we assume, as a benchmark, that the incumbent can commit in period 1 to the terms of the period 2 contract. Second, we assume that also the insurance firm cannot commit to the terms of the period 2 contract. Under both assumptions on commitment, we characterize the equilibrium outcome for a given public disclosure policy and then characterize the one that maximizes ex-ante welfare.

Remark. In setting up the interaction among firms in period 2, we follow Netzer and Scheuer (2014) and allow for a stage in which each firm observes other firms' contracts and can decide to withdraw its own at a small cost $\varepsilon > 0$. We highlight a similarity and a key distinction with respect to their model. As in Netzer and Scheuer (2014), the presence of a withdrawal stage guarantees that there exists a continuation equilibrium in pure strategies in period 2 for any signal m. In particular, the ability to withdraw the offered (menu of) contracts allows for the existence of equilibrium outcomes that feature cross-subsidization. The possibility of withdrawing one's contract after observing the set of contracts offered in equilibrium rules out deviations in which competitors offer a "cream-skimming" contract to the high types. These deviations are at the root of the non-existence of pure strategy equilibria in Rothschild and Stiglitz (1976).

In Netzer and Scheuer (2014), the presence of a strictly positive withdrawal cost, $\varepsilon > 0$, guarantees uniqueness of continuation equilibrium outcomes by ruling out those that generate positive profits for some firms. Our model differs from theirs due to the presence of an informed incumbent. For us, if $\varepsilon > 0$, uniqueness of equilibrium outcomes–and, in fact, even equilibrium existence–additionally requires that the incumbent acts as a Stackelberg leader. This requirement is satisfied necessarily when the incumbent can commit, given that the terms of the contract are predetermined at the beginning of period 2. We maintain the Stackelberg assumption also under lack of firm commitment.[8] We formalize these arguments in Appendix B.2 and B.4 for the cases with and without firm commitment.

[6]The selection rule is without loss of generality, because the incumbent has an informational advantage and can always attract all consumers by offering a little more consumption and still making positive profits. Our selection rule effectively makes the incumbent choice set closed.

[7]If both the incumbent and the consumer can commit to the terms of the contract in period 2 and to staying with the firm, respectively, then the public disclosure policy is irrelevant and $C_t(y^t) = c_{t,am}(y^t) = \frac{Y_1+Y_2}{2}$ and $c_{t,pm}(y^t) = 0$.

[8]An alternative approach would be to set $\varepsilon = 0$ without restrictions on whether the incumbent is a Stackelberg leader or all firms act simultaneously in period 2. The equilibrium outcome we characterize below would correspond to one of the many possible when $\varepsilon = 0$, specifically the one that minimizes period 2 profits for the incumbent.

3 Miyazaki-Wilson value

We start by characterizing the equilibrium value in period 2 that would arise absent an informed incumbent following Netzer and Scheuer (2014). The value associated to this equilibrium outcome will serve as a critical outside option that will shape the contract offered by the informed incumbent.

Consider agents with a signal m, and denote by s the fraction of consumers that have $y_1 = y_H$:

$$s = \frac{\mu\left(m|y_H\right)\pi_1\left(y_H\right)}{\sum_{y_1}\mu\left(m|y_1\right)\pi_1\left(y_1\right)}. \tag{1}$$

To characterize the continuation equilibrium, it is useful to define the maximal expected consumption that can be offered by the outsider to a consumer with history (y_H, m). The outsider chooses a consumption profile for the high-income consumers, (c_{am}, c_{pm}) and expected consumption intended for consumers with history (y_L, m), the low-income consumers, to maximize:

$$C_H^o\left(s\right) = \max_{c_{am}, c_{pm}, C_L^o} c_{am} + \beta\pi\left(y_H|y_H\right)c_{pm} \tag{2}$$

subject to the outsider's nonnegative profit condition,

$$s\left[Y_{2H} - \left(c_{am} + \pi\left(y_H|y_H\right)c_{pm}\right)\right] + (1-s)\left[Y_{2L} - C_L^o\right] \geqslant 0,$$

the incentive compatibility constraint for low-income consumers

$$C_L^o \geqslant c_{am} + \beta\pi\left(y_H|y_L\right)c_{pm},$$

and the participation constraint for low-income consumers,

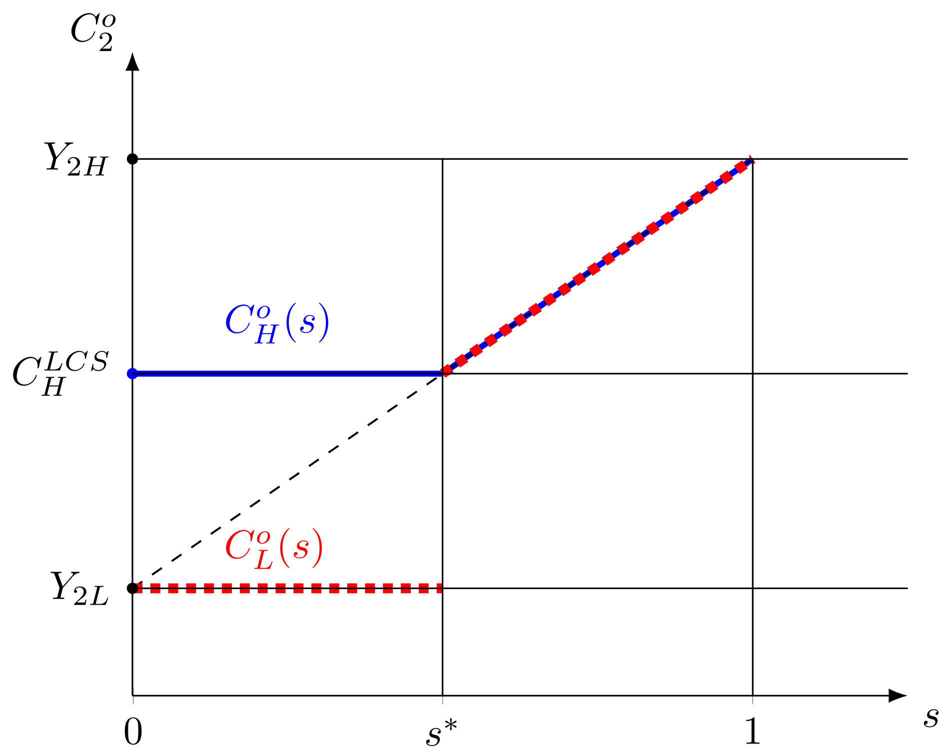
$$C_L^o \geqslant Y_{2L}. \tag{3}$$

We denote by $V_H^o = u\left(C_H^o\right)$ and $V_L^o = u\left(C_L^o\right)$ the associated consumers' values.[9] Throughout the paper, we say that consumption is distorted within the period whenever there is strictly positive consumption in the pm. In our description of (2) we impose two simplifications that are without loss of generality. First, the consumption of low types is undistorted. Second, consumption in the second subperiod after a realization of y_L is always set to 0, hence omitted from the problem.[10]

[9]To ease notation, in what follows we omit the dependence of C^o and V^o on s when the latter is kept fixed.

[10]The first simplification follows from the fact that the IC constraint for the high type is never binding. To see why the second simplification is without loss, suppose consumption in the pm after y_L is strictly

Figure 1: Expected consumption in Miyazaki-Wilson program



The program (2) is what Netzer and Scheuer (2014) term the Miyazaki--Wilson program, after Miyazaki (1977) and Wilson (1977). In Netzer and Scheuer (2014), given the absence of an incumbent with information advantage, the solution to this program characterizes the unique equilibrium outcome of the game in period 2.

We now characterize the solution to the problem. Let

$$\kappa \equiv \left(\frac{\pi(y_H|y_H) - \pi(y_H|y_L)}{\pi(y_H|y_H) - \pi(y_H|y_L)\beta} \right).$$

Lemma 1. *Let $s^* = \beta\kappa < 1$. If $s < s^*$, then the optimal allocation in (2) is the least-cost-separating constract:*

$$c_{am} = \pi(y_H|y_H)(y_H - y_L)(1-\kappa) + y_L, \quad c_{pm} = \kappa(y_H - y_L). \tag{4}$$

and $C_L^o = Y_{2L}$. If $s \geqslant s^*$, the optimal allocation in (2) features pooling:

$$c_{am} = C_L^o = sY_{2H} + (1-s)Y_{2L}, \quad c_{pm} = 0. \tag{5}$$

positive. Then, reducing it by an arbitrary small amount $\delta > 0$ and increasing c_{am} by $\beta\pi(y_L|y_L)\delta$ constitutes a profitable deviation, since it leaves the IC constraints unaffected, it relaxes the nonnegative profit condition, and it increases the value of the objective function.

If the share of high-income consumers is low, the participation constraint for the low-income consumers (3) is binding and the optimal solution is the *least-cost-separating* allocation with no cross-subsidization across types and expected consumption C_H^{LCS} (value $V_H^{LCS} = u\left(C_H^{LCS}\right)$) for the high-income type.[11] Since the participation constraint for low-income consumers binds, $C_L^o = Y_{2L}$ and the allocation for the high-income consumers can be solved by combining incentive compatibility and zero-profits, obtaining (4). The allocation of high-income consumers is distorted. That is, they receive strictly positive consumption in the pm despite that being discounted at a factor $\beta < 1$. This distortion is necessary to prevent low-income consumers – that have a lower probability of having high-income in the pm – to pretend to be high-income type. This is equivalent to imperfect insurance offered to the low risk-type in a standard insurance economy.

If instead the share s is sufficiently high, the solution features cross-subsidization, $C_L^o > Y_{2L}$, as illustrated in Figure 1. This is because it is cheaper to provide a subsidy to the (few) low-income consumers than to distort the allocation for the high-income consumers. In this case, the allocation satisfies perfect pooling across types. The allocation for high-income consumer is not distorted, but her expected consumption is lower than under full information unless $s = 1$.

We further characterize the critical value of s that sustains pooling, $s^* = \beta\kappa$. Intuitively, s^* is increasing in β since more patient agents face a lower cost from delayed consumption, which is necessary to induce separation. Similarly, s^* is increasing in $\pi(y_H|y_H) - \pi(y_H|y_L)$: a larger spread between types relaxes the IC constraint and lowers the adverse selection distortions of a separating contract.

4 One-sided commitment

We now consider the long-term equilibrium contract and the optimal public disclosure policy in the presence of an incumbent firm that can commit to the continuation contract. Consumer can always walk away from the incumbent in period 2 and accept an outsider's offer. We show that the optimal disclosure aims at minimizing the outside option for the consumer that draws y_H in period 1 to maximize the degree of cross-subsidization between types. The disclosure policy that best relaxes the ex-post participation constraint for the high type reveals no information.

Optimal contract We characterize the equilibrium contract for a given (M, μ). Firms in period 1 offer a state-contingent long-term contract $\{c_{1,am}, c_{1,pm}, c_{2,am}(y_1, m), c_{2,pm}(y_1, m)\}$

[11] Formally, the least-cost-separating allocation solves a restricted version of (2), where the participation constraint (3) must hold with equality.

and a withdrawal policy to attract consumers. Competition among firms implies that the equilibrium contract must maximize the consumer's expected utility subject to a zero-profit condition for the firm and ex-post participation constraints in period 2 that arise from outsiders' competition. In particular:

Lemma 2. *The optimal contract* $\{c_{1,am}, c_{1,pm}, c_{2,am}(y_1, m), c_{2,pm}(y_1, m)\}$ *satisfies*

$$c_{2,am}(y_H, m) + \beta\pi_2(y_2|y_H)\, c_{2,pm}(y_H, m) \geqslant C_H^o(s(m)), \tag{6}$$

$$c_{2,am}(y_L, m) + \beta\pi_2(y_2|y_L)\, c_{2,pm}(y_L, m) \geqslant Y_{2L}. \tag{7}$$

Moreover, the incumbent withdraws its offer if outsiders offer a cream-skimming contract intended for the high type.

If conditions (6) and (7) are violated, we can find an alternative contract for the incumbent that satisfies (6) and (7), retains all consumers by delivering the same amount of utility, and makes (weakly) positive profits because the incumbent can use its informational advantage relative to outsiders to offer the same value without distorting allocations to satisfy the incentive compatibility constraint.[12] These extra profits can then be used to increase consumption in period 1. Finally, withdrawing the offer if outsiders offer the cream-skimming contract is optimal because it relaxes constraint (6), as competitors cannot poach high types without also attracting low types.

It is clear that the equilibrium contract features undistorted consumption in period 1, $c_{1,am} = C_1$ and $c_{1,pm} = 0$, and in period 2 contingent on (y_1, m), i.e., $c_{2,pm}(y_1, m) = 0$ for all (y_1, m). Given a public disclosure policy (M, μ), the optimal contract offered in period 1 solves

$$\max_{C_1, C_2} u(C_1) + \sum_{y_1}\sum_{m} \mu(m|y_1)\, \pi_1(y_1)\, u(C_2(y_1, m)) \tag{8}$$

subject to the firm's nonnegative-profit condition,

$$Y_1 - C_1 + \sum_{i=L,H}\sum_{m} \mu(m|y_i)\, \pi_1(y_i)\, (Y_{2i} - C_2(y_i, m)) \geqslant 0,$$

and the ex-post participation constraints for high- and low-income consumers (6) and (7), respectively.

The participation constraint for the low-income consumers (7) is slack because the incumbent wants to subsidize consumption of the low types in period 2.

[12] Notice that we place Y_{2L} on the right-hand side of condition 7 even if the solution to the Miyazaki-Wilson problem features cross-subsidization. This is because if condition 6 is satisfied, all the high types are retained by the incumbent and no cross-subsidizing contract is offered by the outsiders along the equilibrium path.

Optimal disclosure policy We now turn to the design of the optimal public disclosure policy. Before proceeding, we state the following Lemma that will be useful to prove our next Proposition:

Lemma 3. $C_H^o(s)$ *is convex.*

Proof. For $s < s^*$, $C_H^o(s) = C_H^{LCS}$ which is a constant. For $s \geqslant s^*$, $C_H^o(s) = sY_{2H} + (1-s)Y_{2L}$ which is a linear increasing function of s. Since C_H^o is a continuous function, it immediately follows that $C_H^o(s)$ is convex. Q.E.D.

That is, the incumbent's cost of providing utility $V_H^o(s)$ in period 2, $C_H^o(s)$, is convex in the share of high-income consumers. This lemma guarantees that the maximal profits the incumbent firm makes on high-income consumers do not increase if those consumers are associated with more than one signal.

The next proposition shows that the value in (8) is maximized by a disclosure policy that provides no information.

Proposition 1. *When the firm has commitment, the optimal disclosure policy reveals no information.*

Proof. We can equivalently write the problem of choosing the optimal disclosure policy μ as one of choosing the optimal distribution of shares s subject to the Bayesian plausibility constraint,

$$\sum_s p(s)\, s = \pi_1(y_H)\,. \tag{9}$$

That is,

$$\max_{p\in\Delta([0,1])}\ \max_{C_1, V_L(s), V_H(s)} u(C_1) + \sum_s p(s)\left[sV_H(s) + (1-s)V_L(s)\right] \tag{10}$$

subject to

$$C_1 = Y_1 + Y_2 - \sum_s p(s)\left[sh(V_H(s)) + (1-s)h(V_L(s))\right] \geqslant 0$$

$$V_H(s) \geqslant V_H^o(s)\,, \tag{11}$$

and (9) where recall that $h = u^{-1}$.

Suppose by way of contradiction that the optimal disclosure policy induces a distribution $p \neq p^*$, where p^* is such that $p^*(\pi_1(y_H)) = 1$. If $\bar{V}_H = \sum_s p(s)\, sV_H(s)/\pi_1(y_H) \geqslant V_H^o(\pi_1(y_H))$, then by convexity of h, it is possible to deliver $\bar{V}_H$ at a lower cost with no information. Thus, by providing no information, it is possible to improve the ex-ante utility by increasing consumption in period 1 (or by increasing $V_L(s)$). If instead $\bar{V}_H = \sum_s p(s)\, sV_H(s)/\pi_1(y_H) < V_H^o(\pi_1(y_H))$, note that

$$\sum_s p(s)\, sh(V_H(s)) \geqslant \sum_s p(s)\, sh(V_H^o(s)) = \sum_s p(s)\, sC_H^o(s) > \pi_1(y_H)\, C_H^o(\pi_1(y_H))\,,$$

where the first inequality follows from (11), the second, from the definition of h, and the last, from the convexity of C_o^H. Thus, providing no information and delivering $V_H = V_H^o(\pi_1(y_H))$ increase the continuation value for high-income consumers and lower its cost. Thus providing information cannot be optimal. Q.E.D.

