

Self-Fulfilling Debt Crises: A Quantitative Analysis*

Luigi Bocola[†]

Alessandro Dovis[‡]

June 2019

Abstract

This paper investigates the role of self-fulfilling expectations in sovereign bond markets. We consider a model of sovereign borrowing featuring endogenous debt maturity, risk-averse lenders, and self-fulfilling crises à la Cole and Kehoe (2000). In this environment, interest rate spreads are driven by both fundamental and non-fundamental risk. These two sources of risk have contrasting implications for the maturity structure of debt chosen by the government. Therefore, they can be indirectly inferred by tracking the evolution of debt maturity. We fit the model to Italian data and find that non-fundamental risk played a limited role during the 2008-2012 crisis.

Keywords: Self-fulfilling debt crises, rollover risk, maturity choices, risk premia.

JEL codes: F34, E44, G12, G15

*First draft: February 12, 2015. We have benefited from the detailed comments of the editor (Mark Aguiar), two anonymous referees, as well as Pooyan Ahmadi, Fernando Alvarez, Manuel Amador, Cristina Arellano, Juliane Begenau, David Berger, Anmol Bhandari, Javier Bianchi, Hal Cole, Russell Cooper, Satyajit Chatterjee, V.V. Chari, Cosmin Ilut, Patrick Kehoe, Thibaut Lamadon, Guido Lorenzoni, Ellen McGrattan, Gaston Navarro, Monika Piazzesi, Jesse Schreger, Cédric Tille, Mark Wright, and participants at seminars and conferences. Gaston Chaumont, Parisa Kamali, Tommy Khouang, Sergio Armella Olazabal, and Keyvan Eslami provided excellent research assistance. We thank the Federal Reserve Bank of Minneapolis for hospitality while completing this paper.

[†]Stanford University and NBER

[‡]University of Pennsylvania and NBER

1 Introduction

The idea that lenders' pessimistic beliefs about the solvency of a government can be self-fulfilling has often been used by economists to explain fluctuations in sovereign bond yields. For example, it has been a common justification for the sudden increase in interest rate spreads of southern European economies in 2011, and for their subsequent decline upon the introduction of the Outright Monetary Transactions (OMT) bond-purchasing program.¹ According to this view, these interventions were desirable because they protected members of the euro area from inefficient self-fulfilling crises.

However, evaluating whether investors' beliefs are a trigger of debt crises is challenging in practice, and this makes the interpretation of these "lender of last resort" policies like OMT difficult. Going back to the European case, all the countries that experienced an increase in their borrowing costs were also facing deep recessions and a deterioration of their public finances. Thus, an alternative interpretation of these events is that the increase in sovereign risk was purely due to the worsening of economic fundamentals in these economies, and their decline following the establishment of OMT reflected heightened expectations of future bailouts by the European authorities. Clearly, this alternative interpretation leads to a less favorable assessment of the program, as bailout guarantees can induce governments to overborrow and they introduce balance sheet risk for the ECB.

The contribution of this paper is to provide the first quantitative analysis of a benchmark model of self-fulfilling debt crises, and to use it to measure fundamental and non-fundamental fluctuations in interest rate spreads during the eurozone crisis. In the model, the maturity structure of debt chosen by the government responds differently to these two sources of default risk, and it thus provides information on the relative importance of these forces. Our measurement strategy consists of combining the model with data on interest rates, economic fundamentals and observed debt maturity choices to infer the likelihood of a self-fulfilling crisis. After fitting the model to Italian data, we find that 13% of the interest rate spreads during the 2008-2012 period were due, on average, to non-fundamental risk. We then use this decomposition to assess the implications of the OMT program.

We consider the canonical model of sovereign borrowing in the tradition of [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), and [Arellano \(2008\)](#). A government faces shocks to tax revenues and issues debt of multiple maturities to smooth its expenditures. The government lacks commitment over future policies and, as in [Cole and Kehoe \(2000\)](#), it raises new debt before deciding whether to default. This last assumption leads to the possibility of self-fulfilling *rollover* crises. Lenders, in fact, have no incentives to buy new

¹The program, introduced in September 2012, allowed the European Central Bank (ECB) to purchase sovereign bonds in secondary markets without explicit quantity limits. See Section 6.

bonds when they expect the government to default. As the debt market shuts down, the government may find it too costly to service the maturing debt exclusively out of its tax revenues, and it may thus decide to default, validating the lenders' pessimistic expectations. These crises can arise when the stock of debt coming due is sufficiently large and economic fundamentals are weak.

In this setup, interest rate spreads vary over time because of *non-fundamental* and *fundamental* risk. Specifically, they may reflect the self-fulfilling expectations that lenders will not roll over government debt in the near future, or they may be high because investors fear that the government will default just because a recession makes it challenging to service its debt. While these types of risk have similar effects on interest rate spreads, they have different implications for the maturity structure of government debt.

Consider first a scenario where high interest rates mostly reflect the possibility that lenders will not roll over the debt in the future. As originally emphasized in [Cole and Kehoe \(2000\)](#), the government has an incentive to lengthen debt maturity because, by back-loading payments, it can reduce the debt that needs to be rolled over, lowering in this fashion the possibility of a self-fulfilling rollover crisis. Consider now a scenario where high interest rates are not due to the fear of a rollover crisis but rather reflect bad economic fundamentals. In our model, the government wants to shorten debt maturity in this situation: by doing so, it can reduce the equilibrium interest rates at which it borrows from the lenders, and this is valuable for the government because it allows to better smooth its expenditures.²

Because of these properties, changes in the maturity structure of government debt provide information about the importance of rollover risk. *Everything else equal*, observing a government that lengthens maturity during a crisis is interpreted by the model as evidence of a quantitatively sizable role for rollover risk, while a shortening would be evidence that the underlying sources are fundamental.

In practice, however, this simple reasoning does not take into account that other factors that influence the maturity structure of government debt may systematically vary during debt crises. In particular, debt maturity should respond to changes in the term premium, that is, the additional compensation that lenders demand for holding longer-term debt. An increase in the term premium makes long-term debt effectively more expensive for the government, and it incentivizes the issuance of short-term securities. Because debt crises are typically associated to an increase in the term-premium ([Broner, Lorenzoni, and Schmukler, 2013](#)), not controlling for this force could confound our measurement: rollover risk could be

²As emphasized in [Arellano and Ramanarayanan \(2012\)](#) and [Aguiar, Amador, Hopenhayn, and Werning \(2018\)](#), this happens because the lenders anticipate that future governments have fewer incentives to be exposed to default risk when the inherited maturity structure is short, as any increase in interest rates will have larger refinancing costs. Because a shorter maturity structure disciplines the behavior of future governments, the lenders are willing to charge lower default premia if the government shortens maturity today.

driving interest rate spreads and yet we could observe a shortening of debt maturity simply because it is now more expensive for the government to issue long-term debt. To control for this issue, we allow for a time-varying term premium in the model by introducing shocks to the lenders' stochastic discount factor.

After fitting the model to Italian data, we turn to the main quantitative experiment of the paper, which consists of measuring the rollover risk component of observed interest rate spreads during the 2008-2012 crisis. For this purpose, we apply the particle filter to the model and extract the sequence of structural shocks that accounts for the behavior output, the term premium, debt maturity, and interest rate spreads. Equipped with this path, we construct the counterfactual interest rate spreads that would have emerged if the one-period ahead probability of a rollover crisis was zero throughout the episode. The rollover risk component is then the difference between the observed interest rate spreads and the counterfactual ones. We find that this component represents, on average, 13% of the interest rate spreads observed during the episode. The model assigns a limited role to rollover risk because the average maturity of debt decreased substantially during this episode, and the observed increase in the term premium was not large enough to justify such behavior.

We finally discuss the implications of our analysis for the evaluation of the OMT program. We use the model to compute the interest rate spread that would arise in a counterfactual world without rollover risk. If the main effect of the program was to eliminate rollover risk, then we should observe the post-OMT spread to equal this counterfactual spread. Thus, our test consists in comparing the fall in Italian spreads observed after the establishment of OMT to the one we obtain when we eliminate rollover risk from the model. We find that the decline in spreads in the data is larger than the one obtained in the counterfactual. Through the lens of the model, this result suggests that OMT affected spreads over and beyond the elimination of rollover risk, and it lends support to the view that the policy fostered expectations of future bailouts.

Related literature. There is a long literature on multiplicity of equilibria in models of sovereign debt. While the [Eaton and Gersovitz \(1981\)](#) model with short-term debt has a unique equilibrium,³ the seminal papers of [Alesina, Prati, and Tabellini \(1989\)](#) and [Cole and Kehoe \(2000\)](#) show that the government's inability to commit to current repayments can lead to self-fulfilling rollover crises. Starting with [Conesa and Kehoe \(2012\)](#), [Chatterjee and Eyigungor \(2012\)](#), and [Roch and Uhlig \(2014\)](#), recent papers have introduced this feature in models with income shocks. [Aguiar, Chatterjee, Cole, and Stangebye \(2016\)](#) show that the

³See [Aulert and Rognlie \(2016\)](#) for a proof of this result. Multiple equilibria in the [Eaton and Gersovitz \(1981\)](#) model may arise when the government issues long-term debt, see [Stangebye \(2014\)](#) and [Aguiar and Amador \(2018\)](#).

introduction of time-varying rollover risk allows models of sovereign debt to better capture the behavior of spreads for emerging economies. Our paper is complementary to their analysis. Rather than studying the effect of rollover risk on average, we ask the question of how one can quantify its importance in a historical event, such as the European debt crisis. For this purpose, we enrich the workhorse model with maturity choices and risk-averse lenders and propose a measurement strategy based on the joint dynamics of interest rates, economic fundamentals and debt maturity.

Beside the classic rollover problem of [Cole and Kehoe \(2000\)](#), the literature has emphasized other mechanisms through which lenders' beliefs affect interest rate spreads. [Lorenzoni and Werning \(2013\)](#) and [Ayres, Navarro, Nicolini, and Teles \(2018\)](#) build on [Calvo \(1988\)](#) and show that multiple equilibria can arise because of a feed-back between debt accumulation and interest rate dynamics: a shift in investors' beliefs may lead to an increase in borrowing costs for the government and a path of debt accumulation that can raise the risk of a default, which validates the initial shift in beliefs. See also [Aguiar, Chatterjee, Cole, and Stangebye \(2017\)](#) and [Broner, Erce, Martin, and Ventura \(2014\)](#) for other mechanisms. Our analysis is silent on whether these forces contributed to variation in bond yields during the European debt crisis.

From an econometric viewpoint, the environment we consider is an example of an *incomplete* model ([Tamer, 2003](#)), in which regions of the state space are associated to more than one outcome. There are two approaches in the applied literature to analyze this class of models. In the first approach, the researcher conducts inference by characterizing the model's predictions consistent with the full set of equilibria.⁴ In the second approach, the researcher "completes" the model by introducing a rule to select among the potential outcomes. We follow the second approach. Our selection rule builds on [Cole and Kehoe \(2000\)](#), and it has been used extensively in subsequent studies: when outcomes are not unique, an exogenous sunspot determines whether lenders desert the auction or not. This approach allows us to evaluate a likelihood function and to filter the unobserved state variables using techniques routinely applied to models with a unique equilibrium ([Fernández-Villaverde, Rubio-Ramírez, and Schorfheide, 2015](#)).

The idea of using agents' choices to learn about the types of risk they are facing has a long tradition in economics. A classic example is the use of consumption data along with the logic of the permanent income hypothesis to distinguish permanent and transitory income shocks. See [Cochrane \(1994\)](#) for an application on U.S. aggregate data, [Aguiar and Gopinath \(2007\)](#) for emerging markets, and [Guvenen and Smith \(2014\)](#) for a recent study using micro

⁴See [Lubik and Schorfheide \(2004\)](#) for a general treatment of this approach in New Keynesian linear rational expectation models. [Passadore and Xandri \(2015\)](#) pursue this approach in the context of the [Eaton and Gersovitz \(1981\)](#) model.

data. Clearly, this structural approach is not robust to misspecifications of the trade-offs governing the variables used in the measurement—in our case debt maturity. While the literature is scant on systematic studies documenting the motives driving the management of public debt, documents produced by Treasury departments around the world and historical episodes support the idea that governments actively manage debt maturity to prevent rollover crises.⁵

Layout. The paper is organized as follows. We present the model in Section 2 and discuss our measurement strategy in Section 3. We next turn to the quantitative analysis. Section 4 fits the model to Italian data and discusses its properties, while in Section 5 we use the model to measure the importance of rollover risk during the Italian sovereign debt crisis. We analyze the OMT program in Section 6 and discuss the relevance of our results for other debt crises in Section 7. Section 8 concludes.

2 Model

2.1 Environment and recursive equilibrium

Preferences and endowments. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The exogenous state of the world is $s_t \in S$. We assume that s_t follows a Markov process with transition $\mu(\cdot | s_{t-1})$. It is convenient to split the state into two components, $s_t = (s_{1,t}, s_{2,t})$ where $s_{1,t}$ is the *fundamental* component and $s_{2,t}$ is the *non-fundamental* component. The fundamental component affects endowments and preferences, while the non-fundamental component collects coordination devices that are orthogonal to the fundamentals.

The economy is populated by a large number of lenders and a government. The government receives tax revenues every period and decides the path of spending $\{G_t\}_{t=0}^{\infty}$.⁶ Tax revenues are a constant share τ of the output produced in the economy, $Y_t = Y(s_{1,t})$. The government values a stochastic stream of spending according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(G_t), \quad (1)$$

where the period utility function U is strictly increasing and concave.

⁵For instance, the OECD discusses practical issues related to public debt management in its “Sovereign Borrowing Outlook”, see <http://www.oecd.org/finance/financial-markets/oecdsovereignborrowingoutlook.htm>. See also our discussion in Section 7.

⁶Throughout the paper, we refer to G_t as government spending. However, when going to the data, we will interpret G_t more broadly as incorporating also the transfers that the government makes to the private sector.

The lenders value flows using the stochastic discount factor $M_{t,t+1} = M(s_{1,t}, s_{1,t+1})$. Hence, the value of a stochastic stream of payments $\{d_t\}_{t=0}^{\infty}$ at time zero is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} d_t, \quad (2)$$

where $M_{0,t} = \prod_{j=0}^t M_{j-1,j}$. We assume that the economy is small in that $M(s_{1,t}, s_{1,t+1})$ does not depend on the government's choices, but we allow for correlation between the stochastic discount factor and the output in the economy to capture the cyclicity of risk premia.

