Reputation, Bailouts, and Interest Rate Spread Dynamics*

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Abstract

We propose a joint theory for interest rate dynamics and bailout decisions. Interest rate spreads are driven by time-varying fundamentals and expectations of future bailouts from a common government. Private agents have beliefs about whether the government is a commitment type, which never bails out, or an optimizing type, which sequentially decides whether to bail out or not, and learn by observing its actions. The model provides an explanation for why we often observe governments initially refusing to bail out borrowers at the beginning of a crisis even if they eventually end up providing a bailout after the crisis aggravates. In the typical equilibrium outcome, spreads are non-monotonic in fundamentals, and decisions on whether to bail out individual borrowers affect the spreads of other borrowers. These dynamics are consistent with the behavior of bailouts and spreads during the recent US financial crisis and European debt crisis.

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1 Introduction

Expectations about the generosity of bailouts are an important driver for interest rates on defaultable debt. For example, in the recent US financial crisis, CDS spreads of large financial institutions rose sharply after regulators let Lehman Brothers fail, a negative news event about the willingness of the government to bail out financial firms, but reversed course after the announcement of the Troubled Asset Relief Program (TARP), a positive news event, even though fundamentals arguably worsened. Similarly, EU sovereign debt spreads declined significantly after the announcement of the unlimited bond buying program (OMT) by the ECB, which was a positive news event about the willingness to provide government support in the future.

In this paper, we propose a joint theory for interest rate dynamics and bailout decisions. We study a dynamic model in which spreads are driven by fundamentals and time-varying expectations of future bailouts from a common government. Private agents have beliefs about whether the government is a *commitment type*, which never bails out, or an *optimizing type*, which sequentially decides whether to bail out or not, and learn by observing its actions. The model rationalizes why governments initially refuse to bail out borrowers even though they eventually end up bailing out borrowers as the crisis gets more severe. This was indeed the case during the US financial crisis when Lehman Brothers was allowed to fail but eventually the government implemented TARP. This in turn implies that in equilibrium spreads are hump-shaped during a crisis: they start low, then jump at the beginning of a crisis when the government initially refuses to bail out, and eventually fall if the crisis worsens and the government agrees to a bailout. Moreover, the model can generate the contagion and increase in the sensitivity of spreads to fundamentals we observed during the crisis.

We consider a simple economy where borrowers borrow from risk-neutral lenders to invest in a risky project. Borrowers can be interpreted as either banks who raise funds from depositors and lend to the non-financial sector, or sovereign governments who borrow to finance expenditures. Absent a bailout, borrowers default on debt after the realization of a bad idiosyncratic shock. Default imposes a social cost that can be avoided if the government bails out the lenders. The government can be one of two types: a commitment type, which never bails out, or an optimizing type, which trades off the static benefits of bailing out the lenders and avoiding the social default cost, with the dynamic costs of losing future reputation. This type is not observed by private agents, who learn about it over time by observing the government's actions.

The model generates a positive relationship between interest rate spreads and the government's *reputation*, defined as the private agents' prior about facing the commitment type. If the government's reputation is high, lenders expect to be bailed out with low probability and there is a higher probability of default; therefore, lenders need to increase interest rates in order to break even.

The economy is subject to an aggregate shock that changes the distribution over idiosyncratic shocks faced by borrowers and thus affects the fraction of borrowers that need a bailout in order to avoid a default. In normal times, when all borrowers receive high shocks, borrowers do not default; therefore, there is no need for a bailout. As a result, there is no learning about the type of the government, so spreads remain low (and constant) if the initial reputation of the government is low. If the economy is hit with an intermediate shock that affects a small fraction of borrowers adversely, the static incentives to bail out increase, but by choosing not to bail out, the optimizing type can increase its reputation. If the latter effect is large enough, the best response for the optimizing type is to randomize between bailing out and not. If private agents observe no bailout, the reputation of the government increases because observing no bailout is more likely if the government is the commitment type. Thus, private agents assign a larger probability to no bailouts in the future and spreads rise for all borrowers, even those that currently have the high idiosyncratic shock. If the economy is hit by a large negative shock that affects the majority of borrowers, the static bailout benefits eventually dominate the dynamic reputational costs. As a result, the optimizing type government chooses to bail out, which results in a drop in reputation and hence a sharp reduction in spreads despite the fundamentals being worse.

