Reputation, Bailouts, and Interest Rate Spread Dynamics*

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Abstract

In this paper, we propose a joint theory for interest rate dynamics and bailout decisions. Interest rate spreads are driven by time-varying fundamentals and expectations of future bailouts from a common government. Private agents have beliefs about whether the government is a commitment type, which never bails out, or a no-commitment type, which sequentially decides whether to bail out or not, and learn by observing its actions. The model provides an explanation for why we often observe governments initially refusing to bail out borrowers at the beginning of a crisis even if they eventually end up providing a bailout after the crisis aggravates. In the typical equilibrium outcome, spreads are non-monotonic in fundamentals, and decisions on whether to bail out individual borrowers affect the spreads of other borrowers. These dynamics are consistent with the behavior of bailouts and spreads during the recent US financial crisis and European debt crisis.

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1 Introduction

Expectations about the generosity of bailout plans are an important driver for interest rates paid by financial institutions and sovereign governments. For example, in the recent US financial crisis, CDS spreads of large financial institutions rose sharply after regulators let Lehman Brothers fail, a negative news event about the willingness of the government to bail out financial firms, but reversed course after the announcement of the Troubled Asset Relief Program (TARP), a positive news event, even though fundamentals arguably worsened. Similarly, EU sovereign debt spreads declined significantly after the announcement of the unlimited bond buying program (OMT) by the ECB, which was arguably a positive news event about the willingness to provide government support in the future.

In this paper, we propose a joint theory for interest rate dynamics and bailout decisions. We study a dynamic model in which spreads are driven by time-varying expectations of future bailouts from a common government. Private agents have beliefs about whether the government is a commitment type, which never bails out, or a no-commitment type, which sequentially decides whether to bail out or not, and learn by observing its actions. Thus, the model can help rationalize why governments initially refuse to bail out borrowers even though they eventually end up bailing out borrowers as the crisis gets more severe. This was indeed the case during the US financial crisis when Lehman Brothers was allowed to fail but eventually the government implemented TARP. This is turn implies that along the typical equilibrium outcome, spreads are hump-shaped during a crisis: they start low, then jump at the beginning of a crisis when the government initially refuses to bail out, and eventually fall if the crisis worsens and the government agrees to a bailout. Moreover, the model can generate the contagion and the sensitivity of spreads to fundamentals we observed in the crisis.

In our model, borrowers borrow from risk-neutral lenders to invest in a risky project. Absent a bailout, borrowers default on debt after the realization of a bad idiosyncratic shock. We assume that default imposes a social cost. We introduce a government that can be one of two types: a commitment type, which never bails out, and a no-commitment type, which trades off the static benefits of bailing out the lenders and avoids the social default cost with the dynamic costs of losing future reputation. This type is not observed by private agents, who learn about it over time by observing the government's actions. We show that our model generates an increasing relationship between spreads and the government's *reputation*, defined as the private agents' prior about facing the commitment type. If the government's reputation is high, lenders expect to be bailed out with low probability and there is a higher probability of default; therefore, lenders need to increase interest rates in order to break even.

The model is subject to an aggregate shock that changes the distribution over idiosyncratic shocks faced by borrowers and thus affects the fraction of borrowers that need a bailout in order to avoid a default. In normal times, when all borrowers receive high shocks, borrowers do not default; therefore, there is no need for a bailout. As a result, there is no learning about the type of the government, so spreads remain low (and constant) if the initial reputation of the government is low. If the economy is hit with an intermediate shock that affects a small fraction of borrowers adversely, the static incentives to bail out increase, but by choosing not to bail out, the no-commitment type can increase its reputation. If the latter effect is large enough, the best response for the no-commitment type is to randomize between bailing out and not. If private agents observe no bailout, the reputation of the government increases because observing no bailout is more likely if the government is the commitment type. Thus, private agents assign a larger probability to no bailouts in the future and spreads rise for all borrowers, even those that currently have the high idiosyncratic shock. If the economy is eventually hit by a large negative shock that affects the majority of borrowers, the static bailout benefits dominate the dynamic reputational costs. As a result, the government chooses to bail out, which results in a drop in reputation and hence a sharp reduction in spreads despite the fundamentals being worse.

Along the equilibrium path, spreads and debt issuances are less responsive to the state (aggregate and idiosyncratic) when reputation is low. That is, if the probability of facing the no-commitment type is high, then debt prices are mostly unaffected by the state, since lenders expect a bailout in bad states with a high probability. This generates a differential sensitivity of prices to fundamentals that Acharya et al. (2016) and Cole et al. (2016) document in the data.

One message of our paper is that an important driver of spreads is the expectations of future bailouts. In the baseline model, these are driven purely by changes in fundamentals and outcomes of randomization. However, in Section 4 we extend our model to one in which the government learns about the aggregate state of the world from noisy prices, similarly to Nosal and Ordoñez (2016). In this model, we do not need the intermediate shock in order to generate the hump-shaped spread dynamics typical of crises. The reason is that the government's incentives to bail out are driven by its beliefs about the true state of the world, which can be driven by noise rather than changes in fundamentals. At the beginning of a crisis, the government is more uncertain about the state of the world and thus is more likely to find it optimal not to bail out, and increase its reputation.

In the baseline model we show that it is worthwhile to bail out only if a sufficiently large fraction of borrowers will default absent a transfer. This is because the static benefits of a bailout must be large enough to compensate for the loss in reputation. In reality, however, we often observe cases in which governments bail out small banks. One recent example is the case of the Italian government bailing out two mid-sized banks in 2017, in violation of the principles of the European banking union. In an extension of the model, we show that when the borrowers' default decisions are dynamic – in that they consider future profits in deciding whether to default or not – the share of borrowers that ends up receiving the bailout is not a sufficient statistic for the static benefits of a bailout. The government may choose to bail out a few borrowers in order to avoid contagion to other borrowers that would default absent a bailout. The reason for this is that since bailouts (or lack thereof) change private beliefs about the government's future behavior, bailouts affect borrowers' continuation values of not-defaulting. Thus the static benefits of a bailout can be large even if in equilibrium the government ends up bailing out only a small fraction of borrowers.

Finally, we provide empirical evidence to support our model's predictions from three recent crises: the US financial crisis, the recent banking crisis in Italy after the institution of the Single Resolution Mechanism (SRM) within the context of the European banking union, and the European sovereign debt crisis. In particular, we use our model to interpret the movement in spreads after major bailout or non-bailout announcements. We also summarize some recent empirical work that provides further justification for our mechanisms. See Acharya et al. (2016), Veronesi and Zingales (2010), Schweikhard and Tsesmelidakis (2011), Kelly et al. (2016) for the US financial crisis, Neuberg et al. (2018) for the SRM, and Ardagna and Caselli (2014) for the European debt crisis.

Consistent with the predictions of our model, if after an adverse event there is no bailout, the CDS spreads for borrowers not directly affected by the adverse event go up and so does the sensitivity of spreads to fundamentals, which we proxy by the volatility of CDS spreads. The opposite happens when we observe a bailout or announcement of government support for equity and debt holders. Consider for instance the US financial crisis. We show that the mean and cross-sectional standard deviation of CDS spreads for large financial firms rose sharply after the bankruptcy of Lehman Brothers and decreased sharply after the announcement of TARP. Moreover, we show that these features are only true for financial firms and not for non-financial firms, which are arguably not directly affected by these bailout policies.

There has been a lot of discussion among academics and policymakers about whether Lehman Brothers should have been allowed to fail. For example, Ball (2018) argues that the failure of Lehman was a pivotal moment in the financial crisis and that preventing it from declaring bankruptcy might have significantly reduced much of the subsequent disruption. Moreover, he provides convincing evidence that the official reason for the Federal Reserve not intervening, which had to do with Lehman not having sufficient collateral, does not stand up to scrutiny. Our paper suggests that dynamic reputational considerations might be one reason the Fed did not bail out Lehman Brothers. As our model suggests, a bailout of Lehman would have severely reduced the Fed's reputation and led to a reduction of borrowing costs for all banks. This would have incentivized banks to borrow even more in the future, thus increasing the likelihood of future bailouts and implying even greater costs to taxpayers. Moreover, it was precisely the potential cost associated with allowing Lehman Brothers to fail that made it a good time for the Fed to build reputation. This sentiment is echoed by Mishkin (2011), who argues that "letting Lehman fail would serve as a warning to other financial firms that they needed to rein in their risk taking."

Related Literature

Our paper is related to a large literature on repeated games with behavioral types pioneered by Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982). Phelan (2006) uses this framework to study a model of government reputation with hidden types that can stochastically evolve. Our model builds on this framework by embedding it in an investment model with endogenous default and interest rates. A crucial difference between our models is that in Phelan (2006), the temptation for the government to reveal its type is high when reputation is high. In contrast, in our model, the incentives for the government to reveal its type are large when reputation is low. This distinction is important for generating the desired movement in spreads. Dovis and Kirpalani (2017) also use a similar framework to study the efficacy of fiscal rules when private agents are strategic. A key difference between these two environments is that they find conditions so that default is never optimal; so, their model is silent on the behavior of spreads.

Our paper is also related to a literature on reputation and sovereign default, for example Cole et al. (1995), D'Erasmo (2008), and Amador and Phelan (2018). In these models, there is uncertainty about the type of the borrower, while in ours there is uncertainty about the type of the bailout authority (which we call the government). This allows us to study an environment in which spreads are driven by bailout expectations.

Our paper is related to the seminal work of Kareken and Wallace (1978), who study the effects of debt guarantees on ex-ante incentives for borrowers. See Farhi and Tirole (2012), Chari and Kehoe (2015), Davila and Walther (2017), and Chari et al. (2016) for more recent contributions to this literature. Some recent papers including Gourinchas and Martin (2017), Nikolov and Cooper (2018), de Ferra (2017) and Sandri (2015) study the effects of bailouts on the debt accumulation decisions in the European Monetary Union. In contrast to these papers, the bailout decisions in our model are dynamic due to reputation building incentives. This feature is critical to account for the non-monotone behavior of spreadsduring crises.

Nosal and Ordoñez (2016) study a model in which governments learn about the state of the economy through the actions of private agents. In contrast to our model, there is no uncertainty about the type of the government. As mentioned above, uncertainty about the type of the government is crucial to generating the movement in spreads. In particular, their model cannot account for the increase in spreads if there is no bailout observed. In Section C we extend our model to allow for the government to learn about the state through the actions of private agents and prices, and we show how these two channels interact.

Finally, our section on two-sided learning is related to the literature that studies the link between real activity and the ability to learn about fundamentals. Examples of such papers include Veldkamp (2005) and Fajgelbaum et al. (2017). In our model, when reputation is low, prices are less sensitive to fundamentals, which limits the amount the government can learn about the true state of the world. This parallels the idea in these papers that learning is harder in bad economic times when there is low investment.

Outline The rest of paper is organized as follows. In Section 2 we present our baseline model, and in Section 3 we characterize a class of Markov equilibria and derive our main results. In Section 4 we study an extension of the model in which the government learns about the true state of the world from prices, and in Section 5 we show that the government may choose to bail out few borrowers to avoid contagion when default decisions are dynamic. Section 6 reads three events through the lens of our results. Finally, Section 7 concludes the paper.

2 Model

Environment

For much of the main text, we illustrate our results by means of a simple example. We show how these results generalize in Appendix A and C. Time is discrete and is indexed by $\tau = 0, 1, ...$ The economy is populated by a continuum of *borrowers*, a continuum of *lenders*, a continuum of *taxpayers*, and a *government (bailout authority)*. In each period there is a stage game with two sub-periods, $t = 1, 2$.

All borrowers are symmetric ex-ante, risk-neutral, and care only about consumption in sub-period 2. In sub-period 1, borrowers have no capital and must borrow to finance an investment opportunity. If a borrower invests an amount k in sub-period 1, it obtains a return θk^{α} in sub-period 2, where $\alpha \in (0, 1)$ and θ is an idiosyncratic productivity shock. The distribution of θ depends on the aggregate state of the world realized in sub-period 2, s. The aggregate state s is realized according to a distribution P. For illustrative purposes, we assume that the state can take three values: $s \in \{s_L, s_M, s_H,\}$ with probabilities p_L, p_M, and p_H, respectively. Each borrower receives productivity θ drawn from a distribution $H(\cdot|s)$. We assume that θ can take on two values: $\theta_H > 0$ and $\theta_L = 0$. In state s_H, referred to as *normal times*, all the borrowers have high productivity, so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M, referred to as *mild crisis*, $h(\theta_H|s_M) = 1 - \mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L, referred to as *severe crisis*, all borrowers have low productivity, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$. After the realization of θ, each borrower can default on its debt. If the borrower defaults, it loses the claim on investment and its payoff is zero. Default also imposes a social cost (imposed on the government). The government can impose a tax on taxpayers and make transfers to the borrower in sub-period 2 to avoid default and the associated social cost.

Lenders have a large endowment of the final consumption and investment good in the first sub-period. They are risk neutral and have preferences over consumption in sub-periods 1 and 2, x_1 and $x_2(s)$, given by

$$V = x_1 + q \sum_s p_s x_2(s),$$

where q is the discount factor across sub-periods. Taxpayers have linear utility and an endowment E in the sub-period 2, and can be taxed by the government.

The government can be one of two types: *commitment* or *no-commitment*. We assume that the true type of the government evolves according to the transition matrix:

$$\mathbb{P} = \begin{bmatrix} p_c & 1-p_c \\ p_{nc} & 1-p_{nc} \end{bmatrix},$$

where p_c is the probability of transiting from the commitment to the commitment type, and p_{nc} is the probability of transiting from the no-commitment to the commitment type. We assume $p_c > p_{nc} > 0$ so that types are persistent. The type of the government is not observable by private agents (i.e., borrowers and lenders). At the beginning of period 0, private agents share a common prior π_0 that the government is the commitment type. We will refer to private agents' beliefs that the government is the commitment type as the government's *reputation*.

The commitment type never bails out the borrowers, while in each period the no-commitment type decides whether to bail out or not and the size of the bailout. To implement a bailout, it raises lump-sum taxes from taxpayers. The government maximizes the preferences of the lenders and the taxpayers net of the social default costs.[1] Social default

[1] Here we assume that the government cares only about the welfare of lenders and taxpayers, but not of the borrowers. In the context of banking, we can think of the government caring about retail bondholders. In the context of the EU, we can think of the government as representing Germany, who cared about the health of German banks holding Greek bonds.

costs are given by $C(\Delta B)$, where Δ is the fraction of borrowers that default, and B is the average level of debt issued by borrowers in sub-period 1. The function $C(\cdot)$ is increasing and captures the social cost of default. For the purposes of our example, we assume that $C(x) = \psi x$, with $\psi < 1$, but allow for a more general specification in Appendix A. One interpretation of this cost is that the absence of a bailout leads to a reduction in the net worth of the banking sector. This reduction in net worth might have a real cost associated with it, for example reduced investment or fire sales, which is represented by the function $C(\cdot)$. The government discounts utility across periods at rate β.

While we assume that the commitment type never bails out, it is not true in general that the optimal policy with commitment is to never bail out in sub-period 2.[2] For example, if the social default costs are very high, then it might be optimal to bail out, even with commitment. However, if $\psi < 1$, then we can show that the optimal policy with commitment is to never bail out (see footnote 7).

Finally, it is worth noting that our results extend to an environment in which borrowing is driven by consumption smoothing motives, as in much of the sovereign default literature. See Appendix F for further discussion. One can use this alternative setup to think about the impact of the Greek bailout on other southern European countries during the recent crisis in Europe. In Appendices A and C we also show that our results extend to a more general process for θ and allow for persistence in both the aggregate and the idiosyncratic shock.

Markov Equilibria

We now describe in detail the interaction between private agents and the government. The timing of events in each period is as follows. In sub-period 1, borrowers choose the amount of debt to issue given the debt price schedule. In sub-period 2, the state of nature s and the idiosyncratic shocks θ are realized. After observing (s, θ), given the distribution of inherited debt, the government chooses whether to bail out the borrowers and if so, the level of transfers. The commitment type never bails out. The borrower then decides whether to default or not.

We will focus on symmetric Markov equilibria where all histories are summarized by the posterior probability of facing the commitment type, π, and all individual borrowers choose the same level of debt in sub-period 1. As a result, in equilibrium, the distribution of debt and capital facing the government in the sub-period 2 is degenerate, with point mass at B and K, which corresponds to the average level of debt and capital.

We describe the actions and payoffs of the agents starting from the last sub-period. Given the state (π, s) and the distribution of individual debt holdings and capital stock,

[2] See for instance Sandri (2015).

$\Gamma(b,k)$, the government chooses the transfers $T(b,k,\theta)$ to solve

$$W_2(\pi,\Gamma,s) = \max_{T(b,k,\theta)} (1-\Delta)B - \int \sum_\theta h(\theta \mid s) T(b,k,\theta)\, d\Gamma(b,k) - \psi\Delta B + \beta W(\pi'), \quad (1)$$

where Δ is the fraction of borrowers that default given the transfer scheme $T(b,k,\theta)$,

$$\Delta = \sum_\theta h(\theta \mid s) \int \mathbb{I}_{\{\theta k^\alpha - b + T(b,k,\theta) \leqslant 0\}} d\Gamma(b,k),$$

$W(\pi')$ is the continuation value for the government in the next period, given by

$$W(\pi) = e - Q(\pi, B(\pi), K(\pi)) B(\pi) + q \sum_s p_s W_2(\pi, \Gamma(\pi), s), \quad (2)$$

and the new posterior $\pi' = \pi'(\pi,\Gamma,s)$ is defined using Bayes' rule

$$\pi' = \begin{cases} \frac{\pi p_c}{\pi + (1-\pi)\Pr(T(\pi,B(\pi),s)=0)} + \frac{(1-\pi)\Pr(T(\pi,B(\pi),s)=0) p_{nc}}{\pi+(1-\pi)\Pr(T(\pi,B(\pi),s)=0)} & \text{if } T(b,\theta) = 0 \,\forall\, (b,\theta). \\ p_{nc} & \text{if } T \neq 0 \end{cases} \quad (3)$$

The borrower's problem is to choose debt, b, and investment, k, to maximize expected consumption in sub-period 2 given the equilibrium bailout transfers T from the no-commitment type and the pricing schedule Q. Formally,

$$U_1(\pi) = \max_{b,k} \sum_s p_s \sum_\theta h(\theta \mid s) \left[\pi \max\{\theta k^\alpha - b, 0\} + (1-\pi)\max\{\theta k^\alpha - b + T(\theta,b,k), 0\}\right] \quad (4)$$

subject to the budget constraint in sub-period 1:

$$k \leqslant Q(\pi, B(\pi), K(\pi))(b,k)\, b.$$

The pricing schedule Q must satisfy the zero-profit condition for the lenders:

$$\begin{aligned} Q(\pi, B(\pi), K(\pi))(b,k) = q\,[&p_H + p_M(1-\mu) + \\ &+ p_M \mu (1-\pi) \mathbb{I}_{\{T(\pi,B,K,s_M)(b,k,\theta_L) \geqslant b\}} + p_L(1-\pi)\mathbb{I}_{\{T(\pi,B,K,s_L)(b,k,\theta_L) \geqslant b\}}] \end{aligned} \quad (5)$$

if $b \leqslant \theta_H k^\alpha$, where $T(B,K,s)(b,k,\theta_L)$ is the equilibrium transfer rule that lenders expect in state s if all the other borrowers choose (B,K), the given borrower chooses (b,k), and it receives a low idiosyncratic shock θ_L. That is, the price of debt equals the discounted value of payments that lenders receive if the borrower receives a high productivity shock and can repay the debt (the first line on the right side of (5)) plus the payments the lenders receive if the borrower has a low productivity shock and it receives a bailout transfer that enables it to repay (the second line on the right side of (5)).