Market structure. The government can issue a portfolio of non-contingent defaultable bonds of different maturities. Without loss of generality, we consider portfolios of zero coupon bonds (ZCB). In any period t , the government enters with a stock of promised payments $\{b_t^{(n)}\}_{n \geq 1}$ where $b_t^{(n)}$ is the amount of ZCB of maturity $n \geq 1$. Thus, $b_t^{(1)}$ are ZCB that are due at time t , $b_t^{(2)}$ are ZCB that will mature at $t + 1$, and so forth. For computational convenience, we restrict the portfolios of ZCB that the government can choose to follow an exponential rule. That is, there exists (B_t, λ_t) such that $b_t^{(n)} = (1 - \lambda_t)^{n-1} B_t$ for all $n \geq 1$. We can then summarize the whole portfolio of debt – a highly dimensional object – with just two scalars, (B_t, λ_t) . The variable λ_{t+1} captures the maturity of the stock of debt: higher λ_{t+1} implies that the repayment profile is concentrated at shorter maturities. For instance, if $\lambda_{t+1} = 1$, then all the debt is due next period. Given λ_{t+1} , the variable B_{t+1} controls the face value of debt, which is equal to B_{t+1}/λ_{t+1} . This way of modeling maturity composition is similar to the approach used in the literature for modeling long-term debt ([Chatterjee and Eyigungor, 2012](#); [Hatchondo and Martinez, 2009](#)). The difference is that we allow λ_{t+1} to be chosen by the government and vary over time. This allows us to characterize the dynamics of the term structure of promised payments in a parsimonious and tractable way.⁷

The timing of events within the period follows [Cole and Kehoe \(2000\)](#): the government first issues new debt, lenders choose the price for the debt, and then the government decides to default or not, $\delta_t = 0$ or $\delta_t = 1$ respectively. We assume that if the government defaults, it is excluded from financial markets and suffers losses in output. We denote by $\underline{V}(s_{1,t})$ the value of being in default for the government. Lenders that hold inherited or newly issued debt do not receive any repayment.⁸ Differently from the timing in [Eaton and Gersovitz](#)

⁷Our modeling of the maturity choices differs from the formulation of [Arellano and Ramanarayanan \(2012\)](#) and builds on recent work by [Sánchez, Saprizza, and Yurdagul \(2015\)](#) and [Bai, Kim, and Mihalache \(2014\)](#).

⁸The assumption of a zero recovery rate is made for tractability. One could obtain a non-zero recovery rate by modeling the debt restructuring process along the lines of [Benjamin and Wright \(2009\)](#) and [Yue \(2010\)](#). Note that, differently from [Cole and Kehoe \(2000\)](#), the government cannot use the funds raised in the issuance stage if it defaults. Our formulation simplifies the problem and does not change its qualitative features. The same formulation has been adopted in other works; see, for instance, [Aguilar et al. \(2018\)](#).

(1981), the government cannot commit to repay within the current period. As we will see, this assumption opens the door to the possibility of rollover crises.

The budget constraint for the government when it does not default is

$$G_t + B_t \leq \tau Y_t + \Delta_t, \quad (3)$$

where Δ_t is the net amount of resources that the government raises in the period,

$$\Delta_t = \sum_{n=1}^{\infty} q_t^{(n)} \left[(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t \right], \quad (4)$$

and $q_t^{(n)}$ is the price of a ZCB of maturity n issued at time t . In the above expression, if a government enters the period with a portfolio (B_t, λ_t) and wants to exit it with a portfolio (B_{t+1}, λ_{t+1}) , then it must issue additional $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ ZCB of maturity n . When $(1 - \lambda_{t+1})^{n-1} B_{t+1} - (1 - \lambda_t)^n B_t$ is negative, the government is buying back the ZCB of maturity n .

Recursive equilibrium. We consider equilibria with a Markovian structure. We denote by $\mathbf{S} = (B, \lambda, s)$ the state in the current period and by \mathbf{S}' the state next period. A government that has not defaulted first decides the new debt issuances, (B', λ') , anticipating the debt prices, $q^{(n)}(\mathbf{S}, B', \lambda')$, and its default decision at the end of the period, $\delta(\mathbf{S}, B', \lambda')$. Formally, the value for the government and debt issuances decisions solve the following Bellman equation:

$$V(\mathbf{S}) = \max_{B', \lambda', G} \delta(\mathbf{S}, B', \lambda') \{U(G) + \beta \mathbb{E}[V(\mathbf{S}') | \mathbf{S}]\} + [1 - \delta(\mathbf{S}, B', \lambda')] \underline{V}(s_1) \quad (5)$$

subject to the budget constraint

$$\begin{aligned} G + B &\leq \tau Y(s_1) + \Delta(\mathbf{S}, B', \lambda'), \\ \Delta(\mathbf{S}, B', \lambda') &= \sum_{n=1}^{\infty} q^{(n)}(\mathbf{S}, B', \lambda') \left[(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B \right]. \end{aligned}$$

The lenders' no-arbitrage conditions require that

$$q^{(n)}(\mathbf{S}, B', \lambda') = \delta(\mathbf{S}, B', \lambda') \mathbb{E} \left[M(s_1, s'_1) \delta' q^{(n-1)}(\mathbf{S}', B'', \lambda'') | \mathbf{S} \right] \text{ for } n \geq 1, \quad (6)$$

where δ' , B'' , and λ'' are optimal default, debt, and maturity given the state $\mathbf{S}' = (B', \lambda', s')$ and $q^{(0)}(\mathbf{S}, B', \lambda') = 1$. The presence of $\delta(\mathbf{S}, B', \lambda')$ in equation (6) is the key difference between the [Cole and Kehoe \(2000\)](#) framework and the one in [Eaton and Gersovitz \(1981\)](#),

and it implies that new lenders receive a payout of zero in the event of a default today. Because of this feature, the pricing schedule depends not only on the exogenous state s and the portfolio (B', λ') chosen by the government, but also on inherited liabilities because (B, λ) affect the current default decision $\delta(\mathbf{S}, B', \lambda')$.

The optimal default decision of the government at the end of the period must satisfy

$$\delta(\mathbf{S}, B', \lambda') = \begin{cases} 1 & \text{if } V(\mathbf{S}) \geq \underline{V}(s_1) \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

That is, the government decides to repay if and only if the value associated with repayment is weakly higher than the value of defaulting.⁹

A *recursive equilibrium* is a value function for the borrower V , associated decision rules $\{B', \lambda', G, \delta\}$, and a pricing function $q = \{q^{(n)}\}_{n \geq 1}$ such that $\{V, B', \lambda'\}$ are a solution for the government problem (5), the default decision satisfies (7), and q satisfies the no-arbitrage conditions (6).

2.2 Multiplicity of equilibria and Markov selection

This economy features multiple recursive equilibria. Specifically, there are states of the world in which lenders' expectations of a default are self-fulfilling: if lenders expect the government to default today and do not buy new bonds, the government finds it optimal to default, whereas if lenders believe that the government repays and they roll over the maturing debt, the government indeed repays.

To understand how this situation can arise, it is convenient to define the price at which debt would be traded if in state (s, B, λ) the government repays. We refer to it as the *fundamental price*,

$$q^{\text{fund},(n)}(s, B', \lambda') = \mathbb{E} \left\{ M(s_1, s'_1) \delta' q^{(n-1)}(s', B'', \lambda'') \mid \mathbf{S} \right\}, \quad (8)$$

and we denote by Δ^{fund} the amount of resources that the government raises at those prices,

$$\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') = \sum_{n=1}^{\infty} q^{\text{fund},(n)}(s, B', \lambda') \left[(1 - \lambda')^{n-1} B' - (1 - \lambda)^n B \right].$$

We can then partition the state space \mathcal{S} into three different regions: the *default zone*, the *safe zone*, and the *crisis zone*. As we shall see momentarily, indeterminacy of outcomes can

⁹Condition (7) does not allow us to characterize the default decision for off-path histories where debt prices are not equal to (6). However condition (7) is enough to characterize the equilibrium path. We further discuss this issue in the online Appendix A.

arise only in the crisis zone.

In the default zone, the government chooses to default even if the lenders expect a repayment and the bonds are priced according to equation (8). Formally, the default zone is the set of states \mathcal{S}^{def} satisfying

$$\max_{B', \lambda'} \left\{ U \left(\tau Y(s_1) - B + \Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \right) + \beta \mathbb{E} [V(B', \lambda', s') | \mathbf{S}] \right\} < \underline{V}(s_1). \quad (9)$$

The left side of condition (9) is the value to the government of repaying when the lenders expect a repayment today. When this value is lower than the outside option, the unique outcome has the government defaulting on its debt.

In the safe zone, the government decides to repay even if the lenders expect a default and they are not willing to purchase new bonds. Formally, the safe zone is the set of states $\mathcal{S}^{\text{safe}}$ that satisfy

$$\begin{aligned} \max_{B', \lambda'} \left\{ U \left(\tau Y(s_1) - B + \Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \right) + \beta \mathbb{E} [V(B', \lambda', s') | \mathbf{S}] \right\} &\geq \underline{V}(s_1) \quad (10) \\ \text{subject to } \Delta^{\text{fund}}(\mathbf{S}, B', \lambda') &\leq 0. \end{aligned}$$

The left side of condition (10) is the value to the government of repaying when it cannot issue new debt, that is, when $\Delta^{\text{fund}} \leq 0$.¹⁰ If this value is larger than the outside option, then the government always repays its debt. Thus, when $\mathcal{S} \in \mathcal{S}^{\text{safe}}$, the unique outcome has the government repaying.

The *crisis zone*, $\mathcal{S}^{\text{crisis}}$, is the set of states for which neither condition (9) nor condition (10) holds. When this happens, the default decision depends on the beliefs of the lenders. If the lenders expect a repayment and they price bonds according to equation (8), the government will repay (because condition (9) does not hold). If the lenders expect a default and the government cannot issue new debt, then the government will default (because condition (10) does not hold).

To select among these possible outcomes, we assume a rule that describes how lenders coordinate their expectations in the crisis zone. We assume that when $\mathbf{S} \in \mathcal{S}^{\text{crisis}}$, agents coordinate their expectations using the non-fundamental state variables s_2 . Specifically, s_2 is composed of two elements, ζ and π . The variable ζ indicates whether a rollover crisis takes place if the government is currently in the crisis zone: if $\zeta = 0$, lenders roll over government debt and there is no default; if $\zeta = 1$, instead, lenders do not roll over the maturing debt

¹⁰The government has two options when the lenders are not willing to purchase new debt. The first is to just repay the debt coming due. The second is to repay the debt coming due *and* buy back part of the debt that will mature in the future. Under the most pessimistic expectations for the lenders, these buybacks occur at fundamental prices. This explains why the relevant constraint for the government in (10) is $\Delta^{\text{fund}}(\mathbf{S}, B', \lambda') \leq 0$. See online Appendix A for more details.

and there is a default. The variable π is the probability that $\zeta' = 1$. This selection rule is the same as the one employed in [Cole and Kehoe \(2000\)](#), with the exception that π varies over time. Conditional on this selection rule, the outcomes of the debt auctions are unique in the crisis zone.

It is important to stress that the government defaults when a rollover crisis takes place, that is, when $\mathbf{S} \in \mathcal{S}^{\text{crisis}}$ and $\zeta = 1$. Thus, interest rate spreads are not defined when $\zeta = 1$ and the government defaults because it is excluded from financial markets. This does not mean that non-fundamental shocks do not affect the behavior of interest rate spreads. As we shall see in [Section 3](#), the possibility of *future* rollover crises has an impact on current interest rate spreads, implying that shocks to π influence interest rate spreads.

2.3 Discussion

Before continuing, let us discuss some important aspects of the model.

First, our model builds closely on [Cole and Kehoe \(2000\)](#), where the government's inability to commit to repayments within the period leads to the possibility of rollover crises, and variation in lenders' beliefs about these events affects interest rate spreads. More recent papers have proposed other mechanisms through which lenders' beliefs matter for the behavior of interest rate spreads. [Aguiar et al. \(2017\)](#) propose a variant of [Cole and Kehoe \(2000\)](#) in which a crisis results in bond auctions at depressed prices rather than in a run, while [Lorenzoni and Werning \(2013\)](#) present a model where self-fulfilling debt crises are not due to rollover problems but to a feedback between debt accumulation and interest rate spreads as in [Calvo \(1988\)](#). Our quantitative analysis is intended to evaluate the classic framework of [Cole and Kehoe \(2000\)](#), and it is silent about these different approaches.

Second, besides looking at a particular source of indeterminacy, our approach takes a stand on how this indeterminacy is resolved. Specifically, we construct a sunspot equilibrium where lenders coordinate exclusively on the non-fundamental state variables ζ and π . This is the standard approach in the literature, and we consider it a useful benchmark. As we discuss in the next section, however, the key restrictions that we use to quantitatively assess the role of non-fundamental risk are robust to more general selection rules where ζ and π are functions of fundamental state variables.

Third, it is important to stress one difference between our approach and the one typically followed in the sovereign debt literature. Most papers in this literature consolidate the private and the public sector and study the decision problem of a benevolent government that directly chooses the external debt of a country. Implicit in this approach are the assumptions that the government has enough instruments to control the saving behavior of domestic

agents and that the government can discriminate between domestic and foreign bondholders when a default takes place. Under these assumptions, domestic public debt is irrelevant for the decision to default. While both assumptions might be appropriate for an emerging market economy, we believe they are not for a country that belongs to the euro area.¹¹ For this reason, we deviate from this practice and consider the decision problem of a government that faces random tax revenues and chooses total public debt to maximize the value of government spending.¹² Formally, the decision problems of the government in these two approaches are equivalent. However, their predictions apply to a distinct set of variables: total external debt in the canonical approach, and total public debt in our approach. This distinction will be relevant in the quantitative analysis.

3 Measuring rollover risk: the role of maturity choices

In the environment presented in the previous section, interest rate spreads are driven by both fundamental and non-fundamental risk. The goal of our analysis is to measure the relative importance of these two forces. In this section, we discuss this inference problem and explain our approach.

After standard manipulation of equation (6), we can express the difference between the yield of a bond maturing next period, $r_t^{(1)}$, and the risk-free rate, $r_t^{*,(1)} = 1/\mathbb{E}_t[M_{t,t+1}]$, as

$$\frac{r_t^{(1)} - r_t^{*,(1)}}{r_t^{(1)}} = \Pr_t(\delta_{t+1} = 0) - \text{Cov}_t\left(\frac{M_{t,t+1}}{\mathbb{E}_t[M_{t,t+1}]}, \delta_{t+1}\right). \quad (11)$$

Interest rate spreads reflect both the probability of a future default by the government and the compensation that lenders demand for being exposed to this risk.