Along the equilibrium path, spreads and debt issuances are less responsive to the state (aggregate and idiosyncratic) when reputation is low. That is, if the probability of facing the optimizing type is high, then debt prices are mostly unaffected by the state, since lenders expect a bailout in bad states with a high probability. This generates a differential sensitivity of prices to fundamentals that Acharya et al. (2016) and Cole et al. (2016) document in the data.

To generate the hump-shaped dynamics in the baseline model, an economy must reach a severe crisis by first transiting to an intermediate state in which a small fraction of borrowers need a bailout. This is not necessary when the government learns about the aggregate state of the world from noisy prices, similar to Nosal and Ordoñez (2016). The reason is that the government's incentives to bail out are driven by its beliefs about the true state of the world, which can be driven by noise rather than changes in fundamentals.

In the baseline model we show that it is worthwhile to bail out only if a sufficiently large fraction of borrowers will default absent a transfer. This is because the static benefits of a bailout must be large enough to compensate for the loss in reputation. In reality, however, we often observe cases in which governments bail out small banks. One recent example is the case of the Italian government bailing out two mid-sized banks in 2017, in violation of the principles of the European banking union. In an extension of the model, we show that when the borrowers' default decisions are dynamic – in that they consider future profits in deciding whether to default or not – the share of borrowers that ends up receiving the bailout is not a sufficient statistic for the static benefits of a bailout. The government may choose to bail out a few borrowers in order to avoid contagion to other borrowers that would default absent a bailout. The reason for this is that since bailouts (or lack thereof) change private beliefs about the government's future behavior, bailouts affect borrowers' continuation values of not-defaulting. Thus, the static benefits of a bailout can be large even if in equilibrium the government ends up bailing out only a small fraction of borrowers.

Finally, we show that the model can help to account for some patterns in three recent crises: the US financial crisis, the recent banking crisis in Italy after the institution of the Single Resolution Mechanism (SRM) within the context of the European banking union, and the European sovereign debt crisis. In particular, we use our model to interpret the movements in spreads after major bailout and non-bailout announcements. Consistent with the predictions of our model, if after an adverse event there is no bailout, the CDS spreads for borrowers not directly affected by the adverse event go up and so does the sensitivity of spreads to fundamentals. The opposite happens when we observe a bailout or an announcement of government support for equity and debt holders. This is also consistent with Chirinko et al. (2019). They argue that Puerto Rico had been able to borrow at relatively low interest rates despite bad fundamentals because investors anticipated a bailout by the US Treasury. However, after the decision of the US congress to not bail out the city of Detroit, the authors show that the Puerto Rican spreads rose dramatically as our theory predicts. We also summarize some recent empirical work that provides further justification for our mechanisms. See Acharya et al. (2016), Veronesi and Zingales (2010), Schweikhard and Tsesmelidakis (2011), Kelly et al. (2016) for the US financial crisis, Neuberg et al. (2018) for the SRM, and Ardagna and Caselli (2014) for the European debt crisis.

Our model can also shed light on the discussion among academics and policymakers about whether Lehman Brothers should have been allowed to fail.[1] Our paper suggests that dynamic reputational considerations might be one reason the Fed did not bail out Lehman Brothers. As our model suggests, a bailout of Lehman would have severely reduced the Fed's reputation and led to a reduction of borrowing costs for all banks. This would have incentivized banks to borrow even more in the future, thus increasing the likelihood of future bailouts and implying even greater costs to taxpayers. Moreover, it was precisely the potential cost associated with allowing Lehman Brothers to fail that made it a good time for the Fed to build reputation. This sentiment is echoed by Mishkin

[1] For instance, see Ball (2018).

(2011), who argues that "letting Lehman fail would serve as a warning to other financial firms that they needed to rein in their risk taking."

