Credit Market Frictions and Trade Liberalizations*

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Abstract

Are credit frictions a barrier to gains from trade liberalization? We find that the answer to this depends on whether or not the debt limits are endogenous and respond to profit opportunities. If so, exporters expand and non-exporters shrink efficiently allowing for the same percentage gains from reform as with perfect credit markets. If debt limits do not respond, reallocation is reduced and gains are lower. We then use data from a trade liberalization to distinguish between the two models. We find that firm-level changes in export behavior, the growth of new exporters, and the capital distortions of firms that eventually exports are all consistent with a model of endogenous debt limits.

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1 Introduction

Recent work has studied the role of credit constraints in economies undergoing reforms, and has concluded that financial market imperfections limit the gains from undergoing reform.[1] In this paper, we demonstrate that the way that credit constraints are modeled crucially determines their role in reform.[2] In particular, we contrast two commonly used types of debt limits: what we refer to as *forward-looking* debt limits, following Albuquerque and Hopenhayn (2004), and *collateral constraints* or *backward-looking* debt limits. The forward-looking constraint arises endogenously and may respond when non-financial reforms occur in the economy. The backward-looking constraint is an exogenous leverage ratio, modeled as a fixed parameter. Under the forward-looking specification, the debt limits respond to profit opportunities. Thus, after a trade liberalization, exporters expand and non-exporters shrink efficiently allowing for the same percentage gains from reform as with perfect credit markets. In the backward-looking specification instead, debt limits do not respond, reallocation is reduced and gains are lower. We then use a trade liberalization in Colombia to distinguish between these two specifications and find evidence in favor of the forward-looking version.

We extend a dynamic Melitz (2003) trade model to include credit market frictions in the form of debt limits. Our formulation takes both the forward-looking and backward-looking versions as special cases. With forward-looking debt limits, the amount of debt that firms can sustain is limited by the value of continuing to operate the firm (that is, the discounted stream of future income to the firm). With backward-looking debt limits (or collateral constraints), the amount that firms can borrow is at most an exogenous proportion of their assets. The key difference between these specifications is how credit limits are affected by the firm's future profitability. With forward-looking constraints, higher future profits allow firms to sustain more debt. With collateral constraints, future

[1] See, for example, Buera and Shin (2011 and 2013) and Song, Storesletten and Zilibotti (2011).

[2] This is a different question than how much credit market frictions matter for aggregate productivity in steady state, as studied in Midrigan and Xu (2013).

profits do not affect debt limits.

We demonstrate that both specifications of credit frictions are consistent with the empirical relationship between credit and export decisions at the firm level analyzed in a recent literature surveyed in Manova (2010). In particular, both specifications can account for the fact that access to credit affects both export participation and the amount that firms export. In both models, young firms are small and grow over time until they reach their optimal scale. In each, firms generally do not find it optimal to enter export markets when their capital stocks are small.

The main contribution of this paper is to show that these models have different implications for gains from trade reform both at the aggregate and at the firm level. We show that the percentage increase in steady state consumption from a trade reform in the forward-looking specification is *the same* as in a corresponding model with perfect credit markets. The gains are analytically the same in a special case with no endogenous selection into exporting, and are very close in magnitude in more general, calibrated examples. Also the transitional dynamics is similar in both models. However, with collateral constraints, the percentage change in consumption and output are *lower* than with perfect credit markets. Thus the welfare gains from a trade liberalization are lower.

The important difference between the two models of credit constraint is how future profitability affects firms' ability to borrow. In the model with forward-looking debt limits, future exporters are able to sustain higher debt after the trade liberalization than before, even in periods *before* they enter the export market. This allows young, productive firms to start to export earlier. With collateral constraints, entering the export market requires asset accumulation. Non-exporters are less profitable after trade reform (due to increased wages) so they accumulate assets more slowly. Therefore, with collateral constraints productive, young (low net worth) firms are unable to enter export markets, while less productive, old (high net worth) firms are able to enter. This creates perverse selection into the export market that lowers the gains from trade reform. This demonstrates that taking into account the endogenous response of credit markets to reform is important when evaluating the potential gains from policy changes in countries with low quality credit markets.

We use data on Colombian firms from 1981-1991 to test the implications of the two models of financial frictions. Colombia undertook a series of reforms in the mid-1980s that increased the value of exporting relative to domestic production and exhibited a corresponding increase in export activity. We consider three differences in implications between the forward-looking and backward-looking models, and show that the data is consistent with the forward-looking model in all three cases.

First, we show that the increase in export activity is concentrated among young firms in the data, as in the forward-looking model. In the backward-looking model, this is not true. As a second test, we consider the dynamics of firms just after they enter the export market. We find that the growth of capital stock for new exporters in the periods after entering the export market is consistent with the limited enforcement model. Finally, we use a differences-in-differences regression specification and measure how constrained firms are by their marginal product of capital. We find that marginal product of capital falls in both the data and the limited enforcement model, while it rises in the collateral constraint model for future exporters.[3]

Related Literature This paper is related to several strands of literature in international trade and macroeconomics. We build on the seminal contribution of Melitz (2003) and subsequent work, such as Alessandria and Choi (2014), who analyze the gains from trade in a model with heterogeneous firms, and emphasize the role of reallocation and selection into the export market as a driver for the gains from trade. Chaney (2005) and Manova (2008, 2013) introduce credit market frictions into a Melitz (2003) framework. Both papers consider a static environment, and do not address how credit frictions affect the gains

[3]This evidence is consistent with recent work by Li (2015) who considers whether or not firms' one-year-ahead profits affect the levels of firm borrowing using data from Japanese firms. She finds that this has an important level difference in the aggregate losses due to financial market frictions.

from trade, which is the central theme of our paper. Recent papers by Kohn et al (2015) and Gross and Verani (2013) study dynamic trade models with trade frictions but focus on firm-level dynamics and not the effects of trade reform. Caggese and Cunat (2013) study the gains from trade reform with collateral constraints and show that gains are limited due to the extensive margin. We confirm their findings and contrast them with the forward-looking case. In addition we model capital accumulation and how credit frictions affect the response of investment following a trade reform. See Alessandria et al. (2018) for the study of dynamic analysis of trade reform with capital accumulation.

The model presented here is consistent with the growing empirical literature on the relationship between firm-level export behavior and access to credit (see Manova (2010) for a survey). This literature finds that access to credit is an important determinant of export participation (the extensive margin) and the scale of exports (the intensive margin). See Berman and Hericourt (2010), Minetti and Zhu (2011) and Gorodnichenko and Schnitzer (2013). This literature uses measures such as survey responses[4] and leverage ratios to proxy for access to credit. The models of trade and credit frictions developed in the next sections are consistent with both findings from this literature. Amiti and Weinstein (2011) show that shocks to banks impact the export behavior of borrowers.

This paper is also related to the literature that studies how aggregate gains from a trade liberalization are affected by including institutional and technological details in trade models. Arkolakis, Costinot and Rodriguez-Clare (2012) show that a large class of trade models have the same implications for welfare gains from trade given *ex post* realizations of changes in trade flows. Our model does not fall into that class, since our model has capital and firms differ in their markups. The main difference is that we consider an economy with misallocation in inputs and inefficient delays in entry in the export market. A trade liberalization interacts with these factors. Moreover, we are interested in evaluating *ex ante* how a given reduction in tariffs affects welfare with and without credit

[4] For instance, in Minetti and Zhu (2011) they use a firm-level Italian data set that includes answers to the question, "In 2000, would the firm have liked to obtain more credit at the market interest rate?"

market frictions. This is similar in spirit to Atkeson and Burstein (2010), who show that modeling innovation decisions has no effect on aggregate gains from trade.

We model credit market frictions following two specifications widely used in the macroeconomics literature. First, our forward-looking specification extends Albuquerque and Hopenhayn (2004) to a general equilibrium trade model with a discrete choice to export. See Cooley, Marimon and Quadrini (2004) for an application in a closed economy context. Second, we analyze collateral constraints following Evans and Jovanovic (1989), which has been used in many papers, such as Midrigan and Xu (2013). A similar constraint is used in Buera, Kaboski and Shin (2011).

Finally and most importantly, our paper contributes to the literature that analyzes how credit market frictions affect reallocation in economies undergoing reform. Buera and Shin (2013) show that collateral constraints slow down the reallocation process following a reform, because it takes time for productive but low net-worth firm to accumulate sufficient assets to start a business and operate at full scale[5]. Likewise, Song, Storesletten and Zilibotti (2011) consider a similar mechanism for the case of technological growth in China, showing that collateral constraints generate misallocation between constrained, productive private firms and unconstrained, less productive state-owned firms. These results all depend on the backward-looking nature of the financial constraints. If the debt limits have a forward-looking componentthen our results extend to these environments and productive firms can start a business and operate at a larger scale sooner after the reform or technological improvement, and they do not have to accumulate a large stock of assets to do so. Jermann and Quadrini (2007) consider a similar mechanism in the context of news shocks where they show that a signal of future productivity immediately relaxes the firms' enforcement constraints. The second contribution of our paper is to suggest which micro-level evidence can help in telling these two formulations of credit market

[5]Buera and Shin (2011) obtain similar results in an open economy environment (no intratemporal trade) considering debt limits that depend not only on the installed capital stock (collateral constraints) but also on period profits.

frictions apart.

Moreover, it is important to stress that in our model the difference in gains from trade is not transitory but permanent. This is because of the overlapping generations structure of the firm sector. This contrasts with much of the existing literature that considers infinitely-lived firms and financial frictions mainly slow down the transition between stationary equilibria.

2 Model

Time is discrete, denoted by $t = 0, 1, ...$ and there is no aggregate uncertainty. There are two asymmetric countries, home and foreign. Variables for the foreign country are denoted with a superscript f. The home country is populated by a measure μ of identical households. The foreign country is populated by a measure of $1 - \mu$ identical households. In each country there are competitive final good producers and monopolistic competitive firms each producing an intermediate differentiated product. There are no international financial markets and international trade is balanced in every period.

2.1 Household Problem

The stand-in household in each country inelastically supplies 1 unit of labor each period. He chooses final good consumption c_t and bond holdings b_{t+1} to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor and u is increasing, differentiable and concave, subject to the sequence of budget constraints

$$c_t + b_{t+1} \leqslant w_t + R_t b_t + \Pi_t + T_t \quad \forall t \geqslant 0 \tag{2}$$

expressed in terms of the final good in each country. Here w_t is the wage, R_t is the gross interest rate, Π_t is the sum of profits from the operation of firms and T_t are lump-sum transfers from the government (revenue from tariffs). The problem for the stand-in household in the foreign country is similar.

2.2 Final Goods Producers

The final good in the home country is produced using the following CES aggregator:

$$y_t = \left[\omega \int_{I_t} y_{dt}(i)^{\frac{\sigma-1}{\sigma}} di + (1-\omega) \int_{I^f_{xt}} y^f_{xt}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where I_t is the set of active domestic firms at time t, I^f_{xt} is the set of foreign firms that export at t, $y_{dt}(i)$ is the output of firm i in I_t, $y^f_{xt}(i)$ is the output of firm i in I^f_{xt}. The final good in the foreign country is produced analogously. The parameter ω indexes home bias in the production of the final good. The elasticity of substitution among goods is $\sigma > 1$.

Final goods producers are competitive. A representative firm solves

$$\max_{y_t, y_{dt}, y_{xt}} P_t y_t - \int_{I_t} p(i) y_{dt}(i) di - \int_{I^f_{xt}} (1+\tau_t) p(i) y^f_{xt}(i) di \tag{4}$$

subject to (3). One can then derive the inverse demand functions faced by domestic and foreign intermediate good producers:

$$p_{dt}(y_d(i)) = \omega y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}} P_t, \quad p^f_{xt}(y^f_x(i)) = \frac{1-\omega}{1+\tau_t} y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}} P_t \tag{5}$$

Moreover, the inverse demand function faced by domestic exporters is

$$p_{xt}(y_x(i)) = \frac{1-\omega}{1+\tau_t} \left(y^f_t\right)^{\frac{1}{\sigma}} y^f(i)^{-\frac{1}{\sigma}} P^f_t \tag{6}$$

In what follows we are going to normalize the price of the domestic final good to one.

Hence P^f_t is the real exchange rate.

2.3 Intermediate Goods Producers

A mass of monopolistic competitive intermediate goods producers are operated by entrepreneurs in each country. In every period a mass $\delta\mu$ and $\delta(1-\mu)$ of entrepreneurs are born in the home and foreign country respectively. Each operates a firm and is endowed with a new variety of the intermediate good. At birth the entrepreneur draws a type (z, ϕ), where z is the firm's productivity and $\phi \in \{0, 1\}$ indicates if the firm has the ability to export or not[6]. If $\phi = 1$ the firm can pay a fixed cost f_x in any period to enter the export market the following period and it keep such ability by paying a per-period cost ηf_x, while if $\phi = 0$ the firm does not have that option. We can think of this as an extreme form of heterogeneity in the export fixed costs. Moreover, the firms that cannot export stands in for the nontraded sector of the economy. For simplicity, z and ϕ are independently distributed. Productivity z is drawn from a distribution Γ, and the indicator ϕ is a Bernoulli random variable with parameter ρ.[7] The type of the firm remains constant through time[8]. The firm can produce its differentiated variety using the following constant returns to scale technology:

$$y = zF(k, l) = zk^{\alpha}l^{1-\alpha}, \quad \alpha \in (0, 1) \tag{7}$$

where l and k are the labor (in effective units) and capital employed by the firm, and y is total output produced, which the firm splits between domestic and export sales. Every period the production technology owned by the firm becomes unproductive with prob-

[6]This feature of the model is useful to match the fact that there are large, productive firms that are non-exporters, and to generate reallocation after trade reform even if there are no fixed costs.

[7]Note that even if z and ϕ are not correlated, the model generates a positive correlation between productivity and export status because only most productive firms select into the export market.

[8]Our goal is to compare the forward-looking limited enforcement model with the backward-looking collateral constraints model. Adding idiosyncratic uncertainty would require us to also take a stand on the *completeness* of debt contracts.

ability δ. To be able to export, a firm of type $\phi = 1$ must pay a sunk cost f_x in period t and a per-period cost ηf_x to be able to export in all the subsequent period conditional on surviving.

The firm has to borrow to finance its operations each period and to pay the export fixed cost f_x if it is profitable to do so. Firms can save across periods in contingent securities that pay one unit of the final good next period conditional on the firm's survival. All firms start with $a_0(z)$ units of the final good, which are transferred to them by the household. Entrepreneurs are paid a dividend of d_t from the operation of the firm. We are assuming that a_0 is the maximum one-time transfer that the household can make to the firm not subject to the debt limit[9]. That is, in any period it must be that $d_t \geqslant 0$ where d_t are the dividends distributed by the firm. Firms can issue *intra-period* debt at a zero net interest rate.[10] We first present a general formulation, then consider two cases in the next section. The amount that can be borrowed depends on their assets at the beginning of the period:

$$b_t \leqslant \bar{B}^i_t(a_t; z, \phi) \tag{8}$$

We will allow for the degree of financial frictions to be heterogenous across countries.

The firm's problem can be conveniently written recursively using net assets or cash on hand, a, together with its export status and type (z, ϕ) as state variables. The problem of the firm that has already paid to enter the export market can be written as choosing dividend distribution d, new assets a' to solve:

$$V^x_t(a, z, \phi) = \max_{\{d, a'\} \geqslant 0} d + \frac{1-\delta}{R_{t+1}} V^x_{t+1}\left(a', z, \phi\right) \tag{9}$$

$$\text{subject to:} \qquad d + \frac{1-\delta}{R_{t+1}} a' \leqslant \pi^x_t(a, z)$$

[9]Clearly if there was not a bound on such transfers this channel would eliminate the credit friction.

[10]This choice is for notational convenience. The model is equivalent to one in which firms make investment decision one period in advance and borrows across periods.

where $R_{t+1}/(1-\delta)$ is the relevant interest rate for securities contingent on survival of the firm. The production plan y_d, y_x, k, l and the intra-period debt b are chosen to maximize period profits $\pi_t^x(a,z)$ cum un-depreciated capital:

$$\pi_t^x(a,z) = \max_{y_d,y_x,l,b,k} p_{dt}(y_d)y_d + p_{xt}(y_x)y_x - w_t l - b + k(1-\delta_k) - \eta f_x \tag{10}$$

subject to technological feasibility, $y_d + y_x \leqslant zF(k,l)$, the intra-period budget constraint, $(1 + r_t - \delta_k)\, k \leqslant a + b$, and the debt limit, $b' \leqslant \bar{B}_t^x(a; z, \phi)$, where r_t is the rental rate of capital. For a firm that has not yet paid the fixed cost to start exporting, denoted with the superscript nx, the recursive formulation of its problem is the same, with the addition of the discrete decision to export or not:

$$V_t^{nx}(a,z,\phi) = \max_{\{d,a'\}\geqslant 0, x\in\{0,1\}} d + \frac{1-\delta}{R_{t+1}} \left[x\phi V_{t+1}^x\left(a',z,\phi\right) + (1-x)V_{t+1}^{nx}\left(a',z,\phi\right)\right] \tag{11}$$

$$\text{subject to:} \qquad d + \frac{1-\delta}{R_{t+1}} a' + x f_x \leqslant x\pi_t^x(a - f_x, z) + (1-x)\pi_t^{nx}(a,z)$$

and $x \in \{0,1\}$ where x is an indicator variable that takes the value of 1 if the firm pays the fixed cost to export and zero otherwise. Note that a firm can start to export in the same period in which it pays the export fixed cost. The period profits, $\pi_t^{nx}(k,z)$, are given by the following static problem:

$$\pi_t^{nx}(a,z) = \max_{y_d,y_x,l,b,k} p_{dt}(y_d)y_d - w_t l + k(1-\delta_k) - \eta f_x \tag{12}$$

subject to technological feasibility, $y_d \leqslant zF(k,l)$, the intra-period budget constraint, $(1 + r_t - \delta_k)k \leqslant a + b$, and the debt limit, $b' \leqslant \bar{B}_t^{nx}(a; z, \phi)$. We will denote the policy functions of the firms associated with the above problems as $\left\{d_t^{nx}, a_t^{\prime nx}, k_t^{nx}, b_t^{nx}, y_{dt}^{nx}, y_{xt}^{nx}, l_t^{nx}, x_t\right\}_{t=0}^{\infty}$ and $\left\{d_t^x, a_t^{\prime x}, k_t^x, b_t^x, y_{dt}^x, y_{xt}^x, l_t^x\right\}_{t=0}^{\infty}$ for non-exporters and exporters respectively. The definition of competitive equilibrium in this economy is given in the online appendix.