Default risk in the model can be further decomposed:

$$\Pr_t(\delta_{t+1} = 0) = \Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}}) + \Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}) \times \pi_t.$$

First, there is a chance that at $t + 1$ the government will be in the default zone, an event that occurs with probability $\Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{default}})$. Second, there is a chance of a self-fulfilling debt crisis at $t + 1$, an event that occurs with probability π_t if the government is in the crisis zone at $t + 1$.

¹¹First, European rules on capital mobility across borders make it challenging for a euro area government to control the behavior of the private net foreign asset position. Second, almost all public debt of euro area governments was issued under domestic laws, which makes discrimination legally cumbersome.

¹²In a companion paper, we show that this decision problem arises from a more primitive environment in which we model explicitly domestic and foreign lenders. See [Bocola, Bornstein, and Dovis \(2018\)](#).

Ultimately, the goal of our analysis is to measure the component of interest rate spreads that is due to the risk of a future self-fulfilling crisis. Our approach consists of indirectly inferring this component by studying, through the lens of the model, the joint dynamics of interest rate spreads, economic fundamentals and debt maturity. Why is debt maturity useful for this purpose? Because, according to the model, it should behave differently in response to fundamental and non-fundamental risk. Specifically, the government in our model wants to *lengthen* debt maturity when rollover risk increases, while it wants to *shorten* it when default risk is mostly due to a deterioration of economic fundamentals. Because of this property, changes in the maturity structure of government debt provide information on the relative importance of fundamental and non-fundamental risk in accounting for the movements in observed interest rate spreads.

In what follows, we explain the trade-offs that the government faces when choosing debt maturity. Online Appendix B provides a formal analysis of these trade-offs in a three-period version of the model.

Maturity choices and rollover risk. To understand how debt maturity responds to an increase in rollover risk, it is important to note that the government can partly control the risk of facing a rollover crisis next period, $\Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}) \times \pi_t$. By managing its public debt, the government can alter the boundaries of the crisis zone defined by conditions (9) and (10), affecting in this fashion $\Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}})$. Because rollover crises are costly, the government responds to an increase in π_t by taking actions that reduce the risk of being in the crisis zone at $t + 1$. As emphasized in Cole and Kehoe (2000), this can be achieved by lengthening the maturity structure of government debt.

To understand why lengthening debt maturity at time t reduces the exposure of the government to a rollover crisis at $t + 1$, consider a variation in which the government extends the maturity of its debt while keeping constant the amount of resources it raises at time t . This is achieved by decreasing λ_{t+1} and reducing B_{t+1} by the appropriate amount. This variation unambiguously increases the left side of condition (10) whenever $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}$, which has the effect of shrinking the crisis zone.

To illustrate the logic of this result, we can write the left side of condition (10) under the assumption of no buybacks as

$$U(\tau Y_{t+1} - B_{t+1}) + \beta \mathbb{E}_{t+1} [V((1 - \lambda_{t+1})B_{t+1}, \lambda_{t+1}, s_{t+2})]. \quad (12)$$

If $\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}$, we know that the government would choose positive debt issuances if it could borrow.¹³ This means that the marginal utility of consumption at $t + 1$ is higher

¹³This can be shown as follows. Condition (9) implies that the government at \mathbf{S} prefers to repay if it can

than the expected marginal loss in future value due to higher debt. Because extending debt maturity at time t effectively increases $\tau Y_{t+1} - B_{t+1}$ at the expense of higher debt payments in the future, it increases the expression in (12). As this expression increases, the set of states satisfying condition (10) shrinks, and the probability of falling in the crisis zone at $t + 1$ is reduced.

Thus, when π_t increases, the government has an incentive to lengthen debt maturity. Importantly, this discussion assumes that π_t is exogenous and does not respond to the actions of the government. If lenders were to coordinate on fundamental shocks or endogenous variables, the logic that extending debt maturity reduces the risk of a future rollover crisis would go unchanged unless the probability that lenders coordinate on a run is an increasing function of debt maturity, $\pi_t = f(1/\lambda_{t+1})$ with $f(\cdot)$ increasing. In such a case, the probability of a rollover crisis next period would be $\Pr_t(\mathbf{S}_{t+1} \in \mathcal{S}^{\text{crisis}}) f(1/\lambda_{t+1})$. Thus, lengthening debt maturity would on the one hand decrease the probability of being in the crisis zone as described above, but on the other hand it would increase π_t , implying an ambiguous effect. Because there are no particular reasons to assume that π_t is an increasing function of debt maturity, we abstract from this issue.

Maturity choices and fundamental risk. To understand how debt maturity responds to fundamental risk, we consider a version of the model with $\pi_t = 0$ for all t . This is equivalent to adopting the timing convention in Eaton and Gersovitz (1981). The behavior of debt maturity in this environment has been previously studied theoretically by Aguiar et al. (2018), Dovis (2017), and Niepelt (2014), and quantitatively by Arellano and Ramanarayanan (2012), Sánchez, Saprizza, and Yurdagul (2015), and Hatchondo, Martinez, and Sosa Padilla (2016) among others. These papers have emphasized two channels as the main determinants of the maturity composition of debt: the *incentive* channel and the *insurance* channel.

The incentive channel makes short-term debt desirable. Consider the price of a ZCB that matures in $n > 1$ periods in equation (6). This price depends not only on the possibility of a default tomorrow but also on the reselling value of the bond next period, which in turn depends on the issuance decisions of future governments: a higher B'' increases default risk going forward, and it depresses the value of long-term bonds today. This feature creates a time inconsistency problem. Future governments do not internalize the negative effects that new issuances have on the price of long term debt today, and they borrow more than what is optimal from the perspective of the current government. This gives the current government an incentive to shorten debt maturity because, by doing so, it disciplines

freely choose a portfolio at the fundamental prices. Condition (10) states that the government at \mathbf{S} prefers to default when net issuances cannot be positive, $\Delta^{\text{fund}} \leq 0$. Hence, it must be that the maximum on the left side of (9) is attained for a portfolio with $\Delta^{\text{fund}} > 0$, else we would have that $\mathbf{S} \notin \mathcal{S}^{\text{crisis}}$.

the borrowing behavior of future governments. This discipline effect arises because the maturity structure affects borrowing incentives: a government that inherits mostly short-term debt understands that any increase in interest rates raises the costs of rolling over the debt and reduces its consumption; thus, the government has fewer incentives to borrow and be exposed to default risk. Thus, by shortening its debt maturity, the current government aligns the actions of future governments to its preferred spending path and, by doing so, reduces the interest rates at which it can issue debt today.

While the incentive channel generates a motive to issue short-term debt, the insurance channel makes long-term debt desirable because it is a better instrument to provide insurance against shocks. To illustrate this point, consider a situation in which tax revenues decrease. Typically, this shock increases the likelihood of a default and the interest rates on new issuances. If all inherited debt is short term, the government has to refinance its stock of debt at the new high interest rates, and so either its current consumption or its continuation value must decline. If instead part of the inherited debt is long term, only a fraction of the stock of debt has to be refinanced at higher interest rates, and the government will be able to keep higher current consumption and continuation value. The opposite happens in response to a positive shock to tax revenues. Therefore, a risk-averse government prefers issuing long-term debt because this instrument reduces consumption volatility.

The relative strength of the incentive and insurance channels shapes the portfolio choices of the government. For our purposes, it is important to understand how fundamental shocks affect this trade-off. While we are not aware of an analytical characterization of this comparative static exercise in the literature, typical calibrations of sovereign debt models imply that the government shortens its debt maturity when tax revenues decline; see, for example, [Arellano and Ramanarayanan \(2012\)](#). This result, which will be verified in our quantitative analysis, can be justified as follows.

First, when default risk increases, the incentive role of short-term debt becomes more valuable from the government's perspective. States in which default risk is high are also states in which the government would like to issue more debt for consumption-smoothing motives. By shortening the maturity structure of its debt, the government can reduce the interest rates at which it borrows because lenders price in the disciplining role of the maturity structure on future government borrowing. This allows the government to raise more resources today and to better smooth consumption. Second, this shortening of debt maturity does not necessarily come at a cost of less insurance for the government. As discussed in [Dovis \(2017\)](#), the need to issue long-term debt for insurance reasons in this class of models falls when default risk increases.¹⁴

¹⁴This happens because pricing schedules are more sensitive to shocks when the government approaches the default zone. Thus, the conditional volatility of bond prices is higher after bad shocks, implying that the

Given our restriction on issuance policies, the government needs to buy back the outstanding debt of sufficiently large residual maturity when it shortens its portfolio: that is, if $\lambda_{t+1} > \lambda_t$, then eventually $(1 - \lambda_{t+1})^{n+1} B_{t+1} < (1 - \lambda_t)^n B_t$. This result may appear to be at odds with the results in [Aguiar et al. \(2018\)](#), who shows that debt buy backs are not optimal. In their environment, there are no restrictions on the portfolios that the government can issue, so the government can shorten the maturity of the outstanding stock simply by issuing one-period debt. These trades do not satisfy our restrictions, and they are approximated in our model by positive net issuances at short horizons and negative net issuances (buybacks) for long-term bonds. Moreover, their result does not necessarily apply to our economy with output shocks: [Dovis \(2017\)](#) shows that buy backs can be optimal in a sovereign debt model with output shocks.

Summary and quantitative analysis. So far, we have argued that the dynamics of debt maturity provide information on the sources of default risk. In what follows, we build on this insight and use the joint dynamics of interest rate spreads and debt maturity to quantify the importance of rollover risk during the Italian debt crisis of 2008-2012.

Before proceeding, it is important to stress that observed debt maturity depends not only on the government's incentives but also on investors' preferences for the maturity of the bonds they are purchasing. These preferences may vary over time, and they may be a confounding factor in our measurement strategy. For example, a government that is facing high rollover risk may not be willing to lengthen debt maturity if, at the same time, lenders demand high compensation for holding long-term bonds. This view finds support in the data, as [Broner, Lorenzoni, and Schmukler \(2013\)](#) have documented that risk premia on long-term bonds increase during debt crises. In the quantitative analysis that follows, we control for these confounding factors by considering a stochastic discount factor for the lenders that can generate time variation in risk premia on long-term bonds.

4 Quantitative analysis

We now fit the model to Italian data. This section proceeds in four steps. Section [4.1](#) introduces functional forms, and it describes the quantitative strategy. Section [4.2](#) presents the data and reports the numerical values of the model's parameters. Section [4.3](#) studies the fit of the model. Section [4.4](#) discusses the behavior of interest rate spreads and debt maturity conditional on fundamental and non-fundamental shocks. The data Appendix provides detailed information on variables' definitions and sources.

government obtains more insurance holding constant the maturity structure of government debt.

4.1 Functional forms and quantitative strategy

We model the lenders' real stochastic discount factor, $M_{t,t+1} = \exp\{m_{t,t+1}\}$, following [Ang and Piazzesi \(2003\)](#),

$$\begin{aligned} m_{t,t+1} &= -(\phi_0 + \phi_1\chi_t) - \frac{1}{2}\kappa_t^2 + \kappa_t\varepsilon_{\chi,t+1}, \\ \chi_{t+1} &= \rho_\chi\chi_t + \varepsilon_{\chi,t+1} \quad \varepsilon_{\chi,t+1} \sim \mathcal{N}(0,1), \\ \kappa_t &= \kappa_0 + \kappa_1\chi_t, \end{aligned} \tag{13}$$

where $\theta_{\text{sdf}} = [\phi_0, \phi_1, \kappa_0, \kappa_1, \rho_\chi]$ collects the parameters and χ_t is a shock. Depending on the model's parameters, this shock can affect the premia that lenders demand for holding long-term bonds. In order to see that, consider the price of a risk-free ZCB maturing in n quarters, $q_t^{*,(n)}$. This price solves the recursion

$$q_t^{*,(n)} = \mathbb{E}_t \left[\exp\{m_{t,t+1}\} q_{t+1}^{*,(n-1)} \right],$$

with initial condition $q_t^{*,(0)} = 1$. Given the log-normality of $M_{t,t+1}$, we can solve for $q_t^{*,(n)}$ in closed form and compute the difference in yields for bonds with different maturity using the formula that relates bond yields to their prices, $r_t^{*,(n)} = -(1/n) \log(q_t^{*,(n)})$.

For example, the difference in yields on bonds that mature in two periods relative to bonds maturing in one period is

$$r_2^{*,(2)} - r_1^{*,(1)} = \underbrace{\phi_1 \left(\frac{1 + \rho_\chi}{2} - 1 \right) \chi_t}_{\text{Expectation hypothesis}} - \underbrace{\left[\left(\kappa_0 + \frac{\phi_1^2}{2} \right) + \kappa_1\chi_t \right]}_{\text{Risk premia}}.$$

From this expression we can see that the slope of the yield curve for non-defaultable bonds might be different from zero because of two distinct effects. The first is a term that captures the standard expectation hypothesis ([Cochrane, 2009](#)). The second is a term capturing a *risk premium* for holding long-term debt. This term is greater than zero when $m_{t,t+1}$ is negatively correlated with innovations to future bond prices—that is, lenders demand a premium for holding long-term debt when the value of these assets falls in “bad” times (high $m_{t,t+1}$ states). Moreover, this risk premium varies with χ_t when $\kappa_1 \neq 0$ because of movements in the conditional variance of $m_{t,t+1}$. By affecting risk premia on long-term bonds, shocks to χ_t affect the costs for the government to issue long-term securities, capturing in this fashion the confounding forces discussed in the previous section.

The government discounts future flow utility at the rate β . The utility function is

$$U(G_t) = \frac{(G_t - \underline{G})^{1-\sigma} - 1}{1-\sigma},$$

where \underline{G} is a non-discretionary level of public spending. We interpret \underline{G} as capturing the components of public spending that are hardly modifiable by the government in the short run, such as wages of public employees and pensions. This specification helps the model to match the cyclicity of government's debt in the data.

We introduce a utility cost for deviating from a target level of debt maturity \bar{d} ,

$$\alpha \left(\frac{1}{4\lambda'} - \bar{d} \right)^2.$$

This cost serves two purposes. First, it leads to well-defined maturity choices in regions of the state space where the government would be otherwise nearly indifferent over λ' .¹⁵ This ameliorates the convergence properties of the algorithm that we use to numerically solve the model. Second, it gives the model enough flexibility to match the level and the volatility of debt maturity in the sample.

The output process, $Y_t = \exp\{y_t\}$, depends on the factor χ_t and on its innovations,

$$y_{t+1} = \mu_y(1 - \rho_y) + \rho_y y_t + \rho_{y\chi} \chi_t + \sigma_y \varepsilon_{y,t+1} + \sigma_{y\chi} \varepsilon_{\chi,t+1}, \quad \varepsilon_{y,t+1} \sim \mathcal{N}(0, 1). \quad (14)$$

We allow for correlation between χ_t and y_t in order to capture the cyclicity of risk premia.