The optimal public disclosure policy aims at minimizing the outside option for high-income consumers in period 2 to maximize the degree of cross-subsidization between types. Revealing no information is optimal because it maximizes the distortions that the outsiders must impose on the high-income consumers to separate them from the low-income ones. These high distortions lower the value for the high-income consumers that can be offered by the outsiders and allow for greater cross-subsidization.

We can further characterize the optimal allocation under one-sided lack of commitment. The optimal allocation has

$$C_1 = C_2(y_L) \leqslant C_2(y_H) = h(V_H^o), \tag{12}$$

where the inequality is strict if the ex-post participation constraint (11) is binding. As in Harris and Holmstrom (1982) and Thomas and Worrall (1988), the equilibrium allocation has perfect consumption smoothing between periods 1 and 2 after a low income realization.If 11 is binding, consumption must be increased after a high income realization to retain the high-income consumers, consistent with the evidence in Hendel and Lizzeri (2003) and Balke and Lamadon (2022).

5 Two-sided lack of commitment

We next consider the case in which also the firms cannot commit to the terms of the contract beyond the current period. Here we show that, no matter what the public disclosure policy is, it is not possible to cross-subsidize the low type in period 2 and $C_2(y_L, m) = Y_{2L}$. Thus, the public information disclosure does not hinder (or enhance) the cross-subsidization that the low-income consumer can receive in period 2. However, information disclosure does determine the extent of consumption smoothing between period 1 and period 2 conditional on being a high-income consumer. The informational advantage of the incumbent affects the profits it can extract in period 2 on high-income consumers, but, because of competition in period 1, these profits are rebated to the consumer in period 1. Intuitively, the more information is disclosed, the lower the incumbent's ex-post profits and the more consumption is tilted toward period 2.

The optimal disclosure policy can be described by a two-signal system, $M = \{g, b\}$. All high-income consumers receive the good signal g. Low-income consumers receive a good signal with probability $1 - \mu$ and a bad signal with probability μ. Thus, μ measures the degree of informativeness of the disclosure policy. If $\mu = 1$, there is perfect information, while if $\mu = 0$, the outsiders have no information in addition to their prior. We show that, perhaps surprisingly, the amount of information disclosed is increasing in the share of high-income consumers, $\pi_1(y_H)$.

Outcome in period 2

We characterize the outcome by backward induction starting from the terminal period 2. Agents have history y_1 and a signal $m \sim \mu(y_1)$. We model the incumbent as a Stackelberg leader that offers its contract to existing consumers, mimicking the timing in the case with one-sided commitment.

The next lemma characterizes the unique continuation equilibrium outcome for a given public disclosure policy:[13]

Lemma 4. *For any signal m, consumption is undistorted in period 2, that is, $c_{2,pm} = 0$. There is no cross-subsidization from the high-income consumers to the low-income consumers and the latter always consume $C_2(y_L, m) = Y_{2L}$. The consumption of the high-income consumers is*

$$C_2(y_H, m) = h(V_H^o(s(m))).$$

The incumbent must offer a value of $V_H = V_H^o(s(m))$ to consumers with history (y_H, m) to prevent competitors from poaching them. Note that unless the signal is perfectly informative, firms make profits on the high-income consumers because $h(V_H^o(s(m))) < Y_{2H}$. The reason is that the incumbent knows the income realization in period 1, and the contract does not need to satisfy the incentive compatibility constraint in order to exclude the low-income consumers. Thus, the incumbent can offer the same value as the outsiders, but it does so by either fully front-loading consumption in the first subperiod or by not having to cross-subsidize the low-income consumers. In either case, the incumbent is economizing on the cost of delivering $V_H^o(s)$.

Consumers with low income in period 1 always consume their expected value of income Y_{2L}. Thus, firms make no profits on them. Even if consumers with $y_1 = y_L$ may receive a good signal, as consumers with $y_1 = y_H$, because of adverse selection, the former still get to consume Y_{2L}. Thus, their value is independent from the signal and from the disclosure policy. Moreover, other than in the case with perfect information revelation, there is essentially no mobility in period 2: all high-income consumers stay with the

[13] We assume that the insider cannot discriminate among consumers with the same history. In Appendix D, we show how this assumption changes the specifics of the equilibrium allocation without affecting the main message. In particular, the optimal signal structure maintains the same features as in the restricted case.

incumbent, because the incumbent makes strictly positive profits on them due to its informational advantage. Low-income consumers are indifferent between staying with the incumbent and moving to an outsider.

Outcome in period 1

In period 1, firms are competing for consumers. The equilibrium contract maximizes the consumer's expected utility subject to the dynamic zero-profit condition. Firms anticipate making profits in period 2 on the consumers with a high realization in period 1 (unless the signals fully reveal the consumer's type). Because of ex-ante competition, firms distribute such profits to consumers in period 1. It is clear that the optimal contract has no static distortions and consumption is fully front-loaded in the first sub-period, $C_1 = c_{1,am}$ and $c_{1,pm} = 0$. The optimal level of consumption offered in period 1 solves

$$V_1 = \max_{C_1} u\left(C_1\right) + \pi_1\left(y_H\right) \sum_m \mu\left(m|y_H\right) V_H^o\left(s\left(m\right)\right) + \pi_1\left(y_L\right) u\left(Y_{2L}\right) \tag{13}$$

subject to

$$Y_1 - C_1 + \pi_1\left(y_H\right) \sum_m \mu\left(m|y_H\right)\left(Y_{2H} - C_H^o\left(s\left(m\right)\right)\right) \geqslant 0.$$

It immediately follows that period 1 consumption is fully determined by the zero-profit condition holding with equality. We summarize the characterization of the equilibrium outcome given a public disclosure policy (M, μ) in the next lemma:

Lemma 5. *Given a public disclosure policy* (M, μ), *the equilibrium outcome has*

$$C_1 = Y_1 + \pi_1\left(y_H\right) \sum_m \mu\left(m|y_H\right) \Pi\left(m\right) \tag{14}$$

$$C_2\left(y_L, m\right) = Y_{2L} \tag{15}$$

$$C_2\left(y_H, m\right) = Y_{2H} - \Pi\left(m\right), \tag{16}$$

where $\Pi\left(m\right) = Y_{2H} - C_H^o\left(s\left(m\right)\right)$.

5.1 Optimal public disclosure policy

We now characterize the optimal public disclosure policy. Given the characterization in Lemma 5, the choice of disclosure policy does not affect the consumption of the low-income consumer that always consumes Y_{2L} in period 2, but it can affect the consumption of high-income consumers in period 2 and consequently in period 1. In fact, by affecting the share of high-income consumers with a given signal, the information designer can affect the outside options of these consumers in period 2 and therefore the value the incumbent offers them to retain them. Such values can range from the value of the least-cost-separating contract to $u\left(Y_{2H}\right)$, because that is the range of V_H^o, as illustrated in Figure 1.

The optimal disclosure policy depends on whether the following condition is satisfied:

$$C_H^o\left(\pi_1\left(y_H\right)\right) \geqslant Y_1 + \pi_1\left(y_H\right)\left(Y_{2H} - C_H^o\left(\pi_1\left(y_H\right)\right)\right). \tag{17}$$

That is, if under no information disclosure, expected consumption in period 2 after $y_1 = y_H$, $C_H^o\left(\pi_1\left(y_H\right)\right)$, is higher than expected consumption in period 1.

For low values of $\pi_1\left(y_H\right)$, condition (17) holds, the lower bound on high-income consumers' value is high, and the incumbent's profits in period 2 are too low to attain expected consumption equalization. Thus, the best that can be done is to maximize such profits, and the optimal way to do so is by providing no information.

For higher values of $\pi_1\left(y_H\right)$, condition (17) is not satisfied and the disclosure policy is designed to perfectly smooth consumption between period 1 and period 2 after a high income realization in period 1,

$$C_1 = C_2\left(y_H\right). \tag{18}$$

This can be achieved by considering two signals only, $M = \{g, b\}$ (good or bad), with a *bad-signal* structure: All high-income consumers, together with a fraction of low-income consumers, receive a good signal. Only low-income consumers receive a bad signal. It is optimal to have some low-income consumers with a good signal—even if, as we have shown, this does not affect their consumption—to manipulate the value for the high-income consumers and equalize their consumption between period 1 and period 2. This is attained for an intermediate value of signal informativeness between full information revelation and no information disclosure.

The following proposition characterizes the optimal information disclosure:

Proposition 2. *There exists a cutoffs π^* such that:*

1. *If $\pi_1\left(y_H\right) < \pi^*$, then $C_1 < C_2\left(y_H\right)$ and it is optimal to provide no information;*

2. *If $\pi_1\left(y_H\right) \geqslant \pi^*$, then consumption is equalized between periods 1 and 2 after a high income realization, and the optimal disclosure policy has a bad-signal structure, i.e., $M = \{g, b\}$ (good or bad), and $\mu\left(g|y_H\right) = 1$ and $\mu\left(b|y_L\right) \in (0, 1)$, so a bad signal fully reveals the period 1 income.*

First, due to the convexity of $C_H^o(s)$, it is optimal to assign the same signal to all high-income consumers. This is because introducing dispersion for the high-income consumers simply increases the cost of providing utility to such consumers. Thus, without loss of generality, we can consider two signals, $M = \{g, b\}$, and have $\mu\left(g|y_H\right) = 1$. We are only left to choose the fraction of consumers with low income in period 1 that receive the good signal. This is not going to affect their consumption, but it affects the composition of the pool of agents with a good signal and therefore what the incumbent must offer to retain the high-income consumers. The information design problem can then induce any continuation values for the high-income consumer in the range $[V_H^o(\pi_1(y_H)), V_H^o(1)]$.[14]

We can then write the problem as

$$\max_{V_H} u(C_1(V_H)) + \pi_1(y_H) V_H + \pi_1(y_L) u(Y_{2L}) \tag{19}$$

subject to

$$C_1(V_H) = Y_1 + \pi_1(y_H)[Y_{2H} - h(V_H)]$$

$$V_H \in [V_H^o(\pi_1(y_H)), V_H^o(1)]$$

and then recover $s(g)$ and $\mu(b|y_L)$ from

$$V_H = V_H^o(s(g)) \text{ and } s(g) = \frac{\pi_1(y_H)}{\pi_1(y_H) + \pi_1(y_L)(1 - \mu(b|y_L))}. \tag{20}$$

The optimal disclosure policy can then be represented by the share of high-income consumers who received a good signal, $s(g)$. In the appendix, we show that for low levels of $\pi_1(y_H)$, condition (17) holds, or, equivalently, the constraint $V_H \geqslant V_H^o(\pi_1(y_H))$ is binding. It is therefore optimal to provide no information, $s(g) = \pi_1(y_H)$, as illustrated in Figure 2. This is because even by maximizing period 2 profits on high-income consumers, by minimizing their values in equilibrium, we still have that $C_1 < C_2(y_H)$.

For higher values of $\pi_1(y_H)$, the value of expected profits under no information disclosure are higher (as is Y_1), so condition (17) is not satisfied and the V_H that solves (19) is interior.[15] It follows that

$$C_1 = C_2(y_H) = \frac{Y_1 + \pi_1(y_H) Y_{2H}}{1 + \pi_1(y_H)}$$

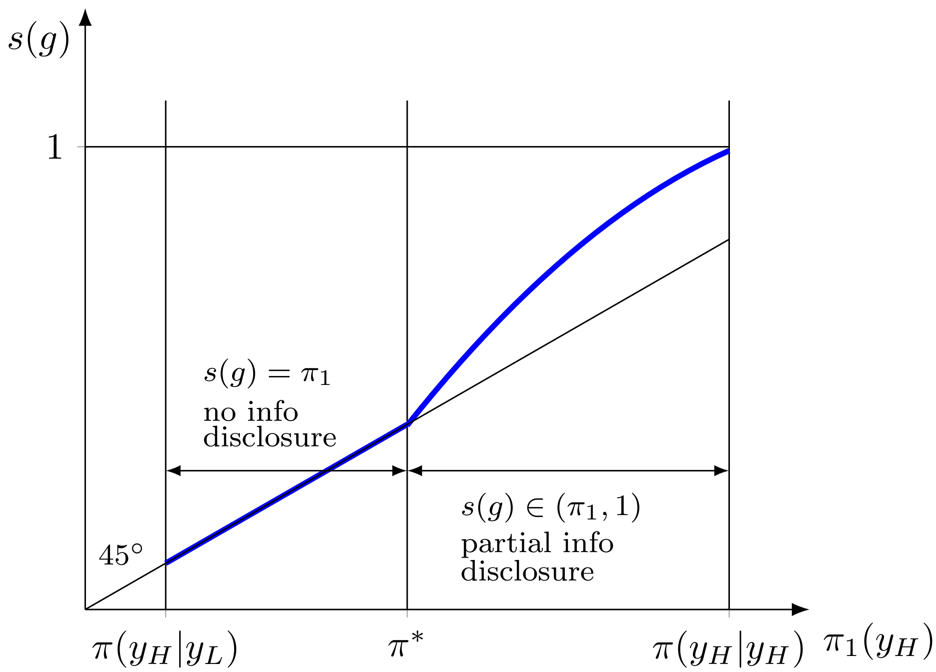
and $s > \pi_1(y_H)$, as shown in Figure 2. Moreover, since in this region $C_2(y_H)$ is increasing in $\pi_1(y_H)$ and $V_H^o(s)$ is increasing in s, then s must be increasing in $\pi_1(y_H)$.

Consider now the expected consumption profile at the optimal public disclosure pol-

[14] The size of the interval $[V_H^o(\pi_1(y_H)), V_H^o(1)]$ is hump-shaped in the degree of income persistence. If income is very persistent, e.g. $\pi_2(y_H|y_H) \to 1$, then it is easy to separate the high-income consumers, and $V_H^o(\pi_1(y_H))$ is close to $V_H^o(1)$. The same is true if income is (close to) i.i.d. over time and y_1 is not informative about y_2. Thus, the optimal public disclosure policy is more effective for intermediate values of income persistence.

[15] For values of $\pi_1(y_H)$ higher than $\pi_2(y_H|y_H)$ violating our requirement that $Y_1 \leqslant Y_{2H}$, condition (17) fails but it is not possible to find a disclosure policy such that consumption in period 1 and 2 are equated. Thus, in this cases it is optimal to provide full information disclosure and consumption is $c_1 = Y_1 > Y_{2H} = c_2(y_H)$. Note that the requirement that $Y_1 \leqslant Y_{2H}$ is never violated in the type interpretation of our economy.