3 Credit Market Frictions

We now turn to two cases for the borrowing constraint that are widely used in the literature. We refer to the first as the *forward-looking* specification, which follows Albuquerque and Hopenhayn (2004), and to the second as the *backward-looking* specification, following Evans and Jovanovic (1989) among others. Intermediate cases have been analyzed in Buera, Kaboski, and Shin (2011) and Li (2015). We choose to consider the two extreme to make our point in the starkest possible way.

3.1 Forward-Looking Specification

In our first specification of debt limits (8), we derive debt limits faced by the firm that arise from the inability of firms to commit to repay their debt obligations. Credit contracts are not enforceable in the sense that every period the entrepreneur can choose to default on their outstanding debt.After default, the entrepreneur can divert a proportion θ of the funds advanced for the next period's capital stock for personal benefits that are consumed immediately. Also with probability $1-\xi$, the entrepreneur loses its production technology. If the technology survives the default, the entrepreneur is able to continue to operate the firm without the assets or debt previously accumulated.[11] The corresponding debt limit $\bar{B}^i$ for $i \in \{x, nx\}$ is implicitly defined by:

$$V_t^i(a) = \theta \left[\bar{B}_t^i(a; z, \phi) + a\right] + \xi v_0(z, \phi) \tag{13}$$

where $v_0(z, \phi) = V_{t+1}^{nx}(0, z, \phi)$. This corresponds to the debt limit being "*not too tight*" in the terminology of Alvarez and Jermann (2000). The parameters θ and ξ index to the quality of financial markets.[12] If $\theta = 0$, then entrepreneurs have nothing to gain from default and credit constraints never bind. In this formulation, firms are able to borrow

[11] Within a stationary equilibrium, this is equivalent to a period of exclusion from financial markets.

[12] As in Jermann-Quadrini (2012), we do not restrict $\theta \in [0, 1]$. This can be interpreted as there being some probability that, following default, the entrepreneur cannot be punished.

even if they have zero assets. For simplicity we set $a_0(z) = 0$.

The key feature of this specification is that debt limits depend on the future profitability of the firm. That is, the higher the present value of the firm, $V^i(a)$, the more debt it can sustain.

3.2 Backward-Looking Specification

The backward-looking specification is a collateral constraint, with a debt limit (8) for $i = x, nx$ given by:[13]

$$\bar{B}_t^i(a; z, \phi) = \frac{1-\theta}{\theta} a \tag{14}$$

for some $\theta \in [0, 1]$. That is, a firm can borrow only up to a multiple $(1-\theta)/\theta$ of its assets. A common interpretation for this formulation is that entrepreneur cannot commit to repay his intra-period debt but the only punishment for doing so is the loss of a fraction $1-\theta$ of the capital stock. In particular, default does not result in the destruction of the firm's technology nor in exclusion from credit markets. In this case, new entrepreneurs must be endowed with some assets in order to begin operation, $a_0(z) > 0$. In particular, we let $a_0(z) = a_0 z^{\sigma-1}$. Again, θ parameterizes the quality of financial markets, where higher values of θ imply lower financial market quality.

The backward-looking debt limits depend only on the amount of profits that the firm has reinvested in the past, a, and not on future profitability. This aspect contrasts with the forward looking case. This difference is crucial for the two specifications to differ in their implications for the response of the economy to a trade reform.

[13] This is equivalent to requiring that $b \leqslant \theta k$.

4 Exporters Dynamics in a Stationary Equilibrium

Before analyzing the effect of a trade reform, we characterize the stationary equilibrium for the economy. We show that both specifications of credit market frictions are able to account for the relationship between export behavior and access to credit documented in the empirical literature: (i) the probability that a firm is an exporter is decreasing with measures of firm-level financial constraints, and (ii) firms' sales and exports grow over time and are decreasing in the credit constraints it faces.

In a stationary equilibrium, all prices and aggregate quantities are constant over time. Therefore, we will drop the dependence on time in this section. First we consider a relaxed problem where the borrowing constraint is dropped. The production decisions are independent of the firm's debt level and solve the following static problem:

$$\pi^*(z,\phi) = \max_{l,k,y_d,y_x,x} \omega y^{1/\sigma} y_d^{1-1/\sigma} + x\phi \frac{1-\omega}{1+\tau} y^{1/\sigma} y_x^{1-1/\sigma} - wl - rk - x\phi \tilde{f}_x \tag{15}$$

subject to $y_d + xy_x \leqslant zF(k,l)$, where $\tilde{f}_x = \left[\left(1 - \frac{1-\delta}{R}\right) + \eta\right] f_x$ is the share of the total export fixed cost paid in the period. Given prices w, q, tariff τ and aggregate final output y, denote the solutions to this problem $\{l^*(z,\phi), k^*(z,\phi), y_d^*(z,\phi), y_x^*(z,\phi), x^*(z,\phi)\}$. These would be the firms' decision rules in a standard Melitz (2003) model. We say that a firm reaches its optimal scale whenever $k = k^*(z,\phi)$.

The following proposition fully characterizes the evolution of a firm over time. The proof is relegated to the appendix.[14]

Proposition 1 *When debt limits are given by (13) or (14) then:*

(i) Firms issue no dividends until they reach their optimal scale[15]*;*

(ii) $\exists$ cut-off productivity level z_x s.t. the firm will eventually export iff $\phi = 1$ and $z \geqslant z_x$;

[14] This characterization extends Albuquerque and Hopenhayn (2004) to an environment with a discrete choice of increasing the number of markets in which the firm operates.

[15] This statement is minorly qualified. The period before the firm reachs its optimal scale it is only required to distribute a low enough level of dividends that it will still be able to operate at full scale in the next period. In that period, zero dividends is optimal, but not uniquely optimal.

(iii) $\forall z \geqslant z_x \; \exists \; \hat{a}(z, 1)$ *s.t. firms export iff* $\phi = 1$ *and* $a \geqslant \hat{a}(z, 1)$*;*

(iv) If $z' > z \geqslant z_x$ *and* $T(z)$ *is the age when a firm starts exporting, then* $T(z') \leqslant T(z)$*.*

Part (i) is consistent with the usual back-loading of incentives that commonly arises in dynamic contracting models. Part (ii) states that only more productive firms will export, as in Melitz (2003), but now with the qualification that they will *eventually* export. In fact, as part (iii) states, the firm's export status depends on both productivity and assets. For each productivity type z, there is an asset cut-off $\hat{a}(z, 1)$ such that it is profitable to start to export only if a firm has assets above that threshold. Firms with low assets are borrowing constrained and their capital stock is too low to make it profitable to pay the fixed cost to export. Finally, part (iv) states that more productive firms enter export markets younger. This is true for two reasons. First, the value of being an exporter is increasing in the productivity of the firm. The minimum amount of assets necessary to justify the fixed cost to be an exporter, $\hat{a}(z, 1)$, is decreasing in z. Second, more productive firms accumulate assets more quickly because they earn higher profits. Moreover, in the forward-looking specification (13), more productive firms are able to borrow more because the value of the firm (which appears on the left hand side of (13)) is increasing in z: For a given value of assets, default is less attractive the higher is the productivity of the firm.

The typical life-cycle path predicted by the model is as follows. After the initial productivity draw there is no uncertainty (except for exogenous exit) and firms are fully characterized by their productivity and their age. The amount of capital that a firm can sustain is initially low, then it increases over time as firms use period profits to accumulate assets (no dividend distributions). Likewise, labor usage and domestic sales (which are the static solutions to (12) above) are also initially low and grow over time with the capital stock. More productive firms eventually find it optimal to pay the fixed cost to enter the export market because they are able to sustain a larger capital stock, which increases the value of being an exporter. Then labor, domestic sales and export sales for a given capital stock are the solution to (10). Again, export sales remain at suboptimal levels as long as the firm's capital stock is constrained below its optimal scale. In finite time, the firm is able to sustain its optimal capital stock, and labor, domestic sales and export sales are constant forever after that. Thus credit market frictions in the form of debt limits (13) or (14) affect firm level export decisions along the extensive and intensive margin. This is consistent with the findings of the empirical literature on the relationship between export behavior and access to credit discussed before.[16]

5 Effects of Trade Liberalization

In this section, we evaluate if credit market imperfections reduce the gains from a bilateral tariff reduction. We show that with forward-looking debt limits the gains from trade are *not* affected by the quality of financial markets, while with backward-looking debt limits the gains from trade are lower than in an economy with perfect credit markets. The key mechanism that we will highlight throughout is how the debt limits that firms face respond to trade reform. With the backward-looking constraint, the amount firms are able to borrow depends only on their history of capital accumulation, and does not directly respond to the reform. However, with the forward-looking constraint, the fact that exporting firms are more profitable makes default less attractive and increases their debt limits.

5.1 Forward-Looking Case: Analytical Result

We first show analytically that the steady state percentage change in output, consumption, and gains from trade are the same in a model with perfect credit markets as they are in a model with forward-looking constraints in a special case with no export fixed costs, $f_x = 0$. In that case, all firms with $\phi = 1$ are exporters both before and after the

[16]Notice that firms with binding debt limits have higher leverage ratios and would identify themselves as constrained in survey responses.

trade liberalization, while all firms with $\phi = 0$ are not. Then because the set of exporting firms is not affected by trade reform, the only margin of adjustment is the intensive margin. With perfect credit markets, trade liberalization causes factors of production to be reallocated from non-exporters to exporters. In principle, financial frictions could be a barrier to that reallocation. The following proposition shows that this is not true with the forward-looking specification of borrowing constraints.

Proposition 2 *Under the forward-looking specification with* $f_x = 0$*, for any change in tariffs the steady state percentage changes in aggregate output and wages are independent of* θ *and* ξ*. Furthermore, firm-by-firm the percentage change in capital usage is independent of* θ *and* ξ*.*

A formal proof of Proposition 2 is provided in the appendix, but here we summarize the intuition. When tariffs are reduced exporting firms are more profitable, so for any debt level and capital stock, the value of not defaulting has increased. Therefore, exporting firms can sustain higher debt levels than before the liberalization allowing the firm to operate at a greater scale. The opposite is true for non-exporters who, because wages have increased, are less profitable after the tariff reduction than before.

This result extends directly to closed economy models with firm-specific distortions that affect the indirect profit function of the firm multiplicatively, such as taxes on revenues or inputs, as well as other types of distortions across firms, as in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) as demonstrated in the Appendix C. That is, the percentage gains of getting rid of firm-specific distortions is independent of credit market distortions when debt limits have the form in (13). Moreover, Proposition 2 also holds for other specifications of the right hand side of (13). For instance, the same result goes through if, instead of the capital stock, the entrepreneur was able to abscond with working capital, period revenues, period profits, or any linear combination thereof. In the appendix, we show that a version of this result also extends to a case with an endogenous entry margin in the domestic market (but no export fixed cost, $f_x = 0$).

If $f_x > 0$ we are not able to prove the analogue of Proposition 2. The presence of the fixed cost breaks down the value function's homogeneity property that is used in the proof. Despite not holding exactly, the numerical results below clearly indicate that the difference in the percentage change in consumption, output, and exports that follows a bilateral tariff reduction between an economy with perfect credit markets and one with debt limits of the form (13) is negligible.

5.2 Quantitative Exercise

To evaluate the effects of a trade liberalization in general equilibrium, we calibrate both specifications of the model and analyze the response to an unforeseen reduction in tariffs.

5.2.1 Calibration

To calibrate our model we make use of the Colombian *Annual Survey of Manufactures* (ASM), which is described in detail in Roberts and Tybout (1997). This dataset covers all manufacturing plants with ten or more employees and provides data on items including sales, exports, input usage (employees, capital and energy), age, and subsidies at the plant level. Plants are classified by 3 digit SIC industry. A trade liberalization occurred in 1985-86 in Colombia, so we calibrate our model to the 1981-84 period. A detailed description of the reform can be found in the next section.

We assume that the foreign country stands in for the rest of the world and it has perfect credit markets. We further assume that the export fixed cost is zero in the foreign country. All the other parameters are set to the values of the domestic economy. The parameters for the domestic economy are chosen to match cross-sectional features of Colombian firms and aggregates. Table 1 lists the parameter values used. The parameters α (the Cobb-Douglas parameter), β (the discount factor), σ (the elasticity of substitution), and δ_k (capital depreciation) are set to standard values.[17] The home bias parameter ω is set

[17]Our choice of β implies a steady state interest rate of 4%. This is lower than the interest rates in Colom-

to 0.5 since it cannot be separately identified from the relative size of the domestic economy, μ. The ratio of per-period to sunk export cost, η, is set to 0.05 as in Alessandria and Choi (2014).[18] The survival probability is set to match the average age of operating firms in the data set during the pre-liberalization period. We assume the productivity distribution Γ is log-normal(0,s). We assume that the utility function is CRRA and we set the intertemporal elasticity of substitution to 2. We set import and export tariffs at the level of Colombian manufacturing tariff rates documented in Attanasio et al (2004). Import tariffs are 50% while export tariffs are 5%.

We calibrate the model with forward-looking and backward-looking constraints separately, with parameter values given in columns (a) and (b) in Table 1. We have six parameters to calibrate in each model:f_x, μ, s, ρ, ξ, and θ under the forward-looking specification and f_x, μ, s, ρ, a_0, and θ in the backward-looking specification. They are set jointly to match six moments from Colombia in the years 1981-1984. These moments are: 1) the fraction of firms that export, 2) exports as a fraction of GDP, 3) the average difference in labor usage between exporters and non-exporters, 4) the standard deviation of (log) MPK, 5) the average annual growth rate in labor usage before age 10,[19] and 6) the proportion of exporters that are below age 5. The values of these moments in the model and data are given in the second panel of Table 1. For comparison, we do a third calibration for the model without credit constraints shown in column (c) in Table 1. Here we only have four parameters to calibrate (f_x, μ, s,and ρ) and we match the first four listed moments.

5.2.2 Results

We consider the effects on the model economy of an unforeseen, bilateral reduction in import and exports tariffs from their pre-reform level in Colombia to zero.[20] The results

bia during the time period considered in this paper. Recall that the interest rate in this paper should correspond to a real risk-free rate as there is no aggregate uncertainty. Since Colombian interest rates reflected a variety of macroeconomic risks, we instead rely on a standard value of β used in the literature.

[18]In Appendix E, we show that our results are roughly invariant to this value.

[19]Age 10 was chosen because that is the first age for which the average growth rate of firms is 0%.

[20]In Appendix E, we show that our conclusions are not altered if one considers an anticipated reform.

are reported in Table 2 and Figure 1. We compare the effects of this trade liberalization in the calibrated version of the model under the forward-looking and backward-looking specifications, and to the economy with perfect credit markets.

We first consider the forward-looking specification. The effect of a trade liberalization is similar to the model with perfect credit markets. Taking into account the transition, the model with forward-looking debt limits generates consumption equivalent variation of 6.66%, while the model with perfect credit markets generates 6.29%.[21] The transitional dynamics for consumption, output, capital, and export over GDP are similar in both cases as shown in Figure 1. The speed of transition is slightly faster with forward-looking debt limits as indicated by the ratio of the overall welfare gains to steady state increase in consumption, which is 0.88 for the forward-looking model and 0.85 for the perfect credit market model. The percentage changes in consumption and output from the initial steady state with high tariffs to the one with no tariffs are essentially indistinguishable between these two case (7.54% versus 7.41% for consumption and 8.20% versus 8.17% for output). We take this finding to mean that the result in Proposition 2 for the economy with $f_x = 0$ holds approximately in an economy with positive export fixed costs and endogenous sorting in the export markets.

Next we consider the backward-looking specification. Here the welfare gains associated with the trade liberalization are lower than those under perfect credit markets by about one percentage point (5.27% versus 6.29%). Along the transition to the new steady state, consumption, output, and capital are uniformly lower with backward-looking constraint and the steady state percentage change in consumption and output are about one percentage point lower (6.44% versus 7.41% for consumption and 7.14% versus 8.17% for output). The speed of transition is slightly slower as the ratio of overall welfare gains to steady state increase in consumption is 0.82 for the backward-looking model and 0.85 for

[21]Consumption equivalent variation is the percentage increase in consumption in the pre-reform steady state that would make the household indifferent, in lifetime utility, between staying in the old steady state and undergoing the reform.

the perfect credit market model. The change in the export to GDP ratio is comparable in the two economies.