If the government defaults, it is excluded from capital markets for a random period of time, and it has a probability of reentering equal to ψ . While in default, the government suffers a loss in tax revenues equal to d_t . This is motivated by evidence that sovereign defaults lead to severe financial and output disruptions (Hébert and Schreger, 2017; Bocola, 2016), and they should therefore imply a loss in the fiscal revenues of the government. These costs are parametrized following Chatterjee and Eyigungor (2012),

$$d_t = \max\{0, d_0 \tau Y_t + d_1 (\tau Y_t)^2\}. \quad (15)$$

We assume the following stochastic process for the sunspot, $\pi_t = \frac{\exp\{\tilde{\pi}_t\}}{1 + \exp\{\tilde{\pi}_t\}}$, and

$$\tilde{\pi}_{t+1} = \pi^* + \sigma_\pi \varepsilon_{\pi,t+1}, \quad \varepsilon_{\pi,t+1} \sim \mathcal{N}(0, 1). \quad (16)$$

¹⁵Maturity choices would not be determined in a version of this model with risk-neutral lenders and no default risk. While these conditions are not met in our model, there are regions of the state space (a small face value of debt and χ_t close to $-\kappa_0/\kappa_1$) where they approximately hold. The utility costs above help to pin down λ' in those regions.

We denote by $\theta_{\text{gov}} = [\sigma, \tau, \underline{G}, \psi, \mu_y, \rho_y, \rho_{y\chi}, \sigma_y, \sigma_{y\chi}, \beta, d_0, d_1, \pi^*, \sigma_\pi, \bar{d}, \alpha]$ the parameters associated with the decision problem of the government.

Our quantitative strategy consists of choosing $\theta = [\theta_{\text{sdf}}, \theta_{\text{gov}}]$ to match a set of moments summarizing the behavior of public finances and interest rates. We proceed in two steps. In the first step, we choose θ_{sdf} to match statistics regarding the term structure of bonds that are free from default risk, measured using German data. In the second step, and conditional on θ_{sdf} , we choose θ_{gov} by matching key facts about Italian public finances. Implicit in the first step is the assumption that lenders are on their Euler equations for both Italian and German government securities. Thus, we can measure their preferences for short- versus long-term bonds by studying the behavior of the term structure of German interest rates. The advantage of this two-step approach is that we can estimate θ_{sdf} without solving the decision problem of the government, which is numerically complex.

4.2 Data and model's parametrization

We employ the method of simulated moments and set θ_{sdf} in order to minimize the distance between a set of empirical targets and the corresponding model-implied moments. We obtain the prices of ZCB issued by the German government from the *Bundesbank online database*. Our analysis focuses on the 1973-2013 period. Because these bonds are nominal, we enrich the stochastic discount factor in equation (13) with a process for inflation. We assume that inflation follows the AR(1) process,

$$\Delta p_t = \mu_p(1 - \rho_p) + \rho_p \Delta p_{t-1} + \sigma_p \varepsilon_{p,t}, \quad (17)$$

where $\varepsilon_{p,t}$ is a standard normal random variable and $\text{cov}(\varepsilon_{\chi,t}, \varepsilon_{p,t}) = \rho_{\chi,p}$. We estimate this process using quarterly data on German inflation, and we set $[\mu_p, \rho_p, \sigma_p]$ to their estimated values.

The empirical targets include the mean and the standard deviation of the yields on a ZCB with a maturity of one quarter, and the correlation between these yields and the inflation rate. We also include statistics measuring the size of risk premia for long-term bonds. Let

$$rx_{t+1}^{\varepsilon,(n)} = \log \left(\frac{q_{t+1}^{\varepsilon,(n-1)}}{q_t^{\varepsilon,(n)}} \right) - r_t^{\varepsilon,(1)}$$

be the *realized* returns from purchasing at time t a nominal bond with residual maturity of n periods and selling it at $t + 1$ relative to the returns one obtains from purchasing at time t a bond maturing in one period. If bondholders were risk-neutral, these excess returns should

be zero on average. Thus, the behavior of $\mathbb{E}_t[rx_{t+1}^{\epsilon,(n)}]$ is effectively a measure of risk premia on long-term bonds. We employ the two-step procedure of [Cochrane and Piazzesi \(2005\)](#) to estimate $\mathbb{E}_t[rx_{t+1}^{\epsilon,(n)}]$, and we include in the empirical targets statistics that control the sample mean, volatility, and autocorrelation of excess returns on a government bond with a maturity of five years.¹⁶ We then select the parameters of the stochastic discount factor and $\rho_{\chi,p}$ to minimize the weighted squared difference between the statistics computed from the data and the same statistics computed on model-simulated data. Panel A of [Table 1](#) reports the point estimates for θ_{sdf} .

After obtaining values for these parameters, we construct the empirical counterpart to χ_t . Specifically, we show in the online [Appendix C](#) that expected excess returns on long term bonds are related to χ_t as follows:

$$\chi_t = \frac{\mathbb{E}_t[rx_{t+1}^{\epsilon,(n)}] - \tilde{A}_n}{\tilde{B}_n}, \quad (18)$$

where \tilde{A}_n and \tilde{B}_n are known functions of the structural parameters. We can therefore construct the time path of χ_t by substituting in the right hand side of [equation \(18\)](#) the estimates of $\mathbb{E}_t[rx_{t+1}^{\epsilon,(n)}]$ obtained using the [Cochrane and Piazzesi \(2005\)](#) methodology.

We next turn to θ_{gov} . A subset of these parameters are set to conventional values in the literature. We fix σ to 2, and ψ to 0.05, a value that implies an average exclusion from capital markets of 5.1 years following a default, in line with the evidence in [Cruces and Trebesch \(2013\)](#). The tax rate is set to 0.41, equal to the sample mean of tax revenues over GDP, and μ_y to 0.89, so that tax revenues are normalized to 1 in a deterministic steady state. We set the spending requirement \underline{G} to 0.68, equal to the sample average of the ratio of wages of public employees and transfers to tax revenues, our measure of non-discretionary spending.

We choose the remaining parameters to match key features of the behavior of Italian public finances. Specifically, we target statistics that summarize the behavior of output, debt, debt maturity and interest rate spreads. We map $\hat{y}_t = (y_t - \mu_y)$ to the log deviations of real GDP from a linear trend. As discussed in [Section 2.3](#), we map B_{t+1}/λ_{t+1} to the face value of the outstanding bonds of the Italian central government.¹⁷ We use monthly data and construct an empirical counterpart for the price of a portfolio of ZCB with decay parameter λ using the approximation

$$Q_t^{\text{ita}}(\lambda) = \lambda \left[\sum_{j=1}^{N-1} (1-\lambda)^{j-1} q_t^{\text{ita},(j)} + \frac{(1-\lambda)^N}{\lambda} q_t^{\text{ita},(N)} \right], \quad (19)$$

¹⁶Online [Appendix C](#) provides a detailed description and a discussion of these steps.

¹⁷We exclude from the computation of public debt direct loans that the government received from financial intermediaries because this category is arguably less subject to the rollover problem studied in this paper.

Table 1: Model parameters

Panel A: Stochastic discount factor		
Parameter	Value	Targets
ϕ_0	0.005	Method of Simulated Moments
ϕ_1	0.002	Method of Simulated Moments
κ_0	0.161	Method of Simulated Moments
κ_1	0.374	Method of Simulated Moments
ρ_χ	0.513	Method of Simulated Moments
Panel B: Government's decision problem		
σ	2.000	Conventional value
ψ	0.050	Cruces and Trebesch (2013)
τ	0.410	Tax revenues over GDP
\underline{G}	0.680	Non discretionary spending over tax revenues
μ_y	0.892	Normalization
ρ_y	0.970	Estimates of equation (14)
σ_y	0.008	Estimates of equation (14)
$\sigma_{y\chi}$	-0.002	Estimates of equation (14)
β	0.980	Method of Simulated Moments
d_0	0.058	Method of Simulated Moments
d_1	0.092	Method of Simulated Moments
α	0.400	Method of Simulated Moments
\bar{d}	6.810	Method of Simulated Moments
π^*	-6.500	Method of Simulated Moments
σ_π	1.650	Method of Simulated Moments

Notes: We reparametrize the $d(\cdot)$ function in equation (15). The parameter d_0 stands for the percentage loss in output after a default when output is three standard deviations below its average value. The parameter d_1 represents the percentage loss in output when the latter is at its average value.

where $q_t^{\text{ita},(j)}$ is the price of an Italian government bond with residual maturity of j periods obtained from *Datastream*, and N is set to 80 quarters. Our indicator of interest rate spreads is the difference between the implied yields on the Italian portfolio with an average maturity of five years and its counterpart constructed using German data.¹⁸

As for debt maturity, we use detailed information at the security level on all outstanding bonds of the Italian central government to compute the redemption profile of public debt. That is, at each time t , we compute the payments— principal *and* coupons— that the central government promised to bondholders at time $t + n$, for all $n \geq 1$. Denoting these payments

¹⁸That is, we compute $r_t^{\text{ita}}(\lambda) - r_t^{\text{ger}}(\lambda) = \frac{\lambda[1 - Q_t^{\text{ita}}(\lambda)]}{Q_t^{\text{ita}}(\lambda)} - \frac{\lambda[1 - Q_t^{\text{ger}}(\lambda)]}{Q_t^{\text{ger}}(\lambda)}$ for $\lambda = 0.05$.

by $C_t^{(n)}$, and the sum of all these payments across n by C_t , we can then define the *weighted average life* of Italian outstanding bonds as

$$\text{wal}_t = \sum_{n=1}^N n \frac{C_t^{(n)}}{C_t}. \quad (20)$$

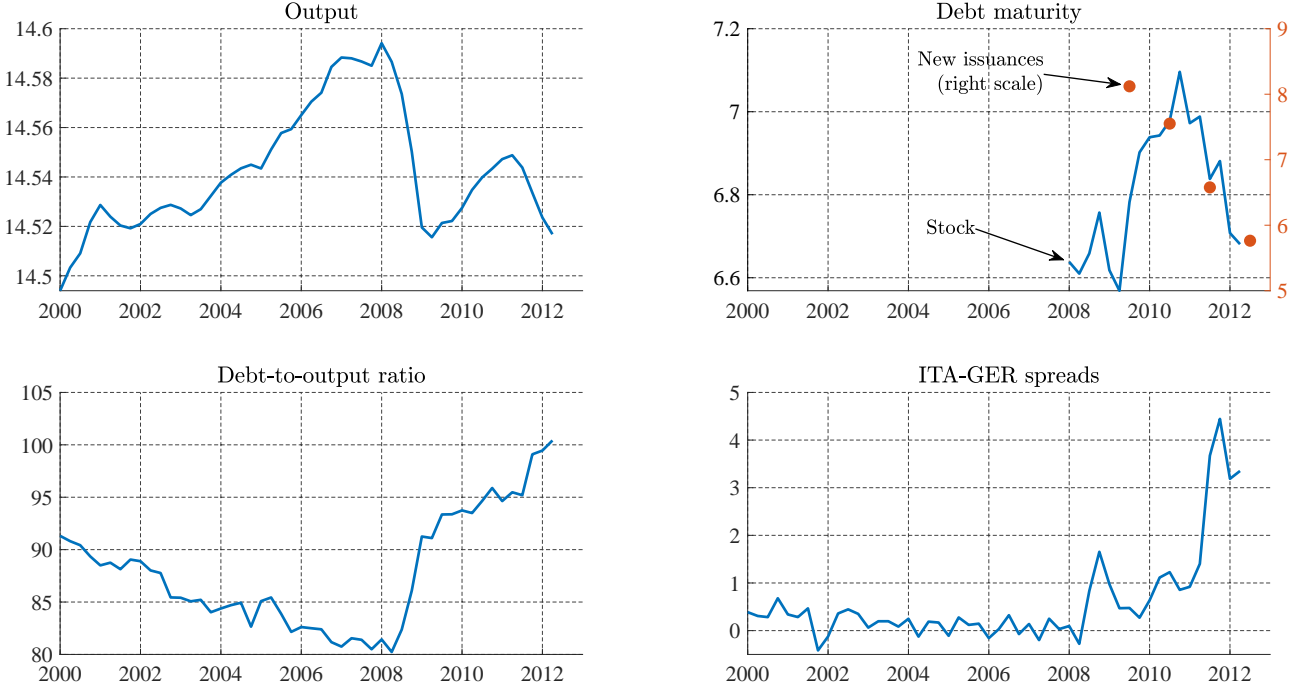
This indicator is constructed for the 2008Q1-2012Q2 period, and it maps exactly to $1/\lambda_t$ in our model.

Figure 1 reports these time series over the sample. During the 2000-2007 period, the Italian economy experienced positive growth and a progressive reduction of the debt-to-output ratio, while interest rate differentials between Italian and German bonds were close to zero, implying that financial markets attached little probability to the possibility of an Italian default. With the global financial crisis of 2008, the Italian economy entered a recession. The fiscal policy response to the crisis was expansionary, with a substantial increase in the debt-to-output ratio. Interest rate spreads became positive, ranging between 100 and 200 basis points. From the second quarter of 2011, the Italian economy experienced a second recession, and a deterioration of public finance indicators: the debt-to-output ratio was now twenty percentage points above the 2008 level, and interest rate spreads exceeded 400 basis points. The figure also reports the behavior of debt maturity in the 2008-2012 period. The dots in the top-right quadrant of Figure 1 reports the average maturity of new issuances. This indicator dropped substantially during the crisis, going from eight to five and a half years between 2009 and 2012. The maturity of the stock increased between 2008 and 2010 because the average maturity of new issuances was higher than that of outstanding bonds. It then fell throughout 2011-2012, as the maturity of the new issuances shortened further.

We use detrended output and the series for χ_t that we obtained earlier to estimate the process in equation (14) for the 2000:Q1-2012:Q2. Because $\rho_{y\chi}$ is not significantly different from zero, we impose the restriction $\rho_{y\chi} = 0$. The point estimates of this restricted model are $\rho_y = 0.970$, $\sigma_{y\chi} = -0.002$, and $\sigma_y = 0.008$.