Definition. A *Symmetric Markov Equilibrium* is an individual debt and investment strategy $b(\pi)$ and $k(\pi)$, aggregate debt $B(\pi)$ and investment $K(\pi)$, a pricing schedule $Q(\pi, B, K)(b, k)$, a transfer strategy for the (no-commitment) government $T(\pi, B, K, s)(b, k, \theta)$, and a law of motion for beliefs, π', such that (i) the debt and investment strategy solves (4); (ii) $b(\pi) = B(\pi)$ and $k(\pi) = K(\pi)$; (iii) the pricing schedule satisfies (5); and (iv) the transfer rule solves (1), where the continuation value W is defined by (2) and the law of motion for beliefs is (3).

3 Bailouts and Spreads Dynamics

In this section, we characterize a class of Markov equilibria and show that the equilibrium outcomes are consistent with the experiences of recent financial and debt crises. We show that there exist equilibria in which all the equilibrium objects are continuous and monotone functions of the government's reputation. In this class of equilibria, investment, debt issuances, the price of debt, and bailout probability are decreasing in the government's reputation. Moreover, the no-commitment type government does not bail out in normal times, mixes during a mild crisis, and bails out for sure in a severe crisis. Thus, if private agents observe no bailout in a mild crisis, their beliefs of facing the commitment type increases; thus, interest rates go up while debt issuances and investment decrease. If the state of the economy worsens and the economy transits to a severe crisis, then the no-commitment type bails out for sure and its reputation collapses; therefore, interest rates decline despite the worse economic fundamentals. The equilibrium outcome can then rationalize the hump-shape of interest rate spreads and why bailouts are often delayed in a crisis. Moreover, we show in an extension that spreads and debt issuances are less responsive to the state when reputation is low; therefore, the cross-sectional volatility of spreads is low when the government has low reputation.

Bailout Decision We begin by characterizing the decision of the government in sub-period 2. One issue that arises is that in a symmetric equilibrium where the distribution of debt holdings is degenerate, the transfer to a borrower that chooses $(b, k) \neq (B, K)$ and deviates from the equilibrium path is not pinned down. This is because each borrower is measure zero; therefore, allowing a single (measure zero) borrower to default does not affect the utility of the government. In principle, it is possible to construct equilibria where transfers off the equilibrium path impose some discipline even absent reputational gains. See Chari et al. (2016), Farhi and Tirole (2012), and Davila and Walther (2017) for related discussions.

Here we select the transfer scheme by considering the limit of the finite borrower case as the number of borrowers converges to infinity. The details of this construction are provided in the Appendix D. In this case, either the no-commitment type government mimics the strategy of the commitment type or it chooses the statically optimal transfer scheme that transfers the minimal amount required to avoid default by all borrowers who would have done so absent the transfer. That is, for all (π, B, K, s), the bailout transfers $T(\pi, B, K, s)(b, k, \theta)$ are either zero for all (b, k, θ) or $T^*(b, k, \theta) = \max\{b - \theta k^\alpha, 0\}$. In the context of our example,

$$T^*(b, k, \theta_L) = b \quad \text{and} \quad T^*(b, k, \theta_H) = 0$$

for all s. Therefore,

$$T \in \{\mathbf{0}, T^*\}, \tag{6}$$

where $\mathbf{0}$ is the function identically equal to zero. From a static perspective, it is always optimal for the government to intervene and avoid default. This is because if the government transfers T^*, it can avoid the social cost of default at no cost because transfers from taxpayers to lenders do not change the utility of the government.

Notice further that this transfer scheme implies that in case of a bailout, a borrower receives its outside option of defaulting and nothing more. This is because any additional transfers make the taxpayers and consequently the government strictly worse off. As a result, given such a transfer policy, the borrower's continuation value is independent of the implemented transfers. Hence, the bailout authority's decisions only affect the borrower through their effect on the interest rates the borrower faces.

Lemma 1. *Given the transfer scheme in (6), the borrower's continuation value is independent of whether the government chooses the statically optimal transfers, T^*, or it mimics the commitment type and chooses $\mathbf{0}$. In particular,*

$$U_2(b, k, \theta) = \max\{\theta k^\alpha - b, 0\}.$$

Given our selection of off-equilibrium transfers, we can summarize the bailout authority's strategy by the probability that it will implement the statically optimal transfer scheme, $\sigma(\pi, B, K, s)$. We will call σ the *bailout policy*. Bailouts generate static benefits and impose dynamic costs on the no-commitment type government. The static value of bailing out is[3]

$$\omega^{\text{bailout}}(B, K, s) = (1 - \Delta(B, K, s))\, B$$

where

$$\Delta(B, K, s) = h(\theta_L \mid s),$$

and we normalize $C(0) = 0$. Here, $(1 - \Delta(B, s))\, B$ denotes the fraction of debt that is paid back absent a bailout. The static value of not bailing out (and allowing default) is

$$\omega^{\text{no-bailout}}(B, K, s) = (1 - \Delta(B, K, s))\, B - \psi(\Delta(B, K, s)\, B),$$

where, (with some abuse of notation) we have dropped T from the argument of Δ. The following Lemma is a direct consequence of the above expression and an increasing cost function.

Lemma 2. *The static incentives to bail out are increasing in* B*, i.e.,*

$$\begin{aligned}\Delta\omega(B, K, s) &\equiv \omega^{\textit{bailout}}(B, K, s) - \omega^{\textit{default}}(B, K, s)\\ &= \psi\Delta(B, K, s)\, B = \psi h(\theta_L|s)\, B\end{aligned}$$

is increasing in B.

As a consequence of bailing out, there is the loss in reputation of the government, as described in (3). We will later show that the equilibrium value for the government, $W(\cdot)$, is increasing in the government's reputation, π; hence, the loss of reputation is costly for the government.

We now characterize the problem of the government. The state variables in sub-period 2 are (B, s, π). Let $\zeta = 1$ denote the event that a bailout occurs and $\zeta = 0$ denote the event of no bailout. Given strategy σ, the law of motion for beliefs satisfies

$$\pi' = \begin{cases} p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma)}(p_c - p_{nc}) & \text{if } \zeta = 0 \\ p_{nc} & \text{if } \zeta = 1 \end{cases}. \tag{7}$$

If the government chooses to bail out, its value is

$$\Omega^{\text{bailout}}(B, K, s, \pi) = \omega^{\text{bailout}}(B, K, s) + \beta W(p_{nc}), \tag{8}$$

[3]Recall that since the government also cares about taxpayers who have linear utility, implementing a bailout lowers welfare by $\Delta(B, s)\, B$, which nets out the additional transfer to the lenders.

and if it chooses not to bail out, its value is

$$\Omega^{\text{no-bailout}}(B, K, s, \pi) = \omega^{\text{no-bailout}}(B, K, s) + \beta W\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma)}\right); \quad (9)$$

so the value in equilibrium is

$$\Omega(B, K, s, \pi) = \max\left\{\Omega^{\text{bailout}}(B, K, s, \pi), \Omega^{\text{no-bailout}}(B, K, s, \pi)\right\}, \quad (10)$$

where the continuation value $W(\pi)$ is defined by

$$W(\pi) = e - Q(\pi) B(\pi) + q \sum_s p_s \Omega(B(\pi), K(\pi), s, \pi), \quad (11)$$

where $B(\pi)$ and $K(\pi)$ are the allocation rules for aggregate debt and capital along the equilibrium path given prior π, respectively, and $Q(\pi) = Q(B(\pi), K(\pi), \pi)(B(\pi), K(\pi))$ is the price of debt in the (symmetric) equilibrium outcome. The optimal strategy for the no-commitment type is then

$$\sigma(\pi, s) = \begin{cases} 0, & \text{if } \psi h(\theta_L|s) B(\pi) \leqslant \beta\left[W(p_{nc} + \pi\Delta p) - W(p_{nc})\right] \\ \tilde{\sigma}, & \text{if } \psi h(\theta_L|s) B(\pi) = \beta\left[W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc})\right], \\ 1, & \text{if } \psi h(\theta_L|s) B(\pi) \geqslant \beta\left[W(p_{nc} + \Delta p) - W(p_{nc})\right] \end{cases} \quad (12)$$

where $\Delta p \equiv p_c - p_{nc}$.

Debt Issuances and Prices We now set up and characterize the decisions for private agents given a bailout policy σ. Given the behavior of the government, the no-arbitrage condition for the lenders (5) specializes to

$$Q(\pi) = q\left\{p_H + p_M(1-\mu) + p_M\mu(1-\pi)\sigma(\pi, s_M) + p_L(1-\pi)\sigma(\pi, s_L)\right\}. \quad (13)$$

Note that $Q(\cdot)$ does not depend on individual debt level and investment, b and k, as long as $b \leqslant \theta_H k^\alpha$.[4] In Section A, we study a more general model in which Q depends on b and k. We can further characterize the private equilibrium by defining a new variable, $\bar{\gamma}$, equal to the probability that an individual borrower will be bailed out conditional on drawing θ_L:

$$\bar{\gamma}(\pi) \equiv \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L}.$$

[4]It is easy to show that in equilibrium $b \leqslant \theta_H k^\alpha$.

We can then define bond prices:

$$Q(\pi) = qP_H + qP_L\bar{\gamma}(\pi).$$

That is, the price of debt is the discounted value of two components: the probability of a repayment absent government interventions, P_H, and the probability of receiving a bailout when there would be a default absent government intervention, $P_L\bar{\gamma}(\pi)$.

Using the fact that Q is independent of b, the problem for the borrower in period 1 can be written as

$$\max_k P_H\left(\theta_H k^\alpha - \frac{k}{Q(\pi)}\right), \tag{14}$$

where $b = k/Q(\pi)$, and $P_H = p_H + p_M(1-\mu)$ and $P_L = p_L + p_M\mu$ are the probabilities that in sub-period 2, the borrower will draw productivities θ_H and θ_L, respectively. The optimal choice of k satisfies

$$K(\pi) = (\alpha\theta_H Q(\pi))^{\frac{1}{1-\alpha}} \tag{15}$$

and

$$B(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q(\pi)^{\frac{\alpha}{1-\alpha}}. \tag{16}$$

Clearly, individual debt issuances and investment are increasing in $Q(\pi)$, and since a decrease in π increases Q, we have the following result:

Lemma 3. *If the function $\sigma(\pi, s)$ is decreasing in π, then the price of debt is decreasing in π in that if $\pi_H \geqslant \pi_L$, then $Q(\pi_H) \geqslant Q(\pi_L)$. Furthermore, $B(\pi_L) \geqslant B(\pi_H)$ and $K(\pi_L) \geqslant K(\pi_H)$.*

Continuous Monotone Equilibria

To show that the set of symmetric Markov equilibria is non-empty, we prove the existence of a class of continuous monotone equilibria that have some desirable properties. The equilibrium objects in this class are continuous and monotone in the government's reputation, π.

Let $\Delta(\pi, s) = \Delta(\pi, B(\pi), K(\pi), s)$ and $\sigma(\pi, s) = \sigma(\pi, B(\pi), K(\pi), s)$. We will show that there exist $W(\pi)$, $\sigma(\pi, s)$, $B(\pi)$, $K(\pi)$, and $Q(\pi)$ that jointly satisfy (11), (12), (13), (15), and (16). The next proposition shows that the set of *continuous monotone equilibria* is non-empty.

Proposition 1. *If p_{nc} is sufficiently small, there exists a continuous monotone equilibrium in which debt issuances, investment, and debt prices are decreasing in the level of reputation, the probability of bailout along the equilibrium path, $\sigma(\pi, s)$, is decreasing in the level of reputation with $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H)$, and the value for the government, $W(\pi)$, is increasing in the level of reputation.*

The proof of this proposition is provided in the Appendix.[5] To establish the result, we show that the equilibrium value in (11), the bailout policy *along the equilibrium path*, the equilibrium debt policy rule $B(\pi)$, and the equilibrium pricing schedule $Q(\pi)$ are a fixed point of an operator and then show that the operator admits a fixed point using Tarski's fixed point theorem.[6] One consequence of this fixed point theorem is that the set of fixed points are ordered. As a result, we can order equilibria by the probability of bailouts. More importantly, our existence proof also provides a characterization of the equilibrium in this class: all equilibrium objects are monotone in the government's reputation and in the aggregate state.

We now discuss the properties for the equilibria in Proposition 1. First, it is immediate from (13) that if $\sigma(\pi, s)$ is decreasing in the government's reputation, π, then the price of debt, Q, is decreasing in the government's reputation and the amount of debt issued is decreasing in the government's reputation (see Lemma 3). This is because the probability of a bailout is decreasing in the government's reputation, π. As a result, for low values of π, lenders expect bailouts with high probability, so they are happy to lend at low interest rates. From (15) and (16) it follows that low interest rates (high Q) incentivize borrowers to increase their indebtedness and investment. Thus debt and capital are decreasing in the government's reputation.

We next discuss why the probability of bailout, $\sigma(\pi, s)$, is decreasing in the government's reputation. First, note that from Lemma 3 a higher reputation reduces the price of debt and in turn reduces the level of debt outstanding. The reduction in outstanding debt reduces the costs of not bailing out the borrowers with low return on investment, $\psi h(\theta_L|s) B(\pi)$. Thus the static costs of not bailing out are low if the government's reputation, π, is sufficiently high. Moreover, since the continuation value for the government, $W(\pi)$, is increasing in the level of reputation, then the dynamic gains from not bailing out,

$$\beta\left[W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma(\pi, s))}(p_c - p_{nc})\right) - W(p_{nc})\right],$$

are increasing in π for a fixed σ. Thus, higher levels of reputation decrease the static losses of not bailing out and increase the dynamic benefits; therefore, the bailout probability, $\sigma(\pi, s)$, is decreasing in π.

Note that the forces just described can give rise to multiple equilibria and *reputation traps.* This is because there is complementarity between debt issuance and bailout probability. If private agents (lenders and borrowers) expect bailouts with high probability,

[5]This result holds for more general environments, as shown in Appendix A, and the proof for this more general result is provided in Appendix B.

[6]Note, when we refer to the bailout policy along the equilibrium path, we mean that our procedure solves for $\sigma(\pi, s) = \sigma(\pi, B(\pi), K(\pi), s)$ evaluated only at the aggregate amount of debt chosen in equilibrium, and not for arbitrary debt and capital holdings.

then debt price will be high and borrowers will find it optimal to borrow more. As described above, higher borrowing in turn induces the government to bail out with high probability, validating private agents' expectations. At the same time, if private agents expect bailouts with low probability, then less debt will be accumulated, reducing the costs of not bailing out and inducing the government not to bail out.

For any level of reputation, the bailout probability is higher in a severe crisis than in a mild crisis and is zero in normal times: $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H) = 0$. That is, the probability of receiving a bailout is increasing in the share of borrowers with a low productivity shock, θ_L. This is because the dynamic benefits of not bailing out are not affected by the current state directly, while the static losses of not bailing out are increasing in the fraction of borrowers with θ_L, $h(\theta_L|s)$. Clearly, if the economy is in normal times, $s = s_H$, there are no static benefits from bailing out, because all borrowers have θ_H, so $\sigma(\pi, s_L) = 0$ for all levels of reputation.

Next, we provide sufficient conditions such that the monotone continuous equilibrium has mixing in a mild recession and a bailout with probability 1 in a severe recession. This turns out to be useful to connect the equilibrium outcome with our motivating evidence.

Assumption 1. *Let* $C(x) = \psi x$ *and define*

$$W^R(\bar{\gamma}) \equiv \frac{e - [\psi(1-\bar{\gamma}) + \bar{\gamma}]\, q P_L \mathbf{b}(\bar{\gamma})}{1-\beta},$$

where $\mathbf{b}(\bar{\gamma}) = (\alpha\theta_H)^{1/(1-\alpha)} [q(P_H + P_L\bar{\gamma})]^{\alpha/(1-\alpha)}$ *is the level of debt that will be issued if borrowers expect to be bailed out with probability* $\bar{\gamma}$. *Assume that*

$$\psi \mathbf{b}(0) > \beta \left[W^R(0) - W^R(1)\right] \tag{17}$$

and

$$\frac{q\beta P_L}{1 - q\beta p_H} > \psi\mu \frac{\mathbf{b}(0)}{\mathbf{b}(1)}. \tag{18}$$

Here, $W^R(\gamma)$ is the value for a fictitious commitment type who follows a fixed bailout policy summarized by sufficient statistic $\bar{\gamma}$ when private agents have beliefs $\pi = 1$. For the commitment type assumed in our model, $\bar{\gamma} = 0$, so $W^R(0)$ is the value for the commitment type in our model when $\pi = 1$.[7] Since $W^R(0) > W(p_c)$ and $W^R(1) < W(p_{nc})$, the difference $W^R(0) - W^R(1)$ is an upper bound on the dynamic gains from not bailing out. This, along with the fact that $B(\pi) \geqslant \mathbf{b}(0)$, implies that condition (17) ensures that the static gains from bailing out dominate the dynamic costs in state s_L when all borrowers would default absent a bailout. The second part of the assumption, (18), provides an

[7] In regard to an earlier discussion on the optimal policy with commitment, it is easy to see that $\frac{\partial W^R(\bar{\gamma})}{\partial \bar{\gamma}} < 0$ if $\psi < 1$. Therefore, the optimal policy is to set $\bar{\gamma} = 0$ and never bail out.

upper bound on μ (the fraction receiving θ_L in s_M) so that it is not the best response for the no-commitment type to bail out with probability 1 in s_M.