We decompose these effects on steady state consumption into three components. Through the lens of an aggregate production function $C = \phi Y = \phi Z K^{\alpha}$, where ϕ is the fraction of output that is consumed and Z is measured TFP.[22] Hence, changes in consumption are driven by changes in measured TFP, changes in the steady state capital stock, and changes in the consumption share of output. As shown in Table 2, the measured TFP component is the most important in all three models, and also accounts for the largest part of the difference in consumption gains between the backward-looking and the perfect credit market cases. This is because the trade liberalization reduces the misallocation of resources in the forward-looking specification but not in the backward-looking one. This also implies that a trade liberalization that generates approximately the same change in trade flows (see the change in export/GDP) generates different welfare gains in the two specifications because of the interaction between the trade liberalization and the allocation of resources in the economy. This is why our economy departs from Arkolakis et al (2012) result.

A key mechanism that hinders the growth in measured TFP in the backward-looking specification is the inability of young firms to borrow sufficiently to enter the export market (see also Caggese and Cunat (2013) on this point). The reason that the extensive margin is important in the backward-looking specification is as follows. All firms start as non-exporters and must accumulate sufficient assets to be able to become exporters. Since trade reform makes non-exporters less profitable, they accumulate assets more slowly after the reform than before. This slows down the entry of young and productive firms in the export market. This is not true in the forward-looking case. There, the fact that the firm will be an exporter in the future allows it to borrow more from the beginning of its life and to pay the export fixed costs early in its life cycle. Therefore, whether or not young firms are able to become exporters is the key factor that determines how financial

[22]Output is the sum of consumption, capital investment and payment of fixed costs. Measured TFP in this model is driven by the export profits of firms, and by the efficiency of input allocation across firms.

frictions affect gains from trade.

6 Distinguishing Between Credit Constraints

In this section, we provide tests to distinguish between the forward-looking and backward-looking specifications of the debt limits using data from a trade reform. As discussed in Section 4, these models are difficult to distinguish using firm level data from a stationary environment because they have very similar implications for firms dynamics but a trade reform provides a means of distinguishing them. We will show that the experience of Colombia in the 1980s provides evidence in favor of the forward-looking specification. In particular, as in the forward-looking specification, in the data after the trade reform firms start to export earlier, the growth rate of new exporters is not affected by the reform, and distortions in the allocation of inputs are reduced for those firms that eventually exports. The backward-looking specification has opposite implications.

6.1 Colombian Reform

We start by briefly describing the reform in Colombia. Through the early 1980s, Colombia had increasingly high tariff rates and quotas (see Roberts (1996)). This trend reversed in 1985, when Colombia agreed to a Trade Policy and Export Diversification Loan from the World Bank. Import tariffs were substantially reduced and trade subsequently increased (see Fernandes (2007)). Though not equivalent to a bilateral trade liberalization analyzed in the previous section, for our purposes, a reduction in import tariffs increases the value of being an exporter compared to being a non-exporter. This is because the reduction in import tariff reduces the competitiveness of local firms in the domestic market. A real exchange rate depreciation is needed to clear the markets. Such depreciation makes exporters more profitable in the foreign market.

To precisely compare the cross-sectional predictions of both models with the outcome of the reform in Colombia, we simulate the effect of a reduction of import tariffs from 50% to the levels required to match the change in export over GDP observed in the data. We take into account that Colombia experienced large aggregate fluctuations in real output in the periods before the reform by incorporating fluctuations in labor productivity to match the variation in real GDP, and we generate a path of tariffs to match aggregate exports in the post-reform period. In Appendix F we describe our procedure in detail. Then we construct synthetic data sets by randomly sampling firms from the corresponding years of the transitions paths of each model of financial frictions that can be directly compared to the data. In all three of the tests that follow, the results from the models are obtained by applying the same empirical techniques to the model-generated data as to the actual Colombian data.

6.2 Extensive margin evidence

As the previous section demonstrates, the important difference between the two specifications of debt limits is whether or not credit constraints restrict the ability of firms to become exporters following trade reform. In the backward-looking case, firms are only able to export once they have accumulated sufficient assets. Since the profitability of young, non-exporting firms is decreased after the reform, it takes longer to accumulate assets and, therefore, credit constraints diminish the extensive margin of exporting. This predicts that the incidence of export activity across firms will be shifted away from young firms (who are more credit constrained) and toward older firms (who are less credit constrained). Under the forward-looking specification, firms that will eventually export are able to borrow more from the beginning of their lifetimes, which allows them to become exporters. Furthermore, since the profitability of exporting has increased, firms may choose to become exporters earlier in their lives.

In the case of Colombia, we document that the increase in export activity after the trade liberalization is more concentrated among youngest firms. In Figure 2, we plot the cumulative distribution function of exporters aged 1 to 20 before and after the reform controlling for industry and year effects. This shows that export activity increased by the most among young firms, independent of overall changes in export activity. We formalize this by using a Kolmogorov-Smirnov test for the equality of the distribution of ages for exporters before and after the reform. This test rejects the null hypothesis that the distributions are the same with a p-value of 0.006.

We make this point more precise by comparing the data with model simulated data. The second panel of Figure 2 shows the results for the forward-looking case, and the third panel for the backward-looking case. The change in the distribution for the forward-looking case is similar to the data: the CDF post reform is first order stochastically dominated by the CDF pre-reform. This is because firms start to export earlier and so the increase in export activity is accounted mainly by younger firms. The KS test also shows that the distribution post reform is different than the one pre-reform with a p-value of 0.002. In the model with backward-looking constraints we do not observe this pattern (the KS test cannot reject that the distribution changed, with a p-value of 0.473). This provides support for the forward-looking case relative to the backward-looking case.

6.3 Intensive margin evidence

We next consider how the intensive margin can be used to distinguish between the two models. The main difference between the two specifications of the debt limits is how future profitability affects the amount of debt that a firm can support. We next show that this aspect have different implications for the growth rate of new exporters. We then show how the distortions for firms that will eventually export respond to the liberalization.

Growth rates vs. Scale We first focus on the impact of the trade liberalization on the size of a firm and its growth rate. In the forward-looking model, the size of a firm, measured by its capital stock, is a function of the net present value of future profits. Since it is not optimal to pay dividends until it reaches its optimal scale, the evolution of the value conditional on survival is given by $V_{t+1}(z) = R/(1-\delta)\, V_t(z)$ so the growth rate of a firm's value is not affected by future profitability or other factors. While constrained, the capital a firm uses is pinned down by (13). Thus it follows that[23]

$$V_t \geqslant \theta k_t + \xi V_0 \Rightarrow \frac{k_{t+1}}{k_t} = \begin{cases} \frac{[R/(1-\delta)]^{t+1}-\xi}{[R/(1-\delta)]^{t}-\xi}, & \text{if } V_{t+1} < V^* \\ 0, & \text{if } V_{t+1} \geqslant V^* \end{cases} \tag{16}$$

That is, the growth rate for a constrained firm is a deterministic function of age and is not affected by the trade liberalization or by the profitability or export status of the firm. The increase in profitability after the reform shows up in an increase in the scale of the firm, not in an increase in its growth rate as illustrated in Figure 3.

The opposite is true in the model with backward-looking constraints. By construction, the initial size of the firm (if constrained) is predetermined by its initial assets and it does not respond to the liberalization. The firm's growth rate instead is affected by the reform to the extent that contemporaneous profits are affected: exporters are more profitable after the reform and can grow faster, and the opposite for non-exporters. Figure 3 illustrates how the trade liberalization changes the dynamics of the size of the firm with backward-looking constraints.

Identifying the effect of entry into the export market on growth may be empirically challenging because of potentially unobserved characteristics that may affect both export decisions and growth. Therefore we use the reform to employ a differences-in-differences specification to take into account that new exporters may be different than other types of firms. To confront the model implications with the data, we run the following regression:

$$\log\left(\frac{k_{t+1}}{k_t}\right) = \beta_0 + \beta_1 \times \text{new exporters}_t + \beta_2 \times \text{new exporters}_t \times \text{post} + \text{controls} \tag{17}$$

[23]Here we are assuming that firms enter the export markets before reaching their optimal domestic scale. We cannot rule out that they reach their optimal domestic scale before they pay the export fixed cost but it never happens in our simulations.

where *new exporter* is an indicator variable that takes value 1 if a firm is exporting for the first time in period t, *post* is an indicator variable that takes value 1 in periods after the reform was implemented, and we are controlling for exporter status (pre and post reform), and industry-year fixed effects. The parameter of interest is β_2 that measures the additional growth for a new exporters after the reform relative to the typical extra growth experienced by new exporters (β_1). In the forward-looking model, we expect the reform to have a small negative effect. Intuitively, the change in growth rates for new exporters is a weighted average of the growth rate of constrained firms and unconstrained firms (which is zero). After the reform the share of unconstrained new exporters is higher, thus the negative effect. In the backward-looking model instead the reform should have a positive impact on the growth rate of new exporters that are smaller when they enter the export markets.

Table 3 confirms these predictions in model simulated data. With forward-looking debt limits, β_2 is -0.016 while with backward-looking limits β_2 is 0.049. When we run the same regression in the data, our estimate for β_2 is -0.017, which is not statistically different than zero but is of similar magnitude to the forward-looking model.

Dispersion in MPK for future exporters We now turn to analysis of how the distortions in capital change after the trade reform. Manipulating the firm's first order conditions, we obtain the following expression for the marginal product of capital (MPK), measured as the firm's revenue to capital ratio,[24]

$$\mathrm{MPK}_t = \frac{p_{dt}y_{dt} + p_{xt}y_{xt}}{k_t} = \frac{1}{\alpha}\frac{\sigma}{\sigma - 1}\left(r_t + \mu_t\right) \tag{18}$$

where α is the capital share in the production function, $\sigma/(\sigma-1)$ is the markup, r_t is the rental capital rate, and μ_t is the multiplier on the debt limit. For unconstrained firms $\mu_t = 0$, for constrained firms instead $\mu_t > 0$. Therefore, changes in the multiplier μ_t

[24]See the appendix for the derivation of this expression.

affects firm-level capital distortions.

We first consider the model with forward-looking constraints. Absent fixed costs, Proposition 2 implies that the cross-sectional dispersion in MPK is unchanged after the trade liberalization for both exporters and non-exporters. With export fixed costs, our numerical simulations suggest that the dispersion in MPK decreases for firms that eventually export after the liberalization. This is because exporters start to export earlier in the firm life-cycle and so can start to borrow more earlier on.

We contrast these results with those of the model with backward-looking debt limits. As shown in Figure 3, for firms that eventually export, the dispersion across age increases after the trade liberalization. This is because the optimal scale of a typical exporter increases but it takes longer to reach the optimal scale. Hence this specification predicts that the multiplier on the debt limits increases after a trade liberalization for firms that eventually export.[25] However, conditional on having not yet paid the export fixed cost we do not find that the effect is large because also optimal scale in the domestic sector is also reduced.

To confront this prediction with the data we focus on future exporters, defined as firms that currently are not exporting but that are exporting 2 years in the future. For these firms, the distortions in the allocation of capital should go up after the liberalization in the model with the backward-looking constraint and go down in the forward-looking specification. As in the previous exercise, we control for potential unobserved differences between firms that will and will not export in the future by using a differences-in-differences approach. In particular, we run the following regression in the data and using data generated from both models:

$$\log \mathrm{MPK}_t = \beta_0 + \beta_1 \times \text{future exporters}_t + \beta_2 \times \text{future exporters}_t \times \text{post} + \text{controls}$$

[25]The opposite happens to non-exporters: their optimal scale is lower after the trade liberalization because of general equilibrium effects and it takes a shorter amount of time to reach the optimal scale.

where *future exporter* in period t is an indicator variable that takes value 1 if a firm is exporting in period $t+2$, *post* is an indicator variable that takes value 1 in periods after the reform was implemented, and we are controlling for exporter status (pre and post reform), and industry-year fixed effects. The results are reported in Table 4. The coefficient of interest is β_2 that measures how the multiplier on the debt limit constraint changes after the reform for firms that eventually export.

In the data, we find a large negative effect of the reform on the distortions for future exporters, -0.067, similarly to what we obtain by running the same regression in model simulated data from the forward-looking specification ($\beta_2 = -0.041$). In the backward-looking specification we find a positive effect ($\beta_2 = 0.066$). Hence, also in this case the data conforms with the forward-looking specification.

7 Conclusion

In this paper we show that the gains from a trade reform under credit market frictions are sensitive to the way credit market frictions are modeled. We consider two polar specifications for credit constraints, what we refer to as the forward-looking specification and the backward-looking or collateral constraint specification. We first show that these two models have importantly different predictions for the gains from undergoing a reform. We argue that the extensive margin is critical in generating such differences. We further show indirect evidence from a trade reform in Colombia that provides evidence in favor of the forward-looking specification. Although the model and data are related to a trade reform, we believe these results are more widely applicable to reforms that remove firm-level distortions.

Our results demonstrate that models with fixed collateral constraints may be misleading when analyzing economies undergoing reform. While a collateral constraint may be a good approximation to an underlying financial market imperfection in a stationary economy, it fails to address the endogenous response of financial markets in economies undergoing change. This may be important in contexts other than trade reform.

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8 Tables and Figures

Table 1: Calibration: Parameter values and targeted moments

Panel 1: Parameters Value

Parameter	Symbol	(a)	(b)	(c)
Discount Factor	β	0.96	0.96	0.96
Cobb-Douglass Parameter	α	0.3	0.3	0.3
Capital Depreciation	δ_k	0.05	0.05	0.05
Elasticity of Substitution among varieties	σ	5	5	5
Intertemporal Elasticity of Substitution		2	2	2
Per-period fixed cost/sunk cost for exporters	η	.05	.05	.05
Pre-Liberalization Tariffs	τ_H	0.50	0.50	0.50
Post-Liberalization Tariffs	τ_L	0.13	0.13	0.13
Export Tariffs	τ^X	0.05	0.05	0.05
Survival Probability	δ	0.07	0.07	0.07
Std of Productivity	s	0.25	0.26	0.26
Relative size of domestic economy	μ	0.38	0.37	0.34
Export fixed cost	f_x	0.29	0.29	0.31
Home Bias	ω	0.50	0.50	0.50
% Firms that can Export	ρ	0.37	0.38	0.37
Enforcement Parameter	θ	1.87	0.27	-
Probability of Starting New Firm	ξ	0.32	-	-
Initial Assets	a_0	-	0.06	-

(a) Calibration for the economy with forward-looking debt limits
(b) Calibration for the economy with backward-looking debt limits
(c) Calibration for the economy with perfect credit markets

Panel 2: Targets

Target	Data	Model		
		(a)	(b)	(c)
Exports/GDP	11%	11%	11%	11%
% Firms Exporters	12%	12%	12%	12%
Average Exporter Size Difference	3.9	3.9	3.9	3.9
St. Deviation of Log(Employees)	1.1	1.1	1.1	1.1
Annual Firm Growth, 1 to 10 years	5%	5%	5%	-
% Exporter Propensity under Age 5	8%	8%	8%	-

Data: IMF IFS and Colombian Annual Survey of Manufacturers using years 1981-1984.