The remaining parameters $[\beta, d_0, d_1, \alpha, \bar{d}, \pi^*, \sigma_\pi]$ are estimated using the method of simulated moments. We include in the empirical targets the sample mean of the debt-to-output ratio, the correlation between the debt-to-output ratio and detrended output, and the mean and standard deviation of interest rate spreads. We also include the sample mean and standard deviation for our indicator of debt maturity. The first moment provides information on \bar{d} , as this parameter controls the average maturity of debt. The second statistic provides information on α : holding the other parameters fixed, a higher α implies a lower standard deviation for debt maturity because it becomes more costly for the government to deviate from the “target” \bar{d} .

Figure 1: Output, debt, maturity and interest rate spreads in Italy: 2000Q1-2012Q2



Notes: Output is log-real GDP, normalized to be equal to zero in 2000:Q1. The debt-to-output ratio is reported in percentages, while debt maturity is the weighted average life defined in (20) and reported in years. Interest rate differentials between Italian and German bonds are reported in annualized percentages. See the text and the data Appendix for variables' definitions and sources.

The literature offers little guidance on the choice of variables that provide information on π^* and σ_π . Our approach consists of targeting the adjusted R^2 of the following regression:

$$\begin{aligned} \text{spr}_t = & a_0 + a_1 \text{gdp}_t + a_2 \text{debt}_t + a_3 \hat{\chi}_t + a_4 \text{wal}_t + a_5 (\text{gdp}_t \times \text{debt}_t) + a_6 (\text{gdp}_t \times \hat{\chi}_t) \\ & + a_7 (\text{gdp}_t \times \text{wal}_t) + a_8 (\text{debt}_t \times \hat{\chi}_t) + a_9 (\text{debt}_t \times \text{wal}_t) + a_{10} (\hat{\chi}_t \times \text{wal}_t) + e_t. \end{aligned} \quad (21)$$

The residual e_t measures variation in interest rate spreads that is orthogonal to the fundamental state variables in the model, and it should therefore discipline the process for π_t . We estimate equation (21) by OLS, obtaining an adjusted R^2 of 82%.¹⁹

The model is solved numerically using a value function iteration algorithm described in the online Appendix D. We compute model implied moments on a long simulations ($T = 100000$), and we weight the distance between sample moments and their model counterpart

¹⁹The high explanatory power is mostly due to output, debt, and their interaction. When including only these three terms in the regression, we obtain an adjusted R^2 of 68%. [Bocola, Bornstein, and Dovis \(2018\)](#) obtain similar results for Spain and Portugal as well. These results are in contrast with the findings in [Longstaff, Pan, Pedersen, and Singleton \(2011\)](#) for emerging markets economies where domestic factors have low explanatory powers for spreads.

by the inverse of the sample moment absolute value.²⁰ We then select the numerical values of $[\beta, d_0, d_1, \bar{d}, \alpha, \pi^*, \sigma_\pi]$ that minimize the distance between the model and the data. These are reported in Panel B of Table 1.

4.3 Model fit

We can verify from Table 2 that the model has a good in-sample fit. The face value of debt is 82.87% of annual GDP on average, close to the 87.87% in the sample. As in the data, the debt-to-output ratio goes down in recessions. Interest rate spreads are on average very close to the data (0.61% in the model vs. 0.63% in our sample), but they are less volatile (the standard deviation is 0.52% in the model vs. 1.01% in the data). The model generates an empirically plausible relation between interest rate spreads and economic fundamentals, as captured by the R^2 of equation (21): 0.79 in the model relative to 0.82 in the data. Finally, debt maturity in model simulations is on average 6.80 years, with a standard deviation of 0.11. In the data, these moments are, respectively, 6.81 years and 0.16.

Table 2: Empirical targets: data vs. model

Statistic	Data	Model
Average debt-to-gdp ratio	87.87	82.81
Correlation debt-to-gdp and output	-0.90	-0.43
Average spread	0.61	0.63
St. dev. of spread	1.01	0.52
R^2 of regression (21)	0.82	0.79
Average debt maturity	6.81	6.80
St. dev. of debt maturity	0.16	0.11

It is important to stress that our parametrization differs from the one typically used in the sovereign debt literature. Earlier studies that have fit this class of models to emerging market economies produce procyclical fiscal policy, with the government borrowing more when hit by positive income shocks. Moreover, in those parametrizations, the government is at risk of a default most of the time, with interest rate spreads being far away from zero even when output is above average.²¹ This pattern would be inconsistent with the Italian data,

²⁰The simulations discard the first 100 periods following a default. These periods in the model are characterized by an upward trend in debt. Because in our sample we did not observe an actual default, we exclude these periods when computing the targets in model-simulated data.

²¹For example, in Chatterjee and Eyigungor (2012) the correlation between the trade balance (equivalent to the government's surplus) and output is -0.44, while annualized interest rate spreads are on average 8%.

as interest rate spreads averaged few basis points over our sample, and the debt-to-output ratio increased by roughly twenty percentage points during the 2008-2012 recession.

As we explain in [Bocola, Bornstein, and DAVIS \(2018\)](#), the main point of departure between our parametrization and the one used in the literature lies in the relative importance of “front-loading” and “consumption-smoothing” motives in the decision problem of the government. In the typical parametrization considered in the literature, β is substantially lower than the market discount factor, which implies that the government uses debt mostly to front-load future consumption. Coupled with the endogenous borrowing limits implied by default risk, this behavior leads to procyclical fiscal policy: in high income states, the debt pricing schedule shifts out and the government borrows more; conversely, low income states are associated with tighter pricing schedules and with less government borrowing. In our parametrization, instead, the higher values of β and the non-homotheticity of the utility function imply that the government uses the debt market mostly to smooth consumption across states of the world, leading to countercyclical borrowing.

This feature has important implications for interest rate spreads in the model. Due to the low front-loading incentives, the government spends most of its time away from the region of the state space in which it is at risk of a default. However, because of the countercyclicity of debt issuances, a string of negative income shocks can induce a large accumulation of debt, exposing the government to the risk of default. It follows that interest rate spreads in the model cluster around zero, and they experience rare and large jumps. These are also features of the Italian data, as we can see from panel (a) of [Figure 2](#), where we compare the unconditional distribution of interest rate spreads in the model with the one in the data.

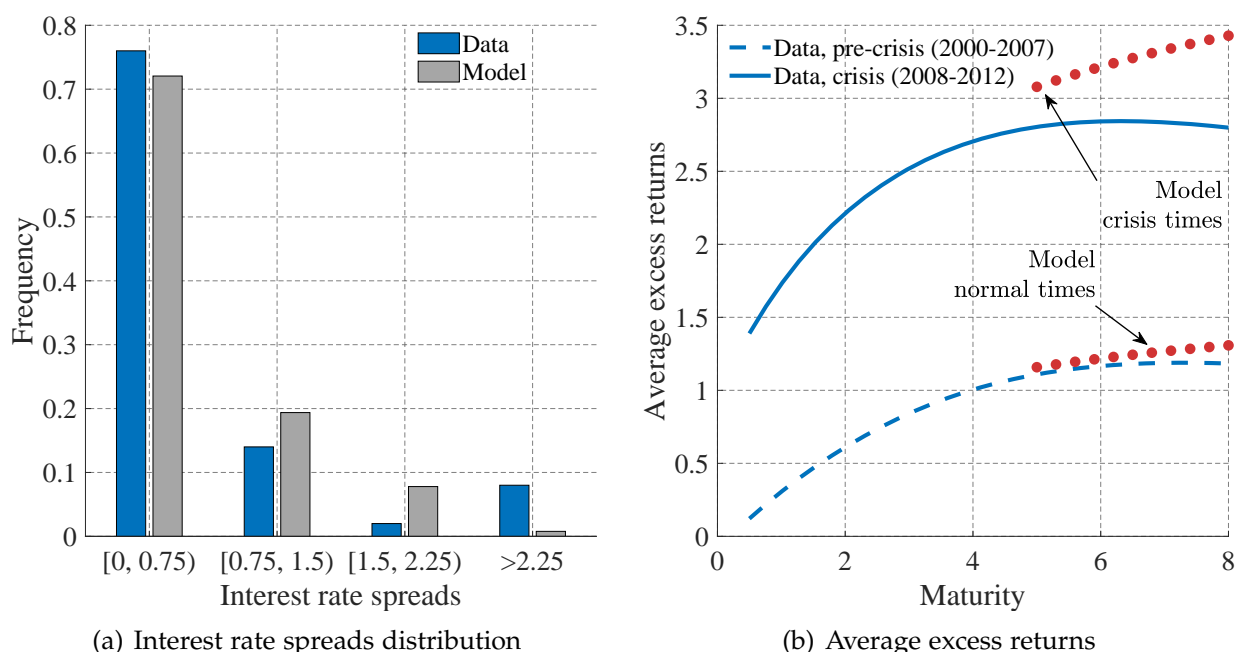
It is also important to verify how well the model captures the level and cyclicity of excess returns on Italian bonds, and how these vary by maturity. For this purpose, we compute realized holding periods excess returns on a λ -type portfolio,

$$\frac{\lambda + (1 - \lambda)Q_{t+1}^{\text{ita}}(\lambda)}{Q_t^{\text{ita}}(\lambda)} - \frac{1}{Q_t^{\text{ger}}(1)},$$

and take sample averages to approximate expected excess returns. Panel (b) of [Figure 2](#) reports average excess returns as a function of the maturity of the portfolio for two sub-samples: a pre-crisis period (2000-2007) and a crisis period (2008-2012). We can verify that, on average, long term Italian government bonds carry a premium relative to the short term risk-free rate, and this premium increases with maturity. Furthermore, this premium increases during the debt crisis, more so for portfolios with longer average maturities.²² Overall, these

²²The increase in average realized returns during the crisis might not only reflect an increase in compensation for risk, but it could also be due to a “peso” problem. That is, we measure positive realized returns because, in our sample, we did not observe the large negative realized returns associated to a government

Figure 2: Spreads and average excess returns: model vs. data



Notes: Panel (a) reports the histogram for interest rate spreads computed using the Italian data and the model-simulated data used to generate Table 2. In panel (b), the solid line reports the sample average of realized returns on portfolios of Italian bonds in excess of the yields on a short term German bond for the 2008-2012 period. The portfolios differ by their average maturity (reported in years in the figure), and the excess returns are reported in annualized percentages. The dashed line reports the same statistics for the 2000-2007 period. The dots in the Figure report the same statistics computed in model-simulated data.

results are qualitatively consistent with the findings of [Broner, Lorenzoni, and Schmukler \(2013\)](#) for emerging market economies.

The figure also reports these statistics in the model. We consider a long simulation, partition it into “crisis” and “normal” times, and compute average holding period returns on portfolios with different maturity, reported as dots in the figure.²³ This out of sample check shows that the model captures well the level and cyclical of risk premia on long term bonds. In normal times, long term bonds carry a risk premium relative to short term bonds that is quantitatively in line with the data. When the government is in a crisis, it faces an empirically plausible increase in risk premia on long term bonds.

default. The comparison between data and model, however, is not affected by this problem because we exclude default events when computing these statistics in the model.

²³Italian spreads in the 2008-2012 varied between 0.85 and 3.5 standard deviations above the sample average. We define a “crisis” in our simulation as a period in which spreads are between these two cut-offs.

4.4 Sources of default risk and maturity choices

Before analyzing the behavior of interest rate spreads and debt maturity in the model, it is instructive to first study how the default, safe and crisis zones vary with the state of the economy.

Panel (a) of Figure 3 plots the boundaries of the default and safe zones— defined respectively by conditions (9) and (10)— in the debt/output space. These boundaries are constructed setting (λ, χ, π) at their ergodic mean value. The area below the safe zone boundary represents the combination of debt and output realizations for which the government never defaults on its debt, while the area above the default zone boundary is the region of the state space in which the government always default. The crisis zone lies in between these two boundaries, and in this region the government is exposed to rollover crises. The contours report the ergodic joint distribution of debt and output. The government spends most of its time close to the boundary of the safe zone, but there is mass in high debt/low output states, in which the government is at risk of a default.²⁴

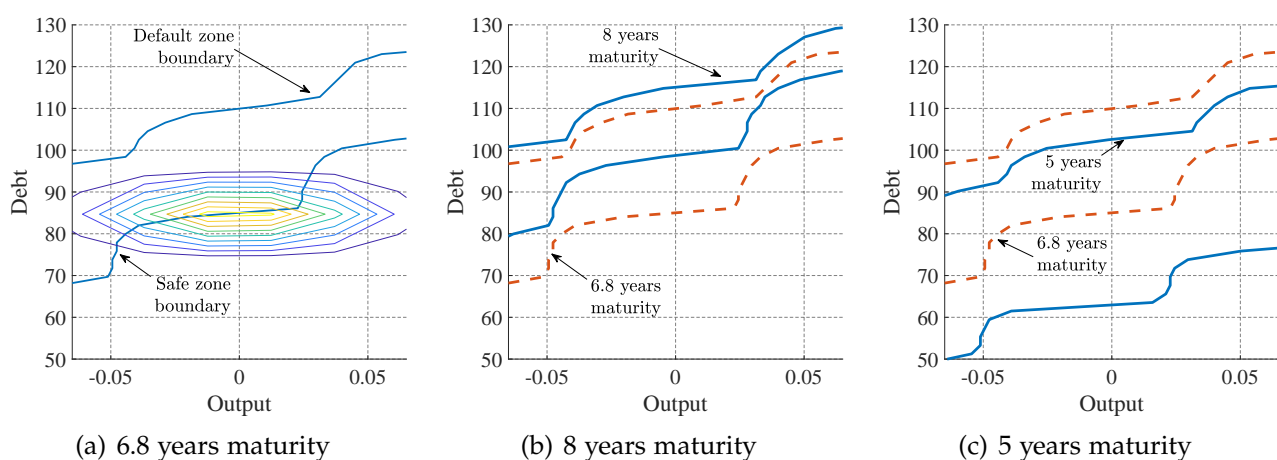
Panel (b) and (c) of Figure 3 show how these boundaries vary with debt maturity. In line with the discussion of Section 3, we can see that a longer maturity structure of public debt shifts up the boundary of the safe zone, and it shrinks overall the size of the crisis zone. We can also see that the effects are quantitatively sizable: when income is at its mean level, having a debt-to-output ratio of 63% is enough to expose the government to rollover crises if average debt maturity is five years; when average debt maturity is eight years, instead, the face value of debt needs to be above 100% of output for the government to be exposed to rollover crises.

Having described the average behavior of the economy, we now study how interest rate spreads and debt maturity respond to shocks that increase the risk of a government default.²⁵ We consider two scenarios. In the first scenario, given by the solid lines in Figure 4, we study the effects of a decline in output of six percent while setting $\pi_t = 0$ for all t along the path. This experiment captures the behavior of interest rate spreads and debt maturity conditional on an increase in fundamental default risk. In the second scenario, instead, we consider a persistent increase in π_t when the economy is currently in the crisis zone. This second experiment approximates the behavior of interest rate spreads and debt maturity conditional

²⁴In Cole and Kehoe (2000) the safe zone is an absorbing state. This happens because in their model the government is not relatively impatient, $\beta(1+r) = 1$. In our parametrization, instead, the government is relatively impatient. This implies that the crisis zone is visited with positive probability even if at date zero the government starts in the safe zone.