Proposition 2. *Under Assumption 1, if* $p_c \to 1$ *and* $p_{nc} \to 0$, *then in any monotone continuous equilibrium it must be that*

- *it is optimal to bail out with probability 1 in a severe recession,* $\sigma(\pi, s_L) = 1$ *for all* π, *and*
- *it is optimal to mix, i.e., there exists a set* $[\pi_1, \pi_2]$ *such that* $\sigma(\pi, s_M) \in (0,1)$ *in state* s_M *for all* π *in* $[\pi_1, \pi_2]$.

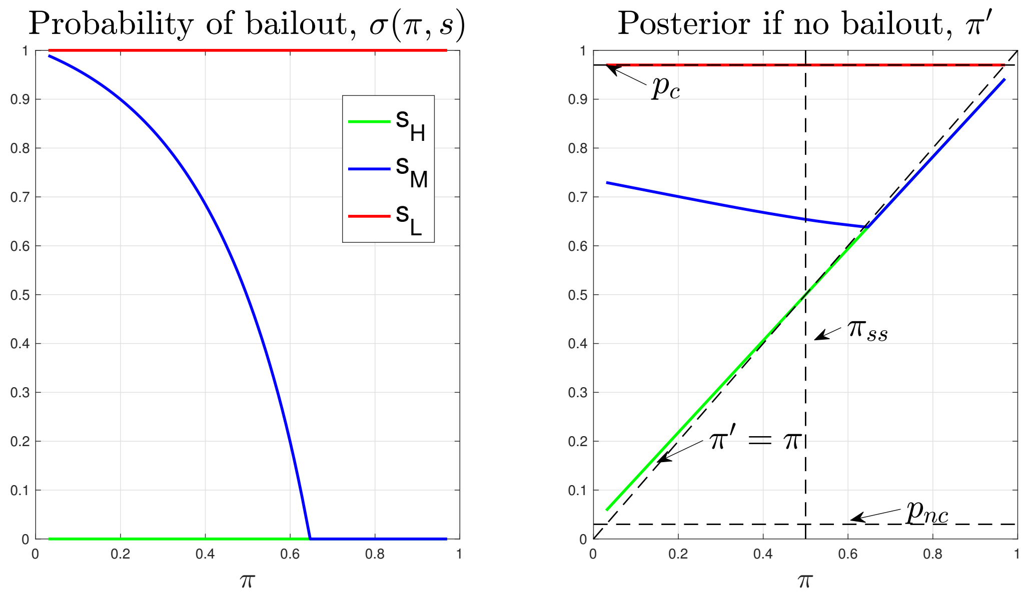
In Figure 1 we use a numerical example to illustrate some of the key properties from the above characterization results. The first panel plots the bailout probability as a function of the government's reputation. Since no borrower defaults in s_H, there are no static benefits of bailing out, so $\sigma(\pi, s_H) = 0$. In state s_L, the static benefits are much larger than the dynamic benefits for all π, so $\sigma(\pi, s_L) = 1$. The plot also shows that there is an interval in which randomizing between bailing out and not is optimal.

The second panel describes how the beliefs about the government type evolve given the equilibrium strategy and conditional on observing no bailout. In state s_H, since private agents believe that the no-commitment type will not bail out, the posterior changes because of the exogenous transition from one type to another. In state s_M, since there is a positive probability of bailout, in the event that there is no bailout, the posterior that the government is the commitment type will rise. Therefore, the state s_M offers the government an opportunity to build its reputation. In state s_L, the dynamic gains from not bailing out are the largest, since private agents believe that the no-commitment type will bail out with probability 1. However, as discussed above, the social costs of default are much larger than these benefits and thus it is not optimal for the no-commitment type to abstain from bailing out the borrowers to build up its reputation.

Equilibrium Outcomes

We now describe an equilibrium outcome that generates the dynamics described in the introduction. This outcome is illustrated in Figure 2. Suppose that the no-commitment type is in charge and the economy has been in normal times for a sufficiently large number of periods so that the prior has converged to $\pi_{ss} = p_{nc} + \pi_{ss}(p_c - p_{nc})$. Suppose now that the economy suffers a mild recession in period t, $s_t = s_M$. Assuming that $\pi_{ss} \in [\pi_1, \pi_2]$, where the bounds of the interval are defined in Proposition 2, then the no-commitment type mixes between bailing out and not. If it turns out that the outcome of the randomization calls for not bailing out, then the private agents' beliefs of facing the commitment type jump above π_{ss} and consequently the aggregate debt level falls and

Figure 1: Equilibrium objects for computed discrete example



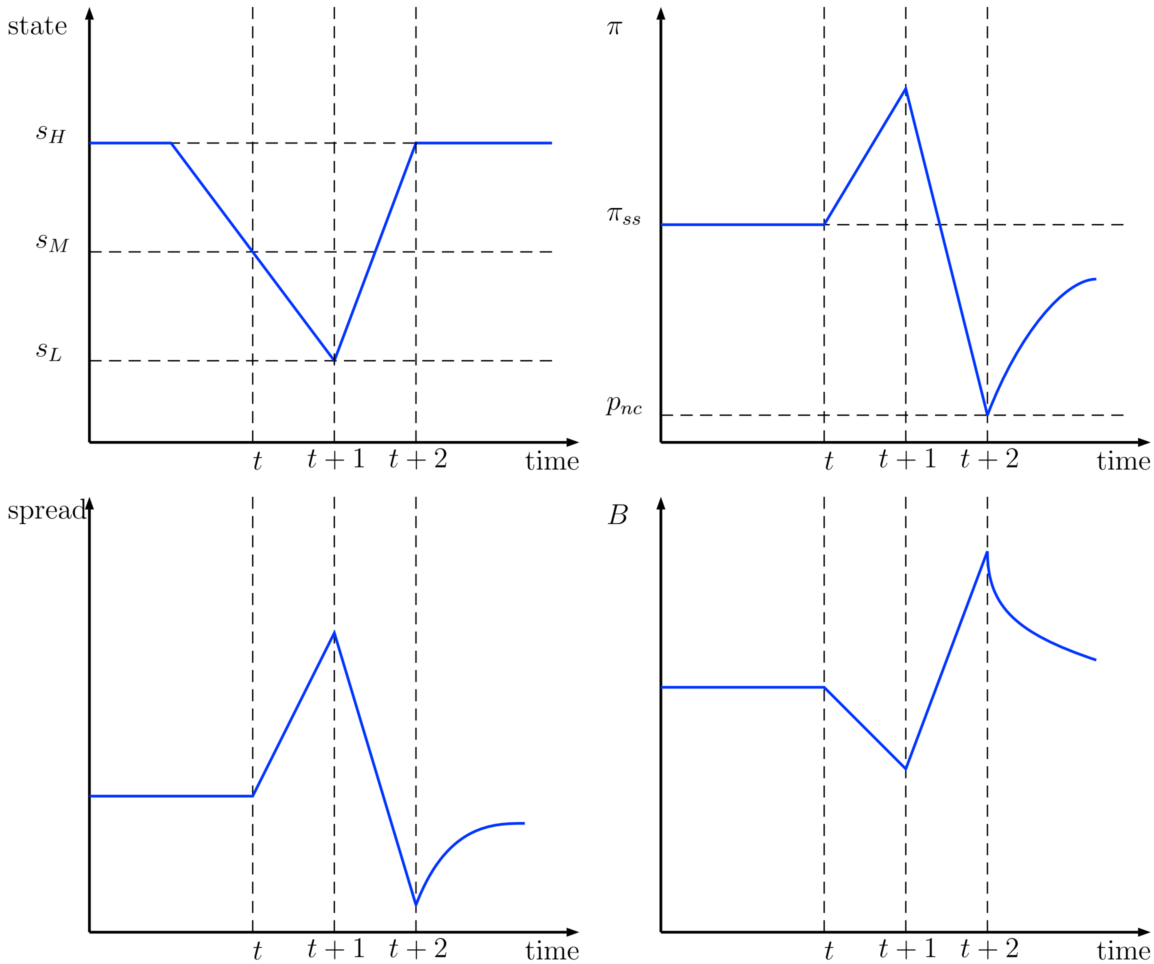
spreads rise the following period. If the economy is then hit by a severe recession in period $t+1$, $s_{t+1} = s_L$, there is a bailout with probability 1; therefore, private beliefs fall to p_{nc}, the aggregate debt levels rise, and spreads fall in subsequent periods due to the high probability of receiving a bailout in the future. The model can then rationalize the hump-shaped dynamics of spreads often observed during crises where spreads are very high at the beginning of a crisis if there is no bailout right away and then fall after a bailout.

The dynamics in Figure 2 are driven by changes in fundamentals. However, they only require that the no-commitment type's *beliefs* about the true state change. In Section 4, we extend our model to one in which the government learns about the state from noisy prices. This model can generate identical dynamics to the baseline without the true state actually changing.

Reputation and Sensitivity of Spreads to Fundamentals

So far we have assumed that borrowers are ex-ante identical, so there is no heterogeneity in their borrowing and investment decisions and in the interest rates at which they borrow. We now consider the case with two types of borrowers that differ in their probability of drawing the low productivity shock in sub-period 2. We show that this extension of our baseline model can generate the differential sensitivity effects documented by Acharya et al. (2016) and Cole et al. (2016). The authors document that the sensitivity of bond yields to fundamentals such as GDP growth increased significantly during the course of the US financial crisis and the European debt crisis, respectively.

Figure 2: Outcome path



We consider the most parsimonious model to make our point. [8] Assume that in each period there are two types of borrowers indexed by $i \in \{m, l\}$. There is a measure μ of type l borrowers that draw the idiosyncratic state θ_L for sure in aggregate state s_L and s_M so $h_m(\theta_L|s_L) = h_m(\theta_L|s_M) = 1$ and $h_m(\theta_L|s_H) = 0$. There is a measure $(1-\mu)$ of type m borrowers that draw θ_L only if $s = s_L$. That is, $h_l(\theta_L|s_L) = 1$ and $h_l(\theta_L|s_H) = h_l(\theta_L|s_M) = 0$. The problem of a type i borrower is then

$$\max_k P_{Hi}\left(\theta_H k^{\alpha} - \frac{k}{Q_i(\pi)}\right),$$

where P_{Hi} is the probability that type i draws θ_H in sub-period 2 and

$$Q_l(\pi) = q\{p_H + p_M(1-\pi)\sigma(\pi, s_M) + p_L(1-\pi)\sigma(\pi, s_L)\} \tag{19}$$

$$Q_m(\pi) = q\{p_H + p_M + p_L(1-\pi)\sigma(\pi, s_L)\}. \tag{20}$$

The next result is immediate:

Proposition 3. *(Sensitivity to fundamentals increasing in reputation) The difference in the price of debt for the low default type,* m*, and the high default type,* l*, is increasing in the reputation of the government; that is,* $Q_l(\pi) - Q_m(\pi)$ *is increasing in* π*. Similarly,* $k_l(\pi) - k_m(\pi)$ *is increasing in the government's reputation.*

Debt prices (and debt issuances) are less responsive to the borrower's fundamentals (its type) when the prior is low. In particular, if the probability of facing the no-commitment type is low, then the lenders are less worried about the type of the borrower since they expect to get bailed out with high probability;therefore, debt prices are not very sensitive to fundamentals.

Proposition 3 has important implications for observables after a bailout. Once we observe a bailout, the government's reputation falls and the cross-sectional volatility in spreads goes down, as prices are less sensitive to fundamentals. Conversely, after we observe a no-bailout event, the reputation of the government goes up and so does the volatility of spreads.

4 Two-Sided Learning

We now extend the baseline model to allow for uncertainty about the aggregate state and sequential bailout requests within a period. The main result of this section is that we

[8] In Appendix C we show that the conclusion of this section can also be obtained by extending our baseline model to allow for persistence in the borrower's shock.

can obtain hump-shaped time series for interest rate spreads without having to rely on a transition through a mild crisis (s_M).

Suppose the aggregate state of the world is $s \in \{s_L, s_H\}$ with probabilities p_L and p_H, respectively. In state s_H, a fraction μ of borrowers draw the low idiosyncratic shock ($h(\theta_L|s_H) = \mu$), while in state s_L all borrowers draw the low shock ($h(\theta_L|s_L) = 1$). We will refer to s_H as normal times and s_L as a systemic crisis. As in the baseline model, we assume that the social default costs are linear, $C(x) = \psi x$. The state of the world s is observed by private agents but is unobservable to the government. The timing in the first sub-period of the stage game is identical to the baseline environment. The second sub-period is further divided into two stages. The timing in the first stage is as follows:

1. Borrowers enter sub-period 2 with debt b and capital k and prior belief π.

2. The state of the world, $s \in \{s_L, s_H\}$, is realized and learned by private agents.

3. Lenders draw the taste shock $\varepsilon \sim G$ and can trade a mutual fund of debt in a secondary market at price $q_2 = Q_2(\pi, B, K, s, \varepsilon|\sigma)$. The taste shock ε is a non-monetary value that lenders attain from holding the portfolio.

4. A measure μ of borrowers ask for a bailout and the government bails outs with probability $\sigma_1(\pi, B, K, q_2)$.

The realization of endowments in sub-period 2 is staggered over the two stages. In the first stage, a fraction μ of borrowers receive the low productivity shock independently of the state. This implies that the government does not learn anything about the state from the fraction requesting a bailout. In the second stage, depending on the state, there is a second wave of bailout requests. If $s = s_H$, no other borrower requests a bailout and the prior is updated to π' . If $s = s_L$, then a fraction $1 - \mu$ of borrowers ask for a bailout and the government bails out with probability $\sigma_2(\pi, B, q_2)$. This implies that the state is perfectly revealed to the government in the second stage.

We now describe the key equilibrium objects. In the second stage, if the state is s_L, the government bails out if and only if

$$\psi(1-\mu)B \geqslant \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)(1-\sigma_2)}\Delta p\right) - W_1(p_{nc})\right].$$

In what follows we suppose that μ is small enough so that it is always optimal to bail out in the second stage if $s = s_L$.[9] Then we have that $\sigma_2 = 1$. Of course, if the economy is in state s_H, there is no borrower to bail out, and trivially $\sigma_2 = 0$ in normal times.

[9] Recall that Assumption 1 guaranteed this when $\mu = 0$.

We now consider the bailout decision in the first stage. Here, regardless of the state, a fraction μ of borrowers require a bailout in order to avoid default. The government does not observe the state, but it can observe and learn from prices in the secondary market. The price of a portfolio of debt in the secondary market at the beginning of sub-period 2 is

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\,\sigma_1(\pi, B, q_2) + \varepsilon, & \text{if } s = s_H \\ (1-\pi)\left[(1-\mu)\,\sigma_2(\pi, B, q_2) + \mu\sigma_1(\pi, B, q_2)\right] + \varepsilon, & \text{if } s = s_L \end{cases}.$$

Thus, the bailout authority's beliefs that the true state is s_H conditional on having observed the secondary market price q_2 is

$$\hat{p}(s_H|\pi, B, q_2) = \frac{p(s_H)\, g_H(q_2)}{p(s_L)\, g_L(q_2) + p(s_H)\, g_H(q_2)},$$

where

$$\begin{aligned} g_L(q_2) &= g\left(q_2 - (1-\pi)\left[(1-\mu)\,\sigma_2(\pi, B, q_2) + \mu\sigma_1(\pi, B, q_2)\right]\right), \\ g_H(q_2) &= g\left(q_2 - \left[(1-\mu) + \mu(1-\pi)\,\sigma_1(\pi, B, q_2)\right]\right), \end{aligned}$$

and $g(\cdot)$ is the probability density function of ε.

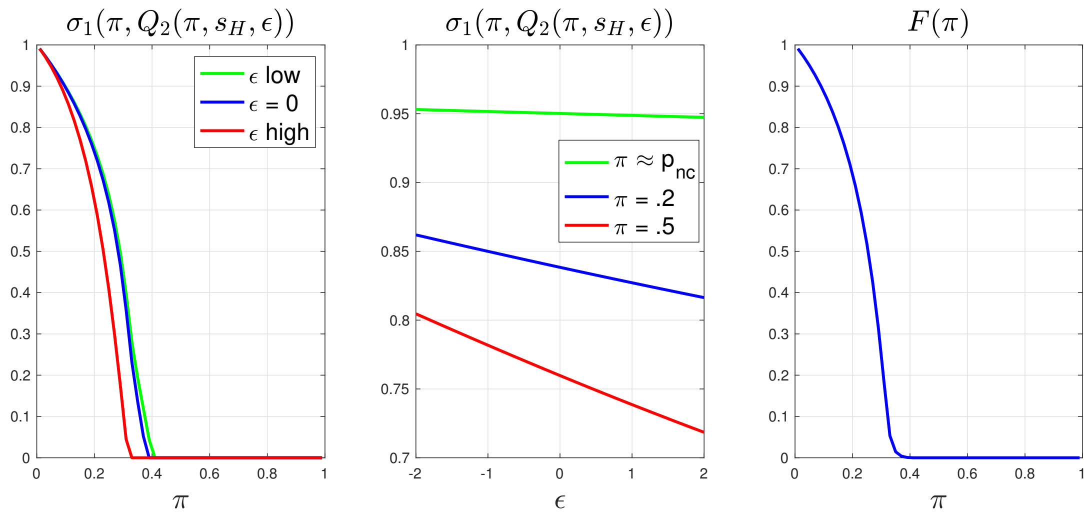
This implies that in the first stage, the no-commitment type bails out if and only if

$$\psi\mu B \geqslant \hat{p}(s_H|\pi, B, q_2)\,\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\Delta p\right) - W_1(p_{nc})\right].$$

Since the state is unknown in the first stage, the dynamic benefits of not bailing out only accrue with probability $\hat{p}(s_H|\pi, B, q_2)$, since the no-commitment type bails out with probability 1 in the second stage if $s = s_L$. All things being equal, a lower value of q_2 lowers the posterior $\hat{p}(s_H|\pi, B, q_2)$. As a result, in contrast to the baseline model, the dynamic gains from not bailing out might change for non-fundamental reasons. In particular, all else equal, a lower realization of ε increases the probability of a bailout in the first stage.