Table 2: Effects of Bilateral Trade Liberalization

	Forward looking debt limits	Backward looking debt limits	Perfect credit markets
Welfare gains (CEV)	6.66%	5.27%	6.29%
Steady state comparison			
Consumption, % change	7.54%	6.44%	7.41%
Output, % change	8.20%	7.14%	8.17%
Capital, % change	8.20%	7.27%	8.71%
Fixed costs, % change	90.44%	94.69%	90.91%
Export/GDP, difference	17.68%	17.68%	17.69%
Steady state decomposition			
Measured TFP	5.51%	4.79%	5.34%
Capital deepening	2.36%	2.11%	2.51%
Consumption share	-0.61%	-0.65%	-0.71%

Note: The steady state decomposition is given by $\Delta \log C = \Delta \log Z + \alpha \Delta \log K + \Delta \log(C/Y)$ *where* Z *is measured TFP and* α *is the capital share in the production function. The second term in this decomposition is called "capital deepening" and the third is "consumption share."*

Figure 1: Transition dynamics after a trade liberalization

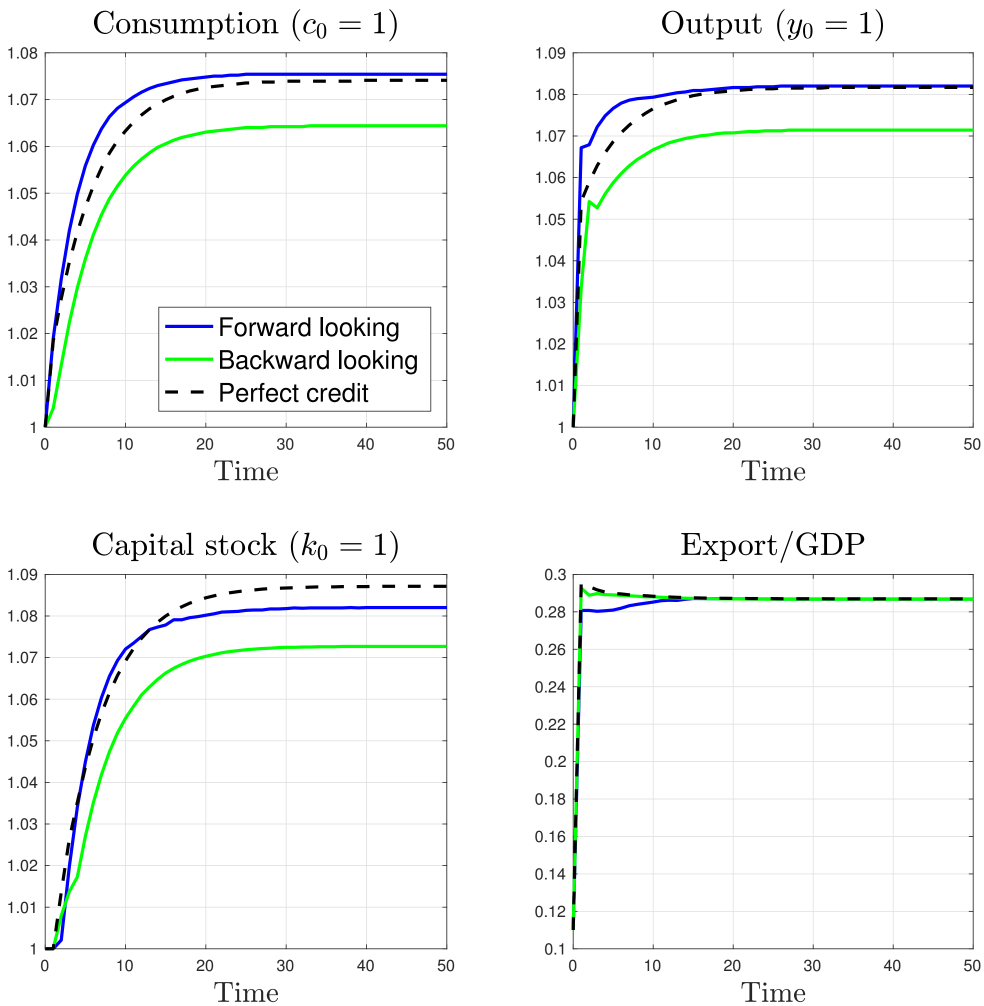


Figure 2: Cumulative distribution function for exporters over age

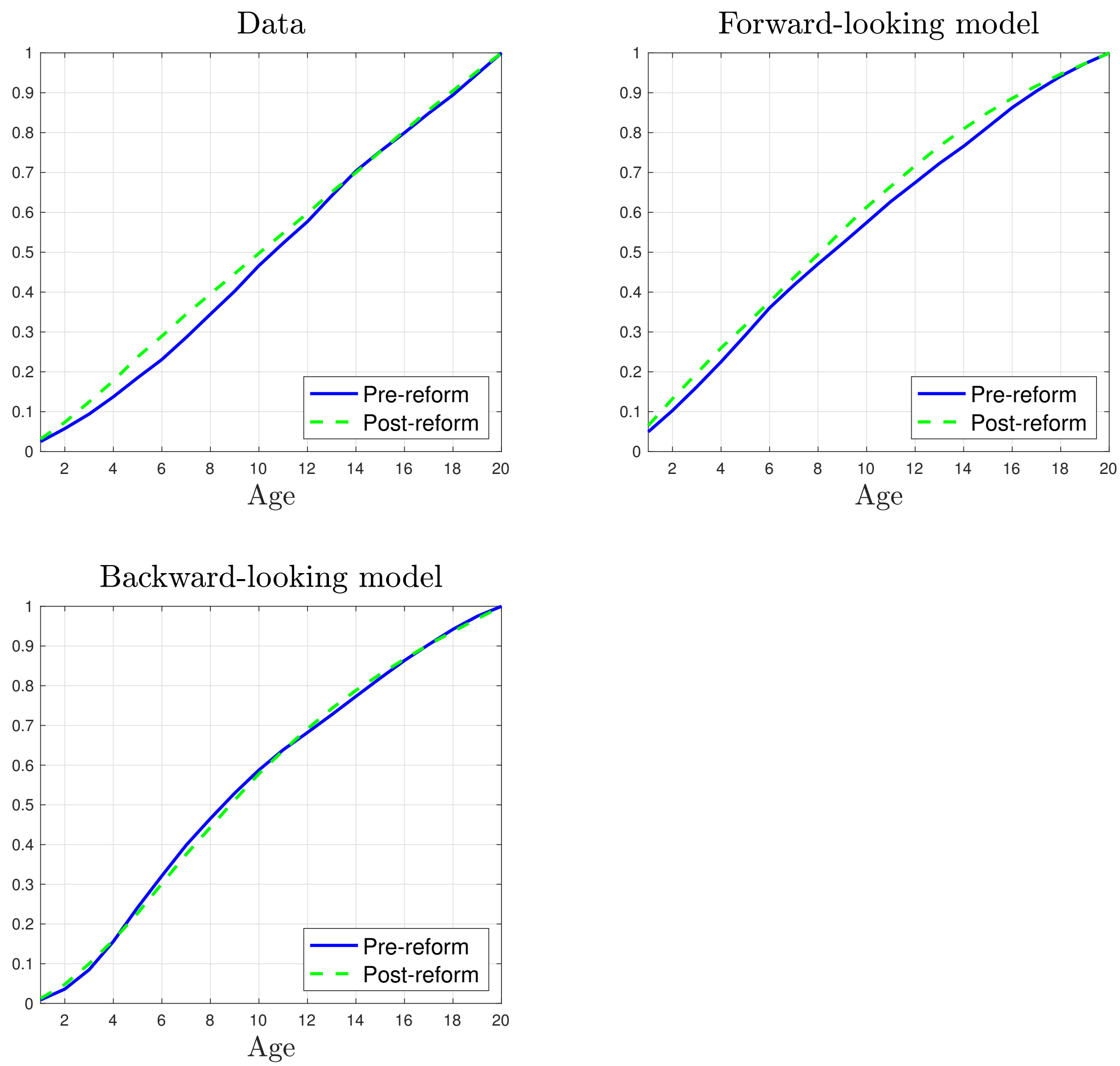
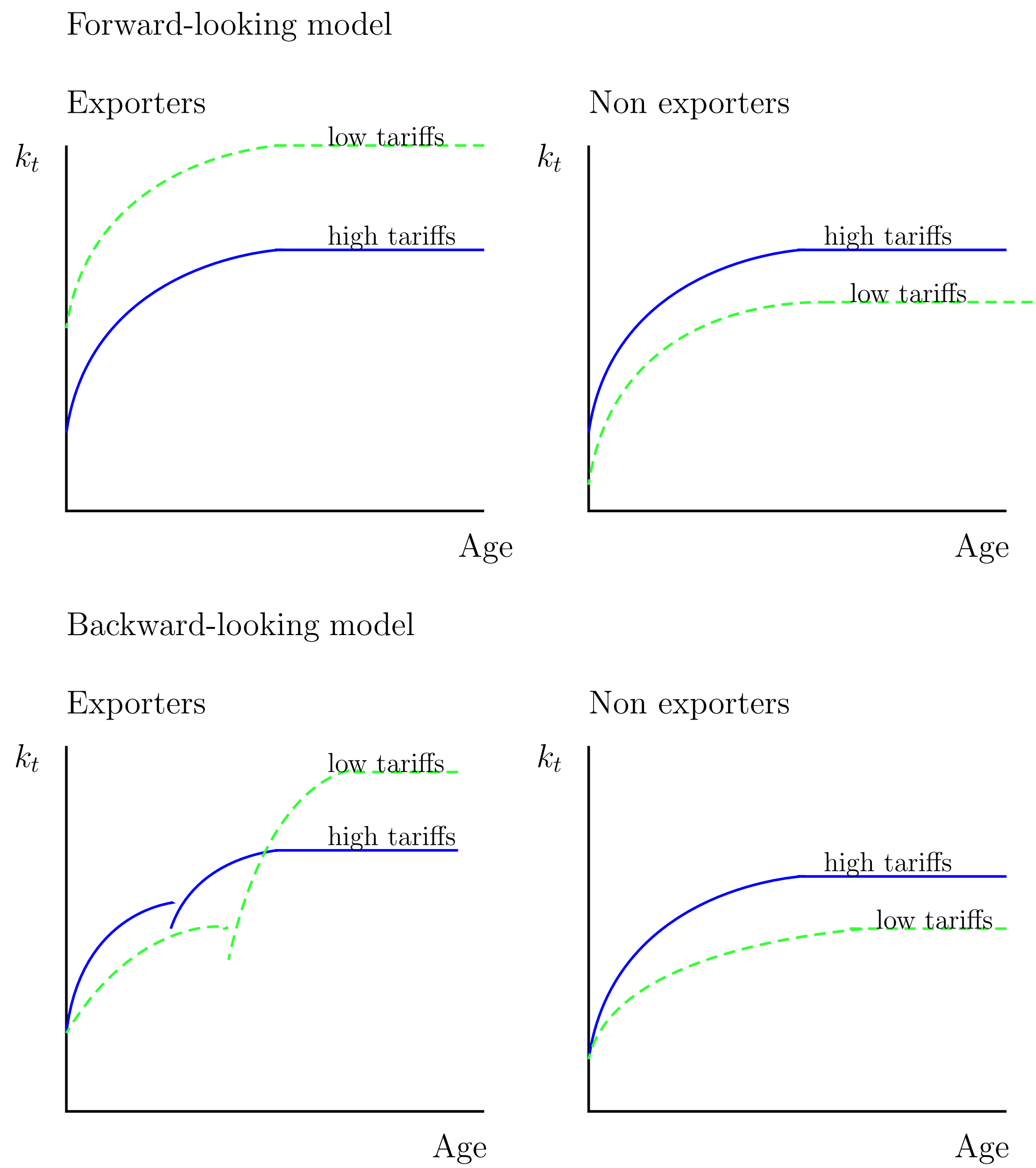


Figure 3: Typical firm dynamics: pre and post reform



Note that the capital stock employed by the firm may decrease in the period in which the firm start to export because now the firm must pay for the sunk-cost of export.

Table 3: Growth rates

	Log-Capital Growth		
	Data	Forward-looking	Backward-looking
New exporter	0.003	0.022	0.255
	(0.035)	(0.005)	(0.012)
Post Reform×New exporter	-0.017	-0.016	0.049
	(0.041)	(0.006)	(0.013)

Controls: Exporter (pre and post), industry-year fixed effects.
Standard errors clustered at the firm level.
The year of reform is excluded.

Table 4: MPK dispersion

	log (MPK)		
	Data	Forward-looking	Backward-looking
Future exporter	0.03	-0.015	-0.201
	(0.027)	(0.006)	(0.066)
Post Reform×Future exporter	-0.067	-0.041	0.066
	(0.031)	(0.007)	(0.013)

Controls: Exporter (pre and post), industry-year and firm fixed effects.
Standard errors clustered at the firm level.
The year of reform is excluded.

Online Appendix

A Definition of Equilibrium

To define an equilibrium for the economy we need to keep track of the evolution of the measure of operating firms over (a, z, ϕ) and export status. Denote by λ_t^{nx} and λ_t^x the measure of non-exporting and exporting firms at the beginning of the period over (a, z, ϕ) respectively after the entry of new firms, and let $\lambda_t = (\lambda_t^{nx}, \lambda_t^x)$. The measure of non exporters evolves over time according to

$$
\begin{aligned}
\text{(19)} \quad \lambda_{t+1}^{nx}(A, Z, \Phi) = (1-\delta) \int 1 \left\{ x_t(a, z, \phi) = 0, a'^{nx}(a, z, \phi) \in A,\ z \in Z, \phi \in \Phi \right\} d\lambda_t^{nx} \\
+ \delta\rho \int_Z 1\{ a_0 \in A, z \in Z,\ 1 \in \Phi\} d\Gamma \\
+ \delta(1-\rho) \int_Z 1\{ a_0 \in A, z \in Z,\ 0 \in \Phi\} d\Gamma
\end{aligned}
$$

where A and Z are sets of asset and productivity respectively. The measure of exporters evolves over time according to

$$
\begin{aligned}
\text{(20)} \quad \lambda_{t+1}^{x}(A, Z) = (1-\delta) \int 1 \left\{ a_t'^{x}(a, z, \phi) \in A,\ z \in Z, \phi \in \Phi \right\} d\lambda_t^x + \\
+ (1-\delta) \int 1 \left\{ x_t(a, z, \phi) = 1, a_t'^{nx}(a; z, \phi) \in A,\ z \in Z, \phi \in \Phi \right\} d\lambda_t^{nx}
\end{aligned}
$$

Market clearing in the final good market requires that

$$
\text{(21)} \qquad y_t = c_t + K_{t+1} - (1-\delta_k)K_t + y_{ft}
$$

where y_{ft} is the total investment in export fixed cost in period t:

$$
\text{(22)} \qquad y_{ft} = f_x \int x_t(a, z, \phi) d\lambda_t^{nx}
$$

Market clearing in the rental capital market requires that

$$K_t = \sum_{i \in \{nx,x\}} \int k_t^i(a, z, \phi) d\lambda_t^i \tag{23}$$

The labor market feasibility is given by

$$\mu A_t = \sum_{i \in \{nx,x\}} \int l_t^i(a, z, \phi) d\lambda_t^x \tag{24}$$

where A_t is the aggregate labor productivity in the domestic economy. For the bond market to clear, it must be that

$$b_t + A_t = K_t \tag{25}$$

where A_t is the aggregate amount of assets held by firms:

$$A_{t+1} = (1-\delta) \sum_{i \in \{nx,x\}} \int a_t'^i(a, z, \phi) d\lambda_t^i + \delta \int_Z a_t'^{nx}(a_0, z, \phi) d\Gamma \tag{26}$$

Analogous conditions hold for the foreign country.[26]

We can then define an *equilibrium* for the economy. Given debt limits $\left\{\bar{B}_t, \bar{B}_t^f\right\}_{t=0}^{\infty}$, aggregate labor productivity $\{A_t\}$, an initial distribution of firms λ_0, λ_0^f, capital stocks K_0, K_0^f, bonds holdings b_0, b_0^f, and a sequence of tariff $\{\tau_t, \tau_t^f\}_{t=0}^{\infty}$ such that $\tau_t = \tau_t^f$, an equilibrium consists of household's allocations $\{c_t, b_{t+1}\}_{t=0}^{\infty}$, prices $\{p_t, w_t, R_t, r_t\}_{t=0}^{\infty}$, inverse demand functions $\{p_{xt}, p_{dt}\}_{t=0}^{\infty}$, firms decision rules $\left\{\left\{d_t^i, b_t'^i, k_t'^i, y_{dt}^i, y_{xt}^i, l_t^i\right\}_{i \in \{nx,x\}}, x_t\right\}_{t=0}^{\infty}$, aggregate capital $\{K_t\}_{t=0}^{\infty}$, measure of firms $\{\lambda_t\}_{t=0}^{\infty}$ and analogous objects in the foreign country such that: 1) the households' allocations solve the problem (1) subject to (2) where the

[26] Without loss of generality we normalize foreign aggregate labor productivity to 1, $A_t^f = 1$.

aggregate dividend distribution is given by

$$\Pi_t = \sum_{i \in \{nx,x\}} \int d_t^i(a,z,\phi) d\lambda_t^i, \tag{27}$$

and the lump-sum transfers are given by[27]

$$T_t = \tau_t \left\{ \sum_{i \in \{nx,x\}} \int p_{xt} \left[y_{xt}^{if}(a,z,\phi) \right] y_{xt}^{if}(a,z,\phi) d\lambda^{if} \right\}; \tag{28}$$

2) the firms' decision rules are optimal for (11) and (9); 3) the inverse demand functions are given by (5)-(6); 4) the rental capital rate is given by $r_t = R_t - (1 - \delta_k)$; 5) the markets for final good, rental capital, labor and bonds clear, that is, (21), (23), (24), and (25) hold; 6) the measures of firms evolve according to (20) and (19).

B Omitted Proofs

B.1 Proof of Proposition 1

(i) Consider first a firm that already paid the fixed cost f_x. We can write the dynamic problem of the firm in steady state, (9), as

$$V^x(a,z) = \max_{d,k',b'} d + \beta(1-\delta)V^x(a',z)$$

subject to

$$d + (1-\delta)qa' \leqslant \pi^x(a)$$

$$d \geqslant 0$$

[27] The distribution λ_t^i records the export status at the beginning of the period t, before new export decisions are made. Therefore, to determine tariff collection, we must sum over non-exporting firm because some of them can pay the export cost and start to export in period t.

since in a steady state it must be that $\beta R = 1$. It can be shown that V^x is differentiable and concave in a and $V^{x\prime}(a) \geqslant 1$ with $V^{x\prime}(a) = 1$ for all $a \geqslant a^*$ and $V^{x\prime}(a) > 1$ for $a < a^*$. Letting λ and η be the multiplier associated with the budget constraint and the non-negativity on dividends respectively, the focs for the problem are:

$$d : 0 = 1 - \lambda + \eta \tag{29}$$

$$a' : 0 = \beta(1-\delta)\lambda - \beta(1-\delta)V^{x\prime}(a') \tag{30}$$

and the envelope condition:

$$V^{nx\prime}(a) = \lambda\pi^{x\prime}(a)$$

We want to show that if $a' < a^*$ then $\eta > 0$. Suppose for contradiction that $\eta = 0$. Then (29) implies that $\lambda = 1$ and in turn (30) implies that

$$1 - V^{x\prime}(a') = 0$$

but $V^{x\prime}(a) > 1$ if $a < a^*$ thus $1 - V^{x\prime}(a') < 0$ yielding a contradiction. Then it must be that $\eta > 0$ and $d = 0$.