Figure 2: Optimal share of high-income consumers conditional on receiving a good signal

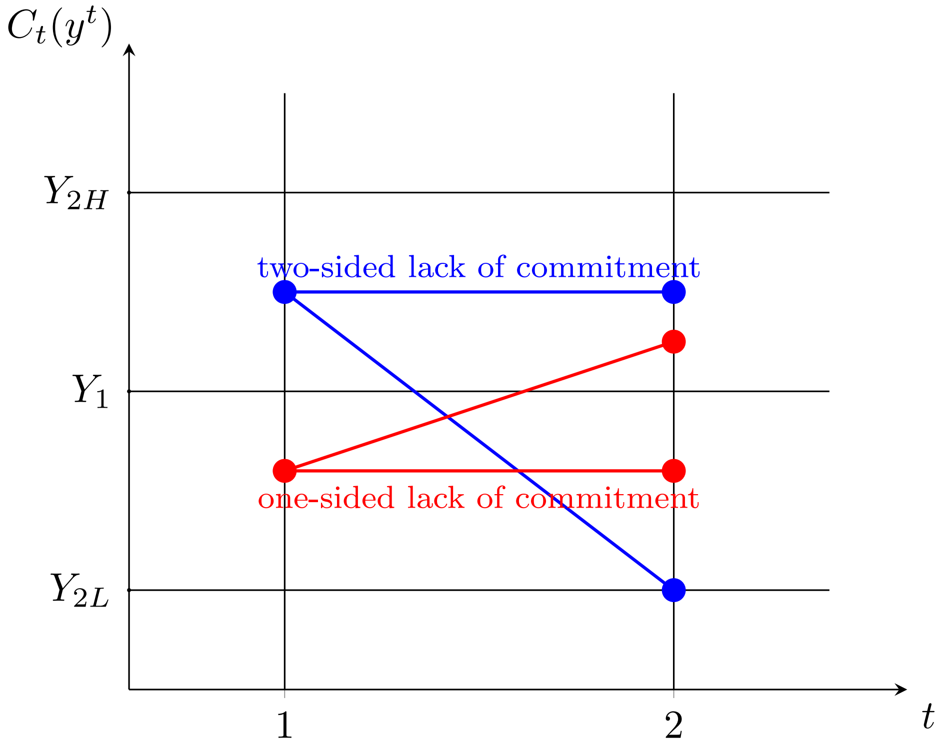


icy. Expected consumption is *front-loaded* and, for high values of $\pi_1(y_H)$, there is perfect insurance between period 1 and period 2 after $y_1 = y_H$:

$$C_1 = C_2(y_H) > C_2(y_L)\,. \tag{21}$$

This is the opposite consumption configuration relative to the standard case with firm commitment (e.g., Harris and Holmstrom (1982) and Thomas and Worrall (1988)). In such a case, consumption is back-loaded and there is perfect insurance between period 1 and period 2 after a bad income realization. The different predictions for the optimal consumption profiles are illustrated in Figure 3. The critical difference is that when the incumbent can commit, it redistributes the profits it can extract from the high-income consumers in period 2 to consumption in period 1 and to low-income consumers in period 2. Consumption smoothing may not be perfect, because the ex-post participation constraint for the high-income consumer may be binding, but consumption is constant otherwise. Without commitment on the firm's side, the incumbent does not provide any transfers to the low-income consumers. (And so do the outsiders because of adverse selection, since they anticipate that only low-income consumers will move to them in equilibrium.) Perhaps surprisingly, the larger the share of high-income consumers in the economy, $\pi_1(y_H)$, the more likely is that consumption is front-loaded across periods and that it is optimal to reveal some information. This result crucially depends on the dynamic nature of the con-

Figure 3: Consumption profile under optimal disclosure policy for high values of $\pi_1(y_H)$



tract and it stands in stark contrast with the traditional static adverse selection economy, as we show in Section 7 below.

The time series of contractual terms can then help to disentangle whether a firm can commit or not. As we showed in Proposition 1 and Proposition 2, the ability of firms to commit to a contract (because of reputation considerations, for example) is a critical factor for the optimal amount of information to disclose. Front-loading of consumption suggests firms' inability to commit and more information must be provided. Back-loading of consumption suggests that firms can commit and less (no) information should be disclosed to competitors.

6 Discussion

We now discuss some of the assumptions of our model and some extensions.

Regulation and commitment We model the choice of the optimal public disclosure as one made by a regulator. The presence of a regulator is not needed for the implementation of the optimal public disclosure policy. A firm in period 1 endowed with a commitment technology for reporting information in period 2 will choose the same disclosure policy as the planner to maximize the consumers' welfare subject to the zero-profit condition; otherwise, another firm can adopt the optimal disclosure policy, offer the same equilibrium value, and make positive profits.

A commitment technology to truthfully reporting according to (M, μ) is necessary. Consider a case in which condition (17) does not hold and it is optimal to provide information. The commitment technology is needed because in period 2 the incumbent would have incentives to provide no information to maximize its profits. To see this, suppose the incumbent offered the optimal disclosure policy from the ex-ante perspective. Consider a deviation where the incumbent provides no information (say it assigns signal g to everybody). In this case the value for the high type is $V_H^o(\pi_1(y_H)) < V^{optimal}$, and the low types get the same consumption as in the original allocation. Thus, the incumbent makes more profits on the high type. Providing no information is optimal ex-post, a commitment disclosure technology is thus necessary. Our required notion of commitment is stronger than the one in Lin and Liu (2022), who impose that the distribution of signals ex-post must be conforming with μ^*. This is because ex-post the incumbent will have incentives to assign the good signal to all low-income consumers—at least as long as it is feasible and not detectable—and a bad signal to the high-income consumers.

Ownership of information We implicitly assume that firms own the information they observe and are the only ones to have access to a commitment reporting technology that allows them to truthfully communicate, according to (M, μ). Clearly, if consumers had access to such a technology in the first period, they would choose the same disclosure policy as the one in the paper. However, if consumers had access to a technology to credibly report information about y_1 at the end of period 1—but they could not commit to a reporting strategy at the beginning of period 1—then full information disclosure would be the only equilibrium outcome, following the logic in Grossman (1981) and Milgrom (1981). In fact, all high-income consumers would have an incentive to fully disclose y_H.

As shown in the previous section, while full information disclosure maximizes the value for the high-income consumers in period 2 without affecting the value of the low-income consumers (if firms cannot commit), it is welfare-reducing, because it destroys the valuable consumption smoothing across periods. In this sense, transferring the ownership of information to the consumers is detrimental, and it is better to have the firm owning the consumers' information.

The optimality of assigning data ownership to firms would typically be true even if firms cannot commit to a disclosure policy. As argued above, the incumbent has incentives to reveal no information ex-post. Consumers instead will reveal all the information. As shown in de Garidel-Thoron (2005), the no-information outcome is preferred to the full-information outcome when $Y_1 = Y_2$.

Connection with information design literature Our environment departs from the traditional information design literature (Kamenica and Gentzkow (2011), GK) along at least three dimensions. First, it is dynamic, in that information in the second period determines the incumbent's profits that are rebated to the consumer in the first period. Second, given an information structure, payoffs are determined by the equilibrium in a game between multiple agents. Third, due to risk aversion and the dynamic nature of the problem, the objective function of the designer/sender typically depends on the whole posterior distribution and not just the posterior mean. Hence, the particular characteristics of our environment do not allow us to cast the information design problem as a variation of the well-known GK framework and its associated graphical representation.

Choosing a disclosure policy is equivalent to choosing a distribution of shares of high-income consumers with a given signal. Feasible distributions p must satisfy the Bayesian plausibility constraint requiring that

$$\sum_s p(s)\, s = \pi_1(y_H)\,; \tag{22}$$

that is, the total share of high-income consumers is $\pi_1(y_H)$.

Define

$$\begin{aligned} w_2(s) &\equiv s V_H^o(s) + (1-s)\, u(Y_{2L})\,, \\ w_1(p) &\equiv u\left(Y_1 + \sum_s p(s)\left(Y_{2H} - C_H^o(s)\right) s\right) \end{aligned}$$

where p is a probability measure over $[0,1]$. In a way similar to GK, we can reformulate the problem to solve for the optimal disclosure policy as

$$\max_{p \in \Delta([0,1])} w_1(p) + \sum_s p(s)\, w_2(s) \tag{23}$$

subject to (22). As it is clear from (23), the posterior mean is not a sufficient statistic for the problem. Even if $w_2(s)$ is convex for s lower than π^*, in Proposition 2 we show that it is optimal to disclose no information because of $w_1(p)$.

Market structure The design of the optimal disclosure policy hinges on the fact that the share of high types with a given signal, s, affects their outside option in equilibrium, $V_H^o(s)$. This is true in the strategic equilibrium notion that we adopt following Netzer and Scheuer (2014), Hellwig (1987), Wilson (1977), and Miyazaki (1977). Our results remain valid in a variant of the model where outsiders are restricted to offer only a single pooling contract, as in Kahn (2013) labor market application, or if we consider mixed strategy equilibria without allowing for contract withdrawal as in Dasgupta and Maskin (1986a), Dasgupta and Maskin (1986b), and Rosenthal and Weiss (1984). By contrast, our conclusions would change under the equilibrium notions in Azevedo and Gottlieb (2017), Bisin and Gottardi (2006), or the competitive search economy of Guerrieri et al. (2010). In these settings, the second-period outcome absent the informed incumbent is the least cost separating allocation that does not depend on the share of high type s. In those papers, $V_H^o(s) = V_H^{LCS}$ for all s, no matter how high, except for $s = 1$. The optimal disclosure problem then collapses to a binary choice: whether to reveal full information or not.

Information structure We assumed that income y_1 is not directly observable by outsiders and is (partially) revealed to them via the signal (M, μ). Here we establish that what is key for a disclosure policy to affect the equilibrium allocation is not the lack of information acquisition by outsiders but the existence of an information advantage held by the incumbent. Consider an alternative economy in which y_1 is some major outcome (e.g., a car accident, a borrower's history default, a worker's history of layoffs) that is publicly observable, while consumers and the incumbent firm also observe a private, non-redundant, and non-verifiable outcome $\tilde{y}_1$ (e.g., driving style, credit usage, performance at work). In the appendix we show that our results naturally extend to the modified environment. In particular, optimal information disclosure is still driven by intertemporal smoothing. The incumbent firm makes ex-post profits on consumers with high realizations of $\tilde{y}_1$, no matter their observed outcome y_1, and rebates such profits as first-period consumption.

Hidden effort In many applications, outcomes are the result not only of innate agents' characteristics but also of individual effort, as in Holmström (1979). For example, a worker's human capital is determined by her intrinsic ability and her investment in the acquisition of skills. By affecting the spread in consumption after good and bad outcomes, information design affects the amount of effort that can be sustained. In the appendix we extend our analysis to this case and show that optimal disclosure policy when both sides lack commitment has the same bad-news structure but provides more information than the base case analyzed earlier. This is because more information allows for more spreading in continuation values that incentivize the agent to exert effort in the first period.

7 Long-term vs. short-term contracts

We next study the value of long-term relationships between firms and consumers. We can do so by comparing the equilibrium outcome in our baseline case to the case where all consumers must change firms in the second period, and there are no long-term relationships. A literal interpretation of the latter arrangement is that all consumers are forced to sign contracts with a new firm (even if the incumbent can offer higher value because of better information). An alternative interpretation is that the incumbent is prohibited from offering different contracts (or menus) to consumers it has a relationship with. Thus, despite having information about the consumers, the incumbent is forced to offer them menus of contracts that satisfy the incentive compatibility constraint. One example of this no-discrimination requirement that is relevant in practice is the prescription of not allowing health insurance providers to condition their offerings on pre-existing health status.[16]

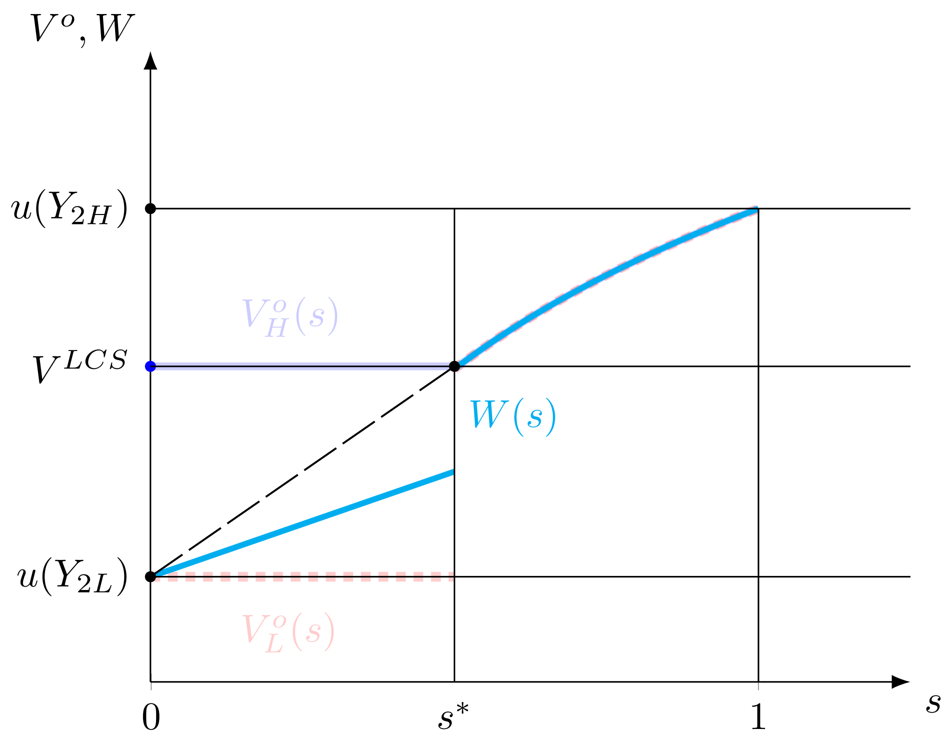
In an economy where firms can commit to the terms of the contract, long-term contracts are valuable because they allow for more insurance. When firms cannot commit, in the second period, low-income consumers consume the expected value of their income with no transfers from the high-income consumers. This is because the presence of an informed incumbent aggravates the adverse-selection problem: new firms know that only low types are willing to switch (absent additional utility benefits) and therefore are not willing to offer anything more than actuarially fair contracts. Thus, only intertemporal smoothing is possible between the first period and the second period in the high-income state.

Consider next the environment without long-term relationships between firms and consumers. In this case, all high-income consumers are forced to switch to new firms. This allows for the possibility of transfers to the low-income consumers in the second period, as shown by Netzer and Scheuer (2014). For a given public disclosure policy, consumption in the first period is Y_1, as there are no future expected profits. In the second period, the spot contract offered by the outsiders with a signal with share s is the Miyazaki-Wilson contract that solves (2). The expected value without long-term relationships is then equal to

$$u(Y_1) + \sum_s p(s) W(s),$$

[16] Another way to attain the same outcome would require departing from the assumption that the incumbent firm is always informed about the realization of y_1 and considering situations in which the consumer has the power to control the information collected. The consumer could then commit to reveal no information to the incumbent.

Figure 4: Expected continuation value with spot contracts, $W(s)$



where $p(s)$ is the distribution of high types' shares induced by the disclosure policy and

$$W(s) \equiv sV_H^o(s) + (1-s)V_L^o(s)$$

is the expected value associated with the Miyazaki–Wilson contract. We remind the reader that when long-term contracts are allowed, high-income consumers in the second period enjoy $V_H^o(s)$, but low-income consumers do not enjoy $V_L^o(s)$ due to the presence of an informed incumbent.

The value W is illustrated in Figure 4 together with V_H^o and V_L^o. As shown in Lemma 1, for $s < s^*$ the value for the low-income consumers is $u(Y_{2L})$ and for the high-income consumers is V_H^{LCS}. Both are independent of s, so W is linear in s over the interval $[0, s^*)$. For shares above s^*, the solution to (2) calls for subsidizing the low types to relax the incentive compatibility constraint. When that happens, both types receive the same (undistorted) consumption allocation, which is equal to $sY_{2H} + (1-s)Y_{2L}$.

The optimal disclosure policy solves

$$\max_{p(s)\in\Delta([0,1])} \sum_s p(s)W(s) \tag{24}$$

subject to $\sum_s p(s)\, s = \pi_1(y_H)$.

Lemma 6. *The optimal disclosure policy is such that:*

1. *If $\pi_1(y_H) < s^*$, then it is optimal to provide partial information disclosure with a bad-signal structure: $M = \{g, b\}$ with $s(g) = s^*$ and $s(b) = 0$.*

2. *If $\pi_1(y_H) \geqslant s^*$, then it is optimal to provide no information.*

The optimal information design problem trades off the cross-subsidization it delivers to the low-income consumers and the distortions associated with the incentive constraint for the high-income consumers. If $\pi_1(y_H) < s^*$, it is optimal to provide partial information disclosure. The share of the high type among the consumers with a good signal is s^*. Mechanically, the partial information disclosure takes advantage of the non-concavity of W over $[0, s^*]$, caused by the upward jump at s^*, and has two signals with shares 0 and s^*.

Intuitively, if $\pi_1(y_H)$ is low, the composition of the consumer pool absent any information disclosure is bad enough that it is optimal for the outsiders to offer a heavily distorted allocation to the high-income consumers and no subsidies to the low-income consumers. Thus, disclosing some information is beneficial to both the high-income consumers that can attain a less distorted allocation and, in expectations, also to the low-income consumers. In fact, with some disclosure, at least a fraction of the low-income consumers get to receive a subsidy. This result is similar to that in Goldstein and Leitner (2018).

When $\pi_1(y_H) > s^*$, it is optimal to provide no information, because that the equilibrium contract features pooling. Under information disclosure, the lower distortions for the high-income consumers are dominated by the reduction in insurance for the low-income consumers.

Lemma 6 implies that at least a fraction of the low-income consumers receive consumption higher than Y_{2L}.[17] Thus, the absence of an informed incumbent has the advantage of delivering higher consumption for the low-income consumers in the second period. There are also costs: no intertemporal smoothing is possible, and there are distortions in the allocation offered to the high-income consumers in the second period in order to screen out the low-income ones.