As a consequence, one can generate similar dynamics to the baseline model with only two aggregate states. For example, in state s_L, a low realization of ε will induce a very low value of q_2, which will lead to a low value of $\hat{p}$, thus inducing a bailout with probability 1 in the first stage. For the same realization s_L, a higher value of ε will result in a larger value of $\hat{p}(s_H)$, which in turn can push the government into the randomization region. Similar to state s_M in the baseline model, in the case when a bailout is not observed, the posterior value of the government being the commitment type rises, which in turn

Figure 3: Equilibrium objects for computed example with two-sided learning



decreases the price of debt on the secondary market at the end of the first stage,

$$Q_3(\pi, B, s, \varepsilon|\sigma) = \begin{cases} 1+\varepsilon & s = s_H \\ (1-\pi)(1-\mu)+\varepsilon & s = s_L \end{cases}.$$

Figure 3 plots the key equilibrium objects for a typical computed numerical example. (In Appendix E we carefully describe the equilibrium conditions.) As we see in the first panel, for a fixed ε, a higher value of π implies a lower probability of bailout in the first stage. This intuition is similar to our baseline model, in which the state is observable. Similarly, for a fixed π, we see that a lower realization of ε induces a higher probability of a bailout in the first stage since the government's posterior of the state being low rises. The expected probability of a bailout in the first stage,

$$F(\pi) = \int \sigma_1(\pi, \varepsilon)\left[p(s_H)\, g(\varepsilon) + p(s_L)\, g(\varepsilon + (1-\mu)\pi)\right] d\varepsilon,$$

is decreasing in π, which in turn implies the price of debt in sub-period 1, Q, is increasing in π. This generates identical spread dynamics to the baseline environment.

An interesting feature of this model is that information conveyed by secondary market prices depends on the government's reputation, π. For low values of π, since lenders expect a bailout with a high probability, the price Q_2 will largely be driven by the taste shock ε, thus conveying little information about the fundamental. In contrast, for high values of π, the price Q_2 will be more sensitive to fundamentals. This implies that for low values of π, a much larger value of Q_2 is needed in order for the government to increase its posterior belief of s_H.

5 Long Lived Borrowers

In the baseline model, we assumed that borrowers live for one period. This implies that their default decision is static. Here we relax this assumption and allow borrowers to be long lived. Consequently, borrowers' default decisions will be dynamic. This allows us to study two interesting applications of our framework. First, we study why a government would bail out a small bank even though the static benefits of doing so might appear small. Second, we study the value for the no-commitment type of a pure announcement of type relative to physical transfers in order to bail out distressed banks.

5.1 Small Bailouts to Avoid Contagion

With static default decisions, we have shown that it was worthwhile to bail out only if a sufficiently large share of borrowers receives a bailout. This is because the static benefits of a bailout must be large enough to compensate for the dynamic reputation losses. In reality, however, we often observe cases in which governments bail out small banks. One recent example is the case of the Italian government bailing out two mid-sized banks in the Veneto region in 2017. Why would governments bail out small borrowers when the static losses associated with them defaulting is small?

In this section, we show that when the borrowers' default decisions are dynamic – in that they consider future profits in deciding whether to default or not – then the measure of borrowers that end up receiving the bailout is not a sufficient statistic for the static benefits of a bailout. The reason for this is that since bailouts (or lack thereof) change private beliefs about the future behavior of the government, bailout announcements affect borrowers' continuation values of not defaulting. In certain circumstances, a bailout to a small number of borrowers is needed in order to avoid contagion to other borrowers. Absent the small bailout, a large number of borrowers might be incentivized to default due to a decrease in future continuation values. By observing a bailout today, these borrowers anticipate higher transfers in the future and therefore choose not to default in order to appropriate these future transfers. Thus the static benefits of a bailout can be large even if in equilibrium the government ends up bailing out only a small fraction of borrowers.

Suppose that $t = 1, 2$ and that borrowers also live for two periods. Borrowers can be one of two types, $i \in \{T, P\}$, with $\Pr(i = P) = \mu$. Types are perfectly observable. Borrowers of type P have perfectly persistent productivity shocks, while borrowers of type T have purely transitory productivity shocks. Let us also assume that government types are perfectly persistent, i.e., $p_c = 1 - p_{nc} = 1$. Finally, we assume that if a borrower defaults in period 1, it is replaced by an identical borrower at the beginning of period 2, which keeps the mass of outstanding borrowers constant. Let's start from the second period. Since there are no dynamic gains, the no-commitment type government will bail out any borrower with θ_L in period 2 so that $\sigma_2 = 1$.

Therefore, the continuation payoff of having access to financial markets in period 2 when the prior is π is given by

$$U_{2i}(\theta_1, \pi') = \max_k P_{Hi}(\theta_1)\left[\theta_H k^\alpha - \frac{k}{Q_{2i}(\pi, \theta_1)}\right],$$

and $Q_{2i} = q\left[P_{Hi} + P_{Li}(1-\pi)\right]$ is the equilibrium price for the debt issued by type i. A simple computation yields

$$U_{2T}(\theta_1, \pi) = P_H\left(q\left(P_H + (1-\pi)P_L\right)\right)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)}\left[1-\alpha\right] \tag{21}$$

so that $U_{2T}(\theta_1, \pi)$ is strictly decreasing in π, while $U_{2P}(0, \pi') = 0$. The value for the no-commitment type government at the beginning of period 2 given a prior π is

$$W_2(\pi) = -qP_L(\alpha\theta_H)^{1/(1-\alpha)}\left(q\left(P_H + (1-\pi)P_L\right)\right)^{\alpha/(1-\alpha)}.$$

Consider a state of the world in which both types of borrowers receive the low endowment in period 1. Absent a bailout, borrowers of type P will default for sure, while borrowers of type T will repay if and only if

$$-B_1 + \beta U_{2T}\left(0, \frac{\pi}{\pi + (1-\pi)\sigma_1}\right) \geqslant 0,$$

where $\pi/\left[\pi + (1-\pi)\sigma_1\right]$ is the posterior of facing the commitment type after observing no bailout. In particular, we can be in a situation in which if $\sigma_1 < 1$, then

$$-B_1 + \beta U_{2T}\left(0, \frac{\pi}{\pi + (1-\pi)\sigma_1}\right) < 0 \tag{22}$$

and

$$-B_1 + \beta U_{2T}(0, p_{nc}) > 0. \tag{23}$$

That is, if there is no bailout, the borrowers with a transitory bad shock also choose to default if they observe no bailout. But if the government bails out the permanent type, the drop in reputation increases the continuation value of the transitory type because the borrowers now expect to extract subsidies from the government in the future. Thus, it is optimal for them to repay their debt even though they do not directly receive transfers in the current period.

Consider now the decision whether to bail out or not for the no-commitment type in period 1 when conditions (22) and (23) hold. If the government does not bail out the permanent types, the high reputation of the government will induce the transitory types to default as well. This is true for any μ arbitrarily close to zero. Thus the static costs of not bailing out are ψB_1 even though only a fraction μ will actually obtain a transfer. Therefore, the government bails out if

$$-\psi B_1 + \beta W_2 \left(\frac{\pi}{\pi + (1-\pi)\,\sigma_1} \right) \leqslant \beta W_2 \left(p_{nc} \right). \tag{24}$$

If ψ is large enough, this is indeed the best response of the no-commitment type.

Assumption 2 provides sufficient conditions on parameters so that inequalities (22), (23), and (24) hold at $\pi = p_c$ and $\sigma_1 = 1$ so that in equilibrium, the government bails out the permanent type borrowers in period 1 even though their measure is arbitrarily small.

Assumption 2. *Suppose that*

$$\beta P_H \left(\frac{1-\alpha}{\alpha} \right) - 1 < 0, \tag{25}$$

$$\beta P_H \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{P_H + (1-p_{nc})\,P_L}{P_H + (1-p_c)\,P_L} \right)^{\alpha/(1-\alpha)} - 1 > 0, \tag{26}$$

and

$$\beta q P_L \left[\frac{(q\,(P_H + (1-p_{nc})\,P_L))^{\alpha/(1-\alpha)}}{(q\,(P_H + (1-p_c)\,P_L))^{\alpha/(1-\alpha)}} - 1 \right] < \psi. \tag{27}$$

The next proposition follows directly from the above analysis.

Proposition 4. *Suppose that Assumption 2 holds. Then there exists a π^* such that for all $\pi \in (\pi^*, p_c]$ and any $\mu > 0$, in the state in which all borrowers receive the low endowment, the no-commitment type bails out type* p *banks with probability 1.*

5.2 Bailouts vs. Announcements

Next, we study the merits of using a pure announcement versus an actual transfer to help distressed banks. Consider the framework described in the previous subsection but assume that all banks are of the transitory type.[10] We drop the subscript T for convenience. Suppose that

$$-b_1 + \beta U_2 \left(p_{nc} \right) > 0$$

so that if the government's reputation (p_{nc}) is sufficiently low, expected future rents are high enough to induce the borrower to repay the debt even if current productivity is zero. Now if we assume that the no-commitment type can credibly announce its type, the above inequality says that even though banks receive the low endowment in period 1,

[10] The analysis would go through as long as types were not perfectly persistent.

the continuation payoffs of participating in financial markets are large enough to induce them not to default. As a result, conditional on wanting to bail out the banks, the best response for the no-commitment type is to set $T = 0$ and announce its type (or promise future bailouts). The reason for this is that a positive transfer provides a subsidy for the banks and, since the government only cares about the lenders and taxpayers, this makes it strictly worse off.

Proposition 5. *Suppose that*

$$\alpha \leqslant \frac{\beta P_H}{1 + \beta P_H}. \tag{28}$$

Then, the no-commitment type either does not bail out or makes an announcement that it will bail out in the next period and chooses $T^* = 0$.

The sufficient condition in the above proposition ensures that just an announcement is sufficient to incentivize the borrowers to repay absent a transfer in period 1.

6 Narrative Analysis

In this section, we use our framework to interpret the dynamics in three recent financial crises: the US financial crisis, the institution of the Single Resolution Mechanism (SRM) within the context of the European banking union, and the European sovereign debt crisis.

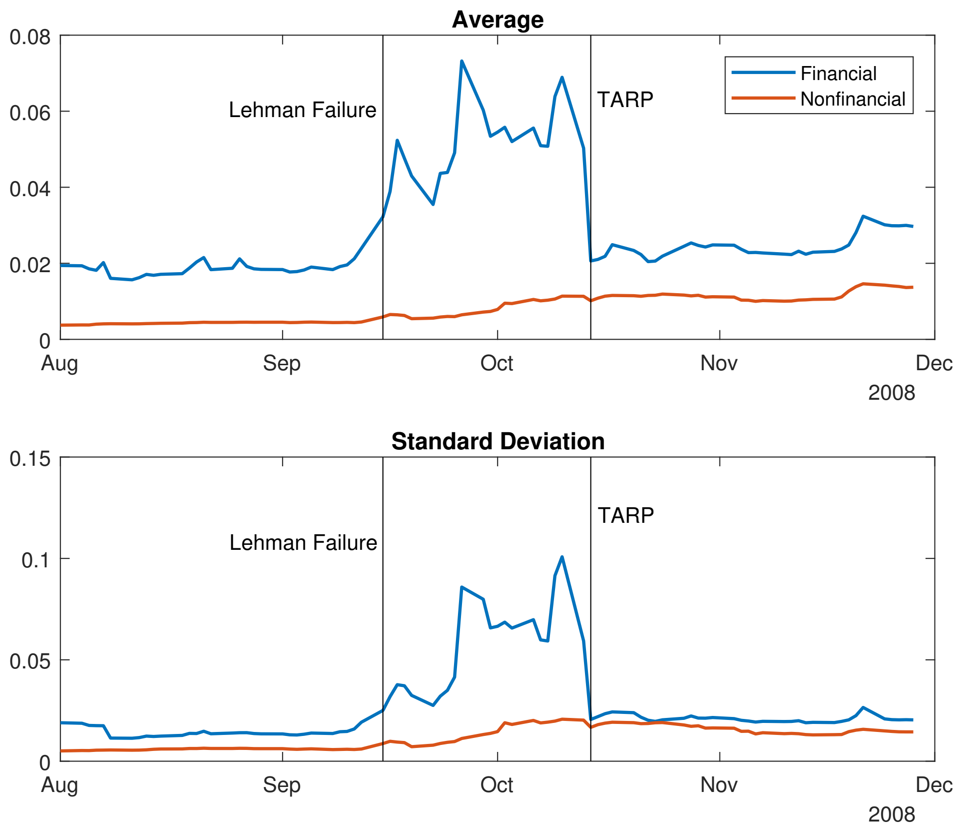
Recall the expression for bond prices,

$$Q(\pi) = qP_H + qP_L \bar{\gamma}(\pi),$$

where $\gamma(\bar{\pi})$ is the probability that an individual borrower will be bailed out conditional on drawing θ_L. The key predictions of the model we are interested in are the following. First, if after an adverse event there is no bailout and lenders are not rescued, our model predicts that $\gamma(\bar{\pi})$ will decrease, which in turn implies that the spreads for borrowers not directly affected by the adverse event will go up. Thus we should observe CDS spreads rising after such events. In the data, movements in counterparty risk in CDS can confound our analysis. Since CDS are traded among financial institutions, a lower probability of bailout could result in higher counterparty risk, which in turn would push CDS spreads down, because agents are less willing to pay for the insurance they provide due to the higher risk not to be repaid. While this concern is valid in theory, studies have shown that counterparty risk is negligible in practice, consistent with the high degree of collateral posted. See Arora et al. (2012).

In addition, as implied by Proposition 3, the sensitivity of interest rates with respect to

Figure 4: CDS spreads of large financial and non-financial firms



Source: Markit. The financial firms are Citigroup, Bank of America, JP Morgan Chase, Wachovia, Wells Fargo, The Bank of New York Mellon, Goldman Sachs, Morgan Stanley, Merrill Lynch. The non-financial firms are the companies included in the Dow Jones Industrial Average.

fundamentals should also rise. While a direct measure of this sensitivity is hard to obtain, we proxy for this by looking at the cross-sectional standard deviation of CDS spreads under the assumption that the underlying distribution of fundamentals is not affected by news about (future) bailout prospects.

Second, or model predicts that the opposite will happen after we observe a bailout or an announcement of future bailouts: $\gamma(\bar{\pi})$ will rise and thus the average level of interest rate spreads will fall and the sensitivity to fundamentals will decrease.

US Financial Crisis We start our analysis by considering the behavior of the CDS spreads of the largest US financial firms around the financial crisis that started in 2007. Figure 4 plots the average and standard deviation of CDS spreads for large financial institutions in the United States. Consistent with our model predictions, these moments sharply rose after the refusal to bail out Lehman Brothers and its consequent bankruptcy and sharply declined after the announcement of the Troubled Asset Relief Program (TARP).[11] Through the lens of our model, we interpret the former event as one in which the government was in the randomization region; thus, after a no-bailout the government's reputation rose, which in turn led to an increase in borrowing rates for banks. In contrast, we interpret the latter event as a bailout event, leading to a drop in reputation and a subsequent decline in borrowing rates.

One potential problem with our interpretation is that spreads for all firms – financial and non-financial – increased during the crisis, as documented by Gilchrist and Zakrajšek (2012). To make sure that dynamic of spreads for financial firms between the Lehman failure and the announcement of TARP is not driven by either changes in fundamentals or the price of risk, we also plot the same moments for a selection of large non-financial firms, which are arguably not directly affected by these bailout policies but can be subject to the same movements in volatility and price of risk. From Figure 4, it is clear that the hump-shaped dynamics of the mean and standard deviation of CDS spreads for financial institutions is not shared by non-financial firms. If anything, spreads for non-financial firms monotonically increased during this period as the recession deepened and aggravated.

This evidence is consistent with previous studies. Veronesi and Zingales (2010) document that CDS spreads increased for other banks after Lehman Brothers filed for bankruptcy. After the announcement of the Paulson plan (a $125 billion equity infusion) in October 2008, spreads for many large banks fell.Schweikhard and Tsesmelidakis (2011) use the difference in the default risk implied by equity prices and CDS spreads to measure the impact of government guarantees in pricing default risk.[12] They find that this difference increases sharply for banks during bailout events, for example TARP and the Bank of America rescue package. See Figure 3 in their paper. Moreover, they show that this difference is relatively unchanged for non-financial firms.

Acharya et al. (2016) find that the risk sensitivity of spreads for large financial firms is substantially weaker than for small and medium financial firms, while there is no such difference for non-financial firms of different size. They also find that "following the collapse of Lehman Brothers in 2008, larger financial institutions experienced greater increases in their spreads than smaller institutions. In contrast, the spreads of large financial institutions also became more risk sensitive after the collapse of Lehman. Following the government's rescue of Bear Stearns in 2008 and the adoption of the TARP and other liquidity and equity support programs, we find that larger financial institutions experienced

[11] Lehman Brothers Holdings Incorporated filed for Chapter 11 bankruptcy protection on September 15, 2008. The US Treasury Department announced TARP on October 14, 2008. See the St. Louis Fed Financial Crisis Timeline at `https://www.stlouisfed.org/financial-crisis/full-timeline`.

[12] Atkeson et al. (2018) use the differences in book and equity values of banks to measure the size of the government guarantees.

greater reductions in credit spreads than smaller institutions; the spreads of large financial institutions also became less risk sensitive." They interpret these results as evidence of "too big to fail" status for large financial institutions. This evidence suggests that our theoretical analysis is more applicable to this subset of firms.

Relatedly, Kelly et al. (2016) provide evidence that during the US financial crisis the expectation of a system-wide bailout to rescue bank equity holders increased. They document that during the financial crisis, the cost of out-of-the-money (OTM) put options for an index of the financial sector was much cheaper relative to OTM put options on the individual banks comprising the index. They argue that the reason for this was the difference in bailout expectations in the case of a single bank failing versus the financial sector collapsing as a whole. Clearly, our model would be able to replicate these dynamics assuming that the non-commitment type government values bank equity holders' utility.