Consider a non-exporter now. If it is never optimal to export, the same logic we used for an exporter goes through (notice that in this case V^{nx} is concave and differentiable). Instead, if it will be optimal to export at some date, V^{nx} is not necessarily concave and differentiable everywhere. Letting T be the period in which a firm with initial cash on hand a will start to export, we can write the problem as follows:

$$V^{nx}(a) = \max_{\{d_t, a_{t+1}\}_{t=0}^T} \sum_{t=0}^T \beta^t (1-\delta)^t d_t + \beta^{T+1}(1-\delta)^{T+1} V^x(a_{T+1})$$

subject to

$$d_t + q(1-\delta)a_{t+1} \leqslant \pi^{nx}(a_t) \quad \text{for} \quad t = 0, ..., T-1$$
$$d_T + q(1-\delta)a_{T+1} \leqslant \pi^{nx}(a_T - f_x) - f_x$$
$$d_t \geqslant 0$$

Lettin $\beta^t(1-\delta)^t\lambda_t$ and $\beta^t(1-\delta)^t\eta_t$ be the multiplier associated with the budget constraint and the non-negativity on dividends respectively, the focs for the problem are:

$$0 = 1 - \lambda_t + \eta_t \quad \text{for } t = 0, ..., T \tag{31}$$
$$0 = \lambda_t - \lambda_{t+1}\pi'(a_{t+1}) \quad \text{for } t = 0, ..., T-1 \tag{32}$$
$$0 = \lambda_T - V^{x'}(a_{T+1}) \tag{33}$$

Starting at $T+1$, suppose that $a_{T+1} < a^*$ and for contradiction that $\eta_T = 0$. Then it must be that $\lambda_T = 1$. This and (33) imply that

$$1 - V^{x\prime}(a') = 0$$

but $V^{x\prime}(a_{T+1}) > 1$ thus $1 - V^{x\prime}(a') < 0$ yielding a contradiction. Hence $\eta_T > 0$ and $d_T = 0$. Now combine (31) at t and $t+1$ with (32) at t we obtain:

$$\eta_t = \lambda_t - 1 = \lambda_{t+1}\pi'(a_{t+1}) - 1 \geqslant \lambda_{t+1} - 1 = \eta_{t+1}$$

Thus, if $\eta_{t+1} > 0$ then $\eta_t > 0$. This is turn implies that as long as any borrowing constraint is binding in the future then there is no dividend distributions as wanted. When no borrowing constraint in the future are binding then the firms optimal dividend policy is indeterminate. Thus, without loss of generality we can set $d = 0$ to characterize the firm's value and policy functions.

To prove the remaining parts of Proposition 1 we will consider the forward and backward looking case separately.

Forward-Looking Constraint In this case it is convenient to write the problem in (9) and (11) using their dual formulation. This can be thought of as an optimal contracting problem between the entrepreneur and competitive, risk-neutral financial intermediaries. Financial intermediaries offer the entrepreneur long-term contracts that specify production plans and the value of the dividends paid to the entrepreneur. It is then straightforward to write this problem recursively using the discounted sum of promised dividend payments v as well as the export status of the firm as state variables. Denote the value functions of the financial intermediaries as $W^{nx}(v, z, \phi)$ and $W^{x}(v, z, \phi)$. The problem of the financial intermediary in steady state can be written as:

$$W^{x}(v, z, \phi) = \max_{k, y_d, y_x, l, d} -rk + p_d(y_d)y_d + p_x(y_x)y_x - wl - d - \eta f_x + \beta(1-\delta)W^{x}(v', z, \phi)$$

subject to

$$\begin{aligned} y_d + y_x &\leqslant z k^{\alpha} l^{1-\alpha} \\ d + \beta(1-\delta)v' &= v \\ v &\geqslant \frac{\theta}{q(1-\delta)}k + \frac{\xi}{q(1-\delta)}v_0 \end{aligned}$$

and for a firm that has not paid the fixed cost already:

$$\begin{aligned} W^{nx}(v, z, \phi) = \max_{k, y_d, y_x, l, d} & -rk + p_d(y_d)y_d + x p_x\left(y_x\right)y_x - wl - d - x f_x\left(1+\eta\right) \\ & + \beta(1-\delta)\left[x W^{x}(v', z, \phi) + (1-x)W^{nx}(v', z, \phi)\right] \end{aligned}$$

subject to

$$
\begin{aligned}
y_d + y_x &\leqslant z k^{\alpha} l^{1-\alpha} \\
d + \beta(1-\delta) &= v \\
v &\geqslant \frac{\theta}{\beta(1-\delta)} k + \frac{\xi}{\beta(1-\delta)} v_0
\end{aligned}
$$

For notational convenience define

$$
\Pi^{x}(v,z) = \max_{y_d, y_x.l,k} p_d(y_d) y_d + p_x(y_x) y_x - wl - rk - \eta f_x
$$

subject to

$$
\begin{aligned}
y_d + y_x &\leqslant zF(k,l) \\
v &\geqslant \frac{\theta}{\beta(1-\delta)} k + \frac{\xi}{\beta(1-\delta)} v_0
\end{aligned}
$$

and

$$
\Pi^{nx}(v,z) = \max_{y_d,l,k} p_d(y_d) y_d + p_x(y_x) y_x - wl - rk
$$

subject to

$$
\begin{aligned}
y_d &\leqslant zF(k,l) \\
v &\geqslant \frac{\theta}{\beta(1-\delta)} k + \frac{\xi}{\beta(1-\delta)} v_0
\end{aligned}
$$

By part (i) we can set $d_t = 0$ without loss for all t and rewrite the intermediary's problem as follows:

$$W^{nx}(v,z,\phi) = \max\left\{\Pi^{nx}(v,z) + q(1-\delta)W^{d}\left(\frac{v}{\beta(1-\delta)},z,\phi\right);\right.$$
$$\left.\Pi^{x}(v,z) - f_x + \beta(1-\delta)W^{x}\left(\frac{v}{\beta(1-\delta)},z,\phi\right)\right\}$$
$$W^{x}(v,z,\phi) = \Pi^{x}(v,z) + \beta(1-\delta)W^{x}\left(\frac{v}{\beta(1-\delta)},z\right)$$

Finally, the minimum equity value for the firm to operate at its efficient scale is given by:

$$v^*(z,\phi) \equiv \min\{\arg\max_{v}\{\max\{\Pi^{nx}(v,z),\phi\Pi^{x}(v,z)\}\}\}$$

A firm will eventually reach v^*, because $v_t = \frac{v_0}{((1-\delta)\beta)^t}$. Then, for $v' \geqslant v^*$ a domestic firm with inside equity value v' will start exporting iff

$$\frac{\Pi^{x*}(z)}{1-(1-\delta)\beta} - \frac{\Pi^{nx*}(z)}{1-(1-\delta)\beta} \geqslant f_x$$

as in a standard Melitz model. Since the LHS is strictly increasing in z, there exists a cut-off z_x s.t. the above condition holds for all $z \geqslant z_x$.

We now prove part (iii) and (iv). To this end consider

$$\begin{aligned}
W^{x}(v,z) - W^{nx}(v,z) &= \Pi^{x}(v,z) + \beta(1-\delta)W^{x}\left(\frac{v}{\beta(1-\delta)},z\right) - \\
&- \max\left\{\Pi^{nx}(v,z) + \beta(1-\delta)W^{nx}\left(\frac{v}{\beta(1-\delta)},z\right); W^{x}(v,z) - fx\right\} \\
&= \min\{\Pi^{x}(v,z) - \Pi^{x}(v,z) + \\
&+\beta(1-\delta)\left(W^{x}\left(\frac{v}{\beta(1-\delta)},z\right) - W^{nx}\left(\frac{v}{\beta(1-\delta)},z\right)\right);\ f_x\Big\} \\
&= \min\left\{\Delta\Pi(v,z) + \beta(1-\delta)\left(W^{x}\left(\frac{v}{\beta(1-\delta)},z\right) - W^{nx}\left(\frac{v}{\beta(1-\delta)},z\right)\right);\ f_x\right\}
\end{aligned}$$

The following lemma shows that the value of becoming an exporter weakly increases with ν.

Lemma 3 *(a) $\forall z\, W^x(\nu,z) - W^{nx}(\nu,z)$ is weakly increasing in ν, and (b) $\forall \nu\, W^x(\nu,z) - W^{nx}(\nu,z)$ is weakly increasing in z.*

Proof. Define $T : C(\mathbb{R}_+ \times \mathbb{R}_+) \to C(\mathbb{R}_+ \times \mathbb{R}_+)$ as

$$Tf(\nu,z) = \min\left\{\Delta\Pi(\nu,z) + \beta(1-\delta) f\left(\frac{\nu}{\beta(1-\delta)}, z\right) ;\ f_x\right\}$$

where $C(\mathbb{R}_+ \times \mathbb{R}_+)$ is the space of continuous and bounded functions. T satisfies the Blackwell's sufficient conditions for a contraction mapping. Then T is a contraction, and $W^x - W^{nx}$ is its unique fixed point.

To prove (a), let $C'(\mathbb{R}_+ \times \mathbb{R}_+)$ be the set of continuous, bounded and weakly increasing function in their first argument. $C'(\mathbb{R}_+ \times \mathbb{R}_+)$ is a closed set, hence by Corollary 3.1 in Stokey, Lucas and Prescott (1989) it suffices to show that $\forall f \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ $Tf \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ to prove that $W^x - W^{nx}$ is increasing in its first argument. Fix z, let $f \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ and $\nu' > \nu$:

$$\begin{aligned} Tf(\nu',z) &= \min\left\{\Delta\Pi(\nu',z) + \beta(1-\delta) f\left(\frac{\nu'}{\beta(1-\delta)}, z\right) ;\ f_x\right\} \\ &\geqslant \min\left\{\Delta\Pi(\nu,z) + \beta(1-\delta) f\left(\frac{\nu}{\beta(1-\delta)}, z\right) ;\ f_x\right\} = Tf(\nu,z) \end{aligned}$$

as wanted, because $\Delta\Pi(\nu,z)$ is increasing in ν, and f is weakly increasing by assumption. Then we established (a). The exact same argument can be used to prove (b) noticing that $\Delta\Pi(\nu,z)$ is increasing in z also.

Thus, if $z \leqslant z_x$ a firm will never export since for all ν $W^x(\nu,z) - W^{nx}(\nu,z) \leqslant W^x(\nu^*,z) - W^{nx}(\nu^*,z) < f_x$. Vice versa, if $z \geqslant z_x$, then the firm will eventually export, proving (ii).

To prove (iii), notice that if $z \geqslant z_x$ the firm will eventually export, and the fact that $W^x(\nu,z) - W^{nx}(\nu,z)$ is increasing in ν implies that there exists a unique threshold $\tilde{\nu}(z)$

such that a firm will export iff $v \geqslant \tilde{v}(z)$.

Lastly, we prove (iv) by showing that if $z' > z$ then $\tilde{v}(z')/v_0(z') \leqslant \tilde{v}(z)/v_0(z)$, implying $\tilde{T}(z') \leqslant \tilde{T}(z)$. Let $z' > z \geqslant z_x$. The fact that $W^{nx}(v, z)$ is strictly increasing in z for all v implies that $v_0(z') > v_0(z)$, since v_0 is such that $W^{nx}(v_0(z), z) = 0$. To prove the proposition it is sufficient to show that $\tilde{v}(z') < \tilde{v}(z)$. By the previous lemma $\forall v$ $W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in z. Thus, if $W^x(\tilde{v}(z), z) - W^{nx}(\tilde{v}(z), z) = f_x$ then $W^x(\tilde{v}(z), z') - W^{nx}(\tilde{v}(z), z') \geqslant f_x$ since $z' > z$, therefore $\tilde{v}(z') \leqslant \tilde{v}(z)$ as wanted.

To relate this to the "cash on hand" formulation, notice that the cash on hand for a firm with value v is given by $W^i(v)$ for $i = x, nx$, which is a monotone relation in v. Hence, all statements about v are also true for a.

Backward-Looking Constraint Proof of part (iii):

Lemma 4 *Consider a restricted problem in which firms can only choose to either pay the fixed cost in the first or second Let* $x(a, z) = 0$ *be the decision to not export in the first period, and* $x(a, z) = 1$ *be the decision to export in the first period. Then* $\exists \hat{a} : \forall a < a, x(a, z) = 0$, *and* $\forall a \geqslant a, x(a, z) = 1$.

Proof. In this restricted problem, the fact that all firms must be exporters after the second period (and the fact that z does not change) implies that the objective of the firm is equivalent to maximizing third period assets. Then the decision to export today or tomorrow yields the following payouts:

If the firm exports today (here assuming all constraints are binding to simplify notation):

$$x(a, z) = 1 \implies \beta(1-\delta)a_x = \pi^x\left(\frac{\pi^x(a - f_x, z)}{1-(1-\delta)\beta}, z\right)$$

and if they export the next period:

$$x(a, z) = 0 \implies \beta(1-\delta)a_{nx} = \pi^x\left(\frac{\pi^{nx}(a, z)}{1-(1-\delta)\beta} - f_x, z\right)$$

The firm then chooses whichever is greater. Define $\Delta(a,z) \equiv \beta(1-\delta)[a_x - a_{nx}]$. Let $F(a,z) \equiv \pi^x(a-f_x,z) - \pi^{nx}(a,z) + (1-\beta(1-\delta))f_x$. Note that $sign(F(a,z)) = sign(\Delta(a,z))$. Then any zero of the function F is also a zero of the function Δ. We can show that F is a strictly increasing function of a:

$$F_1(a,z) \equiv \pi_1^x(a-f_x,z) - \pi_1^{nx}(a,z) > 0$$

which is true because π^x and π^{nx} are concave, and $\forall a, z, \pi_1^x(a,z) > \pi_1^{nx}(a,z)$.

Then notice that $F(f_x,z) < 0$ and (assuming that $z \geqslant z_x$) $F(a^*,z) > 0$. Therefore, $\exists \hat{a} \in [f_x, a^*]$ that has the cutoff properties described in the statement of the lemma.

To complete the proof, we demonstrate that the cutoff found in the restricted problem corresponds to the cutoff in the general problem.

First, consider firms with asset values $a < \hat{a}(z)$. Our claim is that the firm does not export with that level of assets. For contradition, suppose that they did. Then, by the definition of $\hat{a}$ given in the lemma, we know that the firm could generate strictly greater profits by, instead, delaying their decision to export by one period. Hence, exporting this period is not optimal.

Second, consider firms with asset values $a \geqslant \hat{a}(z)$. The next lemma shows that for these firms the restriction on the periods when they can export is not binding.

Lemma 5 *Suppose a firm prefers to export this period instead of one period in the future. Then the firm prefers to export this period rather than any period in the future.*

Proof. We prove this by induction. The base step is true by hypothesis. Let $a^k(t)$ be the asset level of a firm k periods in the future who chooses to enter the export market in period t.

Using the fact that the firm's objective is equivalent to maximizing their assets whenever they are constrained, to complete the proof we need only show that $a^{k+1}(k) < a^{k+1}(1) \implies a^{k+2}(k+1) < a^{k+2}(1)$. Notice that the fact that $a'(a,z)$ is increasing in

a means that $a^{k+1}(k) < a^{k+1}(1) \implies a^{k+2}(k) < a^{k+2}(1)$, so it is sufficient to show that $a^{k+2}(k+1) < a^{k+2}(k)$. But this follows immediately from the previous lemma, the fact that $a \geqslant \hat{a}(z)$, and the fact that $a'(a,z)$ is increasing in a. This completes the proof.

Therefore, $\forall a \geqslant \hat{a}(z)$, the fact that they prefer to export this period rather than the following period implies that they prefer to export this period rather than wait until any other period. Therefore, $\hat{a}(z)$ is the threshold level of assets that determines export status.

Proof of part (iv):

Here we use the fact that $a'(a,z)$ is increasing in z and that $\hat{a}(z)$ is decreasing in z. The fact that $a'(a,z)$ is increasing in z follows immediately from the fact that $\pi^{nx}(a,z)$ is increasing in z. To prove that $\hat{a}(z)$ is decreasing in z we make use of the characterization in the proof to part (iii).

Recall that $\hat{a}(z)$ solves $F(\hat{a}(z),z) = 0$. Then the implicit function theorem implies:

$$\frac{d\hat{a}}{dz} = -\frac{\left[\pi_2^{nx}(a,z) - \pi_2^{x}(a - f_x, z)\right]}{\left[\pi_1^{nx}(a,z) - \pi_1^{x}(a - f_x, z)\right]} < 0$$

The sign follows from the fact that for $j \in \{nx, x\}$, π^j is concave in the first argument, $\pi_{21}^j > 0, \forall a, z, \pi_1^x(a,z) > \pi_1^{nx}(a,z)$ and $\pi_2^x(a,z) > \pi_2^{nx}(a,z)$.

Therefore, starting from assets a_0, firms with higher productivity both have faster asset growth and a lower asset threshold to enter the export market. Hence, $T(z)$ is decreasing in z.

B.2 Proof of Proposition 2

With $f_x = 0$, all firms with $\phi = 1$ always export. Let the aggregate state of the economy be $\mathbf{s} = (y, w, \tau)$ and let $D_0(\tau) = \omega$ and $D_1(\tau) = (\omega^\sigma + (\frac{1-\omega}{1+\tau})^\sigma)^{1/\sigma}$. Let $\Delta y, \Delta w, \Delta D$ and $\Delta(1+\tau)$ be defined by $\Delta x = x\prime/x$, where primes denote post-reform variables.

First we prove some properties for the economy with perfect credit markets. Recall that $k^*(z,\phi;\mathbf{s})$ and $l^*(z,\phi;\mathbf{s})$ are the solution to (15).