Related Literature

Our paper is related to a large literature on repeated games with behavioral types pioneered by Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982). Phelan (2006) uses this framework to study a model of government reputation with hidden types that can stochastically evolve. Our model builds on this framework by embedding it in an investment model with endogenous default and interest rates. A crucial difference between our models is that in Phelan (2006), the temptation for the government to reveal its type is high when reputation is high. In contrast, in our model, the incentives for the government to reveal its type are large when reputation is low. This distinction is important for generating the desired movement in spreads. Dovis and Kirpalani (2020) also use a similar framework to study the efficacy of fiscal rules when private agents are strategic. A key difference between these two environments is that they find conditions so that default is never optimal; so, their model is silent on the behavior of spreads.

Our paper is also related to a literature on reputation and sovereign default, for example Cole et al. (1995), D'Erasmo (2008), and Amador and Phelan (2018). In these models, there is uncertainty about the type of the borrower, while in ours there is uncertainty about the type of the bailout authority (which we call the government). This allows us to study an environment in which spreads are driven by bailout expectations.

Our paper is related to the seminal work of Kareken and Wallace (1978), who study the effects of debt guarantees on ex-ante incentives for borrowers. See Farhi and Tirole (2012), Chari and Kehoe (2015), Davila and Walther (2017), and Chari et al. (2016) for more recent contributions to this literature. Some recent papers including Gourinchas and Martin (2017), Nikolov and Cooper (2018), de Ferra (2017) and Sandri (2015) study the effects of bailouts on the debt accumulation decisions in the European Monetary Union. In contrast to these papers, the bailout decisions in our model are dynamic due to reputation building incentives. This feature is critical to account for the non-monotone behavior of spreads during crises.

Nosal and Ordoñez (2016) study a model in which governments learn about the state of the economy through the actions of private agents. In contrast to our model, there is no uncertainty about the type of the government. As mentioned above, uncertainty about the type of the government is crucial to generating the movement in spreads. In particular, their model cannot account for the increase in spreads if there is no bailout observed. In Section 4 we extend our model to allow for the government to learn about the state through the actions of private agents and prices, and we show how these two channels interact.

Finally, our section on two-sided learning is related to the literature that studies the link between real activity and the ability to learn about fundamentals. Examples of such papers include Veldkamp (2005) and Fajgelbaum et al. (2017). In our model, when reputation is low, prices are less sensitive to fundamentals, which limits the amount the government can learn about the true state of the world. This parallels the idea in these papers that learning is harder in bad economic times when there is low investment.

Outline The rest of paper is organized as follows. In Section 2 we present our baseline model, and in Section 3 we characterize a class of Markov equilibria and derive our main results. In Section 4 we study an extension of the model in which the government learns about the true state of the world from prices, and in Section 5 we show that the government may choose to bail out few borrowers to avoid contagion when default decisions are dynamic. Section 6 examines three events through the lens of our results. Finally, Section 7 discusses the interaction of our mechanism with ex-ante policies and Section 8 concludes the paper.

2 Model

Environment

For much of the main text, we illustrate our results by means of a simple example. We show how these results generalize in Appendix B.5 and B.6. Time is discrete and is indexed by $\tau = 0, 1, \ldots$. The economy is populated by a *government (bailout authority)* and a continuum of *borrowers, lenders*, and *taxpayers*. In each period there is a stage game with two sub-periods, $t = 1, 2$.

Borrowers are ex-ante symmetric, risk-neutral, and care only about consumption in sub-period 2. In sub-period 1, borrowers have no capital and must borrow to finance an investment opportunity. If a borrower invests an amount k in sub-period 1, it obtains a return θk^{α} in sub-period 2, where $\alpha \in (0, 1)$ and θ is an idiosyncratic productivity shock. The distribution of θ depends on the aggregate state of the world realized in sub-period 2, s. The aggregate state s is realized according to a distribution P. For illustrative purposes, we assume that the state can take three values: $s \in \{s_L, s_M, s_H\}$ with probabilities p_L, p_M, and p_H, respectively. We assume that θ can take on two values, $\theta_H > 0$ and $\theta_L = 0$, and let $h(\theta|s)$ denote the probability of drawing θ in state s. In state s_H, referred to as *normal times*, all the borrowers have high productivity, so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M, referred to as *mild crisis*, $h(\theta_H|s_M) = 1 - \mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L, referred to as *severe crisis*, all borrowers have low productivity, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$.