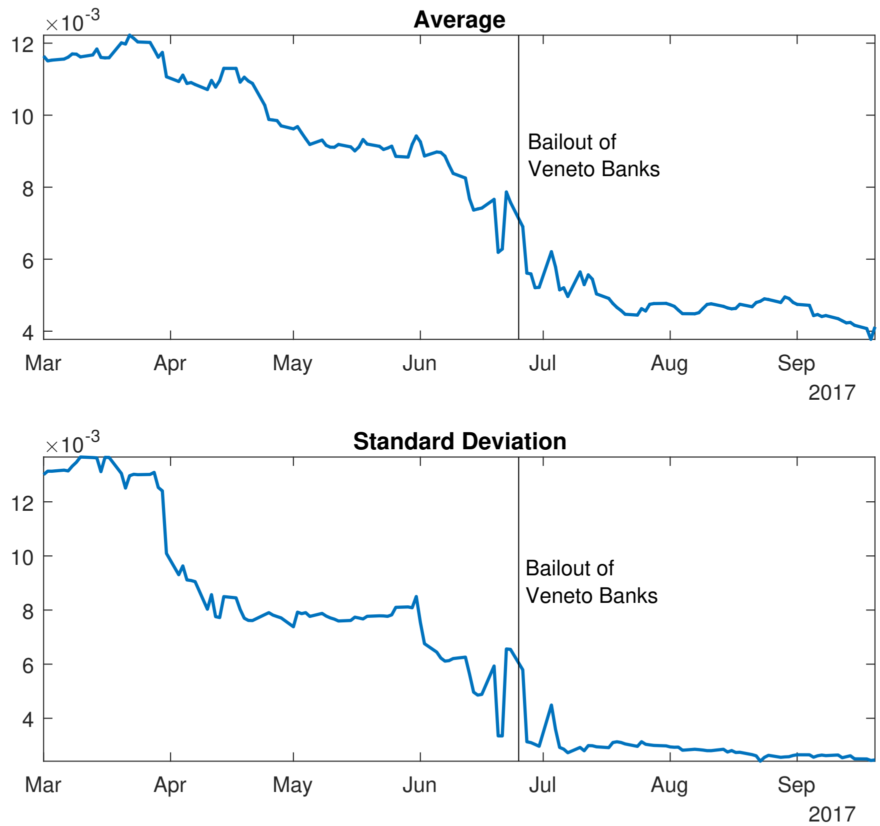
European Banking Union In 2014, member countries of the European Union transferred banks' responsibilities for regulation and resolution to a European authority, the Single Resolution Mechanism (SRM), with the goal of minimizing the cost to taxpayers from bailing out failing banks. The SRM became fully operative in 2016. The governing principle of the SRM is that bailouts should be avoided and replaced by *bail-ins:* shareholders and debt holders should realize losses before any public funds are used.[13] The credibility of the SRM was tested in 2017 with the prospect of bankruptcy for Monte dei Paschi di Siena, Banca Popolare di Vicenza, and Veneto Banca in Italy. The Italian government used taxpayer money to rescue Monte dei Paschi di Siena. For the two banks in the Veneto region, the single resolution board did not perceive them to be a risk to financial stability, but the Italian government intervened. The two banks closed, and the good assets were acquired by Intesa SanPaolo with a government guarantee. Senior debt holders were bailed out and retail investors were compensated for their losses of junior debt. Thus the Italian government avoided applying the bail-in provision of the new European banking regulations.[14]

Through the lens of our model, after the Italian government use of public funds, private agents are less likely to believe that the SRM is credible and thus expect greater government support of banks in the future. Figure 5 plots the mean and standard deviation of CDS spreads for large Italian banks for the period surrounding the bailout of Banca

[13] The same policy was pursued in the US with Title II of Dodd-Frank, which calls for companies in danger of default to be placed in receivership rather than bailed out. In the 2018 David K. Backus Memorial Lecture, Darrell Duffie argues that much of the reason CDS spreads continue to be high compared to pre-crisis levels is due to private beliefs that banks might not be bailed out in the event of a future crisis. He argues that these beliefs are primarily driven by the Lehman episode as well as the regulation in Dodd-Frank. The resolve of US regulators has not been tested yet.

[14] See Cooley (2017) and https://www.economist.com/finance-and-economics/2017/07/01/the-complicated-failure-of-two-italian-lenders

Figure 5: CDS spreads of Italian banks



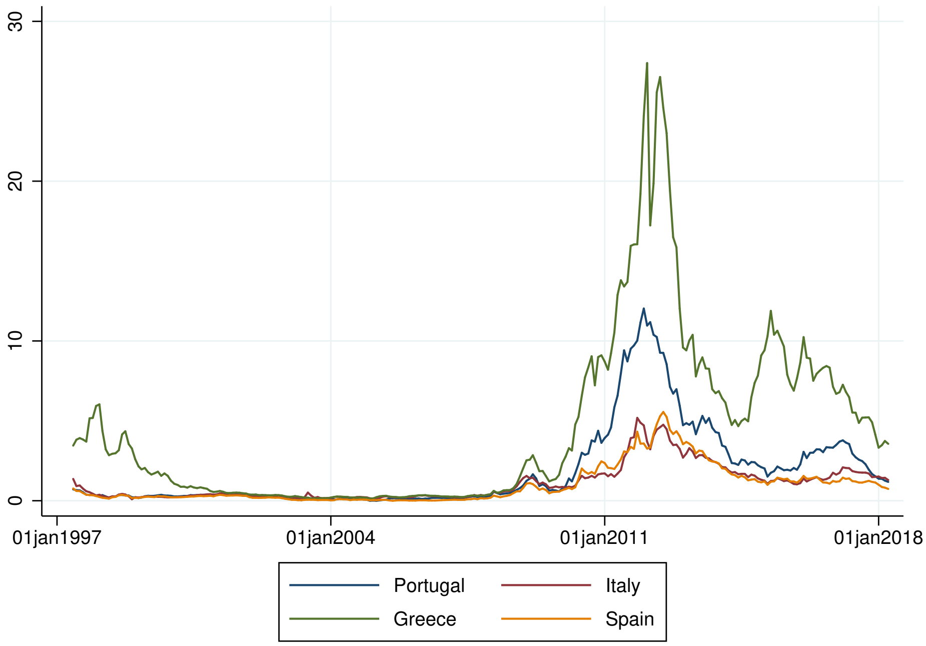
Source: Markit. The banks in the sample are Assicurazioni Generali, Banca Carige, Banca Monte dei Paschi di Siena, Banca Nazionale del Lavoro, Banca Popolare di Milano, Banca delle Marche, Banco BPM, Banco Popolare, Beni Stabili, Intesa Sanpaolo, Mediobanca, UniCredit, UBI Banca, and Unipol Gruppo.

Popolare di Vicenza and Veneto Banca.[15] Consistent with our model, both moments fell right after the announcement.

A recent paper by Neuberg et al. (2018) provides additional empirical evidence for our mechanism. They exploit a change in the contract terms of credit default swaps to measure the market-perceived credibility of the recent financial reforms. In 2014 the terms of CDS contracts were changed to allow for a government intervention event in which bondholders would be compensated if there was a bail-in. Contracts traded prior to 2014 had no such provision and thus bondholders were subject to losses in case of such events. Since contracts under the old terms continued to trade after 2014, the authors use the difference in CDS spreads of the two types of contracts to measure the cost of protecting bondholders against bail-in events. They find that market implied likelihood of a government intervention decreased a little after the announcement of the SRM but increased after it seemed likely that the Italian banks would be bailed out, consistent with our the-

[15] The two Veneto banks were bailed out on 25 June, 2017.

Figure 6: Spreads over 10-year German bonds



Source: FRED

ory.

One can also wonder why the Italian government chose to bail out these two mid-sized banks, thereby damaging the credibility of the SRM guidelines. Our theory suggests that this bailout might have been driven by the fear of contagion to other banks in bad condition, such as Monte Paschi and Carige.

European Debt Crisis After joining the euro area, southern European governments were able to borrow at interest rates similar to Germany, despite differences in fundamentals, until the beginning of the European debt crisis (see Figure 6). The crisis started in Greece but soon spread to other southern European countries. News about the willingness of European institutions to bail out Greece impacted the borrowing rates for other EU member countries. Ardagna and Caselli (2014) document that the spike in five-year Italian bond yields coincided with an announcement that the Greek bailout agreement required them to seek a haircut on outstanding debt to private creditors. They argue that this contagion effect might have worsened prospects for peripheral countries. Figure 7 plots the expected recovery rates for Greek bonds on the left axis and the CDS spreads on 10-year Italian bonds on the right axis. The recovery rate is a measure of how much private lenders expect to recover in the event of default. As the figure shows, the sharp increase in Italian CDS spreads coincides with the drop in expected recovery rates for Greek bonds. As the crisis progressed and fundamentals arguably worsened, the interest

Figure 7: Recovery rates and CDS spreads

Source: Markit

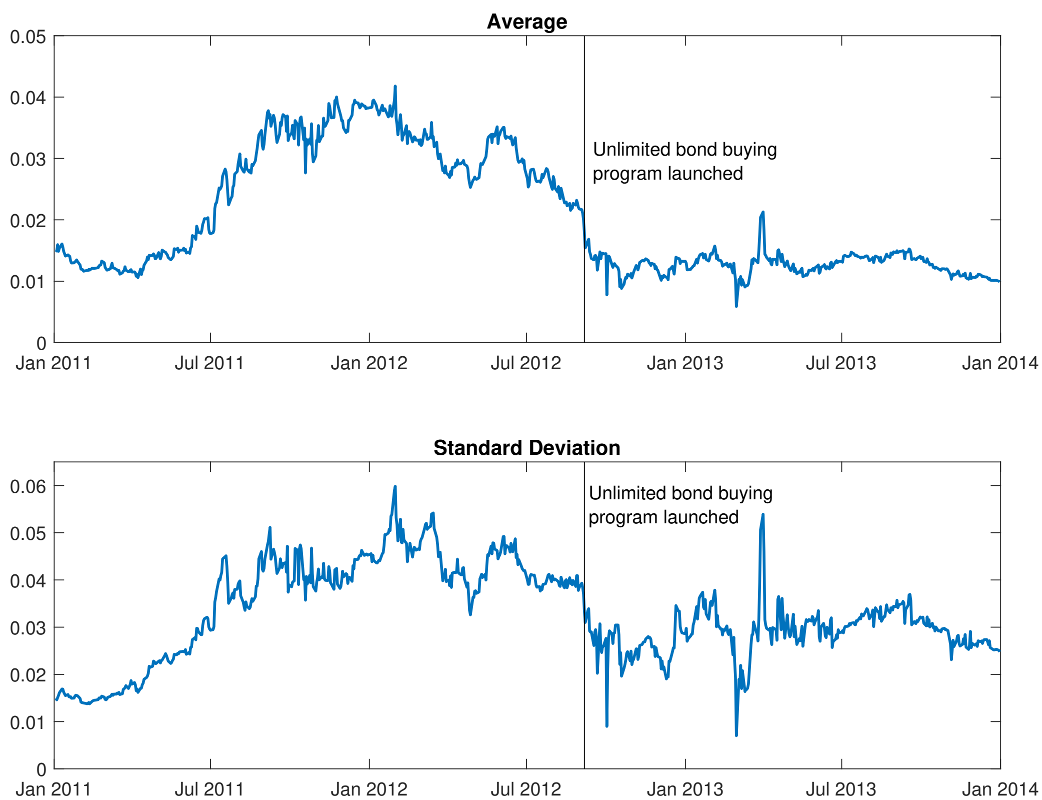
rates at which southern European countries continued to rise until the speech by Mario Draghi about the willingness of the ECB to do "whatever it takes," after which spreads for these countries fell sharply. Figure 8 plots the mean and standard deviation for CDS spreads of EU countries (not including Greece) for the period around the announcement of the unlimited bond-buying program by the ECB. Consistent with our model, both the mean and standard deviation fell after the announcement.

A common alternative explanation for the behavior of spreads in the European debt crisis is that high spreads were driven by a self-fulfilling switch to a bad equilibrium – as in Cole and Kehoe (2000) or Calvo (1988) – and the ECB intervention steered the equilibrium back to one with low interest rates. While this is possible in theory, our narrative suggests that lenders would need to coordinate on news about the generosity of (future) bailouts.

7 Conclusion

In this paper we study a model in which the expectation of future bailouts is an important determinant of interest rate spreads. We jointly characterize these spreads and the optimal bailout decisions of a government that lacks commitment but has incentives to build

Figure 8: CDS spreads of EU countries



Source: Markit. The countries in the sample are Germany, France, Belgium, the Netherlands, Spain, Austria, Cyprus, Estonia, Finland, Ireland, Italy, Latvia, Lithuania, Malta, Portugal, and Slovenia.

reputation. This model can help explain both the behavior of spreads around crises and the delay in intervention we often observe from governments once the crisis has started. Moreover, it can account for the observation that the correlation of spreads for certain borrowers (such as financial firms or governments in the European Monetary Union) is higher than the correlation of their fundamentals because they are linked by a common bailout probability.

There are many interesting extensions worth studying that are outside the scope of this paper. In concurrent work, we study the optimal policy in an environment without commitment but with incentives for building reputation. One application of this would be to characterize the optimal bailout policy in this model.

A second extension would be to allow debt holdings to be a state variable across periods. As discussed earlier, higher reputation has two opposing effects on interest rates. First, since the probability of future private default is larger, interest rates need to rise to allow lenders to break even. Second, a higher cost of borrowing discourages debt accumulation, which pushes down the interest rate. Allowing debt accumulation to be a dynamic choice would likely lead to the first force dominating the other. This is because consumption-smoothing motives would make large changes in debt holds very costly. As a result, this extension is likely to strengthen our result. Finally, a third interesting extension would be to see how much of the movement in spreads can be accounted for by a combination of fundamentals and reputation.

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Appendix

A Generalizing the baseline environment

We now show that the previous results hold in more general environments. We allow both s, θ to be drawn from continuous distributions with densities $P(s)$ and $H(\theta \mid s)$ respectively. Finally, we generalize the social cost function to allow for any increasing function $C(\cdot)$. In this environment, in the event of default, we assume that lenders and borrowers can renegotiate the contract so that borrowers make a partial repayment to the lenders and avoid the default cost. The static value of bailing out is

$$\omega^{\text{bailout}}(B, K, s) = (1 - \Delta(B, K, s))\, B + \tilde{\Delta}(B, s)$$

where

$$\Delta(B, K, s) = \int \mathbb{I}_{\{B > \theta K^{\alpha}\}} dH(\theta|s),$$

$$\tilde{\Delta}(B, K, s) = \int \theta K^{\alpha} \mathbb{I}_{\{B > \theta K^{\alpha}\}} dH(\theta|s)$$

and as before we normalize $C(0) = 0$. Here, $\tilde{\Delta}(B, s)$ denotes the maximal transfer that can be extracted from the borrower such that it is indifferent between defaulting and not. The value of not bailing out (and allowing default) is

$$\omega^{\text{no-bailout}}(B, K, s) = (1 - \Delta(B, K, s))\, B + \tilde{\Delta}(B, K, s) - C(\Delta(B, K, s)\, B)$$

Note that even absent a bailout, since private agents can re-negotiate contracts, lenders will extract $\tilde{\Delta}(B, K, s)$ from borrowers who default. Given this the pricing schedule for debt is

$$\begin{aligned} Q(\pi, B, K)(b) = q &\left\{ \int (1 - \Delta(B, K, s))\, dP(s) + \int \tilde{\Delta}(B, K, s)\, dP(s) \right\} \\ &+ q \left\{ (1 - \pi) \int \sigma(\pi, B, K, s) \left[\Delta(B, K, s)\, B - \tilde{\Delta}(B, K, s) \right] dP(s) \right\} \end{aligned} \tag{29}$$

Given that private contracts can be renegotiated, lenders receive at least $\tilde{\Delta}(B, K, s)$ in the event of default. The expression on the second line denotes the additional transfer received in the event of a bailout. From Lemma 14, the problem for the borrower in period 1 is

$$\max_{b,k} \int \int \max\{\theta k^{\alpha} - b, 0\}\, dH(\theta|s)\, dP(s) \tag{30}$$

subject to

$$K \leqslant Q(B, K, \pi)(b, k)\, b$$

Define $\Theta_+^s(B, K) \equiv \{\theta : \theta K^\alpha - B \geqslant 0\}$. The following Lemma characterizes the private outcome in the stage game given the bailout policy σ if the distribution for θ is continuous:

Lemma 4. *Given* π *and a bailout policy* σ, (B, K, Q) *is a symmetric equilibrium outcome if*

$$K = (QB)^\alpha$$

$$\int_s \int_{\theta \in \Theta_+^s(B,K)} \alpha\theta\,(QB)^{\alpha-1}\left(Q + Q'B\right) dH(\theta \mid s)\, dP(s) - 1 = 0$$

and $Q = Q(B, K, \pi)(B, K)$ *where*

$$Q' = \left.\frac{dQ(B, K, \pi)(b, k)}{dB}\right|_{(b,k)=(B,K)}$$

We show that both the existence and characterization results hold in this environment if the private equilibrium of the stage game satisfies the following condition:

Assumption 3. *For any bailout policy* $\sigma(\pi, B, K, s)$ *which decreasing in* π *for all* (B, K, s)*, the private equilibrium outcome is such* $B(\pi)$ *is a decreasing function.*

The assumption requires the debt issued to be decreasing in π which implies that the equilibrium default probabilities in each state s, to be decreasing in π,. In general, as π decreases there are two effects on the equilibrium price of debt Q. First, since the probability of a bail out is higher, Q increases. However, the resulting increase in borrowing increases the probability of default which might lower Q. The assumption requires the first force to dominate so that in equilibrium the price of issuing debt decreases and thus the debt issued increases. It is easy to see that the example described in the previous section satisfies this assumption.

B Omitted Proofs

B.1 Proof of Proposition 1

Given the setup described above, the general version of Proposition 1 is

Proposition 1 (Generalized). *Under Assumption 3, if* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B(\pi) : [0, 1] \to \mathbb{R}$ *is decreasing,* $K(\pi) : [0, 1] \to \mathbb{R}_+$ *is decreasing,* $\sigma(\pi, s) : [0, 1] \times S \to [0, 1]$ *is decreasing,* $W(\pi) : [0, 1] \to \mathbb{R}$ *is increasing.*

Proof of Proposition 1

Define the following operator: $\mathbb{T} : \Sigma \to \Sigma$ where

$$\Sigma = \{\sigma : [0,1] \times S \to [0,1] : \forall s \in S, \sigma(\cdot, s) \text{ is decreasing, continuous}\}$$

and the operator is defined as follows:

1. Given $\sigma_0 \in \Sigma$, compute $b(\cdot|\sigma_0)$ that solves the borrower's problem in (30) with $Q(\pi, B, K|\sigma_0)(b, k)$ from (29) imposing the representativeness condition so $B(\pi|\sigma_0) = b(\pi|\sigma_0)$.