Lemma 6 $k^*(z, \phi; \mathbf{s})$ *is homogeneous of degree 1 in* y*, degree* σ *in* D*, degree* $\sigma - 1$ *in z, and degree* $(1-\alpha)(1-\sigma)$ *in w;* $l^*(z, \phi; \mathbf{s})$ *is homogeneous of degree 1 in* y*, degree* σ *in* D*, degree* $\sigma - 1$ *in z, and degree* $(\alpha - 1)\sigma - \alpha$ *in w.*

Proof. Letting λ be the Lagrangian multiplier on the constraint, the first order conditions of the unconstrained firm imply:

$$k/l = \frac{\alpha}{1-\alpha}\frac{w}{r}$$

and

$$\lambda = w\frac{1}{1-\alpha}\frac{1}{z}\left(\frac{k}{l}\right)^{-\alpha} = w\frac{1}{1-\alpha}\frac{1}{z}\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{-\alpha} = \text{const} \times \frac{w^{1-\alpha}r^{\alpha}}{z}$$

Then, notice that

$$y_d = (1-1/\sigma)^{\sigma}\omega^{\sigma}\lambda^{-\sigma}y \propto \omega^{\sigma}\left(\frac{w^{1-\alpha}r^{\alpha}}{z}\right)^{-\sigma} y$$

$$y_x = \phi(1-1/\sigma)^{\sigma}\left(\frac{1-\omega}{1+\tau}\right)^{\sigma}\lambda^{-\sigma}y \propto \left(\frac{1-\omega}{1+\tau}\right)^{\sigma}\left(\frac{w^{1-\alpha}r^{\alpha}}{z}\right)^{-\sigma} y$$

Therefore using the production function:

$$y = z\left(\frac{k}{l}\right)^{\alpha} l = z\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha} l$$

it follows that

$$l^*(z, \phi; \mathbf{s}) = y(z, \phi; \mathbf{s})\left[z\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha}\right]^{-1} \propto z^{\sigma-1}D_{\phi}^{\sigma}yw^{(\alpha-1)\sigma-\alpha} \tag{34}$$

$$k^*(z, \phi; \mathbf{s}) \propto z^{\sigma-1}D_{\phi}^{\sigma}yw^{(\alpha-1)\sigma-\alpha+1} = z^{\sigma-1}D_{\phi}^{\sigma}yw^{(\alpha-1)(\sigma-1)} \tag{35}$$

as wanted.

Consider now an economy with limited enforcement. Define

$$v_t(z,\phi;\mathbf{s}) = \sum_{s=0}^{\infty}(\beta(1-\delta))^s d_{t+s}(z,\phi;\varphi) = \frac{v_0(z,\phi;\varphi)}{(\beta(1-\delta))^t}$$

be the present value of future dividends for a firm of age t. The second equality comes from Proposition 1: whenever the borrowing constraint is binding there are no dividends paid. Hence, v_t grows at rate $1/(\beta(1-\delta))$. Let $k(v,z,\phi;\mathbf{s})$ and $l(v,z,\phi;\mathbf{s})$ be the solution to:

$$\pi(v,z,\phi;\mathbf{s}) = \max_{y_x,y_d,l,k} \omega y^{1/\sigma} y_d^{1-1/\sigma} + \phi\frac{(1-\omega)}{1+\tau} y^{1/\sigma} y_x^{1-1/\sigma} - wl - rk \tag{36}$$

subject to the production function $y_d + y_x \leqslant zF(k,l)$ and

$$\beta(1-\delta)v \geqslant \theta k + \xi v_0(z,\phi;\mathbf{s}) \tag{37}$$

For v sufficiently high, the enforcement constraint is not binding and the firm operates at its optimal scale $k^*(z,\phi;\mathbf{s})$ and makes profits $\pi^*(z,\phi;\mathbf{s})$. Define $v^*(z,\phi;\mathbf{s}) = \theta k^* + \xi v_0$ as the smallest value of v needed to sustain optimal scale, and $T^*(z,\phi;\mathbf{s}) = \lceil \log(v_0/v^*)/\log(q(1-\delta)) \rceil$ as the number of periods it takes the firm to reach optimal scale.

Given that the financial sector makes zero expected profits, in equilibrium the initial value of the firm v_0 is the solution to:

$$v_0(z,\phi;\mathbf{s}) = \sum_{t=0}^{\infty}(\beta(1-\delta))^t \pi\left(\frac{v_0(z,\phi;\mathbf{s})}{(\beta(1-\delta))^t}, z,\phi;\mathbf{s}\right)$$

We now prove a series of Lemmas that we will use in the proof of the Proposition.

Lemma 7 $k(v,z,\phi;\mathbf{s})$ *is given by*

$$k(v,z,\phi;\mathbf{s}) = \min\left\{k^*(z,\phi;\mathbf{s}), \frac{\beta(1-\delta)v - \xi v_0(z,\phi;\mathbf{s})}{\theta}\right\} \tag{38}$$

and the indirect profits function is given by

(39)

$$\pi(v,z,\phi;s) = Cw^{\frac{(\alpha-1)(\sigma-1)}{1+\alpha(\sigma-1)}} D_{\phi}^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} k(v,z,\phi;\mathbf{s})^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} - rk(v,z,\phi;\mathbf{s})$$

where C *is a constant.*

Proof. The solution for k is given by

$$k(v,z,\phi;\mathbf{s}) = \min\left\{k^*(z,\phi;\mathbf{s}), \frac{\beta(1-\delta)v - \xi v_0(z,\phi;\mathbf{s})}{\theta}\right\} \tag{40}$$

Combining first order conditions, the solution to the problem is given by the solution to the following equations:

$$y_d = \omega^\sigma(1-1/\sigma)^\sigma y\lambda^{-\sigma} = \omega^\sigma(1-1/\sigma)^\sigma y\left[\frac{(1-\alpha)z}{w}\left(\frac{k}{l}\right)^\alpha\right]^\sigma \tag{41}$$

$$y_x = \left(\frac{1-\omega}{1+\tau}\right)^\sigma(1-1/\sigma)^\sigma y\lambda^{-\sigma} = \left(\frac{1-\omega}{1+\tau}\right)^\sigma(1-1/\sigma)^\sigma y\left[\frac{(1-\alpha)z}{w}\left(\frac{k}{l}\right)^\alpha\right]^\sigma \tag{42}$$

Then I can use the production function to solve for l:

$$\begin{aligned} l &= \left(\frac{(1-1/\sigma)^\sigma\left[\omega^\sigma + \left(\frac{1-\omega}{1+\tau}\right)^\sigma\right] y\left[\frac{(1-\alpha)z}{w}\left(\frac{k}{l}\right)^\alpha\right]^\sigma}{zk^\alpha}\right)^{\frac{1}{1-\alpha}} \\ &= \mathrm{const} \times D_{\phi}^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} w^{\frac{-\sigma}{1+\alpha(\sigma-1)}} k^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} \end{aligned} \tag{43}$$

Plugging these solutions back in the objective function, it follows that when the enforcement constraint is binding we have that:

$$\pi_\phi(v,z;s) = D_\phi(\tau)y_d^{1/\sigma}\left[zk^\alpha l^{1-\alpha}\right]^{1-1/\sigma} - wl - rk \tag{44}$$

Plugging in the above yields

$$\pi_\phi(\nu, z; s) = Cw^{\frac{(\alpha-1)(\sigma-1)}{1+\alpha(\sigma-1)}} D_\phi^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} k(\nu, z, \phi; \mathbf{s})^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} - rk(\nu, z, \phi; \mathbf{s})$$

where C is a constant.

Lemma 8 *With perfect credit market we have that*

$$\Delta_{k,\phi} = \Delta_{D,\phi}^{\sigma} \Delta_y \Delta_w^{(\alpha-1)(\sigma-1)} \tag{45}$$

Proof. It follows directly from (35).

Lemma 9 $\nu_0(z, \phi; \mathbf{s})$ *is homogeneous of degree* $\sigma - 1$ *in* z*,* σ *in* D_ϕ*, 1 in* y*, and* $(\alpha-1)(\sigma-1)$ *in* w*. That is,* $\exists$ *a scalar* $\tilde{\nu}_0$ *such that* $\forall(z, \phi, \mathbf{s})$

$$\nu_0(z, \phi; \mathbf{s}) = \tilde{\nu}_0 z^{\sigma-1} D_\phi^{\sigma} y w^{(\alpha-1)(\sigma-1)} \tag{46}$$

Proof. We now proceed by guess and verify. Suppose that $\nu_0(z, \phi; \mathbf{s})$ takes the form in (46). Then, given the guess $\nu^*(w) = \theta\tilde{k}w^{(\alpha-1)(\sigma-1)} + \xi\tilde{\nu}_0 w^{(\alpha-1)(\sigma-1)} = \tilde{\nu}^* w^{(\alpha-1)(\sigma-1)}$. Hence it follows that

$$k(\nu, z, \phi; \mathbf{s}) = \min\left\{k^*(z, \phi; \mathbf{s}), \frac{\beta(1-\delta)\nu - \xi\nu_0(z, \phi; \mathbf{s})}{\theta}\right\} \propto z^{\sigma-1} D_\phi^{\sigma} y w^{(\alpha-1)(\sigma-1)} \tag{47}$$

Lastly, it can be shown that $\forall t \geqslant 0$

$$\pi(\nu_t(w); w) = \pi\left(\frac{\tilde{\nu}_0}{(\beta(1-\delta))^t}; 1\right) w^{(\alpha-1)(\sigma-1)} \tag{48}$$

by combining (39) and (47). Thus, using (48) in the definition of ν_0 it follows that $\nu_0(z, \phi; \mathbf{s})$ is homogeneous of degree $\sigma - 1$ in z, σ in D_ϕ, 1 in y, and $(\alpha-1)(\sigma-1)$ in w as wanted.

Thus, the above lemmas imply that if the path $\{k_t(z, \phi), b_t(z, \phi), d_t(z, \phi)\}_{t=0}^{\infty}$ for a firm of type (z,ϕ) with aggregate state $\mathbf{s}$, then $\{\Delta_k k_t(z,\phi), \Delta_k b_t(z,\phi), \Delta_k d_t(z,\phi)\}_{t=0}^{\infty}$ is optimal for the aggregate state $s' = (\Delta_y y, \Delta_w w, \tau')$. We are now left to show that labor and good market clear. We denote variables from the perfect credit markets environment with superscript PC and from the forward-looking environment FL. First we prove a lemma that shows that the financial friction induces a distortion across age, but not across productivity levels.

Lemma 10 *Suppose s^{PC} is an equilibrium in the perfect credit markets environment and s^{FL} is an equilibrium in the forward looking environmet. Then $\forall z,\phi$,*

$$l^{PC}(z,\phi;s^{PC}) = \delta\sum_t (1-\delta)^t l^{FL}\left(\frac{\nu_0(z,\phi)}{((1-\delta)\beta)^t}, z,\phi;s^{FL}\right)$$

$$y_d^{PC}(z,\phi;s^{PC})/y^{PC} = \delta\sum_t (1-\delta)^t y_d^{FL}\left(\frac{\nu_0(z,\phi)}{((1-\delta)\beta)^t}, z,\phi;s^{FL}\right)/y^{FL}$$

and

$$y_x^{PC}(z,\phi;s^{PC})/y^{PC} = \delta\sum_t (1-\delta)^t y_x^{FL}\left(\frac{\nu_0(z,\phi)}{((1-\delta)\beta)^t}, z,\phi;s^{FL}\right)/y^{FL}$$

Proof. We only show the case with l as the other cases are analogous. Combining (46) and (43),

$$\forall t,z,\phi, l^{FL}\left(\frac{\nu_0(z,\phi)}{((1-\delta)\beta)^t}, z,\phi;\mathbf{s}\right) \propto z^{\sigma-1}D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha}$$

As above, we already know that

$$\forall z,\phi, l^{PC}(z,\phi;s) \propto z^{\sigma-1}D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha}$$

Then the fact that labor markets clear in both cases implies

$$1 = \rho\int_Z l^{PC}(z,0;s^{PC})d\Gamma(z) + (1-\rho)\int_Z l^{PC}(z,1;s^{PC})d\Gamma(z)$$
$$1 = \delta\int_Z\sum_t (1-\delta)^t\left[\rho l^{FL}\left(\frac{\nu_0(z,0)}{((1-\delta)\beta)^t}, z,0;s^{FL}\right) + (1-\rho)l^{FL}\left(\frac{\nu_0(z,1)}{((1-\delta)\beta)^t}, z,1;s^{FL}\right)\right]d\Gamma(z)$$

Using the above facts yields,

$$1 = l^{PC}(1,0;s^{PC})\left(\rho\int_Z z^{\sigma-1}d\Gamma(z) + \left(\frac{D_1}{D_0}\right)^{\sigma}(1-\rho)\int_Z z^{\sigma-1}d\Gamma(z)\right)$$
$$1 = \delta\sum_t (1-\delta)^t l^{FL}\left(\frac{v_0(1,0)}{((1-\delta)\beta)^t}, 1, 0; s^{FL}\right)\left(\rho\int_Z z^{\sigma-1}d\Gamma(z) + \left(\frac{D_1}{D_0}\right)^{\sigma}(1-\rho)\int_Z z^{\sigma-1}d\Gamma(z)\right)$$

Combining these equations and multiplying through by any value of $z^{\sigma-1}$ or D_ϕ yields the result.

Now we check the labor market clearing condition. Again, let $\mathbf{s} = (y, w, \tau)$. The post-reform labor market clearing condition is:

$$\begin{aligned} 1 &= \rho\int_Z l^{PC}(z,0;s^{PC\prime})d\Gamma(z) + (1-\rho)\int_Z l^{PC}(z,1;s^{PC\prime})d\Gamma(z) = \\ &= \frac{\Delta_y}{\Delta_w^{1+(1-\alpha)(\sigma-1)}}\left[\rho\int_Z l^{PC}(z,0;s^{PC})d\Gamma(z) + \Delta_D^{\sigma}(1-\rho)\int_Z l^{PC}(z,1;s^{PC})d\Gamma(z)\right] \end{aligned}$$

Applying the above lemma we get:

$$\begin{aligned} 1 = & \frac{\Delta_y\delta}{\Delta_w{}^{1+(1-\alpha)(\sigma-1)}}\times \\ & \times\int_Z\sum_t(1-\delta)^t\left[\rho l^{FL}\left(\frac{v_0(z,0)}{((1-\delta)\beta)^t}, z, 0; s^{FL}\right) + \Delta_D^{\sigma}(1-\rho)l^{FL}\left(\frac{v_0(z,1)}{((1-\delta)\beta)^t}, z, 1; s^{FL}\right)\right]d\Gamma(z) \end{aligned}$$

which implies

$$1 = \delta\int_Z\sum_t(1-\delta)^t\left[\rho l^{FL}\left(\frac{v_0(z,0)}{((1-\delta)\beta)^t}, z, 0; s^{FL\prime}\right) + (1-\rho)l^{FL}\left(\frac{v_0(z,1)}{((1-\delta)\beta)^t}, z, 1; s^{FL\prime}\right)\right]d\Gamma(z)$$

Hence, labor market clearing is satisfied for the forward-looking case. The perfect credit markets goods market clearing condition is equivalent to:

$$1 = \int_Z\left[\omega\left(\frac{y_d^{PC}(z,0;s^{PC\prime})}{y^{PC}}\right)^{\gamma} + (1-\omega)\rho\left(\frac{y_x^{PC}(z,1;s^{PC\prime})}{y^{PC}}\right)^{\gamma}\right]d\Gamma(z)$$

which implies

$$1 = \omega\Delta_w^{(1-\sigma)(1-\alpha)} \int_Z \left(\frac{y_d^{PC}(z, 0; s^{PC})}{y^{PC}} \right)^{\gamma} d\Gamma(z) +$$
$$+ (1-\omega)\Delta_w^{(1-\sigma)(1-\alpha)} \Delta_D^{\sigma-1} \rho \int_Z \left(\frac{y_x^{PC}(z, 1; s^{PC})}{y^{PC}} \right)^{\gamma} d\Gamma(z)$$

Applying the above lemma yields:

$$1 = \omega\delta\Delta_w^{(1-\sigma)(1-\alpha)} \left[\int_Z \sum_t (1-\delta)^t \left(\frac{y_d^{FL}(\nu_0(z,0)/(\beta(1-\delta))^t, z, \phi; s^{FL})}{y^{FL}} \right)^{\gamma} d\Gamma(z) + \right.$$
$$\left. + \frac{1-\omega}{\omega} \Delta_D^{\sigma-1} \rho \int_Z \sum_t (1-\delta)^t \left(\frac{y_x^{FL}(\nu_0(z,1)/(\beta(1-\delta))^t, z, 1; s^{FL})}{y^{FL}} \right)^{\gamma} d\Gamma(z) \right]$$

which implies

$$1 = \omega\delta \left[\int_Z \sum_t \delta^t \left(\frac{y_d^{FL}(\nu_0(z,0)/(\beta(1-\delta))^t, z, 0; s^{FL\prime})}{y^{FL\prime}} \right)^{\gamma} d\Gamma(z) + \right.$$
$$\left. + \frac{1-\omega}{\omega} \rho \int_Z \sum_t (1-\delta)^t \left(\frac{y_x^{FL}(\nu_0(z,1)/(\beta(1-\delta))^t, z, 1; s^{FL\prime})}{y^{FL\prime}} \right)^{\gamma} d\Gamma(z) \right]$$

which is goods market clearing in the forward-looking environment. Hence, the changes in prices from the perfect credit markets environment are also the equilibrium changes in prices in the forward-looking environment. This is equivalent to the statement of Proposition 2.