After the realization of θ, each borrower can default on its debt. If the borrower defaults, it loses the claim on investment and its payoff is zero. Default also imposes a social cost, which is internalized by the government. The government can impose a tax on taxpayers and make transfers to the borrower in sub-period 2 to avoid default and the associated social cost. Social default costs are given by an increasing function, $C(\Delta B)$, where Δ is the fraction of borrowers that default, and B is the average level of debt issued by borrowers in sub-period 1. For the purposes of our example, we assume that $C(x) = \psi x$, with $\psi < 1$, but allow for a more general specification in Appendix B.5.[2]

Lenders have a large endowment of the final consumption and investment good in the first sub-period. They are risk neutral and have preferences over consumption in sub-periods 1 and 2, x_1 and $x_2(s)$, given by

$$x_1 + q \sum_s p_s x_2(s),$$

where q is the discount factor across sub-periods. Taxpayers have linear utility and an endowment E in sub-period 2, and can be taxed by the government.

The government can be one of two types: *commitment* or *optimizing*. We assume that the true type of the government evolves according to the transition matrix:

$$\mathbb{P} = \begin{bmatrix} p_c & 1 - p_c \\ p_{nc} & 1 - p_{nc} \end{bmatrix},$$

where $1 - p_c$ is the probability of transiting from the commitment to the optimizing type, and p_{nc} is the probability of transiting from the optimizing to the commitment type. We assume $p_c > p_{nc} \geqslant 0$ so that types are persistent.[3] The type of the government is not observable by private agents (i.e., borrowers and lenders). At the beginning of period 0, private agents share a common prior π_0 that the government is the commitment type. We will refer to private agents' beliefs that the government is the commitment type as the government's *reputation*.

The commitment type never bails out the borrowers, while in each period the optimizing type decides whether to bail out or not and the size of the bailout. To finance a bailout, it raises lump-sum taxes from taxpayers. The government maximizes a weighted sum of

[2]One interpretation of this cost is that the absence of a bailout leads to a reduction in the net worth of the banking sector. This reduction in net worth might have a real cost associated with it, for example reduced investment or fire sales, which is represented by the function $C(\cdot)$.

[3]Besides this restriction p_c and p_{nc} can take on any values, including the symmetric case in which $p_c = 1 - p_{nc}$.

the utilities of lenders, the taxpayers, and the borrowers net of the social default costs. We let λ be the Pareto weight on the borrowers' utility and $1-\lambda$ be the Pareto weight on lenders and taxpayers. We assume that $\lambda \leqslant 1/2$ so that in the absence of social default costs, using bailouts just as a means to transfer resources from taxpayers to borrowers is never optimal.[4] The government discounts utility across periods at rate β.

Discussion of assumptions To help us understand some of the events of the US financial crisis in Section 6, we will interpret the borrowers in this model as banks/financial firms and the lenders as depositors. We follow the common practice in the literature and model a combined financial and non-financial sector in which banks raise funds from the depositors and directly operate the production technology. This is equivalent to a model in which banks raise funds from depositors and make loans to non-financial firms. In crisis times, when the productivity of the non-financial sector is low, they are not able to repay banks and this in turn causes the banks to default. Our theory is particularly applicable to the financial sector because it is the default of banks that arguably result in sizable social default costs, due to spillovers to other banks as well as other parts of the economy.[5] Of course, there are clearly some large non-financial firms that are also likely to generate these default costs and so our model is applicable to these firms as well.

Our results extend to an environment in which borrowing is driven by consumption smoothing motives, as in much of the sovereign default literature. See Appendix B.3 for further discussion. One can use this alternative setup to think about the impact of the Greek bailout on other southern European countries during the recent crisis in Europe.