2. Given σ_0 and $B(\cdot|\sigma_0)$ compute $W(\cdot|\sigma_0)$ as the solution to the following fixed point problem

$$\begin{aligned} \mathbb{T}^W W(\pi) &= w_i(\pi|\sigma_0) \\ &+ \beta \int [1 - \sigma_0(\pi, s)] W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_0(\pi, s))}(p_c - p_{nc})\right) dP(s) \\ &+ \beta \int \sigma_0(\pi, s) W(p_{nc}) dP(s) \end{aligned} \tag{31}$$

where

$$\begin{aligned} w_i(\pi \mid \sigma_0) = [e - Q(\pi|\sigma_0) B(\pi|\sigma_0)] \\ + q \int [1 - \sigma(\pi, s_2)] \omega(\pi, s_2 \mid \sigma_0) dP(s_2|s_1) \\ + q \int \sigma(\pi, s_2) \omega^*(\pi, s_2 \mid \sigma_0) dP(s_2|s_1) \end{aligned}$$

and

$$\omega(\pi, s \mid \sigma_0) = (1 - \Delta(\pi, s)) B(\pi) + \tilde{\Delta}(\pi, s) - C(\Delta(\pi, s) B(\pi))$$

where

$$\tilde{\Delta}(\pi, s) = \int (\theta K(\pi)^\alpha) \mathbb{I}_{\{B(\pi) \geqslant \theta K(\pi)^\alpha\}} dH(\theta|s)$$

and

$$\begin{aligned} \omega^*(\pi, s \mid \sigma_0) &= \int \min\{B(\pi), \theta K(\pi)^\alpha\} dH(\theta|s) \\ &= (1 - \Delta(\pi, s)) B(\pi) + \tilde{\Delta}(\pi, s) \end{aligned}$$

Note that here $\Delta(\pi, s) = \Delta(B(\pi), K(\pi), s)$. The operator $\mathbb{T}^W$ maps the space of continuous and bounded functions into itself. Further, it is easy to see that the mapping is a contraction and thus by the Contraction mapping theorem, there exists a unique fixed point.

3. Compute $\sigma_1 = \mathbb{T}\sigma_0$ as

$$\sigma_1(\pi, s_2) = \begin{cases} 0 & \text{if } \beta\left[W_1\left(p_{nc} + \pi\Delta p \mid \sigma_0\right) - W_1\left(p_{nc} \mid \sigma_0\right)\right] \geqslant \Delta\omega\left(\pi, s \mid \sigma_0\right) \\ \tilde{\sigma} & : \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma})}\Delta p \mid \sigma_0\right) - W_1\left(p_{nc} \mid \sigma_0\right)\right] = \Delta\omega\left(\pi, s \mid \sigma_0\right) \\ 1 & \text{if } \beta\left[W_1\left(p_c \mid \sigma_0\right) - W_1\left(p_{nc} \mid \sigma_0\right)\right] < \Delta\omega\left(\pi, s \mid \sigma_0\right) \end{cases} \tag{32}$$

with $\Delta p = p_c - p_{nc}$ and

$$\Delta\omega\left(\pi, s \mid \sigma_0\right) = \omega\left(\pi, s \mid \sigma_0\right) - \omega^*\left(\pi, s \mid \sigma_0\right)$$

To prove the proposition, we will establish that $\mathbb{T}$ has a fixed point in $\Sigma^{|S_2|}$ using Tarski's theorem.

Step 1: For any $f, g \in \Sigma$ we define a binary relation $\succeq$ where $f \succeq g$ iff $\forall s_2 \in S_2$, $f(\pi, s_2) \geqslant g(\pi, s_2)$ for all π . We want to argue that $(\Sigma, \succeq)$ is a complete lattice. That is, for any arbitrary subset $\tilde{\Sigma}$ of Σ: 1) there exists $\underline{\sigma} \in \tilde{\Sigma}$ such that i) for all $\sigma \in \tilde{\Sigma}$, $\sigma \succeq \underline{\sigma}$ and ii) for all $\sigma' \in \Sigma$ such that $\sigma \succeq \sigma'$ for all $\sigma \in \tilde{\Sigma} \Rightarrow \underline{\sigma} \succeq \sigma'$ ($\underline{\sigma}$ is the greatest lower bound); 2) there exists $\bar{\sigma} \in \tilde{\Sigma}$ such that i) for all $\sigma \in \tilde{\Sigma}$, $\bar{\sigma} \succeq \sigma$ and ii) for all $\sigma' \in \Sigma$ such that $\sigma' \succeq \sigma$ $\Rightarrow \sigma' \succeq \bar{\sigma}$ ($\bar{\sigma}$ is the least upper bound). Clearly for any subset $\Sigma' \subset \Sigma$, for each s_2 these correspond to the lower and upper envelope of functions in the set. In particular, for each s_2 and each π define

$$\bar{\sigma}(\pi, s_2) = \max_{\sigma(\pi, s_2) \in \Sigma'} \sigma(\pi, s_2)$$

and

$$\underline{\sigma}(\pi, s_2) = \min_{\sigma(\pi, s_2) \in \Sigma'} \sigma(\pi, s_2)$$

Notice that both $\bar{\sigma}(\pi, s_2)$ and $\underline{\sigma}(\pi, s_2)$ are continuous, increasing, and satisfy $\bar{\sigma}(0, s_2) = \underline{\sigma}(0, s_2) = 1$. Therefore, $\bar{\sigma}$ and $\underline{\sigma}$ belong to Σ.

Step 2: The value function $W(\pi|\sigma_0)$ that solves (31) is strictly increasing in π. To prove this we will use the corollary to the Contraction Mapping Theorem. Given a weakly increasing, bounded and continuous function $W_0(\pi)$ let $W_1 = \mathbb{T}^W W_0$. We want to show that W_1 is strictly increasing. Consider $\pi_L < \pi_H$ and associated $\sigma_0(\pi_L, s) \leqslant \sigma_0(\pi_H, s)$. Define $\mathcal{S}_1 = \{s : \sigma_0(\pi_L, s) = \sigma_0(\pi_H, s) = 0\}$, $\mathcal{S}_2 = \{s : \sigma_0(\pi_L, s) > \sigma_0(\pi_H, s) = 0\}$, and $\mathcal{S}_3 = \{s : \sigma_0(\pi_L, s) \geqslant \sigma_0(\pi_H, s) > 0\}$ so that in $\mathcal{S}_1$ there are no bailouts under both π_L and π_H, in $\mathcal{S}_2$ there is a positive probability of bailouts under π_L but not under π_H, finally in $\mathcal{S}_3$

bailouts happen with positive probability under both π_L and π_H. Then

$$\begin{aligned} W_1(\pi_L) = {} & [e - Q(\pi_L) B(\pi_L)] \\ & + \int_{\mathcal{S}_1} [q\omega(\pi_L, s) + \beta W_0(p_{nc} + \pi_L \Delta p)] \, dP(s) \\ & + \int_{\mathcal{S}_2} [q\omega^*(\pi_L, s) + \beta W_0(p_{nc})] \, dP(s) \\ & + \int_{\mathcal{S}_3} [q\omega^*(\pi_L, s) + \beta W_0(p_{nc})] \, dP(s) \end{aligned}$$

and

$$\begin{aligned} W_1(\pi_H) = {} & [e - Q(\pi_H) B(\pi_H)] \\ & + \int_{\mathcal{S}_1} [q\omega(\pi_H, s) + \beta W_0(p_{nc} + \pi_H \Delta p)] \, dP(s) \\ & + \int_{\mathcal{S}_1} [q\omega(\pi_H, s) + \beta W_0(p_{nc} + \pi \Delta p)] \, dP(s) \\ & + \int_{\mathcal{S}_3} [q\omega^*(\pi_H, s) + \beta W_0(p_{nc})] \, dP(s) \end{aligned}$$

Then we have,

$$\begin{aligned} & W_1(\pi_L) - W_1(\pi_H) \\ \leqslant & Q(\pi_H) B(\pi_H) - Q(\pi_L) B(\pi_L) \\ & + q \int_{\mathcal{S}_1} [\omega(\pi_L, s) - \omega(\pi_H, s)] \, dP(s) \\ & + q \int_{\mathcal{S}_2} [\omega^*(\pi_L, s) - \omega^*(\pi_H, s)] \, dP(s) \\ & + q \int_{\mathcal{S}_3} [\omega^*(\pi_L, s) - \omega^*(\pi_H, s)] \, dP(s) \\ = & \left\{ -Q(\pi_L) B(\pi_L) + q \int (1 - \Delta(\pi_L, s)) B(\pi_L) \, dP(s) + q \int \tilde{\Delta}(\pi_L, s) \, dP(s) - q \int_{\mathcal{S}_1} C(\Delta(\pi_L, s) B(\pi_L)) \, dP(s) \right\} \\ & - \left\{ -Q(\pi_H) B(\pi_H) + q \int (1 - \Delta(\pi_H, s)) B(\pi_H) \, dP(s) + q \int \tilde{\Delta}(\pi_H, s) \, dP(s) - q \int_{\mathcal{S}_1} C(\Delta(\pi_H, s) B(\pi_H)) \, dP(s) \right\} \\ = & \left\{ -(1 - \pi_L) \int \sigma(\pi_L, s) \mathbb{I}_{B(\pi_L) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi_L) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi_L, s) \, dP(s) \right\} \\ & - \left\{ -(1 - \pi_H) \int \sigma(\pi_H, s) \mathbb{I}_{B(\pi_H) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi_H) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi_H, s) \, dP(s) \right\} \\ < & 0 \end{aligned}$$

where the first inequality follows from

$$W_0(p_{nc} + \pi_L \Delta p) \leqslant W_0(p_{nc} + \pi_H \Delta p)$$

and

$$[\omega(\pi_H, s) + \beta W_0(p_{nc} + \pi_H \Delta p)] \geqslant [\omega^*(\pi_H, s) + \beta W_0(p_{nc})]$$

for $s \in \mathcal{S}_2$, the second equality is algebra, the third equality uses the definition of $Q(\pi)$, finally the last inequality follows since $\sigma(\pi_L, s) \geqslant \sigma(\pi_H, s)$, Assumption 1, and C is an increasing function.

Step 3: Showing that $\mathbb{T}$ maps Σ into Σ. Now we argue that $\sigma_1 = \mathbb{T}\sigma_0$ is decreasing by showing that $\mathbb{T}\sigma_0(\pi, s)$ is decreasing in π for all s. Suppose by way of contradiction there exists $\pi_L < \pi_H$ and $\sigma_1(\pi_L, s) < \sigma_1(\pi_H, s)$ for some s so that the bailout probability is higher if we start from a higher prior. First, note that if $\sigma_1(\pi_L, s) = 0$ then

$$\Delta\omega(\pi_L, s) \leqslant \beta[W_1(p_{nc} + \pi_L \Delta p) - W_1(p_{nc})]$$

but

$$\begin{aligned} \Delta\omega(\pi_H, s) &\leqslant \Delta\omega(\pi_L, s) \\ \beta[W_1(p_{nc} + \pi_L \Delta p) - W_1(p_{nc})] &\leqslant \beta[W_1(p_{nc} + \pi_H \Delta p) - W_1(p_{nc})] \end{aligned}$$

where the first inequality follows from the fact that $B(\pi|\sigma_0)$ is decreasing and the second from W_1 being an increasing function. Therefore

$$\Delta\omega(\pi_H, s) \leqslant \beta[W_1(p_{nc} + \pi_H \Delta p) - W_1(p_{nc})]$$

and $\sigma(\pi_H, s) = 0$, yielding a contradiction. Second, if $0 < \sigma_1(\pi_L, s) < \sigma_1(\pi_H, s) < 1$ then

$$\begin{aligned} \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi_H, s))}\Delta p\right) - W_1(p_{nc})\right] &= \Delta\omega(\pi_H, s) \\ \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi_L, s))}\Delta p\right) - W_1(p_{nc})\right] &= \Delta\omega(\pi_L, s) \end{aligned}$$

so since $\Delta\omega(\pi_H, s) \leqslant \Delta\omega(\pi_L, s)$ and W_1 is increasing it must be that

$$\begin{aligned} p_{nc} + \frac{\pi_L}{\pi_L + (1-\pi_L)(1-\sigma_1(\pi_L, s))}\Delta p &> p_{nc} + \frac{\pi_H}{\pi_H + (1-\pi_H)(1-\sigma_1(\pi_H, s))}\Delta p \\ \iff \frac{(1-\pi_H)}{\pi_H}(1-\sigma_1(\pi_H, s)) &> \frac{(1-\pi_L)}{\pi_L}(1-\sigma_1(\pi_L, s)) \\ \iff 1-\sigma_1(\pi_H, s) &> \frac{(1-\pi_L)}{\pi_L} \Big/ \frac{(1-\pi_H)}{\pi_H}(1-\sigma_1(\pi_L, s)) > 1-\sigma_1(\pi_L, s) \\ &\iff \sigma_1(\pi_L, s) > \sigma_1(\pi_H, s) \end{aligned}$$

obtaining a contradiction. Finally, if $0 < \sigma_1(\pi_L, s) < \sigma_1(\pi_H, s) = 1$ then

$$\beta\left[W_1(p_c) - W_1(p_{nc})\right] < \Delta\omega(\pi_H, s) < \Delta\omega(\pi_L, s)$$

implying $\sigma_1(\pi_L, s) = 1$, again a contradiction. Hence $\mathbb{T} : \Sigma \to \Sigma$ as desired.

Step 4: As an intermediate step we show that if $\sigma_H \succeq \sigma_L$ then $W_1(\pi \mid \sigma_H) \leqslant W_1(\pi \mid \sigma_L)$ for all π. This is because the operator $\mathbb{T}^W$ is decreasing in σ. That is, if $W_{Ln}(\cdot) \geqslant W_{Ln}(\cdot)$ then $W_{Ln+1} = \mathbb{T}_L^W W_{Ln} \geqslant \mathbb{T}_H^W W_{Hn} = W_{Hn+1}$ where $\mathbb{T}_i^W$ is the operator $\mathbb{T}^W$ with $\sigma_0 = \sigma_i$. The proof for this claim is similar to the argument in step 2. Fix π and define $\mathcal{S}_1 = \{s : \sigma_H(\pi, s) = \sigma_L(\pi, s) = 0\}$, $\mathcal{S}_2 = \{s : \sigma_H(\pi, s) > \sigma_L(\pi, s) = 0\}$, and $\mathcal{S}_3 = \{s : \sigma_H(\pi, s) \geqslant \sigma_L(\pi, s) > 0\}$. Then

$$\begin{aligned}
& W_{Ln+1}(\pi) - W_{Hn+1}(\pi) \\
=& Q(\pi|\sigma_H) B(\pi|\sigma_H) - Q(\pi|\sigma_L) B(\pi|\sigma_L) \\
&+ \delta \int_{\mathcal{S}_1} [\omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc} + \pi_L \Delta p)]\, dP(s) - \delta \int_{\mathcal{S}_1} [\omega(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc} + \pi_H \Delta p)]\, dP(s) \\
&+ \delta \int_{\mathcal{S}_2} [\omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})]\, dP(s) - \delta \int_{\mathcal{S}_2} [\omega^*(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc} + \pi_H \Delta p)]\, dP(s) \\
&+ \delta \int_{\mathcal{S}_3} [\omega^*(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})]\, dP(s) - \delta \int_{\mathcal{S}_3} [\omega^*(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc})]\, dP(s) \\
\geqslant& Q(\pi|\sigma_H) B(\pi|\sigma_H) - Q(\pi|\sigma_L) B(\pi|\sigma_L) \\
&+ \delta \int_{\mathcal{S}_1} [\omega(\pi, s|\sigma_L) - \omega(\pi_H, s|\sigma_H)]\, dP(s) \\
&+ \delta \int_{\mathcal{S}_2} [\omega^*(\pi, s|\sigma_L) - \omega^*(\pi, s|\sigma_H)]\, dP(s) \\
&+ \delta \int_{\mathcal{S}_3} [\omega^*(\pi, s|\sigma_L) - \omega^*(\pi, s|\sigma_H)]\, dP(s) \\
=& \left\{ -Q(\pi \mid \sigma_H) B(\pi \mid \sigma_H) + q \int (1 - \Delta(\pi, s) \mid \sigma_H) B(\pi \mid \sigma_H)\, dP(s) + q \int \tilde{\Delta}(\pi, s \mid \sigma_H)\, dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_H)\, dP(s) \right\} \\
&- \left\{ -Q(\pi \mid \sigma_L) B(\pi \mid \sigma_L) + q \int (1 - \Delta(\pi, s) \mid \sigma_L) B(\pi \mid \sigma_L)\, dP(s) + q \int \tilde{\Delta}(\pi, s \mid \sigma_L)\, dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_L)\, dP(s) \right\} \\
=& \left\{ -(1-\pi) \int \sigma_H(\pi, s)\, \mathbb{I}_{B(\pi|\sigma_H) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi \mid \sigma_H) - \left[\theta - u^{-1}(\underline{u}(s, \theta))\right]\right] dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_H)\, dP(s) \right\} \\
&- \left\{ -(1-\pi) \int \sigma_L(\pi, s)\, \mathbb{I}_{B(\pi|\sigma_L) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi \mid \sigma_L) - \left[\theta - u^{-1}(\underline{u}(s, \theta))\right]\right] dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_L)\, dP(s) \right\} \\
\geqslant& 0
\end{aligned}$$

where the first equality simply follows from a definition of $W_{Ln+1}(\pi)$ and $W_{Hn+1}(\pi)$, the second inequality follows from $W_{Hn}(\cdot) \leqslant W_{Ln}(\cdot)$ and $[\omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc} + \pi \Delta p)] \geqslant [\omega^*(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})]$ for $s \in \mathcal{S}_2$, the third equality uses the definition of ω^*, and finally the last inequality follows from Assumption 1. The above argument implies that starting from the same guess we have that

$$W(\pi|\sigma_L) = \lim_{n\to\infty} \left(\mathbb{T}_L^W W_0\right)^n(\pi) \geqslant \lim_{n\to\infty} \left(\mathbb{T}_H^W W_0\right)^n(\pi) \geqslant W(\pi|\sigma_H)$$

as wanted.