C Closed Economy

In this section we show that the implications of Proposition 2 are not peculiar to a trade model and they extend to a closed economy framework where there are firm-specific distortions along the lines of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

Environment. We consider a closed economy version of the monopolistic competitive setting considered above. The problem for the stan-in household is the same as in our trade economy. In particular, recall that they inelastically supply one unit of labor. The final consumption good is produced by competitive firms using a CES aggregatorThe final good in the home country is produced using the following CES aggregator:

$$y_t = \left[\int_I y_t(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\sigma/(\sigma-1)} \tag{49}$$

where I_t is the set of active firms. One can then derive the inverse demand functions faced by producers for the intermediated good i:

$$p_t(y(i)) = \omega y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}} \tag{50}$$

A mass of monopolistic competitive intermediate goods producers are operated by entrepreneurs. In every period a mass $\delta \in (0,1)$ of entrepreneurs is born. Each operates a firm and is endowed with a new variety of the intermediate good. Productivity z is drawn from a distribution Γ and it remains constant through time. The firm can produce its differentiated variety using the following constant returns to scale technology:

$$y = zF(k,l) = zk^{\alpha}l^{1-\alpha}, \quad \alpha \in (0,1) \tag{51}$$

where l and k are the labor and capital employed by the firm, and y is total output produced. Every period the production technology owned by the firm becomes unproductive with probability δ.

The firms are subject to idiosyncratic policy distortions. As in Restuccia and Rogerson (2008), we let τ denote a firm-level tax rate. Its value is revealed once the firm draws its productivity z. We also assume that the value of this tax rate remains fixed for the duration of the time for which the establishment is in operation. The type of a firm is then (z,τ). We assume that τ is drawn from some probability distribution $P(\cdot|z)$. We allow for

z and τ to be correlated. (In our trade model, a tariff may be thought of a negative tax on high productivity firms and a subsidy to low productivity firms). We further assume that eventual revenues of the subsidy are lump-sum rebated to the households (or viceversa they are taxed).

As before, the firm has to borrow to finance its operations each period. We consider a decentralization where firms have access to a rental market for capital. We denote the rental capital rate by r_t. Firms can save across periods in contingent securities that pay one unit of the final good next period conditional on the firm's survival. All firms start with a_0 units of the final good, which are transferred to them by the household. Firms are subject to debt limits and a non-negativity on dividend payouts:

$$b_t \leqslant \bar{B}_t(a_t, z, \tau) \tag{52}$$

$$d_t \geqslant 0 \tag{53}$$

The firm's problem can be conveniently written recursively using assets or cash on hand, a, together with its productivity type (z, τ) as state variable:

$$V_t(a, z, \tau) = \max_{d,a'} d + \frac{1-\delta}{R_t} V_{t+1}(a', z, \tau) \tag{54}$$

subject to

$$d + \frac{1-\delta}{R_t} a' \leqslant \pi_t(a, z, \tau)$$

$$d \geqslant 0$$

where profits $\pi_t(a, z, \tau)$ are given by the following static problem:

$$\pi_t(a, z, \tau) = \max_{y,l,b,k} (1-\tau) y_t^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} - w_t l - b + k\,(1-\delta_k) \tag{55}$$

subject to

$$\begin{aligned} y &\leqslant zF(k,l), \\ (1+r_t)\,k &\leqslant a+b, \\ b &\leqslant \bar{B}_t(a,z,\tau) \end{aligned}$$

We will denote the policy functions of the firms associated with the above problems as $\{d_t, a_t', k_t, b_t, y_t, l_t\}_{t=0}^{\infty}$.

Equilibrium. To define an equilibrium for the economy we need to keep track of the evolution of the measure of operating firms over (a,z,τ). Denote such measure by λ_t. Such measure evolves over time according to

$$\begin{aligned} \lambda_{t+1}(A,Z,T) = (1-\delta)\int 1\left\{a'(a,z,\tau)\in A,\ z\in Z, \tau\in T\right\} d\lambda_t \\ + \delta\rho\int_Z 1\{\ a_0\in A, z\in Z,\ \tau\in T\}\, dP d\Gamma. \end{aligned} \tag{56}$$

Market clearing in the final good market requires that

$$y_t = c_t + K_{t+1} - (1-\delta_k)K_t \tag{57}$$

Market clearing for capital requires that

$$K_t = \int k_t(a,z,\tau)d\lambda_t \tag{58}$$

The labor market feasibility is given by

$$1 = l_t(a,z,\tau)d\lambda_t \tag{59}$$

For the bond market to clear, it must be that

$$b_t + A_t = K_t \tag{60}$$

where A_t is the aggregate amount of assets held by firms, $A_{t+1} = (1-\delta)\int a'_t(a, z, \tau)d\lambda_t + \delta \int_Z a'_t(a_0, z, \tau)dPd\Gamma$.

We can then define a *symmetric equilibrium* for the economy in a way analogous to the one in text for the trade model.

Consider a stationary equilibrium for this economy so we can drop the dependence on time and $\beta R = 1$. Under the forward looking specification of debt limits,

$$V(a, z, \tau) = \theta\left(\bar{B}(a, z, \tau) + a\right) + \xi V(0, z, \tau),$$

let aggregates be denoted by

$$Y = Y(\theta, \xi, P), \quad C = C(\theta, \xi, P), \quad K = K(\theta, \xi, P), \quad w = w(\theta, \xi, P)$$

where we let them depend on (θ, ξ) and P to emphasize the dependence of aggregates from idiosyncratic distortion and level of credit market frictions. A version of Proposition 2 holds in this environment:

Proposition 2′. *Under the forward-looking specification, for any change in distortions* P*, the steady state percentage changes in aggregate output and wages are independent of* θ *and* ξ*. Furthermore, firm-by-firm the percentage change in capital usage is independent of* θ *and* ξ*.*

The proof of this proposition is essentially identical to the one of Proposition 2. Consider an economy with perfect credit markets that undergoes a reform that changes the idiosyncratic distortions from P to P'. Let $\Delta w^* = w'/w$ and $\Delta y = y'/y$ be the steady state change in wages and final output and $\Delta k^*(z, \tau, \tau')$ and $\Delta l^*(z, \tau, \tau')$ be the change in capital and labor inputs used by a firm of productivity z that faces taxes τ pre-reform and τ' post-reform. Note that since we are considering a steady state, the rental rate of capital must equal $r = 1/\beta + \delta$ both pre and post reform. Clearly, since

$$\frac{k}{l} = \frac{\alpha}{1-\alpha}\frac{w}{r} \tag{61}$$

it must the be that

$$\frac{\Delta k^*(z,\tau,\tau')}{\Delta l^*(z,\tau,\tau')} = \Delta w^* \Rightarrow \Delta k^*\left(z,\tau,\tau'\right) = \Delta w^* \Delta l^*\left(z,\tau,\tau'\right). \tag{62}$$

Moreover, the optimal k^* must satisfy

$$\begin{aligned} k^* &= \left[\frac{\sigma-1}{\sigma}(1-\tau)\left(z\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha-1}\right)^{\frac{\sigma-1}{\sigma}}\frac{\alpha}{r}\right]^{\sigma} y \\ \Rightarrow \Delta k^*\left(z,\tau,\tau'\right) &= \left(\frac{1-\tau'}{1-\tau}\right)^{\sigma} \Delta w^{*(\alpha-1)(\sigma-1)} \Delta y^* \\ \Rightarrow \Delta w^* &= \left[\frac{\Delta k^*(z,\tau,\tau')}{\Delta y^*}\left(\frac{1-\tau}{1-\tau'}\right)^{\sigma}\right]^{\frac{1}{(\alpha-1)(\sigma-1)}} \end{aligned} \tag{63}$$

Consider now an economy with imperfect credit markets indexed by (θ,ξ). We now argue that if $\{k(a,z,\tau), l(a,z,\tau)\}$ and w where part of the equilibrium for the economy post reform then $w' = \Delta w^* \times w$ and inputs usage for a firm (a,z,τ') is given by

$$\left\{k\left(a,z,\tau'\right), l\left(a,z,\tau'\right)\right\} = \left\{\Delta k^*\left(z,\tau,\tau'\right) \times k(a,z,\tau), \Delta l^*\left(z,\tau,\tau'\right) \times l(a,z,\tau)\right\}$$

To see that this is the case, it is again convenient to work with the dual formulation of the firm's problem. Let $k(\nu,z,\phi;\mathbf{s})$ and $l(\nu,z,\phi;\mathbf{s})$ be the solution to:

$$\Pi(\nu,z,\tau;\mathbf{s}) = \max_{l,k}(1-\tau)y_t^{\frac{1}{\sigma}}\left[z(1-\tau)F(k,l)\right]^{\frac{\sigma-1}{\sigma}} - w(\mathbf{s})\,l - rk \tag{64}$$

subject to

$$q(1-\delta)v \geqslant \theta k + \xi v_0(z,\tau;\mathbf{s}) \tag{65}$$

Given that the financial sector makes zero expected profits, in equilibrium the initial value of the firm v_0 is the solution to:

$$v_0(z,\tau;\mathbf{s}) = \sum_{t=0}^{\infty} (\beta(1-\delta))^t \Pi\left(\frac{v_0(z,\tau;\mathbf{s})}{(\beta(1-\delta))^t}, z, \tau; \mathbf{s}\right)$$

since on path the evolution of v_t conditional on no exit is given by

$$v_t = \frac{v_0(z,\tau;\mathbf{s})}{(\beta(1-\delta))^t},$$

for our conjecture to be true we just have to verify that v_0 goes up by a factor of $\Delta k^*(z,\tau,\tau')$, i.e.

$$\frac{v_0(z,\tau';\mathbf{s}')}{v_0(z,\tau;\mathbf{s})} = \Delta k^*\left(z,\tau,\tau';\mathbf{s},\mathbf{s}'\right).$$

To see that this is the case, consider Π evaluated at the conjectured new policies:

$$
\begin{aligned}
\Pi' &= (1-\tau')y'^{\frac{1}{\sigma}} \left[zF\left(k',l'\right)\right]^{\frac{\sigma-1}{\sigma}} - w\left(\mathbf{s}'\right)l' - rk' \\
&= (1-\tau')y'^{\frac{1}{\sigma}} \left[zk'\left(\frac{\alpha}{1-\alpha}\frac{w'}{r}\right)^{\alpha-1}\right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha}k' \\
&= (1-\tau')y'^{\frac{1}{\sigma}} \left[z\Delta k^* k\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\left[\frac{\Delta k^*}{\Delta y^*}\left(\frac{1-\tau}{1-\tau'}\right)^{\sigma}\right]^{\frac{1}{(\alpha-1)(\sigma-1)}}\right)^{\alpha-1}\right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha}k' \\
&= (1-\tau')y'^{\frac{1}{\sigma}} \left[z\Delta k^* k\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha-1}\left(\frac{\Delta k^*}{\Delta y^*}\right)^{\frac{1}{\sigma-1}}\left(\frac{1-\tau}{1-\tau'}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha}\Delta k^* k \\
&= (1-\tau')\left(\frac{y'}{\Delta y^*}\right)^{\frac{1}{\sigma}} \left[zk\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha-1}\left(\frac{1-\tau}{1-\tau'}\right)^{\frac{\sigma}{\sigma-1}}\Delta k^{*\frac{\sigma}{\sigma-1}}\right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha}\Delta k^* k \\
&= \left[(1-\tau)y^{\frac{1}{\sigma}}z(1-\tau')k\left(\frac{\alpha}{1-\alpha}\frac{w}{r}\right)^{\alpha-1} - \frac{r}{\alpha}k\right]\Delta k^* = \Pi\Delta k^*
\end{aligned}
$$

as wanted. Note that the first equality is the definition of Π', in the second we used (61), in the third (63), the last steps are simple algebraic manipulations.

To verify this is indeed an equilibrium one is left to show that the market clearing conditions are satisfied. This follows from the fact that initial allocation clears market and so the allocations in the economy with perfect credit markets. The proof is identical to the one provided for Proposition 2.

D Endogenous entry in domestic marrket

Finally, we consider the case with an endogenous entry margin (but still $f_x = 0$). We prove a version of Proposition 2 in this environment.

Proposition 11 *Suppose the productivities of intermediate goods firms are drawn from a Pareto distribution and new entrants must pay f_e units of labor to operate. Then if $f_x = 0$, for any change in tariffs the steady state percentage changes in aggregate output and wages are independent of* θ

and ξ.

Proof. The proof of this modified proposition follows closely the proof of Proposition 2 in the online appendix, which proceeds by guess and verify.

The existence of an entry margin affects the proof in two ways. First, the measure of operating firms enters the market clearing conditions. Second, the entry cutoff changes in response to any change in tariffs. In this version, we guess and verify that the percentage change in the measure of operating firms is the same in model with perfect credit markets and with forward-looking credit constraints. That is, define $\bar{z}(\phi)$ as the productivity of the marginal entrant.

In the perfect credit markets model, $\bar{z}(\phi)$ solves:

$$wf_e = \sum_t (1-\delta)^t q^t \pi_t^* (\bar{z}, \phi)$$

In the forward-looking model, $\bar{z}(\phi)$ solves:

$$wf_e = \max_{\nu_0} \left[\sum_t (1-\delta)^t q^t \pi_t \left(\frac{\nu_0}{[(1-\delta)q]^t}, \bar{z}, \phi \right) - \nu_0 \right]$$

where

$$\pi_t(\nu, z, \phi) = \max_{y_d, y_x, l, k} p_{dt}(y_d) y_d + \phi p_{xt}(y_x) y_x - w_t l - r_t k$$

subject to

$$\begin{aligned} y_d + y_x &\leqslant zF(k, l) \\ \nu &\geqslant \frac{\theta}{\beta(1-\delta)} k + \frac{\xi}{\beta(1-\delta)} \tilde{\nu}_0(z) \end{aligned}$$

where $\tilde{\nu}_0(z, \phi)$ is the value for the entrepreneur of restarting a firm given that the set-up cost f_e has been paid. That is, $\tilde{v}_0(z, \phi)$ is the solution to

$$0 = \sum_t (1-\delta)^t q^t \pi_t \left(\frac{\tilde{v}_0(z,\phi)}{[(1-\delta)q]^t}, z, \phi \right) - \tilde{v}_0(z,\phi)$$

The cutoff $\bar{z}(\phi)$ is the minimal productivity level for which there exists a feasible contract that delivers a net-present discounted value of transfers to the lenders equal to the fixed cost of operating, wf_e. That is, there exists a positive v_0 such that $\sum_t (1-\delta)^t q^t \pi_t \left(\frac{v_0}{[(1-\delta)q]^t}, \bar{z} \right) - v_0 \geqslant wf_e$. For low enough level of productivity, it does not exist a feasible v_0 that allows the lenders to recover the initial entry cost. For $z > \bar{z}$, there is a range of promised values to the entrepreneur that guarantees that the lenders can recover the entry cost. For such productivity values, competition among lenders implies that $v_0(z)$ is determined as the largest solution to

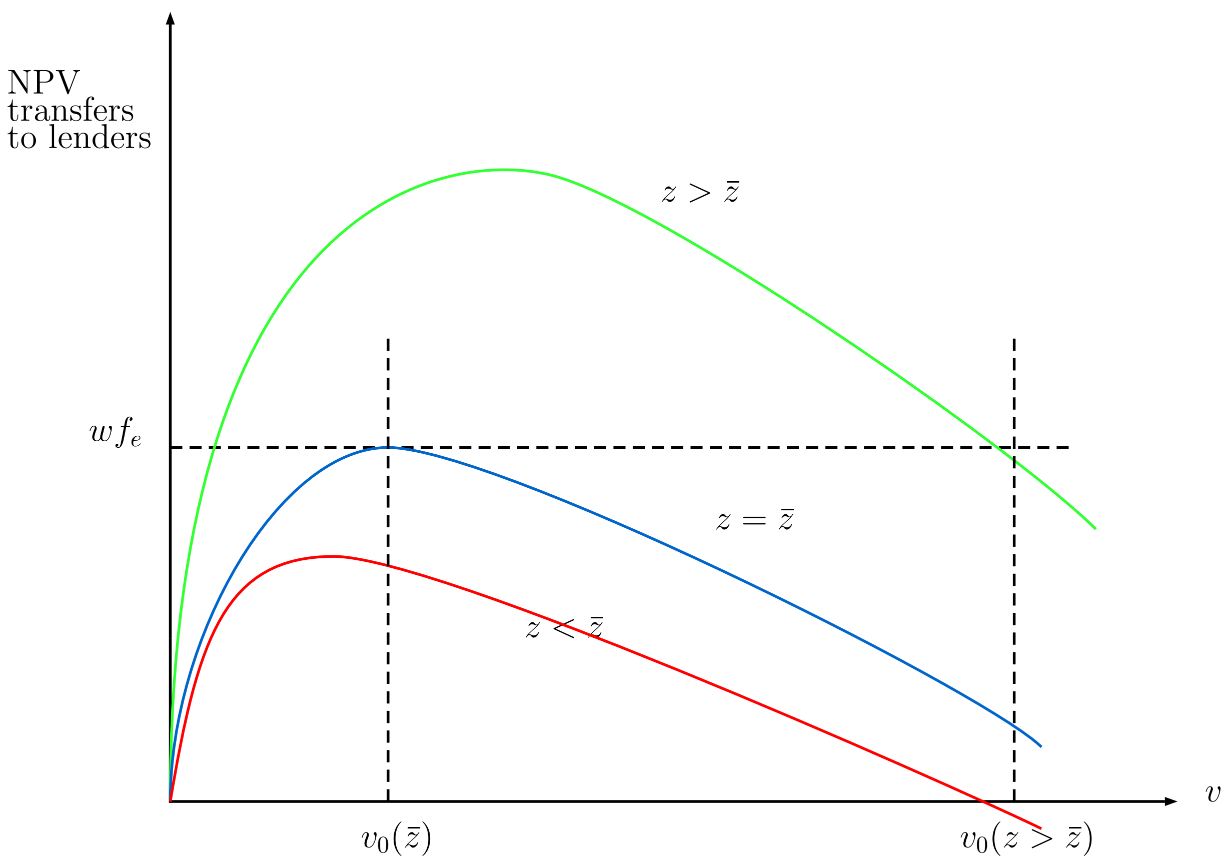
$$wf_e = \sum_t (1-\delta)^t q^t \pi_t \left(\frac{v_0(z)}{[(1-\delta)q]^t}, \bar{z} \right) - v_0(z)$$

as in the baseline model. See Figure 4 for an illustration.