It is then not obvious whether, from an ex-ante perspective, consumers prefer to be in an economy with long-term relationships or one without. The next proposition shows that for $\pi_1(y_H)$ close to 0, long-term relationships are beneficial, while for $\pi_1(y_H)$ close to 1, they are detrimental:

[17] The fraction of the low-income consumers that receive a good signal and consumption above Y_{2L} is $\frac{\pi_1(y_H)}{\max\{\pi_1(y_H), s^*\}} \frac{(1-\max\{\pi_1(y_H), s^*\})}{1-\pi_1(y_H)} > 0$.

Proposition 3. *For $\pi_1(y_H) > 0$ sufficiently close to 0, the expected value with long-term relationships is higher than in the economy with spot contracts only. For $\pi_1(y_H) \leqslant \pi_2(y_H|y_H) < 1$ but sufficiently close to 1, the expected value with long-term relationships is lower than in the economy with spot contracts only.*

To start with, notice that for $\pi_1(y_H) = \{0,1\}$, consumers obtain the same value in both economies. If all consumers have the same income in the first period, there are no asymmetric information in the second period, no scope for intertemporal smoothing, and no distortions in consumption. We can further characterize the ranking of the two economies in a neighborhood of those extreme values of $\pi_1(y_H)$. Let $V^j(\pi)$ be the equilibrium value to consumers at the optimal information disclosure policy given $\pi = \pi_1(y_H)$ and $j = LT, ST$, where LT stands for the economy with long-term contracts and ST for the economy with spot contracts only. In the Appendix, we show that

$$\begin{aligned}
\frac{\partial V^{ST}(\pi)}{\partial \pi}|_{\pi=0} &= u'(y_L)(y_H - y_L) + \left[V_H^{LCS} - u(Y_{2L})\right] + \frac{1-s^*}{s^*}\left[V_H^{LCS} - u(Y_{2L})\right] \\
&< u'(y_L)(y_H - y_L) + \left[V_H^{LCS} - u(Y_{2L})\right] + u'(y_L)\left(Y_{2H} - C_H^{LCS}\right) \\
&= \frac{\partial V^{LT}(\pi)}{\partial \pi}|_{\pi=0}.
\end{aligned}$$

Thus, $V^{LT}(\pi) > V^{ST}(\pi)$ if π is sufficiently close to 0.

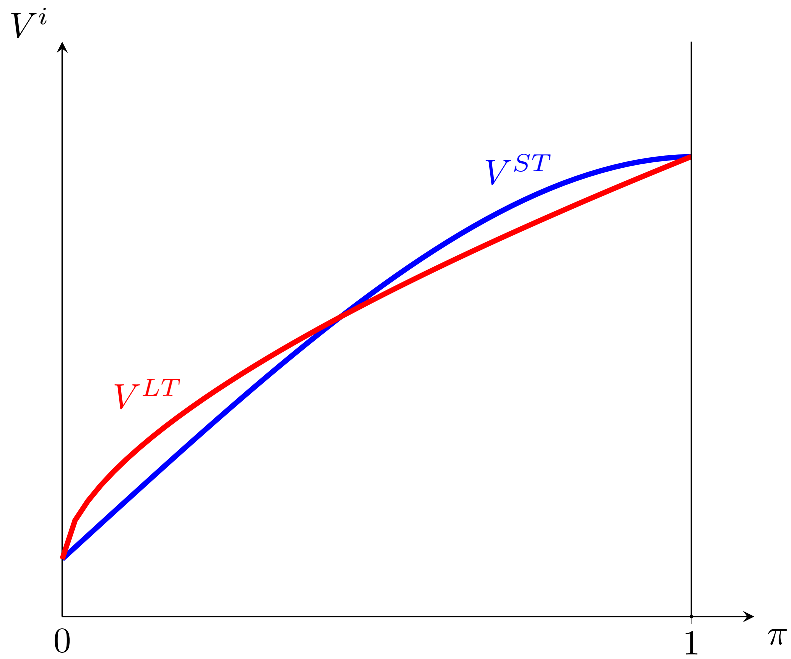
Intuitively, when there are sufficiently few high-income consumers, the second-period allocation entails little cross-subsidization across consumers regardless of the presence of an informed incumbent. However, due to its informational advantage, the incumbent is able to avoid distorting the consumption of the high-income consumers and to distribute the ex-post profits $Y_{2H} - C\left(V_H^{LCS}\right)$ as first-period consumption, valued at the marginal utility $u'(y_L)$. It follows that when adverse selection is severe, long-term relationships are beneficial.

Next consider the other extreme, $\pi = 1$. As we show in the Appendix,

$$\begin{aligned}
\frac{\partial V^{ST}(\pi)}{\partial \pi}|_{\pi=1} &= u'(y_H)(y_H - y_L) + u'(Y_{2H})[Y_{2H} - Y_{2L}] \\
&< u'(y_H)(y_H - y_L) + [u(Y_{2H}) - u(Y_{2L})] = \frac{\partial V^{LT}(\pi)}{\partial \pi}|_{\pi=1}
\end{aligned}$$

where the inequality follows from strict concavity of u. Thus, $V^{ST}(\pi) > V^{LT}(\pi)$ if π is sufficiently close to 1. Intuitively, when the pool of consumers in the economy has sufficiently many high-income consumers, there are large gains from cross-subsidization in the second period, absent the adverse selection induced by the presence of an informed firm. It follows that for a sufficiently large share of high-income consumers, long-term relationships are harmful.

Figure 5: Value of long-term relationships between firms and consumers



Characterizing the benefit of long-term relationships for interior values of π is more challenging. Yet, in all our numerical examples, we find that the intuition we provided for the ranking of values at the extremes of the support of π holds in the interior as well. Hence, there is a cutoff π above which $V^{ST}(\pi) > V^{LT}(\pi)$, as illustrated in Figure 5.

Our result that the quality of the consumer pool modulates the optimality of inducing the reclassification of consumers extends a similar insight in Handel et al. (2015), who restrict attention to spot contracts with symmetrically uninformed firms, to a dynamic environment with asymmetrically informed firms.[18] In both our paper and theirs, prohibiting consumers' reclassification is optimal when adverse selection is not severe (high π_1). However, in our paper with dynamic relationships, a low value of π calls for *partial*—instead of full–information disclosure, since the informed incumbent is able to avoid the adverse-selection distortion and, by making positive profits ex-post, is able to increase consumption ex-ante.

8 Ex-post competition and switchers

So far we have studied the role of information revelation in affecting potential competition since there are no allocative motives for the consumer to switch to a new firm.

[18]See also Finkelstein et al. (2009) for evidence on the redistributive implications of policies that restrict the dependency of contracts on observable consumer characteristics.

In this section, we introduce allocative motives by increasing the utility that the outsiders offer to the consumer who decides to switch, for example due to technological progress embedded into new firms. It is then possible that high-income consumers do switch in equilibrium, and this ameliorates the adverse-selection problem and allows for cross-subsidization of low-income consumers in the second period by new firms. We ask whether the introduction of allocative motives to switch in the second period entails an increase in the optimal amount of information disclosed to the market or whether the possibility of cross-subsidization induced by said switching calls for more secrecy. We show that the answer depends on the quality of the initial pool of consumers. If the pool quality is low—i.e., low $\pi_1(y_H)$—then allocative motives call for more information being revealed to incentivize consumers to switch to a new firm, while if the pool quality is high—i.e., high $\pi_1(y_H)$—then less information should be revealed.

We modify the baseline environment by assuming that consumers who switch obtain a utility benefit $\Delta \geqslant 0$ in addition to the utility from the consumption profile prescribed by the offered contract. Thus, the utility they obtain by switching to the new firm is $u(C) + \Delta$. This additive increase in utility captures the advantage outsiders have over incumbents without affecting the Miyazaki-Wilson contract described in (2). Therefore, it is immediate to see that $V_i^o(s; \Delta) = V_i^o(s) + \Delta$ for $i = H, L$.

We first consider the equilibrium outcome in the second period for a given public disclosure policy. When $\Delta > 0$, all low-income consumers leave the incumbent, breaking the indeterminacy in the baseline case. This is because they can get at least utility $u(Y_{2L}) + \Delta$ from new firms, while the incumbent offers them at most utility $u(Y_{2L})$. However, the critical difference with the baseline case is that when $\Delta > 0$, also some high-income consumers may choose to switch to a new firm if the value offered to them is higher than the best the incumbent can offer them without making negative profits. That is, the incumbent chooses to retain high-income consumers only if $V_H^o(s) + \Delta \leqslant u(Y_{2H})$. For any $\Delta > 0$, we define $\hat{s}(\Delta)$ as the maximal share of the high type with a given signal such that it is optimal for the incumbent to retain high-income consumers with that signal, implicitly defined by $V_H^o(\hat{s}(\Delta)) + \Delta = u(Y_{2H})$. Note that if Δ is sufficiently large, then it is impossible for the incumbent to retain consumers no matter the value of s. This is true for all $\Delta \geqslant \bar{\Delta}$ defined as $V_H^{LCS} + \bar{\Delta} = u(Y_{2H})$.[19] Throughout this section we restrict attention to values of $\Delta \in [0, \bar{\Delta}]$ because for $\Delta > \bar{\Delta}$ the allocation is the same as for $\bar{\Delta}$.

The possibility that consumers switch in the second period if $s \geqslant \hat{s}(\Delta)$ has two important effects. First, if some high-income consumers are not retained, then the incumbent is not making profits on them in the second period and consequently the contractual consumption offered in the first period is going to be lower. Second, switches by high-income

[19]Recall that V_H^{LCS} is the minimum value offered to high-income consumers by outsiders for any disclosure policy.

consumers mitigate the adverse-selection problem and make it possible for low-income consumers to receive a payoff higher than $u(Y_{2L}) + \Delta$ in equilibrium. This is the case if the contract that maximizes the high-income consumers' value calls for subsidizing the low types, that is, if $s \geqslant s^*$.

One can also show that $s^* \leqslant \hat{s}(\Delta)$.[20] Therefore, whenever high-income types switch, that is, for $s > \hat{s}(\Delta)$, low-income types with the same signal as high-income types receive a higher value from consumption than $u(Y_{2L})$.

We now turn to characterizing the optimal disclosure policy. Such policy trades off intertemporal consumption smoothing described in the baseline case ($\Delta = 0$) with the gains from cross-subsidization and allocative efficiency in the second period induced by the utility benefit Δ. To characterize the optimum, it is convenient to first define the expected continuation value for consumers with a signal characterized by the share s:

$$W(s;\Delta) = sV_H^o(s) + (1-s)V_L(s) + \Delta,$$

where

$$V_L(s) = \begin{cases} V_L^o(s) & \text{if } s \geqslant \hat{s}(\Delta) \\ V_L^o(0) = u(Y_{2L}) & \text{if } s < \hat{s}(\Delta) \end{cases}.$$

The following lemma builds on our earlier results to simplify the characterization of the optimal disclosure policy in this environment.

Lemma 7. *The optimal disclosure policy can be implemented with* $M = \{b, i, g\}$ *(bad, intermediate, good), where* b *is a bad signal and* $s(b) = 0$, $s(i) \equiv s_{in} \in (0, \hat{s}(\Delta))$, *so high-income consumers with signal* i *stay with the incumbent, and* $s(g) \equiv s_{out} \in [\hat{s}(\Delta), 1]$, *so high-income consumers with signal* g *switch to an outsider.*

To see why it is optimal to have only one signal associated with consumers that are switching to a new firm, note that the incumbent does not make any profits on them, and, because of the concavity of W, it is optimal to have only one signal with $s \geqslant \hat{s}(\Delta)$. The logic we used in Proposition 2 to show that all good types that stay have the same signal can be replicated to show that there is at most one group of consumers with $s \in (0, \hat{s}(\Delta))$ who are retained by the incumbent.

We can then write the problem for the optimal information disclosure policy as

$$\max_{s_{in}, s_{out}, p_0, p_{in}, p_{out}} u\left(Y_1 + p_{in}s_{in}\left[Y_{2H} - h\left(V_H^o(s_{in}) + \Delta\right)\right]\right) + p_0 W(0,\Delta) + p_{in}W(s_{in},\Delta) + p_{out}W(s_{out},\Delta)$$

[20] To see this, combining the definition of $\hat{s}(\Delta)$ and $\bar{\Delta}$, we have $V_H^o(\hat{s}(\Delta)) + \Delta = u(Y_{2H}) = V_H^{LCS} + \bar{\Delta}$. Thus, it must be that $V_H^o(\hat{s}(\Delta)) - V_H^{LCS} = \bar{\Delta} - \Delta > 0$. Since $V_H^o(s)$ is increasing, it follows that $\hat{s}(\Delta) > s^*$.

subject to

$$
\begin{aligned}
p_0 + p_{in} + p_{out} = 1, \quad p_s \geqslant 0 \\
p_{in} s_{in} + p_{out} s_{out} = \pi_1 (y_H) \\
s_{in} \in (0, \hat{s}(\Delta)), \quad s_{out} \in [\hat{s}(\Delta), 1]
\end{aligned}
$$

Proposition 4. *Full information is never optimal. Moreover, there exist cutoffs $0 < \Delta_L < \Delta_H < \bar{\Delta}$ such that for all $\Delta \leqslant \Delta_L$, we have $p_{out} = 0$; for all $\Delta \geqslant \Delta_H$, we have $p_{in} = 0$.*

We know that for $\Delta = 0$, it is always optimal for the incumbent to retain high-income consumers, as it is making ex-post profits on them. Thus, the information design problem aims at attaining *consumption smoothing* between period 1 and period 2 for the high types. For strictly positive but sufficiently small Δ, this is still the optimal thing to do because the gains from maximizing the continuation value in period 2 by letting the high-income consumers get the utility benefit Δ and allowing a negligible measure of low-income consumers to obtain some cross-subsidization are limited when Δ is small and, therefore, $\hat{s}(\Delta)$ is close to 1. Therefore, it is optimal to have $p_{out} = 0$ and choose s_{in} such that consumption in period 1 equates consumption in period 2 for high-income consumers. If feasible, the $s_{in}(\Delta)$ would solve

$$
Y_1 + \pi_1 (y_H) [Y_{2H} - h (V_H^o (s_{in}) + \Delta)] = h (V_H^o (s_{in}) + \Delta),
$$

otherwise $s_{in}(\Delta) = \pi_1 (y_H)$, as discussed in Proposition 2. Thus, to maximize the gains from consumption smoothing, it is optimal to keep the continuation value for the high type constant as Δ changes, $V_H^o (s_{in}(\Delta)) + \Delta = V_H^o (s_{in}(0))$. So, $s_{in}(\Delta)$ must be decreasing in this range, and the signal for the high type must be less informative, to reduce the value of the outside option and offset the increase in value given by the higher Δ. Thus, for small Δ, the amount of information disclosed is decreasing in Δ.

When Δ is sufficiently high, it is then optimal to let all consumers leave the incumbent, $p_{in} = 0$. The higher utility offered by the outsiders and the subsidy offered in equilibrium to the low type in period 2 outweigh the possible gains from intertemporal consumption smoothing. In this case, since the incumbent makes no profit in period 2, the consumption in period 1 is $C_1 = Y_1$ and the problem reduces to maximizing the static value in period 2:

$$
\max_{p_{out}, s_{out}} (1 - p_{out}) W(0, \Delta) + p_{out} W(s_{out}, \Delta) \tag{25}
$$

subject to

$$
p_{out} s_{out} = \pi_1 (y_H) \text{ and } s_{out} \geqslant \hat{s}(\Delta).
$$

The solution of this problem is the same as the one for the static problem (24) with the additional constraint $s_{out} \geqslant \hat{s}(\Delta)$ in order to guarantee that the high-income types do in fact choose to switch in period 2. In this sense, utility gains from switching to better outsiders provide an additional rationale for the emergence of spot contracts in our dynamic environment. If $\pi_1(y_H) < \hat{s}(\Delta)$, then it is optimal to have a bad signal with $s = 0$ and a good signal with share $\hat{s}(\Delta)$. In this case, some information disclosure is needed to achieve both switching and cross-subsidization of low-income consumers. If instead $\pi_1(y_H) \geqslant \hat{s}(\Delta)$, then it is optimal to provide no information, since outsiders are still able to attract high-income consumers, and providing further information would only hamper cross-subsidization. Moreover, since $\hat{s}(\Delta)$ is decreasing, stronger motives to switch (higher Δ) reduce the amount of information disclosed to the market.