²⁵ We compute nonlinear impulse response functions (IRFs) following Koop, Pesaran, and Potter (1996). Given initial conditions, we compute $2 \times M$ simulations of the model of length T . In the first M simulations, we restrict the innovations of interest to take a particular value, while in the remaining M simulations the innovations are sampled from the unconditional distribution. To obtain the IRFs, we average each set of simulations across M and take the difference between the two paths. We set $M = 50000$ and $T = 40$.

Figure 3: Debt maturity and the crisis zone



Notes: Debt is the face value of debt, reported in percentage of annualized mean output. Output is reported in log-deviations from its mean.

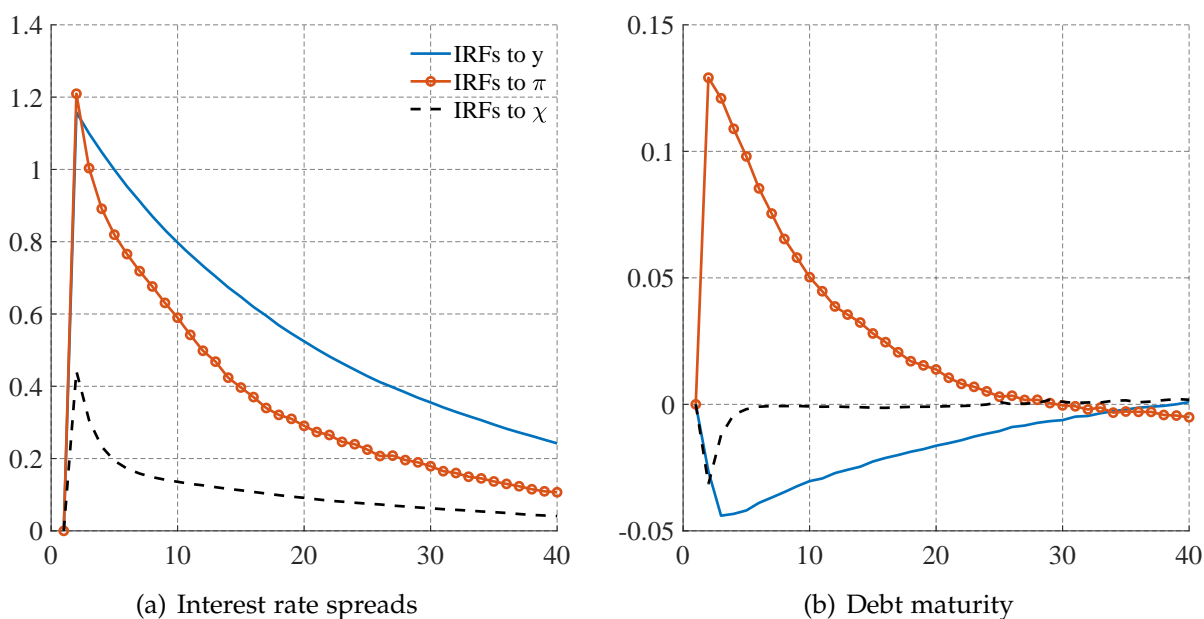
on an increase in rollover risk.

In both experiments, there is an increase in the risk of a government default, as we can see from the increase in interest rate spreads. However, the two impulses have different implications for the maturity structure of government debt. In the first experiment, where default risk is purely due to bad economic fundamentals, the government shortens debt maturity. This is because the incentive benefits of short-term debt become more valuable when the economy approaches the default zone: in our simulations, the average life of outstanding debt drops by 0.05 years on impact following the negative output shock.

In the second experiment, instead, the increase in the risk of a default occurs because of an increase in π_t . The government responds to this shock by lengthening debt maturity: the average life of outstanding debt increases by 0.13 years in our simulations. As explained earlier, lengthening debt maturity reduces the exposure of the government to a future rollover crisis, and it is the optimal response to an increase in the the probability that lenders coordinate on a run. These results confirm, in the parametrized model, the discussion in Section 3. Debt maturity responds differently depending on whether the increase in interest rate spreads is due to bad economic fundamentals or to heightened rollover risk.

The dashed lines in Figure 4 plot the response to an increase in χ_t of three standard deviations. As explained earlier, this shock increases the compensation that lenders demand for holding long-term assets. Accordingly, the government responds to this shock by decreasing the maturity of its debt.

Figure 4: The dynamics of interest rate spreads and debt maturity



Notes: The solid line reports impulse response functions (IRFs) of interest rate spreads (in annualized percentages) and debt maturity (in years) to a 6% decline in y . The circled line reports the response to a 2.25 standard deviations increase in π , assuming a decaying rate of 0.97 for the shock. The dashed line reports the IRFs to a 3 standard deviations increase in χ . The IRFs to χ and y are constructed by setting $\pi = 0$ throughout the simulations, and they are initialized by setting the state variables at their ergodic mean. The IRFs to π_t are initialized by setting the state variables at their mean conditional on the government being in the crisis zone. See footnote 25 for the calculation of the IRFs.

5 Decomposing Italian spreads

We now turn to the main experiment of the paper and measure the importance of rollover risk during the debt crisis of 2008-2012. Specifically, we combine the model with the data in order to retrieve the path for the non-fundamental shock $\{\pi_t\}$. We then use this path to measure the rollover risk component of interest rate spreads.

The model defines the nonlinear state-space system

$$\begin{aligned} \mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t), \end{aligned} \quad (22)$$

with \mathbf{Y}_t being a vector of observable variables, $\mathbf{S}_t = [B_t, \lambda_t, y_t, \chi_t, \pi_t]$ the state vector, and ε_t the vector collecting the structural shocks.²⁶ The vector η_t contains uncorrelated Gaussian measurement errors, and it captures any deviation between the data \mathbf{Y}_t and $\mathbf{g}(\mathbf{S}_t)$. The functions $\mathbf{g}(\cdot)$ and $\mathbf{f}(\cdot)$ are obtained using the model's numerical solution.

²⁶In our sample we did not observe a default. Thus, we can drop from ζ_t from \mathbf{S}_t because ζ_t does not affect the endogenous variables conditional on repayment. See online Appendix D.

The vector of observables includes detrended output, the data counterpart to χ_t constructed using equation (18), and our indicators of debt maturity and interest rate spreads. Given the time path of these variables over the 2008:Q1-2012:Q2 period, we estimate the realization of the state vector using the relation between states and observables implied by the system in (22). Technically, we carry out this step by applying the particle filter to the above state-space model, see the online Appendix E for a description. We set the variance of the measurement errors $\eta_{y,t}$ and $\eta_{\chi,t}$ to zero. This implies that the path of the fundamental shocks in the model coincides with the one in the data. This leaves us with two additional variables in \mathbf{Y}_t , interest rate spreads and debt maturity, and only one additional stochastic variable in \mathbf{S}_t , π_t . Because of that, we set the variance of the measurement errors associated to debt maturity and interest rate spreads equal to 1% of their respective sample variance. In Figure 6 we conduct a sensitivity analysis of the result with respect to these values.

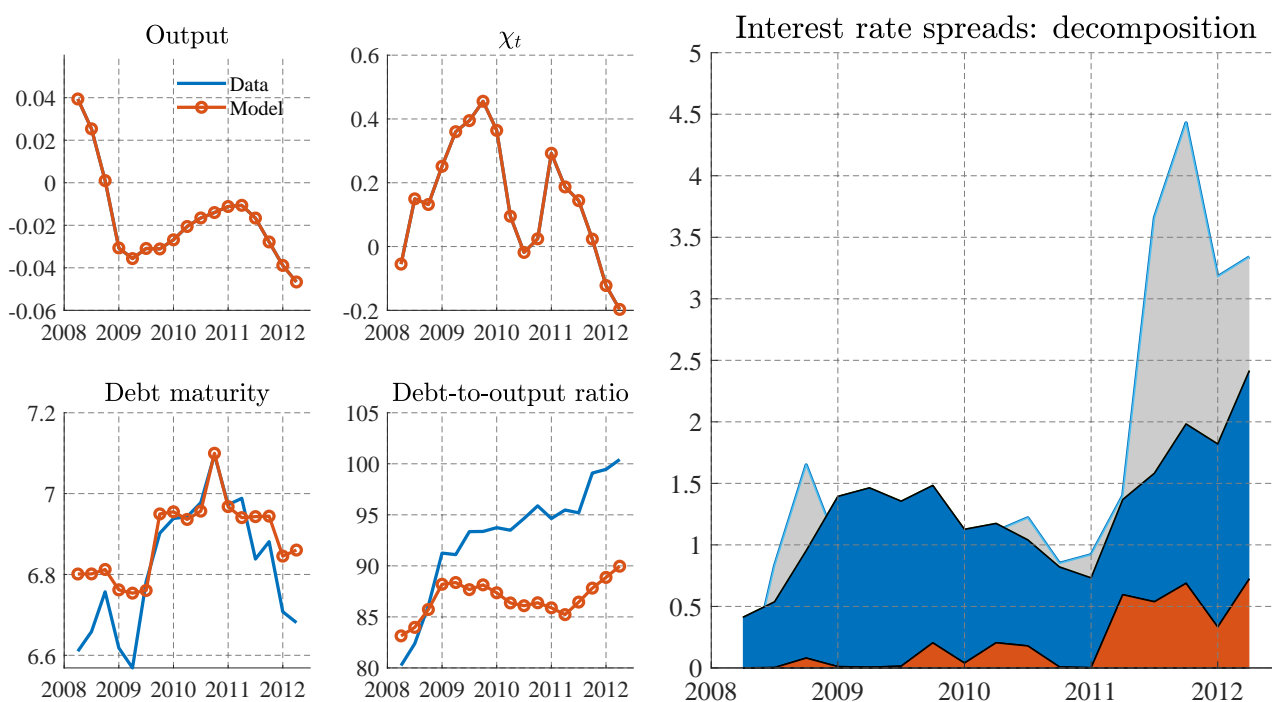
Equipped with the path for the exogenous shocks, we next measure the contribution of rollover risk to interest rate spreads. To do so, we feed the pricing function of the model with the filtered state and control variables, with the exception that π_t is set to zero for all t in the sample. We label the implied interest rate spread series from this counterfactual as the *fundamental component* of interest rate spreads because, by construction, the one-step-ahead probability of a rollover crisis is zero in every period.²⁷ The difference between the filtered interest rate spread series and the counterfactual one nets out the impact of rollover risk. Accordingly, we label this difference the *rollover risk component* of interest rate spreads. Importantly, the model-implied interest rate spreads are not necessarily equal to the one in the data because the system in (22) has more observables than structural shocks. Any difference between the observed interest rate spreads and the one generated by the model is captured by $\eta_{\text{spread},t}$.

Figure 5 reports the results of this experiment. The four panels to the left report the behavior of the fundamental state variables in the model and in the data. By construction, the model tracks perfectly the time path for y_t and χ_t , and it replicates fairly accurately the time path for the weighted average life of public debt. We can also see that the model generates an empirically plausible increase in the debt-to-output ratio following the 2008 and 2011 recessions, even though we did not include this variable in \mathbf{Y}_t . Differently from the data, though, the model predicts a decline in debt during the 2009-2011 period because of the recovery in output. Due to this discrepancy, the model understates the overall increase in the debt-to-output ratio during the event.

The right panel of Figure 5 reports interest rate spreads in the data along with their de-

²⁷An alternative would be to adjust the endogenous state variables to their implied value at $\pi_t = 0$, and compute bond prices conditional on this counterfactual path for the endogenous state variables. This alternative decomposition gives very similar results to the one reported in the paper.

Figure 5: Decomposition of interest rate spreads



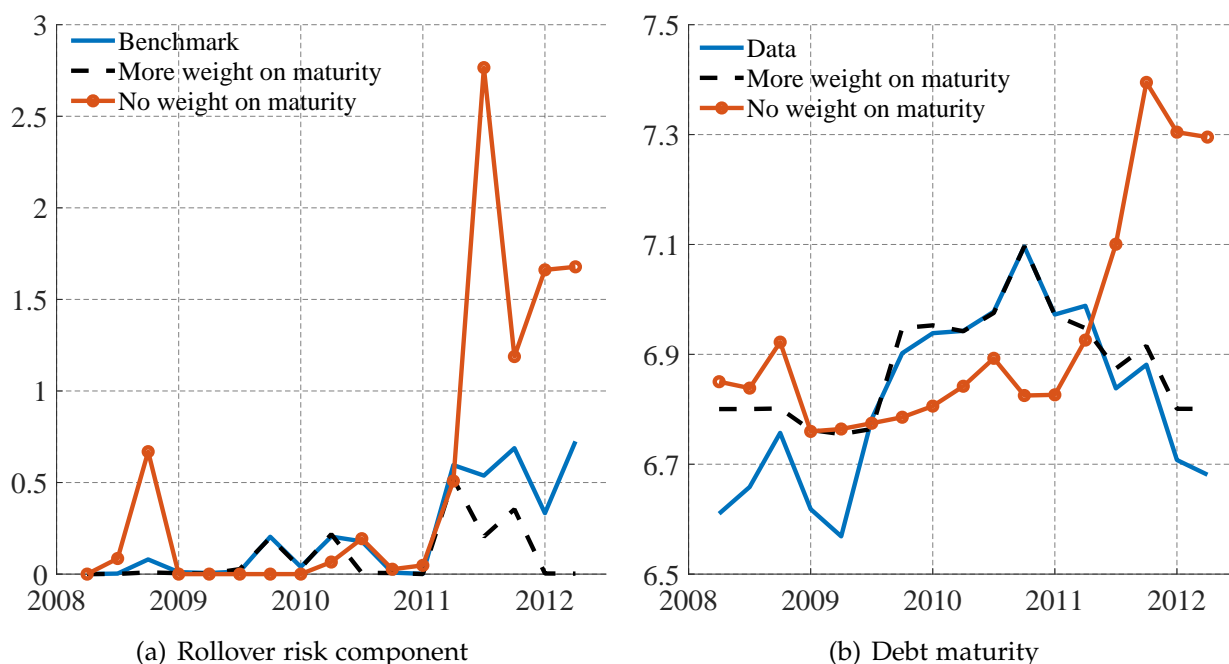
Notes: The left panels report detrended output, the χ_t series, the weighted average life of outstanding bonds, and the debt-to-output ratio in the data (solid lines) and associated filtered ones (circled lines). The right panel plots the interest rate spreads indicator along with its decomposition.

composition into the fundamental component, the rollover risk component, and the residual component that we attribute to $\eta_{\text{spread},t}$. The model fits well the dynamics of Italian interest rate spreads during the event, with the exception of the sharp increase observed during the second half of 2011.