As we do in the proof of Proposition 2 for the baseline case, let D be ω for a firm with $\phi = 0$ and $(\omega^\sigma + (\frac{1-\omega}{1+\tau})^\sigma)^{1/\sigma}$ for a firm that can export. It is straightforward to verify that $\bar{z}$ is proportional to $w^{((1-\alpha)(1-\sigma)-1)/(1-\sigma)}$, $y^{1/(1-\sigma)}$, and $D^{\sigma/(\sigma-1)}$ in both cases. Therefore, if the guess (that all aggregates change by the same percentage in both cases) is correct, then the cutoff $\bar{z}$ also changes by the same percentage with full enforcement or the forward-looking debt limit.

The second modification to the proof is that all market clearing conditions integrate over the set of operating firms, which changes with the tariff regime. When productivities are Pareto-distributed, notice that the ratio of pre-reform market clearing conditions to post-reform market clearing conditions are proportional to $\bar{z}'/\bar{z}$ in both the perfect credit markets and forward-looking constraints cases, where $\bar{z}$ is the marginal firm in the pre-reform environment and $\bar{z}'$ is the marginal firm in the post-reform environment. As

Figure 4: Determination of entry cutoff and initial promised value to entrepreneur $v_0(z)$ for $z \geqslant \bar{z}$



demonstrated above, these ratios are equal in these two environments. Therefore, the ratios in the integrals in the market clearing conditions are the same in the two environments. Combined with the arguments from the previous version of the proof, this is sufficient to show that the guess is verified and completes the proof.

E Robustness exercises

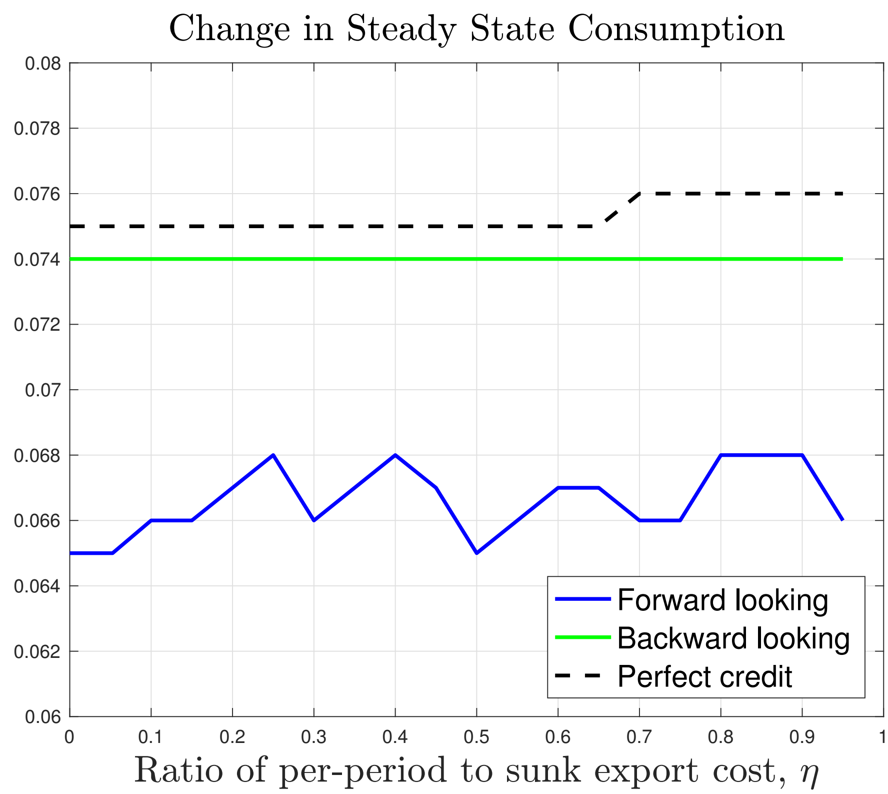
Because the decision to enter the export markets is key to determine the predictions of the two models with debt limits, one may then worry that our results can be driven bythe ratio of the per-period export costs to the sunk cost of export f_x, η, a non calibrated parameter. Here we show that our conclusions are unaffected by different values for η.

Note first that in the model with forward looking constraint (and with perfect credit markets), the total discounted export fixed cost, $F_x = f_x + \sum_{t=0}^{\infty} [(1-\delta)/R]^t \eta f_x$, is a sufficient statistic for the firm's maximixation problem. Thus, modulo general equilibrium effects,[28] with forward-looking debt limits (and perfect credit markets) our simulations are not affected by the way the total exports fixed costs are splitted between sunk and per-period costs.

This is not the case for the model with backward-looking debt limits. Everything else equal, a larger share of sunk costs can delay entry in the export markets. To match the age distribution of exporters (fraction of firms that export before age 10) our calibration procedure would call for more moderate financial frictions. This could in principle affects our results.

[28] Two economies with the same F_x are not exactly equivalent because the average export fixed costs paid by firms in a given period differs from the discounted expectations $F_x = f_x + \sum_{t=0}^{\infty} [(1-\delta)/R]^t \eta f_x$ the interest rates are strictly positive. Hence the resource constraints are not the same in the two economies and so factor prices can differ.

Figure 5: Comparative statics with respect to ratio of per-period to sunk export cost, η



We then recalibrate the three versions of the model for different values of η. Figure 5 shows how the percentage changes in consumption from the high tariffs steady state is affected by various levels for η. The percentage change in consumption is essentially constant across η for all the specifications. For the case with perfect credit markets and with forward-looking debt limits, this simply signifies that the aforementioned general equilibrium effect is negligible. For the collateral constraint model, one would expect that a higher share of per-period fixed costs should make the model collateral constraint behave more like the perfect credit market specifications. However, for the model to match our target moments, in particular the fraction of firms that export before age 10, higher values for the share of per-period costs must be compensated by either i) more inefficient credit markets or ii) higher total export costs (sunk plus per-period). These two changes have a countervailing effect on the welfare gains and it turns out that these two effects essentially cancel each other out.

Finally, one may conjecture that our results may be different if the trade liberalization were anticipated by the firms. This is particularly important for the model with forward-looking constraints. The mere announcement of a future liberalization can improve the static allocation of resources - and so increase TFP - because higher future profits allow productive firms (exporters) to borrow more and get closer to the optimal scale while the opposite happens for low productivity firms. This mechanism is similar to Jermann and Quadrini (2007).

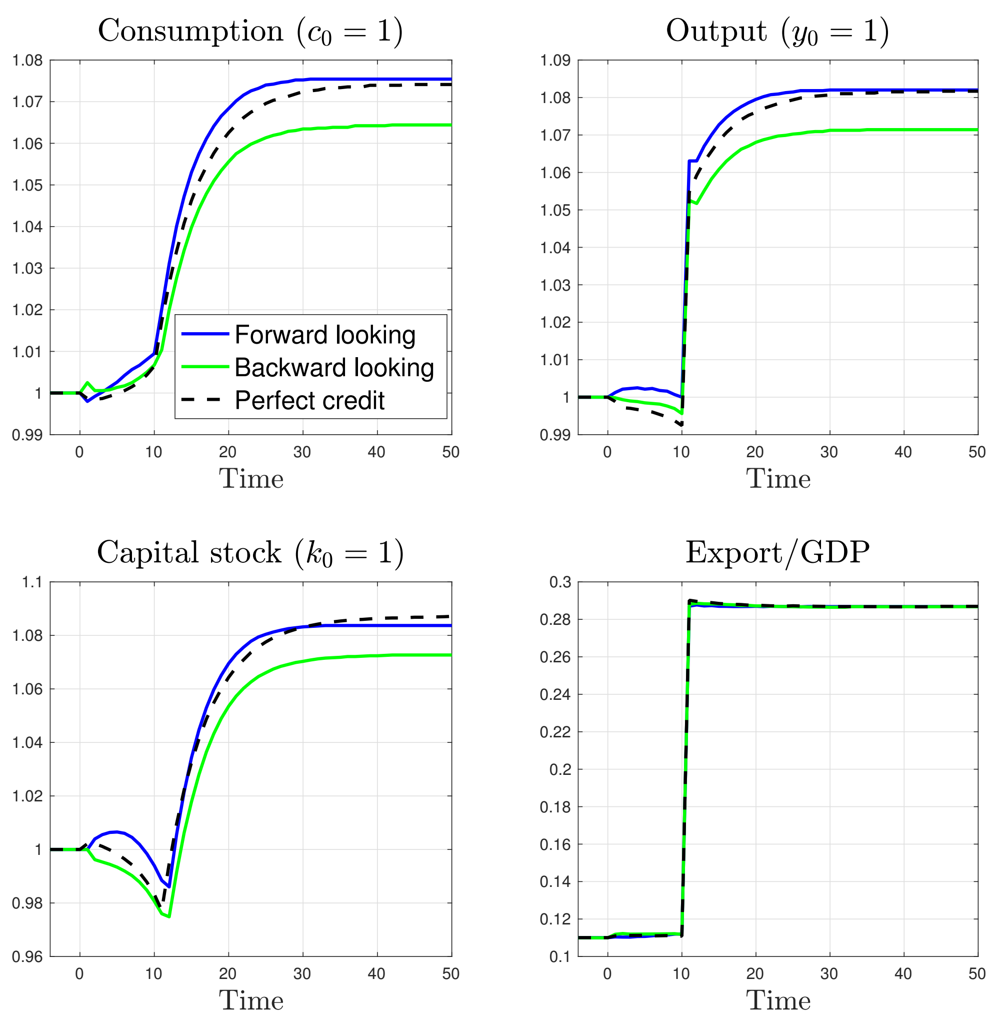
We do not expect large changes for the case with backward-looking constraints. This is because when solving the model we always assume that firms accumulate as much as financial asset as possible in order to give the backward-looking specification the best shot at performing well after a liberalization. So the announcement cannot help the gains in the backward looking specifications by allowing future exporters to accumulate more assets knowing about the increase in future profitability.

As an example, Figure 6 shows the transitional dynamics assuming that the tariff reduction is announced in period 0 and implemented in perios 10. The message is the same as our baseline case. The dynamics of the economy with forward looking debt limits is very similar to the one with perfect credit markets.[29] The only difference is a small increase in output in the economy with forward-looking debt limits between the announcement and the effective liberalization. This is because the future liberalization allows exporters (high productivity firms) to borrow more even before the liberalization as described above. Moreover, note that the capital stock in all three specifications is decreasing from period 0 to period 10. This is because households want to smooth consumption and are anticipating higher future income so they optimally choose to consume less. Under forward-looking debt limits capital is actually hump-shaped between period 0 and period 10. This is because of two countervailing forces: the consumption smoothing force

[29]The gains for the simulated reform in terms of permanent increase in consumption are 4.48% for the forward-looking case, 3.67% for the backward-looking case, and 4.16% under perfect credit markets. The gains are lower than the baseline case because of the delay in the implementation of the liberalization.

calls for reducing investments and the better allocation of resources for high productivity firms increases the returns on investment calling for higher investment rates. The second effects initially dominates while the second effects dominates as period 10 approaches.

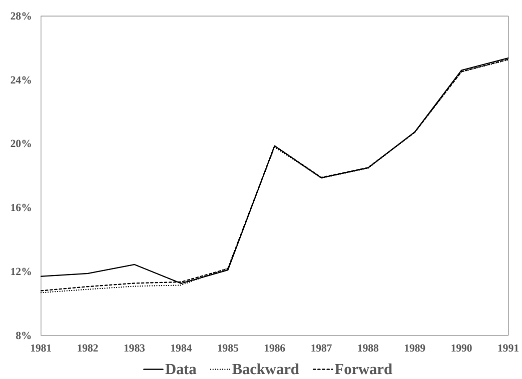
Figure 6: Transition dynamics after an anticipated trade liberalization



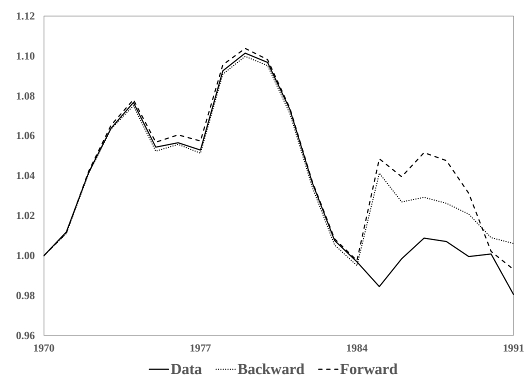
F Details of Aggregate Fluctuations

In Section 6, we simulate the aggregate fluctuations in real GDP in Colombia in the period immediately before the reform. Real GDP grew at an average rate of 2.2% both from 1960-

Figure 7: Targeted Aggregates over Transition



(a) Exports as a fraction of output



(b) Real GDP

1970 and from 1995-2010. Therefore, we first detrend real GDP from the data by 2.2% in the period from 1970-1984. To construct the firm-level model-generated data we feed in both the backward-looking and forward-looking versions of the model sequences of labor productivity shocks and tariff to match aggregates in Colombia. To be precise, the economy begins in an initial steady state. In period 1970, all agents learn of the sequence $\{A_t\}$ of efficiency fluctuations that the economy will experience from 1971-1984, and they believe that $\{A_t\}$ will be constant forever after that and that tariffs will never change. In 1985, they are surprised with a sequence of unforeseen tariffs. In the periods 1985-1991, these tariff values are chosen to replicate the exports as a fraction of total value added in the Colombian data, and after 1991 is constant at its final value. The series for real GDP, and exports over value added are reported in Figure 7.

G Derivation of equation (18)

Consider the programming problem (10). Let λ, χ, and $\mu/(1+r)$ be the multipliers associated with the technological feasibility constraint, the intra-period budget constraint, and the debt limit constraint respectively. The first order conditions with respect to k, b,

y_x, and y_d are

$$0 = (1-\delta_k) + \lambda z \alpha \left(\frac{k}{l}\right)^{\alpha-1} - \chi\left(1+r-\delta_k\right)$$
$$0 = -1 + \chi - \mu/\left(1+r\right)$$
$$0 = p_x\left(y_x\right) + p_x'\left(y_x\right) y_x - \lambda$$
$$0 = p_d\left(y_d\right) + p_d'\left(y_d\right) y_d - \lambda$$

Using the second equation to substitute for χ in the first condition we obtain:

$$r + \mu = \lambda z \alpha \left(\frac{k}{l}\right)^{\alpha-1}$$
$$= \alpha\lambda\frac{y_d + y_x}{k}$$

where the second equality follows from the production function $y_d + y_x = zk^{\alpha}l^{1-\alpha}$. Using that $\lambda = p_d\left(y_d\right) + p_d'\left(y_d\right) y_d = p_x\left(y_x\right) + p_x'\left(y_x\right) y_x$ from the last two first order conditions we can write

$$r + \mu = \alpha\frac{\left[p_d\left(y_d\right) + p_d'\left(y_d\right) y_d\right] y_d + \left[p_x\left(y_x\right) + p_x'\left(y_x\right) y_x\right] y_x}{k}$$

Using (5), (6) and

$$p_d'\left(y_d\right) = -\frac{1}{\sigma}\frac{\omega}{1+\tau}\left(y\right)^{\frac{1}{\sigma}} y_d^{\frac{1}{\sigma}-1}$$
$$p_x'\left(y_x\right) = -\frac{1}{\sigma}\frac{1-\omega}{1+\tau}\left(y_f^f\right)^{\frac{1}{\sigma}} y_x^{\frac{1}{\sigma}-1} P^f,$$

to substitute for p_d, p_x, p_d', and p_x' we obtain

$$r + \mu = \alpha\frac{\sigma-1}{\sigma}\frac{p_d y_d + p_x y_x}{k} = \alpha\frac{\sigma-1}{\sigma}\frac{\text{revenues}}{k}$$

Thus, defining $\mathsf{MPK} = \frac{\text{revenues}}{k}$ and rearranging we obtain

$$\mathsf{MPK} = \frac{1}{\alpha}\frac{\sigma}{\sigma - 1}\left[r + \mu\right]$$

as wanted. A similar derivation applies for firms that do not export.

References for online appendix

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Stokey, N., Lucas, R. and Prescott, E. (1989), *Recursive Methods in Economic Dynamics*, (Cambridge, MA: Harvard University Press).