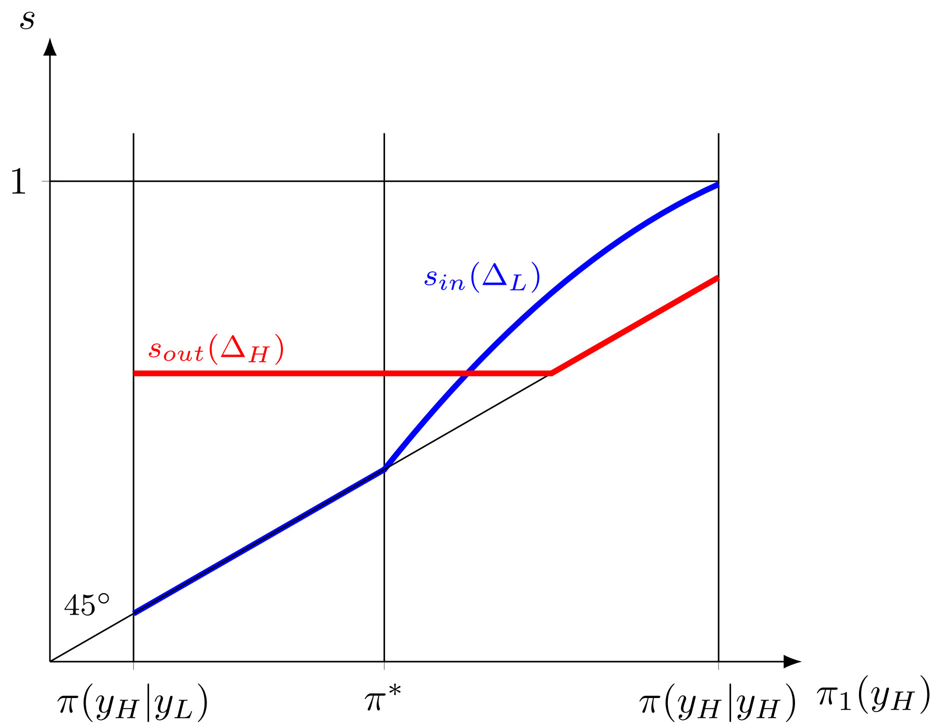
Figure 6 compares the information content in the two extreme cases: when Δ is high enough and in our baseline case with $\Delta = 0$. These two cases feature an opposite relationship between the pool composition π_1 and the optimal amount of information disclosure. If $\pi_1(y_H)$ is sufficiently small, then $\pi_1(y_H) < \pi^*$ and $\pi_1(y_H) < s_{in}(\Delta_L = 0) < s_{out}\left(\Delta_H \approx \bar{\Delta}\right) = s^*$. With Δ_L, intertemporal smoothing requires no information disclosure, while, with Δ_H, a better consumer pool composition than $\pi_1(y_H)$ is required to attain some cross-subsidization, and more information is disclosed. If $\pi_1(y_H)$ is close to the upper bound $\pi(y_H|y_H)$, then $s_{in}(\Delta_L = 0) \approx 1 > s_{out}\left(\Delta_H \approx \bar{\Delta}\right) = \pi_1(y_H)$, since the intertemporal smoothing motive calls for some information revelation, while with Δ_H it is optimal to provide no information.

For intermediate values of Δ, it might be optimal to have two signals with a positive measure of high-income consumers. The intermediate quality signal is associated with high-income consumers who do not switch, who enable intertemporal consumption smoothing; the highest-quality pool is associated with high-income consumers who switch, who enable cross-subsidization of low-income consumers, all of whom sign a contract with outsider firms.

Full information is never optimal. This is because either full information is not necessary in order to induce high-income consumers to switch—in which case insurance motives call for an interior pool composition—or full information is needed for high-income consumers to switch—but that would only be true if $\Delta = 0$, which is our benchmark case.

Remark. Note that here it is critical that firms cannot impose an exit fee agreed upon in period 1 to high-income consumers who leave the firm in period 2 (such as a prepayment penalty in a lending environment) and that firms lack commitment to the continuation contract. This is why our result differs from that of Mukherjee (2008), who finds that full information is optimal in an economy with $\Delta > 0$ when exit fees can be imposed and firms can commit. Under those conditions, firms can extract resources from the high-income consumers without distorting their choice about staying with the incumbent or

Figure 6: How information content varies by Δ for different consumer pool compositions



leaving. High-income consumers can then enjoy the higher utility offered by the outside firms, and the incumbent can collect a fee and make positive profits in period 2 on the high types. Perfect insurance is then attained by paying a severance payment to the low-income consumers who leave in period 2, and possibly by giving a transfer in period 1. In this ideal world, full information maximizes the amount of resources available to the incumbent who optimally distributes them across time and space. See also Cochrane (1995) for a similar insurance mechanism.

When exit fees, severance payments, or both are not available, or firms' commitment is not possible, then full information is not optimal.[21] Without commitment, changes in information disclosure affect not only the amount of resources available to the incumbent but also the way they are distributed. Full information is not optimal because the additional resources extracted from high-income consumers cannot be redistributed to the state in which they are most valued. For sufficiently high s, reducing such misallocation of resources dominates with respect to increasing their total quantity, making full information suboptimal.

[21]One can also show that with only commitment or only exit fees, full information is never optimal. In the first case, full information lowers the resources available to be distributed to low-income consumers. In the second case, full information eliminates cross-subsidization by the only firms that provide it, i.e., outsiders.

9 Application: Bank Lending and Open Banking

The model presented so far applies to numerous markets in which incumbent firms obtain an informational advantage over all other firms (e.g. labor markets, insurance markets, etc). One prominent application is the bank lending market, both in the context of corporate loans and personal credit.

This application is particularly relevant for two reasons. First, recent work provides direct evidence that incumbent banks are better informed about a borrower's creditworthiness than the rest of market. For example, Beyhaghi et al. (2024) use regulatory data on banks' internal assessment of corporate loan probability of default and loss given default. They find those measures to predict future stock and bond returns, and earning surprises relative to analyst estimates or what predicted by publicly available data. Interestingly, their result are stronger for small firms but hold for public companies as well.

Second, our analysis speaks directly to the recent policy debate about *open banking*. Under the open banking directive in the EU, currently discussed in the US, if a consumers chooses so, incumbent financial firms must share their client information with other financial firms. The consequences of open banking on bank competition and entry have been studied in recent empirical (Babina et al. (2024)) and theoretical (He et al. (2023)) work. Both papers restrict attention to static contracts, hence neglect intertemporal smoothing and its interaction with cross-subsidization. Our optimal disclosure policy can then inform the debate about how much information banks should be forced to disclose.

To map our model to the lending environment, assume that consumers are borrowers and firms are lenders. The contract between a borrower and lender is (b, R), where b is the loan size and R is the gross interest rate charged. Borrowers repay the loan if they have a high-income realization, y_H, otherwise the bank receives a recovery value y_L. Let

$$b = c_{am} \text{ and } R = \frac{y_H - c_{pm}}{b}.$$

It is easy to verify that the zero-profit condition in our general formulation implies that the expected return on the loan is equal to 1 either statically for the contract offered by the outsiders in the second period or in present-value for the incumbent.

Under two-sided lack of commitment, the model predicts that the interest rate offered in the contract starts low and eventually increases: the bank subsidizes the initial interest rate to compensate for the profits it makes in the second period on the high-income borrowers. This is consistent with Ioannidou and Ongena (2010) that show that newer loans carry a lower interest rate, and that banks subsequently increase the cost of lending to firms with whom they already have a relationship.

In our model extension that includes switching motives to more productive outsiders, the pool of switchers is typically worse than the pool of stayers due to adverse selection.[22] This result echoes the finding in the personal loan market by Di Maggio and Yao (2021) who document higher default rates among borrowers that switch to a fintech firm, but decreasingly so with time spent by the fintech firm in the market. Moreover, hard information like credit score and loan size explains 80% of the variation in interest rates for fintech loans but only 59% for bank loans.

We finally turn to the implications for open banking. Our analysis contains three main messages. First, unrestricted open banking, in the sense of full information sharing, is never optimal. Importantly, this conclusion holds even if open banking requires borrowers to opt in, as in the majority of proposals being discussed. Thus, assigning property rights over information to consumers is not optimal. Second, when banks lack commitment, a partial form of open banking can be welfare-improving, in particular in markets in which the typical firm repays and only a few default (high π_1), as shown in Proposition 2. In fact, absent any information disclosure, high values of π_1 trigger excessive front-loading of loan quantities, and interest rates that rise too much within the relationship. Third, when new technologies emerge that deliver additional value to customers, information sharing should actually be reduced when the pool of borrowers is sufficiently good, as shown in Proposition 4. When the introduction of new technologies induces creditworthy borrowers to move away from the incumbent, the adverse selection problem is less severe and it is possible to cross-subsidize less creditworthy lenders by providing less information to the new firms. This is valuable in our economy because borrowers are risk-averse and prefer an allocation that insures them against low income realizations. Alternatively, the same result would arise in an economy where the government has redistributive motives.

10 Conclusion

We have studied optimal public disclosure of information in a dynamic insurance economy in which the incumbent firm has an informational advantage over its competitors. We showed that if the incumbent firm has commitment, it is optimal to disclose no information. Competition from outsiders limits the amounts of insurance that can be sustained, as more information allows outsiders to bid for high-income consumers more aggressively, hence limiting the transfers available to low-income consumers. If the incumbent firm has no commitment, the presence of competition in the second period disciplines the ex-post behavior of the incumbent firm. While no cross-subsidization can be

[22] For example, in the equilibrium in Section 8, all low-income borrowers switch if $\Delta > 0$ while only a subset of high-income borrowers do.

sustained due to adverse selection, partial information disclosure allows intertemporal smoothing between consumption in the first period and in the second period after a high income realization. Full information is never optimal, as it erases the ex-post incumbent profits that are rebated in the first period.

Our analysis highlights a key trade-off between spot and long-term contracts: the former allow for cross-subsidization, while the latter remove the screening distortions due to adverse selection. We show that spot contracts are optimal only when the pool of consumers is sufficiently good to guarantee a transfer to low-income consumers absent an informed incumbent. In an extension of the model that includes utility benefits provided by outsider firms, information is increasing in the strength of those benefits only when necessary to induce high-income consumers to switch firms, but information is otherwise decreasing in order to preserve cross-subsidization. Yet, also in this modified environment, full information is never optimal.

We see two main avenues for further research. From a theoretical perspective, our model abstracts from endogenous firms' entry and innovation. The presence of an information monopoly and a set of agents that are locked-in with their incumbent may disincentivize entry and the introduction of innovative technologies that deliver higher utility, as in the case analyzed in Section 8. Modeling entry and studying how it interacts with the provision of insurance is a fruitful area for future research.[23] From an empirical perspective, measuring the model fundamentals in specific settings, like the banking sector we introduced in the previous section, would help quantify the forces behind information policy reforms such as open banking. This is especially true because, as we showed, optimal disclosure depends on the consumer's incentives to switch and on the composition of the relevant population, both of which are likely to vary systematically across market segments.

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[23] This aspect is analyzed by Jin and Vasserman (2021) in the context of the adoption of a monitoring technology in the car insurance market. They show in a policy counterfactual that forcing firms to share data will reduce the incentives to elicit such data and lead to lower welfare in equilibrium. Although via different mechanisms, both their paper and ours show that erasing ex-post profits by fully revealing consumer information is never optimal.

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Appendix

A Model

A.1 Type interpretation

Here we show that the pure exchange economy can be interpreted as one with types. Suppose that a consumers can be one of two types, θ_H and θ_L. The type affects the probability distribution of income. In particular, income in the second subperiod of period 1 and 2 can take on two values, $y_t \in \{y_L, y_H\}$ with $y_H > y_L$. θ_H consumers are more likely to have income than θ_L consumers, $\pi_L(\theta) = \Pr(y_L|\theta)$ and $\pi_H(\theta) = \Pr(y_H|\theta)$ with $\pi_H(\theta_H) > \pi_H(\theta_L)$.

Both the consumer and the firms do not know the consumer's type at the beginning of the period and learn about it through the observations of the history of income realization. Let $\rho(\theta)$ the common prior of being θ type at the beginning of period 1. After observing a high realization of income in period 1, the prior increases while it decreases after the realization of a low income realization:

$$\rho(\theta_H|y_H) = \frac{\rho(\theta_H)\,\pi_H(\theta_H)}{\sum_\theta \rho(\theta)\,\pi_H(\theta)} > \rho(\theta_H), \quad \rho(\theta_H|y_L) = \frac{\rho(\theta_H)\,\pi_L(\theta_H)}{\sum_\theta \rho(\theta)\,\pi_L(\theta)} < \rho(\theta_H). \tag{26}$$

Then, the probability that a consumer with history y_1 draws $y_2 = y_H$ is

$$\pi_H(y_1) = \rho(\theta_H|y_1)\,\pi_H(\theta_H) + \rho(\theta_L|y_1)\,\pi_H(\theta_L)$$

and the expected income is $\mathbb{E}(y_2|y_1) = \rho(\theta_H|y_1)\,\mathbb{E}_H y + \rho(\theta_L|y_1)\,\mathbb{E}_L y$. Thus, expected income in period 2 is higher after a high income realization in period 1 than after a low income realization:

$$\mathbb{E}(y_2|y_H) > \mathbb{E}(y_2|y_L).$$

This economy is equivalent to the one considered in the main text with

$$\begin{aligned}
\pi_1(y_H) &= \rho(\theta_H)\,\pi_H(\theta_H) + (1-\rho(\theta_H))\,\pi_H(\theta_L) \\
\pi_2(y_2 = y_H|y_H) &= \rho(\theta_H|y_H)\,\pi_H(\theta_H) + (1-\rho(\theta_H|y_H))\,\pi_H(\theta_L) \\
\pi_2(y_2 = y_H|y_L) &= \rho(\theta_H|y_L)\,\pi_H(\theta_H) + (1-\rho(\theta_H|y_L))\,\pi_H(\theta_L)
\end{aligned}$$

and

$$\begin{aligned}Y_1 &= \rho(\theta_H)\,\mathbb{E}_H y + (1-\rho(\theta_H))\,\mathbb{E}_L y\\ Y_{2H} &= \mathbb{E}(y_2|y_H)\\ Y_{2L} &= \mathbb{E}(y_2|y_L).\end{aligned}$$

As we describe in the text when discussing Proposition 2, providing full information is never optimal. This is because in this type formulation it must be that

$$Y_1 = \pi_1(y_H)\,Y_{2H} + (1-\pi_1(y_H))\,Y_{2L} = Y_2$$

because $\rho = \pi_1(y_H)\,\rho(\theta_H|y_H) + (1-\pi_1(y_H))\,\rho(\theta_H|y_L)$. Thus, we cannot be in a situation in which constraint $V_H \leqslant V_H^o(1)$ is binding in problem (19) because at $s=1$ we have

$$Y_1 < h(V_H^o(1)) = Y_{2H}.$$

[24]In most numerical examples we find a cutoff ρ^* such that for $\rho < \rho^*$ it is optimal to provide no information as condition (17) holds, while for $\rho > \rho^*$ it is optimal to provide partial information and have $C_1 = C_2(y_H)$ because condition (17) does not hold.

B Omitted proofs

B.1 Proof of Lemma 1

The IC constraint in problem 2 must be binding or the solution of the problem would be $c_{am} = Y_{2H}$ and $c_{pm} = 0$, which would clearly violate either the IC or the budget constraint. The budget constraint is binding by monotonicity of preferences. We can then substitute for C_L^o in the budget constraint using the IC constraint and further substitute for c_{am} in the objective function using the budget constraint. The objective function simplifies to

$$sY_{2H} + (1-s)\,Y_{2L} + [-s\pi(y_H|y_H) - (1-s)\,\beta\pi(y_H|y_L) + \beta\pi(y_H|y_H)]\,c_{pm}.$$

The coefficient on c_{pm} is negative iff $s \geqslant s^* \equiv \beta\kappa$. In that case, the solution to the problem entails $c_{pm} = 0$ and $c_{am} = C_L^o = sY_{2H} + (1-s)\,[Y_{2L}]$. If $s < s^*$, the objective is increasing

[24]Another way to see the issue is that it is not possible for $\pi_1(y_H)$ to go above $\pi_2(y_H|y_H)$ because even if $\rho \to 1$ then $\pi_1(y_1) \to \pi_H(\theta_H)$ with constant expected income in period 1 and 2.

in c_{pm}. Expressing C_L^o as a function of c_{pm}, we obtain

$$C_L^o = sY_{2H} + (1-s)\,Y_{2L} - s\left[\pi\left(y_H|y_H\right) - \beta\pi\left(y_H|y_L\right)\right]c_{pm}$$

which is strictly decreasing in c_{pm}. It follows that the participation constraint is binding, $C_L^o = Y_{2L}$, and that $c_{pm} = \kappa\left(y_H - y_L\right)$. From the IC constraint we further obtain $c_{am} = \pi\left(y_H|y_H\right)\left(y_H - y_L\right)(1-\kappa) + y_L$. Q.E.D.