Step 5: Showing that $\mathbb{T}$ is monotone increasing. To apply the Tarski theorem we need to show that $\mathbb{T}$ is increasing in that if $\sigma_H \succeq \sigma_L$ then $(\mathbb{T}\sigma_H) \succeq (\mathbb{T}\sigma_L)$. Suppose not. Suppose first that for some s and π we have that $0 < \tilde{\sigma}_H = \mathbb{T}\sigma(\pi, s) < \mathbb{T}\sigma(\pi, s) = \tilde{\sigma}_L < 1$. First, note that $\Delta\omega(\pi, s \mid \sigma_H) \geqslant \omega(\pi, s \mid \sigma_L)$. Then from the definition of $\mathbb{T}$ we have that

$$\begin{aligned} 0 =& \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] - \Delta\omega(\pi, s \mid \sigma_H) \\ =& \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] - \Delta\omega(\pi, s \mid \sigma_L) \end{aligned}$$

This can be written as

$$\begin{aligned} & \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] \\ & - \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] \\ =& \Delta\omega(\pi, s \mid \sigma_L) - \Delta\omega(\pi, s \mid \sigma_H) < 0 \end{aligned} \tag{33}$$

where the inequality follows from the fact that $\Delta\omega(\pi, s \mid \cdot)$ is increasing in the expected probability of a bailout σ. The left side of (33) is

$$\begin{aligned} & \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] \\ & - \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] \\ >& \beta W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - \beta W_1(p_{nc} \mid \sigma_L) \\ & - \beta W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_H\right) + \beta W_1(p_{nc} \mid \sigma_H) \\ \geqslant& \beta W_1(p_{nc} \mid \sigma_H) - \beta W_1(p_{nc} \mid \sigma_L) \end{aligned}$$

where the first inequality follows from the contradiction hypothesis, $\tilde{\sigma}_L > \tilde{\sigma}_H$, and $W_1(\cdot|\sigma)$ being increasing, the second inequality follows from the fact that $W_1(\pi \mid \sigma_H) \leqslant W_1(\pi \mid \sigma_L)$ as shown in step 4. Finally, if we take the limit as $p_{nc} \to 0$ then, the above expression converges to zero but the right side of (33) is strictly less than zero which is a contradiction. Therefore, for p_{nc} small enough, by continuity it must be that $\tilde{\sigma}_H \geqslant \tilde{\sigma}_L$.

Next, suppose that $0 \leqslant \tilde{\sigma}_H = \mathbb{T}\sigma_H(\pi, s) < \mathbb{T}\sigma_L(\pi, s) = 1$. Then we have

$$\begin{aligned} \Delta\omega(\pi, s \mid \sigma_H) > \Delta\omega(\pi, s \mid \sigma_L) & \geqslant \beta\left[W_1(p_c \mid \sigma_L) - W_1(p_{nc} \mid \sigma_L)\right] \\ & \geqslant \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_L)\right] \end{aligned}$$

where the last inequality follows from $W_1(p_c \mid \sigma_L) \geqslant W_1(p_c \mid \sigma_H)$. Therefore, for p_{nc} close enough to zero we have that

$$\Delta\omega(\pi, s \mid \sigma_H) > \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_L)\right] = \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right]$$

which implies that $\mathbb{T}\sigma_H(\pi, s) = 1$, a contradiction.

Finally, suppose that $0 = \mathbb{T}\sigma_H(\pi, s) < \mathbb{T}\sigma_L(\pi, s) < 1$. Then,

$$\beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right] > \Delta\omega(\pi, s \mid \sigma_H) \geqslant \Delta\omega(\pi, s \mid \sigma_L)$$

As in the previous case, we know that $W_1(p_c \mid \sigma_H) > W_1(p_c \mid \sigma_L)$. Therefore, for p_{nc} small enough

$$\beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_L) - W_1(p_{nc} \mid \sigma_L)\right] \geqslant \beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right] > \Delta\omega(\pi, s \mid \sigma_L)$$

which implies that $\mathbb{T}\sigma_L(\pi, s) = 0$, a contradiction.

We then verified all the conditions needed to apply the Tarski's fixed point theorem to establish that set of fixed points of $\mathbb{T}$ is in Σ and is non-empty. Q.E.D.

B.2 Proof of Proposition 2

We first show that under condition (17) in Assumption 1 we have $\sigma(\pi, s_L) = 1$ for all π. To this end, note that any equilibrium $B(\pi) = \mathbb{B}(\bar{\gamma}(\pi)) \geqslant \mathbb{B}(0)$. Moreover, note that the dynamic gains from bailing out, $W(p_c) - W(p_{nc})$, is bounded by $W^R(0) - W^R(1)$ in that

$$W(p_c) - W(p_{nc}) \leqslant W^R(0) - W^R(1)$$

because $W^R(0) = W^R \geqslant W(p_c)$ since the value of the Markov equilibrium is lower than the value of the Ramsey plan, and $W(p_{nc}) \geqslant W^R(1)$ because along the equilibrium path private agents believe that with some probability they are facing the commitment type. Hence we have that

$$\psi B(\pi) \geqslant \psi\mathbb{B}(0) > \beta\left[W^R(0) - W^R(1)\right] \geqslant \beta\left[W(p_c) - W(p_{nc})\right]$$

and so it is optimal to bailout with probability one if $s = s_L$.

Next, we show that it is optimal to mix in a mild recession under assumption (18). Suppose by way of contradiction that $\sigma(\pi, s_M) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the no-commitment type for $\pi = 1$ is

$$W(1) = e + qp_H\left[0 + \beta W(1)\right] + qp_M\left[0 + \beta W(0)\right] + qp_L\left[0 + \beta W(0)\right]$$

and for $\pi = 0$, since $\bar{\gamma}(0) = 1$ we have

$$W(0) = e - \mathbb{Q}(1)\mathbb{B}(1) + q\int \Delta(0,s)\mathbb{B}(1)\,dP(s) + qp_H[0 + \beta W(0)] + qp_M[0 + \beta W(0)] + qp_L[0 + \beta W(0)]$$

Moreover,

$$\mathbb{Q}(1)\mathbb{B}(1) - q\int \Delta(0,s)\mathbb{B}(1)\,dP(s) = q\mathbb{B}(1) - q[p_H + p_M(1-\mu)]\mathbb{B}(1) = q\mathbb{B}(1)[1 - [p_H + p_M(1-\mu)]] = q\mathbb{B}(1)P_L$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(1) - W(0) = q\mathbb{B}(1)P_L + q\beta p_H[W(1) - W(0)]$$

or

$$W(1) - W(0) = \frac{q\mathbb{B}(1)P_L}{1 - q\beta p_H}$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the government prefers not to incur the default costs

$$\psi\mu B(\pi = 1) = \psi\mu\mathbb{B}(0)$$

than to obtain the dynamic gains $\beta[W(1) - W(0)]$. Note that we use that inherited debt at $\pi = 1$ under the contradiction hypothesis because private agents expect a bailout with probability zero, $\bar{\gamma} = (1-\pi) = 0$. Thus it must be that

$$\frac{\beta q\mathbb{B}(1)P_L}{1 - q\beta p_H} < \psi\mu\mathbb{B}(0)$$

But this contradicts condition (18) in Assumption 1. Hence it must be that $\sigma(\pi, s_M) < 1$ for some π. In particular, $\sigma(1, s_M) < 1$ because of monotonicity of σ.

We are now left to show that we cannot have that $\sigma(\pi, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_M) = 0$. Hence it must be that

$$\bar{\gamma} = \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L} = \frac{p_L(1-\pi)}{p_L + p_M\mu}$$

and the posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi (p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and finally

$$\mu B (\bar{\gamma}) \leqslant \beta [W (p_{nc}) - W (p_{nc})]$$

but this is a contradiction since

$$0 < \mu \mathbb{B} (\bar{\gamma}) \leqslant \beta [W (p_{nc}) - W (p_{nc})] = 0$$

Hence we cannot have that $\sigma (\pi, s_M) = 0$ for all π. Therefore there is mixing for some interval. Q.E.D.

B.3 Proof of Proposition 4

Evaluating (21) and (16) at $\pi = p_c = 1$, (22) can be written as

$$(q (P_H))^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} (\alpha)^{\frac{1}{1-\alpha}} \left[\beta P_H \frac{[1-\alpha]}{\alpha} - 1\right] < 0$$

where the inequality follows from (25). By continuity, this inequality continues to hold for a neighborhood around p_c. Similarly, evaluating (21), at p_{nc} and (16) at p_c, inequality (23) follows directly from (26). Finally using (16) and the expression for $W_2 (\pi)$ evaluated at p_c and (27) it is straightforward to see that the inequality (24) holds. Q.E.D.

B.4 Proof of Proposition 5

We now want to show that

$$-b_1 + \beta U_2 (p_{nc}) \geqslant 0$$

Note that

$$b_1 \leqslant b_2 (0) = (\alpha \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)}$$

Thus it is sufficient to show that

$$-(\alpha \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\right] \geqslant 0$$

or

$$\theta_H^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} \left\{\beta P_H \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\right] - \alpha^{1/(1-\alpha)}\right\} \geqslant 0$$

or

$$\beta P_H - (1 + \beta P_H)\,\alpha \geqslant 0$$

But this follows directly from (28). Q.E.D.

C Persistent shocks: Contagion and Shock Sensitivity

In the model with iid shocks, there is no heterogeneity among borrowers at the beginning of any period. As a result the model cannot generate the contagion effects described in the introduction. By the contagion effect, we mean the increase in the price of debt for a country not directly affected by an adverse fundamental shock. Moreover, since the price of debt depends only on π and not the state, it is not possible to generate the differential effect of reputation on the sensitivity of prices to fundamentals unless we introduce multiple types of borrowers. To show that our framework can generate such features we extend the baseline model to allow for persistent of aggregate and idiosyncratic states. As a result, the distribution functions of idiosyncratic and aggregate shocks are now $h(\theta'|s', s, \theta)$ and $P(s'|s)$.

Let's consider our simple example. The aggregate state s follows a Markov chain

$$P(s'|s) = \begin{bmatrix} p_{HH} & p_{HM} & p_{HL} \\ p_{MH} & p_{MM} & p_{ML} \\ p_{LH} & p_{LM} & p_{LL} \end{bmatrix}$$

As before, in state s_H, all borrowers draw θ_H and in state s_L all borrowers draw θ_L, i.e. $h(\theta_H|s_H, \theta) = 1$ and $h(\theta_L|s_L, \theta) = 1$ for all θ. We assume that in the medium state, a fraction μ of borrowers have the low output θ_L and

$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_L) = \rho_L$$
$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_H) = \rho_H$$

with $\rho_L\mu + \rho_H(1-\mu) = \mu$. Thus the productivity shock is persistent in the medium state.

Let $\mathbf{z}_- = (s_-, \theta_-)$, $\mathbf{z} = (s, \theta)$ and $\nu(\mathbf{z}_-)$ denote the fraction of type $\mathbf{z}_-$. Next, let $P_H(\mathbf{z}_-)$ and $P_L(\mathbf{z}_-)$ be probabilities of a high and low idiosyncratic endowment respectively, conditional on history $\mathbf{z}_-$. Therefore,

$$P_H(\mathbf{z}_-) = p_{s_-H} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\left(1-\rho_{\theta_-}\right) + \left(1-\mathbb{I}_{s_-=s_M}\right)(1-\mu)\right]$$

$$P_L(\mathbf{z}_-) = p_{s_-L} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right]$$

Next, define

$$\bar{\gamma}(\mathbf{z}_-) \equiv \frac{p_{s_-L}(1-\pi)\sigma(\pi, s_-, s_L) + p_{s_-M}p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right](1-\pi)\sigma(\pi, s_-, s_M)}{P_L(\mathbf{z}_-)}$$

to be the probability that an individual borrower with history $\mathbf{z}_-$ will be bailed out conditional on drawing θ_L. As in the i.i.d case this serves as a useful sufficient statistic to characterize private decisions. The price of debt in this environment is

$$\mathbb{Q}(\mathbf{z}_-, \bar{\gamma}) = qP_H(\mathbf{z}_-) + qP_L(\mathbf{z}_-)\bar{\gamma}(\mathbf{z}_-)$$

and the optimal debt level $\mathbb{B}(\mathbf{z}_-, \bar{\gamma})$ is given by

$$\mathbb{B}(\mathbf{z}_-, \bar{\gamma}) = (\alpha\theta_H)^{\frac{1}{1-\alpha}}\mathbb{Q}(\mathbf{z}_-, \bar{\gamma})^{\frac{\alpha}{1-\alpha}} \tag{34}$$

Define the $\bar{\mathbb{B}}(s_-, \bar{\gamma})$ to be aggregate level of debt where

$$\begin{aligned}
\bar{\mathbb{B}}(s_L, \bar{\gamma}) &\equiv \mathbb{B}((s_L, \theta_H), \bar{\gamma}) = \mathbb{B}((s_L, \theta_L), \bar{\gamma}) \\
\bar{\mathbb{B}}(s_M, \bar{\gamma}) &\equiv \mu\mathbb{B}((s_M, \theta_L), \bar{\gamma}) + (1-\mu)\mathbb{B}((s_M, \theta_H), \bar{\gamma}) \\
\bar{\mathbb{B}}(s_H, \bar{\gamma}) &\equiv \mathbb{B}((s_H, \theta_H), \bar{\gamma}) = \mathbb{B}((s_H, \theta_L), \bar{\gamma})
\end{aligned}$$

Next, we characterize a set of continuous monotone equilibria for the economy for an arbitrary transition matrix P and provide sufficient conditions so that the characterization results for the iid case extend to this more general environment. Assumption 4 is the analog for Assumption 1 for the case with persistent endowments.

Assumption 4. *Let* $C(x) = \psi x$. *Define* $W^R(s, \bar{\gamma})$ as be the solution to

$$\begin{aligned}
W^R(s_-, \bar{\gamma}) = e - \sum_{\theta} \nu(s_-, \theta)\left[\bar{\gamma}(s_-, \theta) + \psi(1-\bar{\gamma}(s_-, \theta))\right] qP_L(s_-, \theta)\mathbb{B}((s_-, \theta), \bar{\gamma}(s_-, \theta)) \\
+ \beta\sum_s p_{s_-s}W^R(s, \bar{\gamma})
\end{aligned}$$

Assume that

$$\psi\bar{\mathbb{B}}(s_-, 0) > \beta\left[W^R(s, 0) - W^R(s, 1)\right] \text{ for all } s \tag{35}$$

and

$$\mathbf{A}^{-1} \cdot \mathbf{x} > \mathbf{G} \tag{36}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1-p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1-p_{HH} \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} q\left(p_{LM}\mu + p_{LL}\right)\bar{\mathbb{B}}\left(s_L, 1\right) \\ q\mu\left(p_{MM}\rho_L + p_{ML}\right)\mathbb{B}\left(\left(s_M, \theta_L\right), 1\right) + q\mu\left(p_{MM}\rho_H + p_{ML}\right)\mathbb{B}\left(\left(s_M, \theta_H\right), 1\right) \\ q\left(p_{HM}\mu + p_{HL}\right)\bar{\mathbb{B}}\left(s_H, 1\right) \end{bmatrix}$$

and

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}\left(s_L, 0\right) \\ \psi\left[\mu\rho_L\mathbb{B}\left(\left(s_M, \theta_L\right), 1\right) + \left(1-\mu\right)\rho_H\mathbb{B}\left(\left(s_M, \theta_H\right), 1\right)\right] \\ \psi\mu\bar{\mathbb{B}}\left(s_H, 0\right) \end{bmatrix}$$

Proposition 6. *For an arbitrary transition matrix* P*, if* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π*,* $\sigma\left(s_-, \pi, s\right) : S \times [0,1] \times S \to [0,1]$ *is decreasing in* π*,* $W\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is increasing in* π*, debt price* $Q\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π *for all* s_-*, and*

$$W\left(s_L, \pi\right) < W\left(s_M, \pi\right) < W\left(s_H, \pi\right)$$

Furthermore, under Assumption 4, if $p_c \to 1$ *and* $p_{nc} \to 0$ *then it must be that:*

- *It is optimal to bailout with probability one in a severe recession,* $\sigma\left(s_-, \pi, s_L\right) = 1$ *for all* π *and* s_-
- *It is optimal to mix in a mild recession for some values of* π *for all* s_-

Proof. The proof of the first part is identical to the i.i.d case. To see the second, we first show that under condition (35) in Assumption 4 we have $\sigma\left(\pi, s_-, s_L\right) = 1$ for all $\left(\pi, s_-\right)$. To this end, note that any equilibrium $\mathbb{B}\left(\mathbf{z}_-, \bar{\gamma}\right) \geqslant \mathbb{B}\left(\mathbf{z}_-, 0\right)$. Moreover, note that that the dynamic gains from bailing out, $W\left(s, p_c\right) - W\left(s, p_{nc}\right)$, are bounded by $W^R\left(s, 0\right) - W^R\left(s, 1\right)$ in that

$$W\left(s, p_c\right) - W\left(s, p_{nc}\right) \leqslant W^R\left(s, 0\right) - W^R\left(s, 1\right)$$

because $W^R\left(s, 0\right) \geqslant W\left(s, p_c\right)$, and $W\left(s, p_{nc}\right) \geqslant W^R\left(s, 1\right)$. Hence we have that

$$\psi B\left(s_-, \pi\right) \geqslant \psi\bar{\mathbb{B}}\left(s_-, 0\right) > \beta\left[W^R\left(s, 0\right) - W^R\left(s, 1\right)\right] \geqslant \beta\left[W\left(s, p_c\right) - W\left(s, p_{nc}\right)\right]$$

and so it is optimal to bailout with probability one if $s = s_L$.

Next we show that it is optimal to mix in a mild recession under assumption (36). Suppose by way of contradiction that $\sigma\left(\pi, s_-, s_M\right) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the no-commitment type in state s for $\pi = 1$ is

$$W\left(s, 1\right) = qp_{sH}\left[0 + \beta W\left(s_H, 1\right)\right] + qp_{sM}\left[0 + \beta W\left(s_M, 0\right)\right] + qp_L\left[0 + \beta W\left(s_L, 0\right)\right]$$

and for $\pi = 0$, since $\bar{\gamma}(0) = 1$ we have for $s = \{s_H, s_L\}$

$$W(s,0) = -q\left(p_{sM}\mu + p_{sL}\right)\bar{\mathbb{B}}(s,1) + qp_{sH}\beta W(s_H,0) \\ + qp_{sM}\beta W(s_M,0) + qp_{sL}\beta W(s_L,0)$$

and for $s = s_M$

$$W(s_M,0) = -q\mu\left(p_{MM}\rho_L + p_{ML}\right)\mathbb{B}((s_M,\theta_L),1) - q(1-\mu)\left(p_{MM}\rho_H + p_{ML}\right)\mathbb{B}((s_M,\theta_H),1) \\ + qp_{MH}\beta W(s_H,0) + qp_{MM}\beta W(s_M,0) + qp_{ML}\beta W(s_L,0)$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(s,1) - W(s,0) = x_s + q\beta \sum_{s'} p_{ss'}\left[W(s',1) - W(s',0)\right]$$

for some constant x_s. Hence we can write

$$\mathbf{A} \cdot \mathbf{W} = \mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1-p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1-p_{HH} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W(s_L,1) - W(s_L,0) \\ W(s_M,1) - W(s_M,0) \\ W(s_H,1) - W(s_H,0) \end{bmatrix}$$

and

$$\mathbf{x} = \begin{bmatrix} q\left(p_{LM}\mu + p_{LL}\right)\bar{\mathbb{B}}(s_L,1) \\ q\mu\left(p_{MM}\rho_L + p_{ML}\right)\mathbb{B}((s_M,\theta_L),1) + q\mu\left(p_{MM}\rho_H + p_{ML}\right)\mathbb{B}((s_M,\theta_H),1) \\ q\left(p_{HM}\mu + p_{HL}\right)\bar{\mathbb{B}}(s_H,1) \end{bmatrix}$$

and so

$$\mathbf{W} = \mathbf{A}^{-1} \cdot \mathbf{x}$$

The static gains of bailing out in the medium state if $\pi = 1$ is given by

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}(s_L,0) \\ \psi\left[\mu\rho_L\mathbb{B}((s_M,\theta_L),1) + (1-\mu)\rho_H\mathbb{B}((s_M,\theta_H),1)\right] \\ \psi\mu\bar{\mathbb{B}}(s_H,0) \end{bmatrix}$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the government prefers not to incur the default costs, or

$$\mathbf{G} \geqslant \mathbf{A}^{-1} \cdot \mathbf{x}$$

which contradicts in Assumption 1. Hence it must be that $\sigma(\pi, s_-, s_M) = 1 < 1$ for some π.