Most of the increase in interest rate spreads during the episode is attributed to fundamental shocks. At the beginning of the period, in 2008:Q2, interest rate spreads were around 0.5%, and fully accounted by the fundamental component. At the end of the period, in 2012:Q2, the model generates a spread of 2.5%, with the fundamental component accounting for 1.75%. This pattern is the result of two main developments. First, the Italian economy experienced a prolonged major recession during this period: output went from being 4% above trend in 2008:Q1 to being 5% below trend at the end of the sample. Second, the Italian government increased its debt during the crisis, a fact that our model captures. Both of these developments push the government closer to the default zone, increasing in this fashion the fundamental component of interest rate spreads.

The model assigns a more limited role to rollover risk, on average 13% of the model-implied interest rate spreads, despite the fact that π_t is not directly observed and could in principle be used to fit all the variation in interest rate spreads not explained by the

Figure 6: Rollover risk and debt maturity



Notes: Panel (a) plots the rollover risk when we exclude debt maturity from Y_t (No weight on maturity), and when we reduce the variance of $\eta_{mat,t}$ by 50% relative to the benchmark (More weight on maturity). Panel (b) plots the filtered debt maturity series in these two experiment.

fundamental shocks. For example, the model has hard time capturing the jump in spreads observed in 2011:Q3 with the fundamental shocks because y_t and χ_t barely moved between 2011:Q2 and 2011:Q3. However, the model attributes this jump to the measurement error rather than to an increase in π_t .

In principle, this result could have two explanations. First, it might be that the Italian economy was far from the crisis zone in 2011, in which case shocks to π_t would have limited effects on interest rate spreads. Second, it might be that the increase in π_t necessary to fit interest rate spreads would have counterfactual implications for debt maturity.

To further explore this issue, we repeat this analysis excluding debt maturity from the set of observables. When doing so, the model tracks more closely the dynamics of interest rate spreads in 2011, and most of the improvement in fit is due to an increase in the rollover risk, see the circled line in panel (a) of Figure 6. Panel (b) of the figure plots the model-implied behavior for debt maturity in this experiment along with the data counterpart. We can observe that heightened rollover risk in 2011 is associated with an increase in the average life of outstanding debt of 0.5 years, which is at odds with the data because this indicator declined by 0.2 years during the same period.

This experiment clarifies the role of maturity choices in our measurement strategy. Absent

data on debt maturity, the model has limited restrictions to discipline the time path of π_t , and it attributes to this term variation in interest rate spreads that is not explained by the fundamental shocks. By conditioning on the observed path of debt maturity, instead, we discipline empirically the rollover risk component. Realizations of the state vector for which rollover risk accounts for a sizable fraction of spreads in 2011 imply an increase in the maturity of Italian debt. This variable, however, follows the opposite pattern in the data. Because of that, our measurement assigns a more limited role to this component.

Figure 6 also reports the rollover risk component of interest rate spreads and the path for debt maturity that we obtain when repeating the experiment with a smaller measurement error on debt maturity relative to what we considered in the benchmark experiment of Figure 5. We can verify that the model now tracks more closely the decline in debt maturity observed after 2011 relative to the benchmark. To achieve this, the model needs smaller values for π_t and, consistently, we can see a reduction of the rollover risk component of interest rate spreads in the latest part of the sample, as the comparison between the dashed and solid line in panel (a) of Figure 6 shows.

Given that debt maturity plays an important role in our measurement of rollover risk, online Appendix F performs a sensitivity analysis of the results. In the first exercise, we measure the welfare gains that the government obtains from lengthening debt maturity under the assumption that rollover risk was a key driver of interest rate spreads in our event. Small gains would signal that our results could be easily overturned by other determinants of debt maturity that we omitted from the analysis. We do find large welfare gains instead, suggesting strong incentives to lengthen debt maturity in presence of a sizable role for rollover risk. In the second exercise, we study how varying the utility cost α affects the measurement of rollover risk. The worry here is the following: by decreasing α , debt maturity would respond more to income shocks, implying that the model would generate a much larger decline in debt maturity than what observed in the data in 2011-2012. The particle filter would then need a higher level of π_t in order to keep debt maturity close to the data. In the experiment reported in the online Appendix we find that this concern is not warranted, because the rollover risk component measured in our procedure varies little with α .

6 Evaluating OMT announcements

We now turn to analyze the effects of the Outright Monetary Transactions (OMT) program through the lens of the model. As a response to soaring interest rate spreads in the euro-area periphery, the Governing Council of the European Central Bank (ECB) announced during the summer of 2012 that it would consider outright bond purchases in secondary sovereign

bond markets. The technical framework of these operations was formulated on September 6 of the same year. The OMT program replaced the Security Market Program as a mean through which the ECB could intervene in sovereign bond markets.²⁸

Even though the ECB as of today did not purchased government bonds within the OMT framework, the mere announcement of the program had significant effects on interest rate spreads of peripheral countries. [Altavilla, Giannone, and Lenza \(2014\)](#) estimate that OMT announcements decreased the Italian and Spanish two year government bonds by 200 basis points. This decline in interest rate spreads was widely interpreted by economists and policy makers as a reflection of the success of this program in reducing belief-driven inefficient fluctuations in sovereign bond markets of euro area peripheral countries. Here we use the model to evaluate this interpretation.

We model OMT as a commitment by the central bank to buy government bonds in secondary market at a chosen price, and conditional on the government respecting a borrowing limit, see the online Appendix G. There, we show that with these instruments the central bank can uniquely implement the *fundamental equilibrium*, defined as the equilibrium outcome that arises if $\pi_t = 0$ in all possible realizations, or equivalently the equilibrium outcome that arises with the timing in [Eaton and Gersovitz \(1981\)](#). Intuitively, by committing to intervene in secondary markets, the central bank is setting a floor on bond prices. This in turn allows the government to always access financial markets and repay the maturing debt, thereby eliminating the self-fulfilling aspect of rollover crises. The quantity limits on debt issuances are needed to guarantee that the government does not choose a B' that is higher than the one arising in the fundamental equilibrium.²⁹

The drop in interest rate spreads of southern European economies observed after the introduction of the OMT program is consistent with this interpretation. However, it is also consistent with other interpretations. For example, a decline in interest rate spreads following the OMT announcements may signal that the policy raised bondholders' expectations of future bailouts for euro area peripheral countries. To understand this point, suppose that the central bank is committed to keeping the price of debt in a given state above the fundamental price. The announcement of this policy leads to an increase in bonds' prices today (equivalently, a reduction in interest rate spreads).

We can use the model to evaluate whether the reduction in interest rate spreads observed

²⁸OMTs consist in direct purchases of sovereign bonds of members of the euro area in secondary markets. These operations are considered by the ECB once a member state asks for financial assistance, and upon the fulfillment of a set of conditions. There are two main characteristics of these purchases. First, *no ex ante quantitative limits* are set on their size. Second, OMTs are *conditional* on the country being in a European Financial Stability Facility/European Stability Mechanism macroeconomic adjustment or precautionary program.

²⁹Under OMT, the government acts as a price taker and has an incentive to borrow more relative to the fundamental equilibrium outcome.

Table 3: **Change in spreads relative to 2012:Q2: data vs. fundamental equilibrium**

	Data	Fundamental equilibrium
2012:Q4	-0.88	-1.69
2013:Q1	-1.37	-1.10
2013:Q2	-2.15	0.60

Notes: The spreads in the fundamental equilibrium are constructed as follows. We first obtain the decision rules from the fundamental equilibrium by solving the model with π_t identically equal to zero. The spread is then obtained by feeding the decision rules with the y_t and χ_t observed in the data. The initial conditions for the face value of debt and its maturity are set to their filtered level at 2012:Q2.

after the OMT announcements solely reflects the elimination of rollover crises. Suppose that the central bank credibly commits to a policy that uniquely implements the fundamental equilibrium. The announcement of this intervention eliminates rollover risk in every state of the world, and interest rate spreads jump to their value in the fundamental equilibrium. These spreads are different from the fundamental component in Figure 5: in that decomposition, we were setting to zero the one-period-ahead probability of rollover crisis while here we set rollover risk to zero in all periods and states.

The spreads in the fundamental equilibrium represent a lower bound on the post-OMT spreads under the hypothesis that the program was directed exclusively to prevent rollover crises. Thus, we can compare them to the spreads observed in the data after the OMT announcements. If the ones in the data are below the one implied by the model, it would mean that the policy did not only operate through a reduction in rollover risk.

The results of this exercise are in Table 3. The first column reports the change in Italian spreads between 2012:Q2 and the following quarters, that is before and after the establishment of OMT. The second column reports the difference between the model implied spread in 2012:Q2 and the spreads in the fundamental equilibrium. We can see that spreads in the data fell gradually, reaching a 215 basis points reduction in the second quarter of 2013. In the model, the spreads initially fell by 169 basis points due to the elimination of rollover crises, but they eventually came back to their pre-OMT levels due to the further deterioration of economic conditions in Italy. Thus, our calculations suggest that the decline in interest rate spreads observed after the OMT announcements cannot be fully justified by a reduction in rollover risk, and it provides evidence consistent with the view that the policy partly operated by fostering bailout expectations.

7 Relevance for other debt crises

One key insight of this paper is that debt maturity provides information on the importance of rollover risk: *everything else equal*, a government facing rollover problems would have an incentive to lengthen debt maturity. Previous studies by [Broner, Lorenzoni, and Schmukler \(2013\)](#) and [Arellano and Ramanarayanan \(2012\)](#) have shown that on average the maturity of new issuances in emerging markets shortens around default crises, and examples of governments extending the life of their debt in turbulent times are not well documented in the literature. One might be tempted to conclude from this evidence that rollover risk is not important for emerging markets. In this section, we argue that such a conclusion is not warranted.

We consider the same set of emerging markets studied in [Broner, Lorenzoni, and Schmukler \(2013\)](#) over the 1995-2009 period.³⁰ First, we study how debt maturity varies between “crisis” and “normal” times, as defined in [Broner, Lorenzoni, and Schmukler \(2013\)](#). That is, for each country i we estimate the following relation:

$$\text{debt maturity}_{i,t} = \alpha + \beta \times \text{crisis}_{i,t} + e_{i,t}, \quad (23)$$

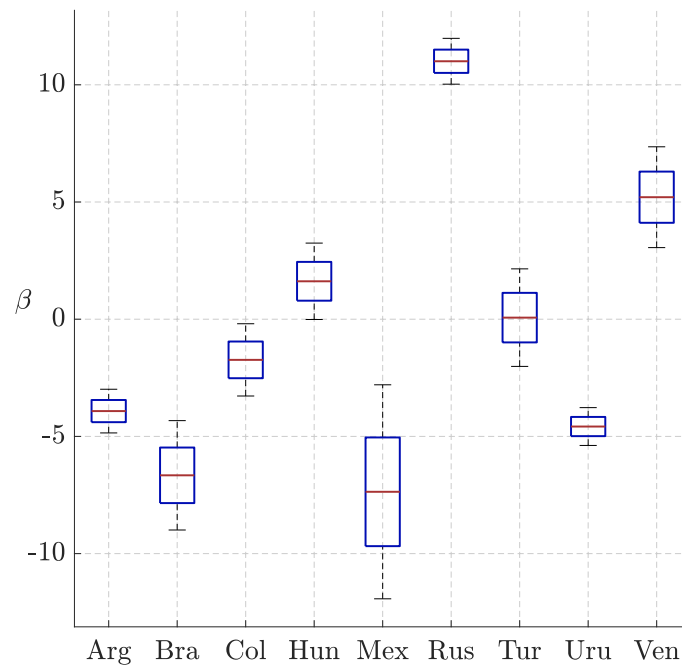
where $\text{debt maturity}_{i,t}$ is the average maturity of new issuances in period t for country i and $\text{crisis}_{i,t}$ is a dummy variable equal to 1 if country i is in a crisis in period t . The coefficient β measures the difference in the maturity of new issuances between crisis and non-crisis periods. [Broner, Lorenzoni, and Schmukler \(2013\)](#) estimate a β of -3.6 when pooling all countries (see Table 6A in their paper). That is, the maturity of new issuances in a typical emerging market crisis is on average 3.6 years lower than that during normal times, a number that is close to the Italian experience documented in Section 4.2. Figure 7 reports the point estimates for the coefficient β for each country in their dataset, along with a 95% confidence interval.

We can see that there is substantial heterogeneity across countries in the behavior of debt maturity. For Argentina, Brazil, Colombia, Mexico, and Uruguay, the maturity of new issuances shortens during crises, as for these countries the estimated β is negative and statistically significant.³¹ However, we can also see that for Russia, Venezuela, and Hungary, this coefficient is positive and statistically significant. In the case of Russia, for example, the large and positive β reflects two issuances of eurobonds with a 30 years maturity in June

³⁰We thank the authors for kindly sharing their data with us.

³¹Interestingly, while the maturity of new issuances in Argentina fell around the 2001 crisis, the Argentinian government took actions to lengthen the maturity of its debt coming due by offering a debt swap agreement, the so-called *megacanje* (megaswap). See [Sturzenegger and Zettelmeyer \(2006\)](#), p. 173-177 and Figure 8.3. These trades, while relevant for our analysis, are not considered in the statistics on new issuances.

Figure 7: Estimates of β in equation (23)



of 1998, just few weeks before the default event. These issuances allowed the government to effectively redeem short-term maturing bonds and postpone payments to the future in view of what was perceived by the public at the time as a rollover problem (Pinto, Kharas, and Ulatov, 2001). Thus, examples of governments attempting to lengthen the maturity of their debt during a crisis are not rare. In the online Appendix H, we document in greater detail one such example, the case of Italy in the early 1980s, a period in which the Treasury department took several measures to lengthen debt maturity in the face of what was arguably a rollover problem. While only suggestive, these findings imply that rollover risk is more important than what one could infer from the average experience of emerging market economies.