B.2 Equilibrium outcome with one-sided commitment

As we describe in the text, for a given disclosure policy, the optimal contract offered in period 1 solves

$$\max_{C_1, C_2} u\left(C_1\right) + \sum_{y_1}\sum_{m} \mu\left(m|y_1\right)\pi_1\left(y_1\right)u\left(C_2\left(y_1, m\right)\right) \tag{27}$$

subject to the firm's nonnegative-profit condition,

$$Y_1 - C_1 + \sum_{i=L,H}\sum_{m} \mu\left(m|y_i\right)\pi_1\left(y_i\right)\left(Y_{2i} - C_2\left(y_i, m\right)\right) \geqslant 0,$$

and the ex-post participation constraints for high- and low-income consumers (6) and (7), respectively.

The participation constraint for the low-income consumers (7) is slack because the incumbent wants to subsidize consumption of the low types in period 2. Thus, the solution to problem (27) has $C_1 = C_2\left(y_L, m\right)$ for all m, while the consumption for consumers with history $\left(y_H, m\right)$ is given by $C_2\left(y_H, m\right) = \max\left\{C_1, C_H^o\left(s\left(m\right)\right)\right\}$, and the level of C_1 is given by the nonnegative-profit condition with equality:

$$C_1\left(1 + \pi_1\left(y_L\right)\right) + \pi_1\left(y_H\right)\sum_{m}\mu\left(m|y_H\right)\max\left\{C_1, C_H^o\left(s\left(m\right)\right)\right\} = Y_1 + Y_2.$$

This equilibrium outcome is the unique equilibrium when firms can commit because the solution to (27) is unique.

B.3 Proof of Lemma 4

The logic of Lemma 2 implies that the incumbent will always offer contracts that satisfy (6) and (7). We are going to show that the following is the unique equilibrium outcome: the incumbent offers a contract with no distortion and value $V_H^o\left(s\left(m\right)\right)$ to the consumers with history $\left(y_H, m\right)$ and a contract with no distortion and $c_{2,am}\left(y_L, m\right) = C_2\left(y_L, m\right) = Y_{2L}$ to the consumers with history (y_L, m).

Suppose first that in equilibrium low-income consumers receive a payoff of $u(Y_{2L})$. It is then clear that the insider will offer a contract with full insurance and value $V_H^o(s(m))$ to the consumers with history (y_H, m). Fully front-loading consumption in the am (no distortions) is optimal to minimize the cost of delivering such a level of utility. Note that the incumbent is making positive profits because $h(V_H^o(s(m))) \leqslant Y_{2H}$, with equality only if the signal is fully revealing. Offering a value $V_H^o(s(m))$ is optimal because if the incumbent offers less, then the outsiders can attract all high-income consumers and erase all the incumbent's profits. Offering more is not optimal, because it only reduces profits.

We are left to show that it is optimal to offer a contract with no distortions with value $u(Y_{2L})$ to the low-income consumers. Clearly, offering less is not feasible, because any outsider can always offer a contract with no distortions and value $u(Y_{2L})$. Offering more has no advantages, because the outside options for the high-income consumers are constant in the value offered by the incumbent to the low-income consumers. Q.E.D.

B.4 Equilibrium outcome with two-sided lack of commitment

As we show in Lemma 4 and 5, given a public disclosure policy (M, μ), the equilibrium outcome is given by

$$C_1 = Y_1 + \pi_1(y_H) \sum_m \mu(m|y_H)\, \Pi(m) \tag{28}$$

$$C_2(y_L, m) = Y_{2L} \tag{29}$$

$$C_2(y_H, m) = C_H^o(s(m)), \tag{30}$$

where $\Pi(m) = Y_{2H} - C_H^o(s(m))$.

As discussed in the remark in Section 2, if the withdrawal cost is strictly positive and the informed incumbent acts as a Stackelberg leader, this is the unique equilibrium outcome. To understand the role played by the Stackelberg leader assumption, suppose that $s(m) > s^*$ so that $C_L^o(s(m)) > Y_{2L}$. When the informed incumbent is a Stackelberg leader, it offers $C_H^o(s(m))$ to high types to prevent outsiders from offering a menu $\{C_H^o(s(m)), C_L^o(s(m))\}$ and poaching high-income consumers.

Consider instead the case where all firms act simultaneously. If the menu $\{C_H^o(s(m)), C_L^o(s(m))\}$ is offered by at least one outsider, the informed incumbent then offers $C_H^o(s(m))$ to the high types to retain them, since it makes strictly positive profits on them because of its informational advantage. If the withdrawal cost is strictly positive, this cannot be an equilibrium: offering $\{C_H^o(s(m)), C_L^o(s(m))\}$ is optimal for an outsider if the informed incumbent offers $C_H^o(s(m))$ to the high types. This is because all high types stay with the incumbent, while all the low types accept the outsider's offer, if not withdrawn. Thus, this strategy earns the outsider strictly negative profits,

$$\max\{-\varepsilon, -\pi_1(y_L)\,\mu(m|y_L)\,(C_L^o(s(m)) - Y_{2L})\} < 0.$$

Offering nothing is then preferable for the outsiders. If the incumbent offers less than $C_H^o(s(m))$ to the high types because it expects the outsiders to offer less than that, then it is optimal for the outsiders to offer $\{C_H^o(s(m)), C_L^o(s(m))\}$. Then no pure strategy equilibrium exists.

If the withdrawal costs ε are equal to zero, the outcome in Lemma 5 is also an equilibrium outcome when all firms act simultaneously. To support this equilibrium outcome, outsiders offer the menu $\{C_H^o(s(m)), C_L^o(s(m))\}$ and withdraw their offer if the informed incumbent offers $C_H^o(s(m))$ to the high types. The outsiders then attains a payoff of 0, so they are indifferent between this choice and not offering anything. However, if $\varepsilon = 0$, there can be multiple equilibria even absent the informed incumbent, as shown in Netzer and Scheuer (2014). The equilibrium outcome we consider is the one that minimizes the incumbent's period 2 profits.

B.5 Proof of Proposition 2

First we show that there exists a π^* such that condition (17) holds if and only if $\pi_1(y_H) \leqslant \pi^*$. To simplify the notation, we write $\pi_1 = \pi_1(y_H)$ when not ambiguous. Letting

$$K(s) \equiv C_H^o(s) = h(V_H^o(s)),$$

we can re-write condition (17) as

$$f(\pi_1) \equiv K(\pi_1) - [\pi_1 y_H + (1-\pi_1)\,y_L + \pi_1(Y_{2H} - K(\pi_1))] \geqslant 0$$

Note that

$$\begin{aligned} f'(\pi_1) &= (1+\pi_1)\,K'(\pi_1) - (y_H - y_L) - (Y_{2H} - K(\pi_1)) \\ f''(\pi_1) &= (1+\pi_1)\,K''(\pi_1) + 2K'(\pi_1) \end{aligned}$$

so f is convex in π_1 as $K'' \geqslant 0$ and $K' \geqslant 0$. Furthermore, evaluating at $\pi_1 = 0$ and $\pi_1 = \pi_2(y_H|y_H)$ we have

$$\begin{aligned} f(0) &= K(0) - y_L > 0 \\ f(\pi_2(y_H|y_H)) &= -(1+\pi_2(y_H|y_H))\,(Y_{2H} - K(\pi_2(y_H|y_H))) < 0. \end{aligned}$$

Thus, since $f(0) > 0$, $f(\pi_2(y_H|y_H)) < 0$ and f is convex, f must cross 0 only once between $(0, \pi_2(y_H|y_H))$, and we denote such point by π^*. Then for $\pi_1 \in (0, \pi^*)$ we have $f(\pi_1) > 0$ and for $\pi \in (\pi^*, \pi_2(y_H|y_H))$ we have $f(\pi_1) < 0$.

Suppose that $\pi_1(y_H) \geqslant \pi^*$ so condition (17) does not hold. First, we show that all high-income consumers receive the same signal. Suppose by way of contradiction that the optimal disclosure policy, (M^*, μ^*), is such that there are two signals, m_1 and m_2, with $\mu^*(m_1|y_H) > 0$ and $V_H^o(s(m_1)) \neq V_H^o(s(m_2))$. Let

$$\bar{V} = \sum_m \mu^*(m|y_H) V_H^o(s(m)).$$

Suppose that $\bar{V} \geqslant V_H^o(\pi_1(y_H))$. In this case, there exists an alternative disclosure policy with $M = \{g, b\}$ with $\mu(g|y_H) = 1$ and $\mu(b|y_L) \in [0,1)$ and $V_H^o(s(g)) = \bar{V}$. Due to concavity of the utility function, this alternative disclosure allows the incumbent to save resources in period 2 and still deliver expected utility $\bar{V}$ to high-income consumers. Because of competition in period 1, these additional profits are rebated to the consumer in the first period. Thus, this alternative disclosure policy is an improvement because it delivers the same expected utility in period 2 but higher utility in period 1, reaching a contradiction.

Consider now the case with $\bar{V} < V_H^o(\pi_1(y_H))$. Consider an alternative policy that discloses no information. The value in the second period for the high-income type is higher than $\bar{V}$. Also utility in period 1 is higher. In fact, note that

$$\begin{aligned}
&u\left(Y_1 + \pi_1(y_H) \sum_{m \in M} \mu(m|y_H)[Y_{2H} - h(V_H^o(s(m)))]\right) + \pi_1(y_H)\bar{V} \\
<&u\left(Y_1 + \pi_1(y_H)\left[Y_{2H} - h\left(\bar{V}\right)\right]\right) + \pi_1(y_H)\bar{V} \\
<&u(Y_1 + \pi_1(y_H)[Y_{2H} - h(V_H^o(\pi_1(y_H)))]) + \pi_1(y_H) V_H^o(\pi_1(y_H))
\end{aligned}$$

where the first inequality follows from the convexity of h, and the second inequality from the concavity of u, convexity of h and condition (17) not holding. To see this last step, note that

$$F(V) \equiv u(Y + \pi_1(y_H)[Y_{2H} - h(V)]) + \pi_1(y_H) V$$

is increasing in V for $V < V_H^o(\pi_1(y_H))$ under (17). In fact,

$$\begin{aligned}
F'(V) &\equiv \pi_1(y_H)\left[1 - u'(Y_1 + \pi_1(y_H)[Y_{2H} - h(V)])\, h'(V)\right] \\
&> \pi_1(y_H)\left[1 - u'(Y_1 + \pi_1(y_H)[Y_{2H} - h(V_H^o(\pi_1))])\, h'(V_H^o(\pi_1))\right] \\
&= \pi_1(y_H)\left[1 - \frac{u'(Y_1 + \pi_1(y_H)[Y_{2H} - h(V_H^o(\pi_1))])}{u'(h(V_H^o(\pi_1)))}\right] \\
&> 0
\end{aligned}$$

where the first inequality follows from u being concave, h increasing and convex and $V_H^o(\pi_1) > V$; and the last inequality from condition (17) not holding that implies

$$u'(Y + \pi_1(y_H)[Y_{2H} - h(V_H^o(\pi_1))]) < u'(h(V_H^o(\pi_1))).$$

Thus, even in this case the alternative disclosure policy improves upon the original allocation yielding a contradiction.

We established that under condition (17) all consumers with high income in period 1 receive the same signal. Without loss of generality we can consider $M = \{g, b\}$ and have $\mu(g|y_H) = 1$. We are only left to choose the fraction of consumers with low income in period 1 that receive the same "good" signal. This is not going to affect their consumption but it affects the composition of the pool of agents with a good signal and therefore the continuation value of the high income. The set of implementable continuation values for the high income consumer is $[V_H^o(\pi_1(y_H)), V_H^o(1)]$. We can then write the problem as

$$\max_{c_1, V_H} u(c_1) + \pi_1(y_H) V_H + \pi_1(y_L) u(Y_{2L}) \tag{31}$$

subject to

$$Y_1 - c_1 + \pi_1(y_H)[Y_{2H} - h(V_H)] \geqslant 0$$

$$V_H \in [V(\pi_1(y_H)), V_H^o(1)]$$

and then recover $\mu(b|y_L)$ from

$$V_H = V_H^o\left(\frac{\pi_1(y_H)}{\pi_1(y_H) + \pi_1(y_L)(1 - \mu(b|y_L))}\right). \tag{32}$$

If the last constraint in problem (31) does not bind and the optimal V_H is interior, it is clear that $u'(c_1) = u'(c_{2H})$ and so

$$C_1 = C_{2H} = \bar{C} \equiv \frac{Y_1 + \pi_1(y_H) Y_{2H}}{1 + \pi_1(y_H)} > Y_1$$

Thus, $\mu(b|y_L)$ solves

$$V_H^o\left(\frac{\pi_1(y_H)}{\pi_1(y_H)+\pi_1(y_L)(1-\mu(b|y_L))}\right)=u(\bar{C})$$

Then we just have to check if $u(\bar{C})\in[V_H^o(\pi_1(y_H)),V_H^o(1)]$. Because condition (17) is not satisfied, $V_H^o(\pi_1(y_H))\leqslant u(\bar{C})$. Moreover, since $\pi_1\leqslant\pi_2(y_H|y_H)$ then $u(\bar{C})\leqslant V_H^o(1)$. Thus, the last constraint does not bind.

Finally, suppose that $\pi_1(y_H)<\pi^*$ so condition (17) is satisfied. In this case, it is not possible to equalize consumption in period 1 and period 2 if $y_1=y_H$ by assigning the same signal to all high-income consumers. This is because the last constraint in (31) binds and $u'(C_1)>u'(C_2(y_H))$. It might then be optimal to assign different signals to the high-income consumers in order to reduce their expected continuation value, economize on resources used in period 2, increase the profits rebated in period 1. This is not feasible since $K(s)=h(V(s))$ is convex. Suppose by way of contradiction that the optimal disclosure policy, (M^*,μ^*), is such that there are two signals, m_1 and m_2, with $\mu^*(m_1|y_H)>0$ and $V_H^o(s(m_1))\neq V_H^o(s(m_2))$ with

$$\bar{V}=\sum_m\mu^*(m|y_H)V_H^o(s(m))<V_H^o(\pi_1(y_H)).$$

(Clearly, if $\bar{V}>V_H^o(\pi_1)$ an argument similar to the one above shows that assigning multiple signals to high income consumers is not optimal.) The period 1 consumption associated with this plan is

$$\begin{aligned}&Y_1+\pi_1(y_H)\left(Y_{2H}-\sum_m\mu^*(m|y_H)K(s(m))\right)\\<&Y_1+\pi_1(y_H)\left(Y_{2H}-K\left(\sum_m\mu^*(m|y_H)s(m)\right)\right)\\\leqslant&Y_1+\pi_1(y_H)(Y_{2H}-K(\pi_1(y_H)))\end{aligned}$$

where the first inequality follows from the convexity of K, and the second from the observation that $\sum_m\mu^*(m|y_H)s(m)\geqslant\pi_1(y_H)$ and K is increasing. Thus, disclosing no information increases both the expected continuation value in period 2 for the high-income consumers and consumption in period 1. Thus, the original allocation cannot be optimal and it must be optimal to assign the same signal to all high-income consumers. Therefore, problem (31) characterizes the full problem. Since the last constraint is binding, one way to obtain the optimum is to provide no information so $C_2(y_H)=h(V_H^o(\pi_1))$ and

$$C_1=Y+\pi_1(Y_{2H}-h(V_H^o(\pi_1)))<C_2(y_H).$$

Q.E.D.

B.6 Proof of Lemma 6

Problem 24 can be solved using the graphic approach in Kamenica and Gentzkow (2011). The value of the objective function is the convex hull of $W(s)$ evaluated at $\pi_1(y_H)$. We report the graph of $W(s)$ in Figure 7 for convenience. The key step is showing that the convex hull of $W(s)$ is equal to the line connecting $W(0)$ to $W(s^*)$ for $s < s^*$ and it is equal to $W(s)$ for $s \geqslant s^*$. It then follows that for $s < s^*$ the optimal information disclosure entails two signals with composition $s = 0$ and $s = s^*$, while for $s \geqslant s^*$ the optimal disclosure has a single pool of quality $\pi_1(y_H)$, which concludes the proof.