We are now left to show that we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_-, s_M) = 0$. Hence it must be that

$$\bar{\gamma}(\mathbf{z}_-) = \frac{p_L(1-\pi)\,\sigma(\pi, s_L) + p_M \mu (1-\pi)\,\sigma(\pi, s_M)}{P_L(\mathbf{z}_-)} = \frac{p_{s_- L}(1-\pi)}{P_L(\mathbf{z}_-)}$$

and the posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi(p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and finally for $s_- \in \{s_H, s_L\}$

$$\mu \bar{\mathbb{B}}(s_-, \bar{\gamma}) \leqslant \beta\,[W(s, p_{nc}) - W(s, p_{nc})]$$

but this is a contradiction since

$$0 < \mu \bar{\mathbb{B}}(s_-, \bar{\gamma}) \leqslant \beta\,[W(s, p_{nc}) - W(s, p_{nc})] = 0$$

Hence, we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Therefore there is mixing for some interval of π. A similar argument holds for $s_- = s_M$. Q.E.D.

Thus a continuous monotone equilibrium exists when shocks are persistent and the economy displays similar dynamics to the i.i.d case. We now show that the introduction of persistence can generate the contagion effects described previously. In the i.i.d case, there is only a single type of borrower in each period. However, with persistent shocks, if $s_- = s_M$, then in the following period there are two types of borrowers: (s_M, θ_L) and (s_M, θ_H). If there is no bail out and a subsequent rise in reputation, the interest rates faced by both types rise due to the presence of a common government. This provides an explanation as to why the CDS spreads for Italy rose after the the perceived recovery rates for Greek bonds declined. The announcement that private creditors were expected to receive haircuts on Greek bonds signaled that EU countries were less likely to receive the benefit of a full bail out in case of default in the future. As a result, the cost of borrowing for other countries that might have been considered at risk of default rose as well.

Proposition 7. *(Contagion) If the reputation of the government increases after observing no bail out in state s_M, then the price of debt for types (s_M, θ_H) decreases.*

The proofs follows from the observation that the pricing function Q depends positively on π.
We next show that this model is capable of generating higher sensitivity to fundamentals when reputation is high.

Proposition 8. *(Sensitivity) For any s_-, the difference in the price of debt for a $\theta_- = \theta_H$ borrower and a $\theta_- = \theta_L$ borrower is increasing in the reputation of the government. That is, $Q((s_-, \theta_{H-}), \pi) - Q((s_-, \theta_{L-}), \pi)$ is increasing in π. Similarly, for any θ_-, the differences $Q((s_H, \theta_-), \pi) - Q((s_M, \theta_-), \pi)$ and $Q((s_M, \theta_-), \pi) - Q((s_L, \theta_-), \pi)$ are increasing in π for π large enough.*

Debt prices (and debt issuances) are less responsive to the state s_- when the prior is low. That is, if the probability of facing the no-commitment type is low then lenders are less worried about the state of the world since they expect to get bailed out with high probability and therefore, debt prices are not sensitive to the state. These effects are illustrated in Figure 9. As the fourth plot illustrates, the difference between the price of debt across the different states is increasing in π. At $\pi = 0$, the prices are identical and equal to the risk-free rate since lenders expect to be bailed out with probability one. At $\pi = 1$, prices are driven exclusively by the probability of default and since the states are persistent, the difference in prices is large.

D Model with N borrowers

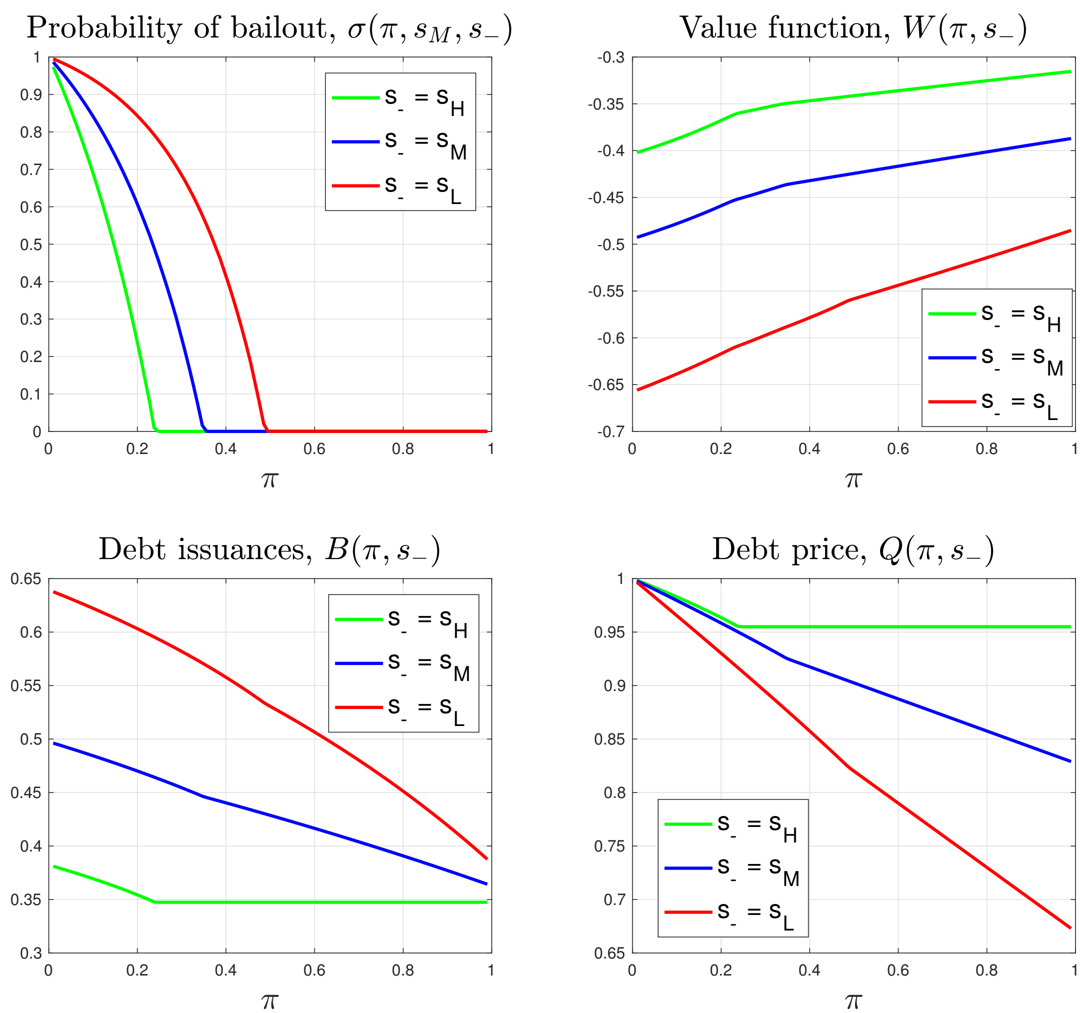
Consider an environment with N borrowers. The problem for each borrower is identical to main text, except that now the internalize the effect of their choices on the policies of the government. The first order condition for the borrower is

$$\int_s \int_{\theta \in \Theta_+^s(B,K)} \alpha\theta (Qb_i)^{\alpha-1} \left(Q + Q_{b_i} b_i\right) dH(\theta \mid s)\, dP(s) - 1 = 0 \tag{37}$$

and $Q = Q(\mathbf{B}, \mathbf{K}, \pi, \sigma)(b_i, k_i)$ where $\mathbf{B} = (b_1, ..., b_N)$, $\mathbf{K} = (k_1, ..., k_N)$ and

$$Q(\mathbf{B}, \mathbf{K}, \pi, \sigma)(b, k) = q \left[\int (1 - \Delta(b, k, s))\, dP(s) + \int \tilde{\Delta}(b, k, s)\, dP(s) \right] + q(1-\pi) \int \sigma(\pi, \mathbf{B}, \mathbf{K}, s) \int \left[\Delta(b, k, s)\, b - \tilde{\Delta}(b, k, s)\right] dP(s)$$

Figure 9: Equilibrium objects for computed discrete example with persistent shocks



$$Q_b = q\frac{\partial \int (1-\Delta(b,k,s))\,dP(s) + \int \tilde{\Delta}(b,k,s)\,dP(s)}{\partial b}$$
$$+ q\frac{\partial (1-\pi)\int \sigma(\pi,\mathbf{B},\mathbf{K},s)\int \left[\Delta(b,k,s)\,b - \tilde{\Delta}(b,k,s)\right] dP(s)}{\partial b}$$

where each borrower internalizes the effect of b on $\sigma(\pi,\mathbf{B},\mathbf{K},s)$.

Let's consider now the incentives for the government in sub-period 2. Notice that since N is finite, the optimal transfers will satisfy $T \in \{\mathbf{0}, T^*\}$ since choosing any other level only imposes costs on the government. We will normalize $C(0) = 0$ so if the government bails out, its static value is

$$\omega^*(\mathbf{B},\mathbf{K},s) = \sum_i \frac{1}{N}\left[(1-\Delta(b_i,k_i,s))\,b_i + \tilde{\Delta}(b_i,k_i,s)\right]$$

since transfers are a wash. The value if it allows default is

$$\omega^*(\mathbf{B},\mathbf{K},s) = \sum_i \frac{1}{N}\left[(1-\Delta(b_i,k_i,s))\,b_i + \tilde{\Delta}(b_i,k_i,s)\right] - C\left(\frac{\sum_i \Delta(b_i,k_i,s)\,b_i}{N}\right)$$

Consider the derivative

$$\begin{aligned}\frac{\partial}{\partial b}\Delta\omega(\mathbf{B},\mathbf{K},s) &= \frac{\partial}{\partial b_i}\frac{1}{N}C\left(\frac{\sum_i \Delta(b_i,k_i,s)\,b_i}{N}\right)\\ &= \frac{1}{N^2}C'\left(\frac{\sum_i \Delta(b_i,k_i,s)\,b_i}{N}\right)\frac{\partial \Delta(b_i,k_i,s)\,b_i}{\partial b_i}\end{aligned}$$

which converges to 0 as $N \to \infty$. Note the dynamic benefits are independent of b_i. Therefore,

$$\lim_{N\to\infty}(1-\pi)\int\left(\frac{\partial}{\partial b_i}\sigma(\pi,\mathbf{B},\mathbf{K},s)\right)\int\left[\Delta(b_i,k_i,s)\,b_i - \tilde{\Delta}(b_i,k_i,s)\right] dP(s) = 0$$

and so in the limit the first order condition for the borrower is

$$\int_s \int_{\theta\in\Theta_+^s(B,K)} \alpha\theta\,(Qb)^{\alpha-1}\,(Q + Q_b b)\,dH(\theta \mid s)\,dP(s) - 1 = 0$$

where

$$Q_b = q\frac{\partial \int (1-\Delta(b,k,s))\,dP(s) + \int \tilde{\Delta}(b,k,s)\,dP(s)}{\partial b}$$
$$+ q(1-\pi)\int \sigma(\pi,\mathbf{B},\mathbf{K},s)\int \frac{\partial\left[\Delta(b,k,s)\,b - \tilde{\Delta}(b,k,s)\right]}{\partial b}dP(s)$$

E Learning Model

We can simplify the analysis by noting that since $\sigma_2 = 1$, the price in the secondary market simplifies to

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\sigma_1(\pi, B, q_2) + \varepsilon & s = s_H \\ (1-\pi)\left[(1-\mu) + \mu\sigma_1(\pi, B, q_2)\right] + \varepsilon & s = s_L \end{cases}$$

It follows that if $Q_2(\pi, s_H, \varepsilon_H) = Q_2(\pi, s_L, \varepsilon_L)$ then

$$\varepsilon_L = \varepsilon_H + (1-\mu)\pi$$

If we assume that $\text{supp}(g) = (-\infty, +\infty)$ we can then make a change of variable and express all the equilibrium objects as a function of the realization of ε in state s_H. Define

$$F(\pi) \equiv \int \sigma_1(\pi, \varepsilon)\left[p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon + (1-\mu)\pi)\right] d\varepsilon \tag{38}$$

to be ex-ante probability of a bailout in the first stage of the sub-period two given prior π. Then, the price of issuing debt in the first sub-period is

$$Q(\pi) = q\left[p(s_H)(1-\mu) + p(s_L)(1-\mu)(1-\pi) + \mu(1-\pi)F(\pi)\right] \tag{39}$$

and so the optimal choice of debt satisfies

$$B(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q(\pi)^{\frac{\alpha}{1-\alpha}}. \tag{40}$$

The value for the government is given by

$$\begin{aligned} W(\pi) = {} & -Q(\pi)B(\pi) + \\ & + qp(s_H)\Big\{(1-\mu)B(\pi) \\ & + \int \sigma_1(\pi, \varepsilon)\beta W(p_{nc}) g(\varepsilon) d\varepsilon \\ & + \int \left[1-\sigma_1(\pi, \varepsilon)\right]\left[-c\mu B(\pi) + \beta W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi, \varepsilon))}\Delta p\right)\right] g(\varepsilon) d\varepsilon\Big\} \\ & + qp(s_L)\left\{\int \left[1-\sigma_1(\pi, \varepsilon + (1-\mu)\pi)\right]\left[-c\mu B(\pi)\right] g(\varepsilon + (1-\mu)\pi) d\varepsilon + \beta W(p_{nc})\right\} \end{aligned} \tag{41}$$

Finally, the probability of a bailout in the first stage $\sigma_1(\pi, \varepsilon)$ is given by

$$\sigma_1(\pi,\varepsilon) = \begin{cases} 0, & \text{if } c\mu B(\pi) \leqslant \beta\hat{p}_H(\pi,\varepsilon)\left[W(p_{nc}+\pi\Delta p) - W(p_{nc})\right] \\ \tilde{\sigma}, & \text{if } c\mu B(\pi) = \beta\hat{p}_H(\pi,\varepsilon)\left[W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc})\right] \\ 1, & \text{if } c\mu B(\pi) \geqslant \beta\hat{p}_H(\pi,\varepsilon)\left[W(p_{nc}+\Delta p) - W(p_{nc})\right] \end{cases} \tag{42}$$

where

$$\hat{p}_H(\pi,\varepsilon) = \frac{p(s_H)}{p(s_H)\,g(\varepsilon) + p(s_L)\,g(\varepsilon + (1-\mu)\pi)}$$

Thus, (38)–(42) define a set of functional equations that can be solved for the equilibrium objects $F(\pi)$, $Q(\pi)$, $B(\pi)$, $W(\pi)$, and $\sigma_1(\pi, \varepsilon)$.

F A Consumption Smoothing Model

In this section we show how all our results go through in a model in which the desire to borrow arises from consumption smoothing motives. The rest of the model is unchanged.

All borrowers are one-period lived and symmetric ex-ante. In sub-period 1, borrower i has income $Y_{i1} = Y_1$ and can borrow b_i from risk neutral lenders to finance consumption c_{i1}. In sub-period 2, the aggregate state s is realized according to a distribution P. As in our baseline model, assume that the state can take three values: $s \in \{s_L, s_M, s_H\}$ with probabilities p_L, p_M, and p_H respectively. Each borrower receives stochastic income θ drawn from a distribution $H(\cdot|s)$. We assume that θ can take on two values: θ_H and θ_L. In state s_H, all the borrowers receive high endowment so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M instead, $h(\theta_H|s_M) = 1-\mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L all borrowers receive the low endowment, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$. After the realization of θ, each borrower can default on its debt.

Let $c_{i2}(s,\theta)$ denote the consumption of borrower i in sub-period 2 given (s,θ). The preferences of borrower i are given by

$$u(c_{i1}) + \delta \sum_s p_s \sum_\theta h(\theta \mid s)\, u(c_{i2}(s,\theta)) \tag{43}$$

where $u(\cdot)$ is increasing, concave, and differentiable, and δ is the borrower's discount factor across sub-periods. The budget constraint of the borrower in sub-period 1 is

$$c_{i1} = Y_1 + Qb_i$$

where b_i is the debt issued by the borrower and Q is the price of the debt. In sub-period

2, if the borrower does not default, its budget constraint is

$$c_{i2}(s,\theta) = \theta - b_i + T_i$$

where T_i are transfers from the government. We assume that the private cost of default to the borrower defaults, $\underline{u}(s,\theta)$ is given by

$$\underline{u}(s,\theta) = \begin{cases} u(0) & \text{if } \theta = \theta_H \\ u(\theta_L) & \text{if } \theta = \theta_L \end{cases}$$

which implies that these costs exhibit a high degree of convexity. Thus, it is always optimal for borrowers to repay debt if $\theta = \theta_H$ while if $\theta = \theta_L$ there is repayment only if debt issued is zero or there is a transfer equal to at least b. Therefore, the fraction of borrowers defaulting is given by $\Delta = \sum_\theta h(\theta \mid s)\, \mathbb{I}_{\{u(\theta - b + T(B,s)(b,\theta)) < \underline{u}(s,\theta)\}}$ and the optimal transfer is $T \in \{\mathbf{0}, T^*\}$ where

$$T^*(B,s)(b,\theta_L) = b$$

Given this setup, it is easy to see that all the results in the baseline model apply here as well.