Finally, while we assume that the commitment type never bails out, it is not true in general that the optimal policy with commitment is to never bail out in sub-period 2.[6] For example, if the social default costs are very high, then it might be optimal to bail out, even with commitment. However, if ψ is sufficiently small and α is sufficiently large, then we can show that the optimal policy with commitment is to never bail out (see footnote 10). We will restrict attention to economies where these conditions are met so that bailouts are never optimal ex-ante but are optimal ex-post. Under this assumption, it is natural to assume that the commitment type never bails out. Alternatively, instead of considering behavioral types, we could consider payoff types where governments experience different levels of the social default cost. In this case the commitment type will correspond to

[4] All our results are valid if we assume that the government cares only about the welfare of lenders and taxpayers, but not of the borrowers. In the context of banking, we can think of the government caring about retail bondholders. In the context of the EU, we can think of the government as representing Germany, who cared about the health of German banks holding Greek bonds. The assumption that lenders and taxpayers are weighted equally is just for convenience and does not affect our results.

[5] See for instance Gertler and Kiyotaki (2010).

[6] See for instance Sandri (2015).

the case in which ψ is small. Therefore, even though this type can bail out ex-post, it will optimally choose not to. (See Appendix B.4.)

In Appendices B.5 and B.6 we also show that our results extend to a more general process for θ and allow for persistence in both the aggregate and the idiosyncratic shock and private renegotiation of debt contracts.

Markov Equilibria

We now describe in detail the interaction between private agents and the government. The timing of events in each period is as follows. In sub-period 1, borrowers choose the amount of debt to issue given the debt price schedule. In sub-period 2, the state of nature s and the idiosyncratic shocks θ are realized. After observing s, given the distribution of inherited debt, the optimizing type government chooses whether to bail out the borrowers and if so, the level of transfers. The commitment type never bails out. The borrower then decides whether to default or not.

We will focus on *symmetric Markov equilibria* where all histories are summarized by the posterior probability of facing the commitment type, π, and all individual borrowers choose the same level of debt in sub-period 1.

We describe the actions and payoffs of the agents. We start with the borrower's problem in sub-period 1. The borrower's problem is to choose debt, b, and investment, k, to maximize expected consumption in sub-period 2 given the pricing schedule Q and the bailout transfer function of the optimizing type, $T(b, k, \theta|\pi, \Gamma, s)$ where (b, k, θ) and (π, Γ, s) are the borrower's and the aggregate states in the second sub-period and Γ is the joint distribution of individual debt holdings and capital stock across borrowers. Since we are focusing on symmetric equilibria where all borrowers make the same choices, in equilibrium, the distribution has point mass over the aggregate levels $(B(\pi), K(\pi))$. The borrower's maximization problem is

$$\max_{b,k} \sum_{s} p_s \sum_{\theta} h(\theta \mid s) \left[\pi \max\{\theta k^{\alpha} - b, 0\} + (1-\pi) \max\{\theta k^{\alpha} + T(b, k, \theta|\pi, \Gamma, s) - b, 0\}\right] \tag{1}$$

subject to the budget constraint in sub-period 1,

$$k \leqslant Q(b, k|\pi, \Gamma)\, b.$$

To understand the objective function, note that the borrower has positive consumption if the returns on the investment project plus the bailout transfer T – which is expected with probability $1-\pi$ – are higher than the debt contracted in the first sub-period, b. In all the other cases the borrower has a payoff of zero because of limited liability. In the budget constraint, $Q(b,k|\pi,\Gamma)$ refers to the price of debt for the borrower given choices (b,k), the prior π, and the aggregate distribution Γ.

The pricing schedule Q must satisfy the zero-profit condition for the lenders:

$$Q(b,k|\pi,\Gamma) = q\left[p_H + p_M(1-\mu) + \right. \\ \left. + p_M\mu(1-\pi)\mathbb{I}_{\{T(\theta_L,b,k|\pi,\Gamma,s_M)\geqslant b\}} + p_L(1-\pi)\mathbb{I}_{\{T(\theta_L,b,k|\pi,\Gamma,s_L)\geqslant b\}}\right] \tag{2}$$

if $b \leqslant \theta_H k^\alpha$. That is, the price of debt equals the discounted value of payments that lenders receive if the borrower receives a high productivity shock and can repay the debt (the first line on the right side of (2)) plus the payments the lenders receive if the borrower has a low productivity shock and receives a bailout transfer that enables repayment (the second line on the right side of (2)).