Since $W(s)$ is strictly concave for $s > s^*$, we need to show that the right derivative of $W(s)$ at s^* is smaller than the slope of the line connecting $W(0)$ to $W(s^*)$. The former is equal to $u'(Y(s^*))(Y_{2H} - Y_{2L})$, while the latter is equal to

$$\frac{u(Y(s^*)) - u(Y_{2L})}{s^*}$$

where $Y(s) \equiv sY_{2H} + (1-s)Y_{2L}$. Strict concavity of u then implies

$$\frac{u(Y(s^*)) - u(Y_{2L})}{Y(s^*) - Y_{2L}} > u'(Y(s^*)) \Rightarrow \frac{u(Y(s^*)) - u(Y_{2L})}{s^*} > u'(Y(s^*))(Y_{2H} - Y_{2L})$$

Q.E.D.

B.7 Proof of Proposition 3

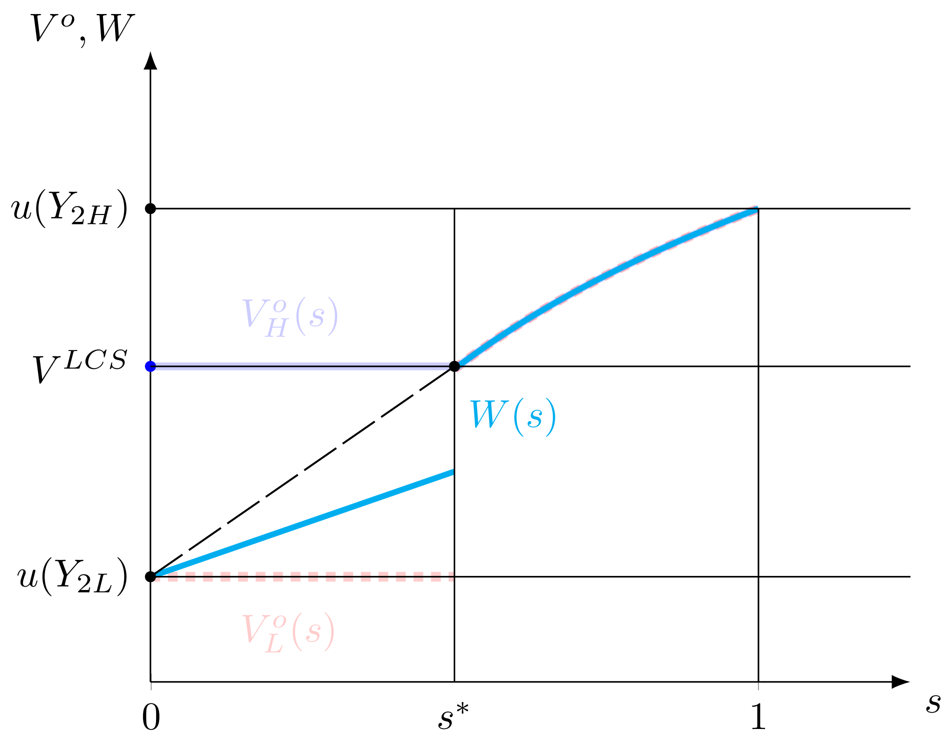
As argued in the text, for $\pi_1(y_H) = \{0, 1\}$, consumers obtain the same value in both economies. If all consumers have the same income in the first period, there are no asymmetric information in the second period, no scope for intertemporal smoothing since we assumed that $Y_{2L} \leqslant Y_1 \leqslant Y_{2H}$, and no distortions in consumption.

We characterize the ranking of the two economies in a neighborhood of those extreme values of $\pi_1(y_H)$. Let $V^j(\pi)$ be the equilibrium value to consumers at the optimal information disclosure policy given $\pi = \pi_1(y_H)$ and $j = LT, ST$, where LT stands for the economy with long-term contracts and ST for the economy with spot contracts only.

If π is close to 0 then $\pi < s^*$. Under short-term contracts:

$$\begin{aligned} V^{ST}(\pi) &= u(\pi y_H + (1-\pi) y_L) + \pi u\left(C_H^{LCS}\right) + (1-\pi)\left[\mu u\left(C_H^{LCS}\right) + (1-\mu) u(Y_{2L})\right] \\ &= u(\pi y_H + (1-\pi) y_L) + \pi u\left(C_H^{LCS}\right) + \pi \frac{1-s^*}{s^*} u\left(C_H^{LCS}\right) + \left(1 - \pi - \pi \frac{1-s^*}{s^*}\right) u(Y_{2L}) \end{aligned}$$

Figure 7: Expected continuation value with spot contracts, $W(s)$



where $\mu = \frac{\pi}{1-\pi}\frac{1-s^*}{s^*}$ is the share of low-income consumers' with a good signal so that the pool composition has share s^*. Thus,

$$\frac{\partial V^{ST}(\pi)}{\partial \pi} = u'(Y_1)(y_H - y_L) + \left(V_H^{LCS} - u(Y_{2L})\right) + \frac{1-s^*}{s^*}\left(V_H^{LCS} - u(Y_{2L})\right)$$

and, evaluating at $\pi = 0$,

$$\frac{\partial V^{ST}(\pi)}{\partial \pi}|_{\pi=0} = u'(y_L)(y_H - y_L) + \left[V_H^{LCS} - u(Y_{2L})\right] + \frac{1-s^*}{s^*}\left(V_H^{LCS} - u(Y_{2L})\right)$$

Consider next long-term contracts. Since $\pi < \pi^*$ there is no information being disclosed and

$$V^{LT}(\pi) = u\left(\pi y_H + (1-\pi)y_L + \pi\left(Y_{2H} - C_H^{LCS}\right)\right) + \pi u\left(C_H^{LCS}\right) + (1-\pi)u(Y_{2L})$$

Thus,

$$\frac{\partial V^{LT}(\pi)}{\partial \pi} = u'(Y_1)(y_H - y_L) + \left(V_H^{LCS} - u(Y_{2L})\right) + u'(Y_1)\left(Y_{2H} - C_H^{LCS}\right)$$

and, evaluating at $\pi = 0$,

$$\frac{\partial V^{LT}(\pi)}{\partial \pi}|_{\pi=0} = u'(y_L)(y_H - y_L) + \left(V_H^{LCS} - u(Y_{2L})\right) + u'(y_L)\left(Y_{2H} - C_H^{LCS}\right)$$

Hence, to prove that

$$\frac{\partial V^{ST}(\pi)}{\partial \pi}|_{\pi=0} < \frac{\partial V^{LT}(\pi)}{\partial \pi}|_{\pi=0}$$

we must show that

$$\frac{1-s^*}{s^*}\left(V_H^{LCS} - u(Y_{2L})\right) < u'(y_L)\left(Y_{2H} - C_H^{LCS}\right).$$

Note that

$$V_H^{LCS} - u(Y_{2L}) < u'(Y_{2L})\left(C_H^{LCS} - Y_{2L}\right) \leqslant u'(y_L)\left(C_H^{LCS} - Y_{2L}\right)$$

Thus, it suffices to show that

$$\frac{1-s^*}{s^*}\left(C_H^{LCS} - Y_{2L}\right) \leqslant \left(Y_{2H} - C_H^{LCS}\right)$$

which is true since $C_H^{LCS} = s^* Y_{2H} + (1-s^*) Y_{2L}$. Thus, $V^{LT}(\pi) > V^{ST}(\pi)$ if π is sufficiently close to 0.

Next consider the other extreme, $\pi = 1$. With short-term contracts, since $\pi > s^*$, there is pooling in the second period, so

$$V^{ST}(\pi) = u(\pi y_H + (1-\pi) y_L) + u(\pi Y_{2H} + (1-\pi) Y_{2L})$$

Hence,

$$\frac{\partial V^{ST}(\pi)}{\partial \pi} = u'(\pi y_H + (1-\pi) y_L)(y_H - y_L) + u'(Y_{2H})[Y_{2H} - Y_{2L}]$$

and, evaluating at $\pi = 1$,

$$\frac{\partial V^{ST}(\pi)}{\partial \pi}|_{\pi=1} = u'(y_H)(y_H - y_L) + u'(Y_{2H})[Y_{2H} - Y_{2L}]$$

With long-term contracts, since π is close to 1 (and to $\pi_2(y_H|y_H)$) there is perfect consumption smoothing between period 1 and period 2 for the high types, so

$$V^{LT}(\pi) = (1+\pi) u(C(\pi)) + (1-\pi) u(Y_{2L})$$

where

$$C(\pi) = \frac{Y_1 + \pi Y_{2H}}{1+\pi} = \frac{\pi y_H + (1-\pi) y_L + \pi Y_{2H}}{1+\pi}.$$

Hence,

$$\frac{\partial V^{LT}(\pi)}{\partial \pi} = (1+\pi)\, u'(C(\pi))\, C'(\pi) + u(C(\pi)) - u(Y_{2L})$$
$$C'(\pi) = \frac{(1+\pi)(Y_{2H} + y_H - y_L) - (Y_1 + \pi Y_{2H})}{(1+\pi)^2}$$

and, evaluating at $\pi = 1$ and using that $Y_{2H} = y_H$ in such case, we have

$$C'(1) = \frac{2(Y_{2H} + y_H - y_L) - (y_H + Y_{2H})}{2^2} = \frac{y_H - y_L}{2}$$

and

$$\begin{aligned}
\frac{\partial V^{LT}(\pi)}{\partial \pi}\Big|_{\pi=1} &= u'(y_H)(y_H - y_L) + u(Y_{2H}) - u(Y_{2L}) \\
&> u'(y_H)(y_H - y_L) + u'(Y_{2H})(Y_{2H} - Y_{2L}) \\
&= \frac{\partial V^{ST}(\pi)}{\partial \pi}\Big|_{\pi=1}
\end{aligned}$$

where the inequality follows from strict concavity of u. Thus, $V^{ST}(\pi) > V^{LT}(\pi)$ if π is sufficiently close to 1. Q.E.D.

B.8 Proof of Lemma 7

The optimal public disclosure policy solves

$$\max_{C_1, p(s) \in \Delta([0,1])} u(C_1) + \sum_s p(s)\, W(s;\Delta)$$

subject to

$$\sum_s p(s)\, s = \pi_1$$
$$C_1 = Y_1 + \sum_s p(s)\, s\, [Y_{2H} - \min\{h(V_H^o(s,\Delta)), Y_{2H}\}]$$

Suppose by way of contradiction that p^* that solves the problem above assigns positive probabilities to more than one shares above $\hat{s}(\Delta)$. That is, there are $\{s_1, s_2, ..., s_N\}$ with $N \geqslant 2$ such that $p^*(s_n) > 0$ and $s_n \geqslant \hat{s}(\Delta)$ for all $n = 1, ..., N$. Consider an alternative solution that only assigns probability $p(\bar{s}) = \sum_{n=1}^{N} p^*(s_n)$ to $\bar{s} = \sum_{n=1}^{N} \frac{p^*(s_n)}{p(\bar{s})} s_n$ for $s \geqslant \hat{s}(\Delta)$ and $p(s) = p^*(s)$ for all $s < \hat{s}(\Delta)$. Since the incumbent does not make any profits on consumers with $s \geqslant \hat{s}(\Delta)$ then this alternative solution does not change C_1. However, by concavity of $W(s;\Delta)$ for $s \geqslant \hat{s}(\Delta)$ we have that

$$\begin{aligned}\sum_{n=1}^{N} p^*(s_n) W(s_n;\Delta) &= p(\bar{s}) \sum_{n=1}^{N} \frac{p^*(s_n)}{p(\bar{s})} W(s_n;\Delta) \\ &< p(\bar{s}) W\left(\sum_{n=1}^{N} \frac{p^*(s_n)}{p(\bar{s})} s_n;\Delta\right) = p(\bar{s}) W(\bar{s};\Delta)\end{aligned}$$

so the alternative solution attains a higher value. Hence we have a contradiction.

The logic we used in Proposition 2 to show that all good types that stay have the same signal can be replicated to show that there is at most one group with $s \in (0, \hat{s}(\Delta))$ that are retained by the incumbent. Q.E.D.

B.9 Proof of Proposition 4

First we show that there exists a $\Delta_L > 0$ such that $p_{out} = 0$ for $\Delta \leqslant \Delta_L$. Suppose by way of contradiction that $p_{out} > 0$. Let V be the value at the optimum

$$\begin{aligned}V = {} & u(Y_1 + p_{in} s_{in} \Pi(s_{in})) + p_{in}[s_{in} V_H^o(s_{in}) + (1 - s_{in}) V_L^o(0)] \\ & + p_{out}[s_{out} V_H^o(s_{out}) + (1 - s_{out}) V_L^o(s_{out})] + p_0 W(0) + \Delta.\end{aligned}$$

Consider a deviation to an alternative policy $(s'_{in}, p'_0, p'_{in}, p'_{out} = 0)$ that delivers the same expected value to the high-income consumers as the one with $p_{out} > 0$, $\sum_s p(s) s V_H(s) = \pi_1 V_H(s'_{in})$. For Δ close to 0, such policy exists and $s'_{in} < s^*(\Delta)$. Then notice that, by (strict) convexity of $K(s)$,

$$\sum_s p(s) s h(V_H^o(s) + \Delta) > p'_{in} s'_{in} h(V_H^o(s'_{in}) + \Delta)$$

As $\Delta \to 0$ we have that $s_{out} \to 1$ and $h(V_H(1)) = Y_{2H}$, so the expected profits in the original allocation are

$$\begin{aligned}p_{in} s_{in} \Pi(s_{in}) &= p_{in} s_{in} Y_{2H} - p_{in} s_{in} h(V_H^o(s_{in})) \\ &= \pi_1 Y_{2H} - \sum_s p(s) s h(V_H^o(s)) \\ &< \pi_1 Y_{2H} - \sum_s p'_{in} s'_{in} h(V_H^o(s'_{in})) \\ &= p'_{in} s'_{in} \Pi(s'_{in})\end{aligned}$$

where the second equality follows from $p_{out}s_{out}(Y_{2H} - h(V_H(1))) = 0$. Then,

$$V' - V = u(Y_1 + p'_{in}s'_{in}\Pi'(s'_{in})) - u(Y_1 + p_{in}s_{in}\Pi(s_{in})) > 0$$

since, by construction, the continuation value of the high-income consumers is equal across policies, and all low-income consumers receive $u(Y_{2L})$ under both policies. As the deviation is profitable, we have reached a contradiction. Thus, for Δ close enough to 0 it is optimal to have $p_{out} = 0$. The intuition is that when Δ is small enough, a positive p_{out} is akin to providing full information about a subset of high types in our baseline model, which we established not to be optimal.

Next, we show that there exist $\Delta_H < \bar{\Delta}$ such that $p_{in} = 0$ for $\Delta \geqslant \Delta_H$. This is the case since $p_{in} = 0$ for $\Delta \geqslant \bar{\Delta}$. By definition of $\bar{\Delta}$, $V_H^{LCS} + \bar{\Delta} = u(Y_{2H})$ so for all $s \leqslant s^*\left(\bar{\Delta}\right)$ the incumbent makes no profits on the high-income consumers it retains. Then $C_1 = Y_1$. We want to show that if Δ is sufficiently close to $\bar{\Delta}$ then it is optimal to have $p_{in} = 0$. Suppose by way of contradiction that $p_{in} > 0$. The value is

$$V = u(Y_1 + p_{in}s_{in}\Pi(s_{in})) + p_{in}W(s_{in}) + p_{out}W(s_{out}) + p_0W(0)$$

Consider a perturbation that decreases p_{in} by $\varepsilon > 0$, increases p_{out} by ε, and decreases s_{out} by $\varepsilon(s_{out} - s_{in})/p_{out}$ so that the perturbation is feasible. The marginal value of such perturbation is

$$\begin{aligned} dV = &-s_{in}\Pi(s_{in})u'(Y_1 + p_{in}s_{in}\Pi(s_{in})) - W(s_{in}) \\ &+ W(s_{out}) - W'(s_{out})(s_{out} - s_{in}) \end{aligned}$$

As $\Delta \to \bar{\Delta}$ we have $\Pi(s_{in}) \to 0$ so

$$\begin{aligned} dV &\to W(s_{out}) - W(s_{in}) - W'(s_{out})(s_{out} - s_{in}) \\ &\geqslant W(s_{out}) - \bar{W}(s_{in}) - W'(s_{out})(s_{out} - s_{in}) > 0 \end{aligned}$$

The first inequality follows from $W(s_{in}) \leqslant \bar{W}(s_{in})$ where $\bar{W}$ is the concavification of $W(s)$. The second inequality follows from $s_{out} \geqslant \hat{s}(\Delta) \geqslant s^*$, $s_{out} \geqslant \hat{s}(\Delta) > s_{in}$, and strict concavity of W over $[s^*, 1]$ as graphically illustrated in Figure 4. Thus, the perturbation is welfare improving for Δ sufficiently close to $\bar{\Delta}$, reaching a contradiction. Then it must be that $p_{in} = 0$ for all Δ close to $\bar{\Delta}$.