It is also important to stress that our approach does not infer the importance of rollover risk only from the behavior of debt maturity, but it is a more complex inference problem that controls for other determinants of maturity choices, such as variation in the risk premium over long-term bonds. For emerging market crises, these movements are extremely relevant. Broner, Lorenzoni, and Schmukler (2013) find that the difference between the average realized excess returns of 9 year bonds and 3 year bonds is more than 20 percentage points during crisis while it is approximately zero in normal times (see Table 3 in their paper). For comparison, this increase in the term premium is much larger than what we observed during the crisis in Italy (see Panel (b) of Figure 2). The typical emerging market crisis is

thus characterized by a large increase in excess returns on long-term government bonds. As we have suggested in this paper, an increase in the risk premium on long term bonds may induce a shortening of debt maturity even when rollover problems are important. Thus, a researcher that wants to assess the role of rollover risk for emerging market crises should not only use the information content of debt maturity but study it in conjunction with economic fundamentals and the term structure of interest rates, as we have advocated in this paper.

Finally, while the behavior of debt maturity is informative about current expectations of future rollover problems, it is not useful to detect whether an actual default event was due to a rollover crisis. In our framework, a rollover crisis corresponds to a shock to ζ_t which, as shown in [Cole and Kehoe \(2000\)](#), does not generate an incentive to change maturity.

8 Conclusion

This paper has proposed a strategy to bring to the data the classic model of self-fulfilling debt crises of [Cole and Kehoe \(2000\)](#). We applied this framework to Italian data during the debt crisis of 2008-2012, and documented that rollover risk accounted for a modest fraction of the increase in government's bond yields. This finding suggests that the sharp reduction in spreads observed upon the establishment of the OMT program was not motivated by a reduction in rollover risk.

Our approach is not limited to sovereign bond markets, and it could be applied in other environments where self-fulfilling expectations may be important drivers of default risk. For example, one could use changes in the liability and asset structure of financial intermediaries in periods such as the Great Depression to assess whether bankruptcies of these institutions were driven by insolvency or due to "bank runs" à la [Diamond and Dybvig \(1983\)](#) or [Gertler and Kiyotaki \(2015\)](#). We leave this application to future research.

References

- Aguiar, Mark and Manuel Amador. 2018. "Self-fulfilling Debt Dilution: Maturity and Multiplicity in Debt Models." Manuscript, Federal Reserve Bank of Minneapolis.
- Aguiar, Mark, Manuel Amador, Hugo Hopenhayn, and Iván Werning. 2018. "Take the Short Route: Equilibrium Default and Debt Maturity." Manuscript, Princeton University.
- Aguiar, Mark, Satyajit Chatterjee, Hal Cole, and Zachary Stangebye. 2016. "Quantitative Models of Sovereign Debt Crises." Forthcoming, Handbook of Macroeconomics, vol. 2.

- Aguiar, Mark, Satyajit Chatterjee, Harold Cole, and Zachary Stangebye. 2017. "Self-Fulfilling Debt Crises, Revisited: The Art of the Desperate Deal." Working Paper 23312, National Bureau of Economic Research.
- Aguiar, Mark and Gita Gopinath. 2006. "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics* 69 (1):64–83.
- . 2007. "Emerging Market Business Cycles: The Cycle Is the Trend." *Journal of Political Economy* 115 (1):69–102.
- Alesina, Alberto, Alessandro Prati, and Guido Tabellini. 1989. "Public Confidence and Debt Management: A Model and a Case Study of Italy." Working Paper 3135, National Bureau of Economic Research.
- Altavilla, Carlo, Domenico Giannone, and Michele Lenza. 2014. "The Financial and Macroeconomic Effects of OMT Announcements." Working Paper 1707, European Central Bank.
- Ang, Andrew and Monika Piazzesi. 2003. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics* 50 (4):745–787.
- Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review* 98 (3):690–712.
- Arellano, Cristina and A. Ramanarayanan. 2012. "Default and the Maturity Structure in Sovereign Bonds." *Journal of Political Economy* 120 (2):187–232.
- Auclert, Adrien and Matthew Rognlie. 2016. "Unique equilibrium in the Eaton–Gersovitz model of sovereign debt." *Journal of Monetary Economics* 84:134–146.
- Ayres, Joao, Gaston Navarro, Junapa Nicolini, and Pedro Teles. 2018. "Sovereign Defaults: The Role of Expectations." *Journal of Economic Theory* 175:803–812.
- Bai, Yan, Seon Tae Kim, and Gabriel Mihalache. 2014. "The Maturity and Payment Schedule of Sovereign Debt." Manuscript, University of Rochester.
- Benjamin, David and Mark LJ Wright. 2009. "Recovery before redemption: A theory of delays in sovereign debt renegotiations."
- Bocola, Luigi. 2016. "The Pass-Through of Sovereign Risk." *Journal of Political Economy* 124 (4):879–926.
- Bocola, Luigi, Gideon Bornstein, and Alessandro Dovis. 2018. "Quantitative Sovereign Default Models and the European Debt Crisis."

- Broner, Fernando, Aitor Erce, Alberto Martin, and Jaume Ventura. 2014. "Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-Out Effects." *Journal of Monetary Economics* 61 (c):114–142.
- Broner, Fernando A., Guido Lorenzoni, and Sergio L. Schmukler. 2013. "Why Do Emerging Economies Borrow Short Term?" *Journal of the European Economic Association* 11 (s1):67–100.
- Calvo, Guillermo A. 1988. "Servicing the Public Debt: The Role of Expectations." *American Economic Review* 78 (4):647–661.
- Chatterjee, Satyajit and Burcu Eyigungor. 2012. "Maturity, Indebtedness and Default Risk." *The American Economic Review* 102 (6):2674–2699.
- Cochrane, John H. 1994. "Permanent and Transitory Components of GNP and Stock Prices." *Quarterly Journal of Economics* 109 (1):241–265.
- . 2009. *Asset Pricing*. Princeton university press.
- Cochrane, John H. and Monika Piazzesi. 2005. "Bond Risk Premia." *American Economic Review* 95 (1):138–160.
- Cole, Harold L. and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crises." *Review of Economic Studies* 67 (1):91–116.
- Conesa, Juan Carlos and Timothy J. Kehoe. 2012. "Gambling for Redemption and Self-Fulfilling Debt Crises." Research Department Staff Report 465, Federal Reserve Bank of Minneapolis.
- Cruces, Juan J. and Christoph Trebesch. 2013. "Sovereign defaults: the price of haircuts." *American Economic Journal: Macroeconomics* 5:85–117.
- Diamond, Douglas W. and Philip H. Dybvig. 1983. "Bank runs, deposit insurance, and liquidity." *Journal of Political Economy* 91 (3):401–419.
- Dovis, Alessandro. 2017. "Efficient Sovereign Default." Forthcoming.
- Eaton, Jonathan and Mark Gersovitz. 1981. "Debt with Potential repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* 48 (2):289–309.
- Fernández-Villaverde, Jesús, Juan Rubio-Ramírez, and Frank Schorfheide. 2015. "Solution and Estimation Methods for DSGE Models." Manuscript, University of Pennsylvania.
- Gertler, Mark and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review* 105 (7):2011–2043.

- Guvenen, Fatih and Anthony A. Smith. 2014. "Inferring Labor Income Risk and Partial Insurance From Economic Choices." *Econometrica* 82 (6):2085–2129.
- Hatchondo, Juan Carlos and Leonardo Martinez. 2009. "Long Duration Bonds and Sovereign Defaults." *Journal of International Economics* 79:117–125.
- Hatchondo, Juan Carlos, Leonardo Martinez, and César Sosa Padilla. 2016. "Debt Dilution and Sovereign Default Risk." *Journal of Political Economy* 124 (5):1383–1422.
- Hébert, Benjamin and Jesse Schreger. 2017. "The Costs of Sovereign Default: Evidence from Argentina." *American Economic Review* 107 (10):3119–3145.
- Koop, Gary, M Hashem Pesaran, and Simon M Potter. 1996. "Impulse response analysis in nonlinear multivariate models." *Journal of econometrics* 74 (1):119–147.
- Longstaff, Francis A, Jun Pan, Lasse H Pedersen, and Kenneth J Singleton. 2011. "How sovereign is sovereign credit risk?" *American Economic Journal: Macroeconomics* 3 (2):75–103.
- Lorenzoni, Guido and Iván Werning. 2013. "Slow Moving Debt Crisis." Manuscript, Northwestern University.
- Lubik, Thomas A. and Frank Schorfheide. 2004. "Testing for Indeterminacy: An Application to U.S. Monetary Policy." *American Economic Review* 94:190–217.
- Nelson, Charles R. and Andrew F. Siegel. 1987. "Parsimonious Modeling of Yield Curves." *Journal of Business* 60:473–489.
- Niepelt, Dirk. 2014. "Debt Maturity Without Commitment." *Journal of Monetary Economics* 68:S37–S54.
- Passadore, Juan and Juan Pablo Xandri. 2015. "Robust Conditional Predictions in Dynamic Games: An Application to Sovereign Debt." Manuscript, MIT.
- Pinto, Brian, Homi Kharas, and Sergei Ulatov. 2001. "An Analysis of Russia's 1998 Melt-down: Fundamentals and Market Signals." *Brookings Papers on Economic Activity* 2001 (1):1–50.
- Roch, Francisco and Harald Uhlig. 2014. "The Dynamics of Sovereign Debt Crises and Bailouts." Manuscript, University of Chicago.
- Sánchez, Juan M., Horacio Sapriza, and Emircan Yurdagul. 2015. "Sovereign Default and Maturity Choice." Manuscript, University of Rochester.
- Stangebye, Zachary. 2014. "Dynamic Panics: Theory and Application to the Eurozone." Manuscript, University of Pennsylvania.

- Sturzenegger, Federico and Jeromin Zettelmeyer. 2006. *Debt defaults and lessons from a decade of crises*. MIT press.
- Svensson, Lars E. O. 1994. "Estimating and Interpreting Forward Rates: Sweden 1992 1994." NBER Working Paper No. 4871.
- Tamer, Elie. 2003. "Incomplete Simultaneous Discrete Response Model with Multiple Equilibria." *Review of Economic Studies* 70 (1):147–165.
- Yue, Vivian Z. 2010. "Sovereign Default and Debt Renegotiation." *Journal of international Economics* 80 (2):176–187.

DATA APPENDIX

Real gross domestic product (GDP). *OECD Quarterly National Accounts*, GDP expenditure approach, volume estimates (reference year 2010), 2000:Q1-2013:Q4.

Debt-to-output ratio. Debt is the face value of outstanding debt securities of the central government obtained from *OECD Quarterly Public Sector Debt*, expressed in millions of euros at current prices. We obtain this series for the period 2000:Q1-2013:Q4, seasonally adjust it, and scale it by GDP at current prices.³²

Debt maturity. We use detailed information on outstanding bonds issued by the Italian central government to construct an indicator of debt maturity for the 2008:Q1-2013:Q4 period. Every quarter, the Italian Treasury publishes a list of all outstanding bonds issued by the central government.³³ We can classify these bonds into four main categories: i) *Buoni ordinari del Tesoro* (BOT); ii) *Certificati del Tesoro Zero Coupon* (CTZ); iii) *Buoni del Tesoro poliannuali* (BTP); iv) *Certificati di credito del Tesoro* (CCT).

The first two categories are zero coupon bonds with a maturity of up to two years. BTP are fixed coupon bonds, with a scheduled payment occurring every six months. CCT are variable coupon bonds, with a scheduled payment occurring every six months. The coupon per unit of principal is computed as a deterministic function of the prevailing yield on BOT. Specifically, letting r_{BOT} to be the annualized yield on the last auction of a BOT. The coupon on the CCT is $r_{\text{BOT}} \times 0.5 + \text{spread}$, where the spread is specified in the contract (typically 15 basis points).

At a given quarter t , we use this information to construct a sequence of payments (principal and coupons) that the government has promised to make for any future date. We denote by $C_t^{(1)}$ the payments due within a year, $C_t^{(2)}$ those due between 1 and 2 years, etc. This calculation does not require an approximation for BOT, CTZ and BTP, because we have information on the principal due at maturity and the series of coupons that each instrument pays over its life. For CTZ, instead, we need to infer the prevailing yield on BOT at future dates in order to compute future coupon payments. We approximate those yields using the time t yield on BOTs with a residual maturity of 1 year.

³²To seasonally adjust the series, we estimate a linear regression

$$b_t = \gamma t + \sum_{j=1}^4 \delta_{j,t} + e_t,$$

where $b_{j,t}$ is outstanding debt (in logs) at time t quarter j , and $\delta_{j,t}$ are quarterly dummies. The seasonally adjusted series is then $\tilde{B}_t = \exp\{b_t\} - \exp\{\sum_{j=1}^4 \delta_{j,t}\}$.

³³The list can be downloaded at http://www.dt.tesoro.it/en/debito_pubblico/dati_statistici/scadenze_titolli_suddivise_anno/index.html.

After computing the sequence of payments we calculate the weighted average life of principal and coupon payments as

$$\sum_{n=1}^N n \frac{C_t^{(n)}}{C_t},$$

where $C_t = \sum_{n=1}^N C_t^{(n)}$. This indicator maps exactly to $1/\lambda_t$ in our model.

We can also use these data to construct the average maturity of new issuances. Specifically, we can define net issuances between period t and $t + 1$ for a given maturity n as $\Delta_t^{(n)} = C_{t+1}^{(n)} - C_t^{(n+1)}$, and the average maturity of new issuances is then

$$\sum_{n=1}^N n \frac{\Delta_t^{(n)}}{\Delta_t},$$

where $\Delta_t = \sum_{n=1}^N \Delta_t^{(n)}$.

Term structure of Italian interest rates. Data on the term structure of Italian government bonds is obtained from *Datastream*. Datastream provides an estimate of the Italian yield curve by fitting a polynomial on the yields on several government securities that differ by residual maturity.³⁴ We use the parameters of this curve to generate nominal bond yields for all maturities between $n = 1$ and $n = 80$ quarters for the 2000:M1-2013:M12 period. We convert yields into bond prices, and construct $Q_t^{\text{ita}}(\lambda)$ using equation (19).

Term structure of German interest rates. Data on the term structure of ZCB for German federal government securities is obtained from the *Bundesbank online database*. We collect monthly data on the parameters of the [Nelson and Siegel \(1987\)](#) and [Svensson \(1994\)](#) model for the period 1973:M1-2013:M12, and we generate nominal bond yields for all maturities between $n = 1$ and $n = 80$ quarters. These data are used to estimate the stochastic discount factor, and to construct $Q_t^{\text{ger}}(\lambda)$ as explained in Section 4.2.

³⁴In the Italian case, Datastream uses BTP with a maturity of up to 30 years. The fitting curve is a polynomial of 3rd degree, estimated by OLS on daily data. The series mnemonic are GVIL03(CM05) for a bond with residual maturity of 5 years, GVIL03(CM10) for a bond with residual maturity of 10 years, etc.