Finally, to see that full information is never optimal, simply note that a feasible solution is the solution to (25) with $s_{out} = \max\{\hat{s}(\Delta), \pi_1\}$, $p_{out} = \pi_1/s_{out}$, $s_{in} = 0$. The value associated with this feasible solution is

$$
\begin{aligned}
&u\left(Y_1\right)+\left[\left(1-p_{out}\right)W\left(0,\Delta\right)+p_{out}W\left(s_{out},\Delta\right)\right] \\
>&u\left(Y_1\right)+\left[\left(1-\pi_1\right)W\left(0,\Delta\right)+\pi_1 W\left(1,\Delta\right)\right]
\end{aligned}
$$

since the feasible policy maximizes the value in period 2. Thus, full information is never optimal. Q.E.D.

C Observed outcome and unobserved signals

In the main text we assume that y_1 is not observable by the outsiders. Here we sketch how our analysis extends to the case in which y_1 is observable to outsiders but the incumbent and the consumer receive a signal $\tilde{y}_1$ in period 1 that is informative about the distribution of income in period 2. Assume that $\tilde{y}_1 \in \{\tilde{y}_H, \tilde{y}_L\}$ and let $p\left(\tilde{y}_1|y_1\right)$ be the probability of getting signal $\tilde{y}_1$ given y_1. Further, let $\pi_2\left(y_2|y_1,\tilde{y}_1\right)$ be the probability of y_2 given $\left(y_1,\tilde{y}_1\right)$.[25]

The outside option for the consumer with a high-signal and observable income in period 1 y_z is

$$
V_z^o\left(s\right)=\max_{c,V_L}\sum_{y_2}\pi_2\left(y_2|y_z,\tilde{y}_H\right)u\left(c\left(y_2\right)\right)
$$

subject to

$$
\begin{aligned}
s\left[Y_{2sH}-\sum_{y_2}\pi_2\left(y_2|y_z,\tilde{y}_H\right)C\left(y_2\right)\right]+\left(1-s\right)\left[Y_{2zL}-h\left(V_L\right)\right]&\geqslant 0 \\
\sum_{y_2}\pi_2\left(y_2|y_z,\tilde{y}_H\right)u\left(C\left(y_2\right)\right)&\leqslant V_L \\
V_L&\geqslant u\left(Y_{2zL}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
Y_{2zH}&\equiv\sum_{y_2}\pi_2\left(y_2|y_z,\tilde{y}_H\right)y_2 \\
Y_{2zL}&\equiv\sum_{y_2}\pi_2\left(y_2|y_z,\tilde{y}_L\right)y_2.
\end{aligned}
$$

It is straightforward to show that under two-sided lack of commitment for an arbitrary $\left(M,\mu\right)$ we have:

[25] Appendix E offers another example where the incumbent and the consumer/worker see a private signal about the return on the private investment in general human capital.

Lemma. *Given a public disclosure policy* (M, μ), *the equilibrium outcome has*

$$c_1(y_1) = Y_1 + \sum_z \pi_1(y_z)\, p(\tilde{y}_H|y_z) \sum_m \mu(m|y_z, \tilde{y}_H)\, \Pi_z(m) \tag{33}$$

$$c_2(y_z, \tilde{y}_L, m, y_2) = Y_{2zL} \tag{34}$$

$$c_2(y_H, m, y_2) = Y_{2zH} - \Pi_z(m) \tag{35}$$

where $\Pi_z(m) = Y_{2zH} - h(V_z^o(s(m)))$.

Thus, the optimal disclosure policy can be found as the solution to the analog of problem (19):

$$\max_{c_1, V_{HH}, V_{LH}} u(c_1) + \pi_1(y_H)\left[p(\tilde{y}_H|y_H) V_{HH} + p(\tilde{y}_L|y_H)\, u(Y_{2HL})\right] + \pi_1(y_L)\left[p(\tilde{y}_H|y_L) V_{LH} + p(\tilde{y}_L|y_L)\, u(Y_{2LL})\right]$$

subject to

$$c_1 = Y_1 + \sum_z \pi_1(y_z)\, p(\tilde{y}_H|y_z)\left[Y_{2zH} - h(V_{zH})\right]$$

$$V_{zH} \in \left[V_z^o(p(\tilde{y}_H|y_z)), V_z^o(1)\right], \quad z = H, L$$

and then recover $s_z(g)$ and $\mu(b|y_z, \tilde{y}_L)$ from

$$V_{zH} = V_z^o(s_z(g)) \text{ and } s_z(g) = \frac{p(\tilde{y}_H|y_z)}{p(\tilde{y}_H|y_z) + p(\tilde{y}_L|y_z)(1 - \mu(b|y_z, \tilde{y}_L))}. \tag{36}$$

D Discrimination across consumers

So far we have assumed that the incumbent firm cannot discriminate among consumers with the same history. We now relax this assumption and show that the optimal signal structure maintains the same features as in the restricted case. Our main interest is in the differences in the contract structure induced by discrimination. We show that when releasing information is optimal, the incumbent firm provides $C_2(y_H) < h(V_H^o(s(m)))$ to almost all consumers with $y_1 = y_H$.

First consider the case $V_H^o(s(m)) = V_H^{LCS}$. Then the allocation coincides with that in Lemma 5,

$$C_2(y_H, m) = C_H^{LCS},$$

and lack of discrimination has no impact on the allocation. The second and more interesting case is one in which $V_H^o(s(m)) > V_H^{LCS}$. In this case, discrimination allows the incumbent firm to offer less than $V_H^o(s(m))$ to almost all high types by exploiting the fact that lower offers from outsiders attract a worse pool of agents. Let s^* be the largest share of high income consumers s such that

$$V_H^o(s) = V_H^{LCS}.$$

We then order all high income consumers and index them by $i \in [0,1]$. We define $i^*(m)$ to be the measure of high types with signal m that generates the share s^*, for given information policy $\mu(m|y_L)$. The value $i^*(m) \in (0,1)$ identifies two groups of consumers. For $i \in [0, i^*(m)]$, the value consumer i receives is

$$V(i) = V_H^{LCS}.$$

This is because the least cost separating contract constitutes a lower bound for all high income consumers. For $i \in [i^*(m), 1]$,

$$V(i) = V_H^o(s_i(m))$$

where

$$s_i(m) = \frac{\pi_1(y_H)\int_0^i d\tilde{i}}{\pi_1(y_H)\int_0^i d\tilde{i} + (1-\pi_1(y_H))\,\mu(m|y_L)}$$

is the share of high income consumers in a pool that includes all low types with signal m and all high types with signal m and index smaller than i. Next we solve for the optimal disclosure policy.

Optimal public disclosure

The introduction of discrimination modifies the allocation associated to a given signal structure in two ways. First, as long as the value of the Miyazaki-Wilson contract is higher than V_H^{LCS}, high-type consumers with the same signal receive unequal consumption in the second period. In particular, their utility in the second period belongs to the interval $\left[V_H^{LCS}, V_H^o(s(m))\right]$. Second, firms are able to extract additional profits in the second period and rebate them to the agent in the first period. We show that these two features do not fundamentally change the nature of the optimal disclosure policy.

Proposition 5. *The optimal disclosure policy has a bad-signal structure i.e.* $M = \{g, b\}$ *(good or bad) and* $\mu(g|y_H) = 1$ *and* $\mu(b|y_L) \in [0,1]$*. i) For* π_1 *sufficiently low, it is optimal to provide no information; ii) For all* π_1*, more information is disclosed under discrimination–and strictly so if* π_1 *is sufficiently high.*

Proof. Proposition 5 shares most predictions with Proposition 2. Part i) follows from

$y_L < h\left(V_H^{LCS}\right)$ since $h\left(V_H^{LCS}\right)$ is the minimum consumption that must be guaranteed to high-income consumers. Part ii) hinges on the fact that the incumbent firm earns more profits in the second period under discrimination, hence consumption is typically more front-loaded for all interior disclosure policies.

To easy notation, we replace $\mu(b|y_L)$ with μ, and, with slight abuse of notation, we replace the dependence of $i(.)$ and $s_i(.)$ on m with that on the associated μ. Formally, the optimal signal under discrimination solves

$$\max_{\mu\in[0,1]} u\left(Y + \pi_1(y_H)\Pi(\mu)\right) + \pi_1(y_H)\left[i^*(\mu)V_H^{LCS}(y_H) + \int_{i^*(\mu)}^1 V_H^o(s_i(\mu))\,di\right]$$

where

$$\Pi(\mu) = Y_{2H} - \left[i^*(\mu)h\left(V_H^{LCS}(y_H)\right) + \int_{i^*(\mu)}^1 h\left(V_H^o(s_i(\mu))\right)di\right].$$

If $\mu = 1$, the result is trivial as that corresponds to full information disclosure. If $\mu < 1$,

$$-u'(C_1)\int_{i^*(\mu)}^1 \partial_\mu h\left(V_H^o(s_i(\mu))\right)di + \int_{i^*(\mu)}^1 \partial_\mu V_H^o(s_i(\mu))\,di \leqslant 0.$$

Since

$$u'(C_1)h'\left(V_H^o(s_1(\mu))\right)\int_{i^*(\mu)}^1 \partial_\mu V_H^o(s_i(\mu))\,di \geqslant u'(C_1)\int_{i^*(\mu)}^1 \partial_\mu h\left(V_H^o(s_i(\mu))\right)di$$

by convexity of h, then

$$u'(C_1)h'\left(V_H^o(s_1(\mu))\right) \geqslant 1.$$

Let μ^* be the optimal signal structure in the economy without discrimination. Suppose by contradiction that $\mu^* > \mu$. Then

$$u'(C_1(\mu)) \geqslant \frac{1}{h'\left(V_H^o(s_1(\mu))\right)} > \frac{1}{h'\left(V_H^o(s_1(\mu^*))\right)} = u'\left(C_1^*(\mu^*)\right)$$

where the second inequality follows by convexity of h and the last equality from interior optimality since $\mu^* > 0$. It follows that $C_1(\mu) < C_1^*(\mu^*)$ or, equivalently, $\Pi(\mu) < \Pi^*(\mu^*)$. Since $V_H^o(s_i(m)) \leqslant V_H^o(s_1(m))\ \forall i$, then $\Pi(\tilde{\mu}) \geqslant \Pi^*(\tilde{\mu})\ \forall\tilde{\mu}$, which implies $\Pi^*(\mu) < \Pi^*(\mu^*)$. Since Π^* is a decreasing function, the latter inequality constitutes a contradiction. Finally, if $V_H^o(s(m)) > V_H^{LCS}$, then $\Pi(\mu) > \Pi^*(\mu)$ which implies that $\mu > \mu^*$. Q.E.D.

E Effort

In this section, we extend the benchmark model to incorporate unobservable effort in the first period of the contract. In order to highlight the effect of this extension on optimal information design, we intentionally keep the environment as close as possible to our benchmark.

E.1 Environment

Consider a model in which first period output y_1 is constant and predetermined. At the end of the first period, the borrower exerts effort e at cost $v(e)$. At the beginning of the second period, the incumbent and the borrower jointly observe the outcome of the effort in the form of productivity $z \in \{z_H, z_L\}$ where $z \sim f(z|e)$. Effort is an investment-like good that does not affect first period output, but contributes to the formation of general productivity. Productivity in turn affects the distribution of second subperiod output, $y_2 \in \{y_L, y_H\}$ with $y_2 \sim p(y_2|z)$. We assume that while effort is privately known only by the borrower, productivity is observed by both the borrower and the incumbent lender, but not by outsiders. Outsiders only observe a signal m about the value of productivity, $m \sim \mu(z)$.

An allocation is a contract offered by the insider, $\{C_1, e, C_2(z, m)\}$, and a menu of contracts offered by the outsiders.

Additional assumptions and definitions We define $E(y_2|z) = \sum_s p(y_s|z) y_s$, with $E(y_2|z_H) > E(y_2|z_L)$, and $E(y_2|z) = \sum_z f(z|e) E(y_2|z)$. To make the environment comparable to the economy in the text, we assume that y_1 is equal to Y_1 in the benchmark model. We also assume that $E(y_2|z_s) = Y_{2s}$ and $E(y_2|e) = Y_2$ which requires us to specify a given level of effort. Hence we assume that effort can take on two values, e and 0, and in equilibrium we guess (and verify) that is optimal to exert effort e. Furthermore, we assume that $f(z_H|e) = \pi_1(y_H)$ and $p(y_2|z_s) = \pi_2(y_2|y_s)$ for $s = H, L$.

In this economy, productivity replaces first period income as the source of information about future income that both the borrower and the incumbent have access to and that the designer conveys a signal about. The key difference with our benchmark model is that the determination of productivity z is influenced by an action the borrower performs and the incumbent cannot observe.[26] Thus, the two economies are equivalent except for the existence of an incentive compatibility constraint that guarantees that the borrower exerts effort e.

[26]Human capital does not fully reveal the amount of effort the worker exerts. The distribution f might be induced by either unobserved worker type or by pure lack. What matters for our results is that the source of uncertainty underlying f is uncorrelated with that behind p.

Equilibrium and optimal public disclosure We solve for the equilibrium under a given signal structure. Most of the results follow directly from what we showed in the text, hence we focus on the new features that originate from the introduction of effort. We discuss the economically interesting case in which it is efficient to induce strictly positive effort. Due to adverse selection, a borrower with a low realization of productivity always consumes her expected output, $E(y_2|z_L)$, which is also the only contract the outsiders are willing to offer. However, in order to induce the outsiders not to offer a contract that would attract borrowers with z_H, the incumbent offers them

$$V(z_H, m) = V_H^o(s(z_H|m; e))$$

where $s(h_H|m; e)$ is the share of agents with productivity z_H with signal m given the equilibrium effort level e,

$$s(z_H|m; e) = \frac{\mu(m|z_H) f(z_H|e)}{\sum_z \mu(m|z) f(z|e)}.$$

The key difference with the previous model is the endogeneity of productivity. For the borrower to exert the effort level e, it has to be that

$$(f(z_H|e) - f(z_H|0)) [V(z_H, m) - u(Y_{2L})] \geqslant v(e) - v(0)$$

or

$$[V(z_H, m) - u(Y_{2L})] \geqslant \frac{v(e) - v(0)}{(f(z_H|e) - f(z_H|0))}. \tag{37}$$

If this incentive compatibility constraint is satisfied, the equilibrium consumption allocation is identical to the one in the benchmark model, otherwise borrowers exert no effort.

Notice that the information design has no effect on the value received by borrowers with low productivity. The only way to induce effort is to provide additional information about borrowers with high productivity, hence increasing their equilibrium value. We summarize this result in the following proposition:

Proposition 6. *Let μ^* be the optimal signal structure in the economy without hidden effort. i) If (37) holds, then μ^* is optimal in the economy with effort. ii) If (37) does not hold when evaluated at μ^*, then the optimal signal structure is: $M = \{g, b\}$ (good or bad) and $\mu(g|z_H) = 1$ and $\mu(b|z_L) \in (0, 1)$ such that*

$$V_H^o\left(\frac{f(z_H|e)}{f(z_H|e) + (1 - \mu(b|z_L)) f(z_L|e)}\right) = \frac{v(e) - v(0)}{(f(z_H|e) - f(z_H|0))} + u(Y_{2L})$$

Moreover, $\mu(b|y_L) > \mu^*(b|z_L)$. That is, the optimal signal with hidden effort is more informative *than the one without hidden effort.*

Proof. Part $\mathrm{i})$ is straightforward since μ^* satisfies the IC constraint and it is optimal in its absence. Part $\mathrm{ii})$ follows from concavity of (13), the binding incentive constraint (37) and $V_H^o(s)$ being strictly increasing in s. Q.E.D.