We now consider the problem of the optimizing government in sub-period 2. The state that this government confronts is given by the prior π, the exogenous state s, and the distribution of individual debt holdings and capital stock, $\Gamma(b,k)$. The optimizing government chooses the transfers $T(b,k,\theta)$ to each borrower. These transfers can affect the borrower's utility and thus the fraction of borrowers who default and the default cost. Given the state (π,Γ,s) and an arbitrary transfer scheme $T(\cdot)$, the fraction of borrowers that default is given by the measure of borrowers with not enough resources to repay their debt:

$$\Delta(\pi,\Gamma,s,T(\cdot)) = \sum_\theta h(\theta \mid s)\int \mathbb{I}_{\{\theta k^\alpha - b + T(b,k,\theta)\leqslant 0\}} d\Gamma(b,k). \tag{3}$$

The optimizing government's static payoff in sub-period 2 can then be written as

$$\lambda\left[\int\sum_\theta h(\theta|s)\max\{\theta k^\alpha + T(b,k,\theta) - b, 0\}\,d\Gamma\right] \\ + (1-\lambda)\left[(1-\Delta(\pi,\Gamma,s,T))\,B - \int\sum_\theta h(\theta \mid s)\,T(b,k,\theta)\,d\Gamma(b,k)\right] - \psi\Delta(\pi,\Gamma,s,T)\,B \tag{4}$$

where the first term in square brackets is the average borrower's utility (recall that λ is the Pareto weight on the borrower), the second term in square brackets is the lenders' payoff in the second subperiod, $(1-\Delta(\pi,\Gamma,s,T))\,B$, net of the costs of raising resources for the bailout transfers from the tax-payers, and the third term denotes the costs of default $\psi\Delta(\pi,\Gamma,s,T)\,B$. The optimizing government chooses the transfers $T(b,k,\theta)$ to maximize the sum of the static payoff plus the discounted continuation value, $W(\pi')$:

$$W_2(\pi, \Gamma, s) = \max_{T(b,k,\theta)} \lambda \left[\int \sum_\theta h(\theta|s) \max\{\theta k^\alpha + T(b,k,\theta) - b; 0\} d\Gamma \right] \tag{5}$$
$$+ (1-\lambda) \left[(1-\Delta) B - \int \sum_\theta h(\theta \mid s) T(b,k,\theta) \, d\Gamma(b,k) \right] - \psi \Delta B$$
$$+ \beta W(\pi')$$

where Δ is given by (3) and the new posterior $\pi' = \pi'(\pi, \Gamma, s)$ is defined using Bayes' rule

$$\pi' = \begin{cases} \frac{\pi p_c}{\pi + (1-\pi) \Pr(T(\pi,\Gamma,s)=0)} + \frac{(1-\pi) \Pr(T(\pi,\Gamma,s)=0) p_{nc}}{\pi + (1-\pi) \Pr(T(\pi,\Gamma,s)=0)} & \text{if } T(b,\theta) = 0 \, \forall (b,\theta). \\ p_{nc} & \text{if } T \neq 0 \end{cases} \tag{6}$$

That is, if the optimizing type chooses to not disburse any bailout transfers – like the commitment type – then the new posterior is determined by Bayes' rule which implies the term in the first line. If instead it chooses $T \neq 0$ and deviates from the commitment type's policy, then private agents learn that they are facing the optimizing type for sure and the posterior is p_{nc}, the probability that the government's type switches from the optimizing type to the commitment type.

The optimizing government's value in sub-period 1 with prior π is given by

$$W(\pi) = (1-\lambda) [e - Q(B(\pi), K(\pi) | \pi, \Gamma(\pi)) B(\pi)] + q \sum_s p_s W_2(\pi, \Gamma(\pi), s), \tag{7}$$

where $(1-\lambda) [e - Q(B(\pi), K(\pi) | \pi, \Gamma(\pi)) B(\pi)]$ is the lenders' consumption in the first sub-period and $\Gamma(\pi)$ has mass one at $(B(\pi), K(\pi))$.

Definition. A *Symmetric Markov Equilibrium* is an individual debt and investment strategy $b(\pi)$ and $k(\pi)$, aggregate debt $B(\pi)$ and investment $K(\pi)$, a pricing schedule $Q(b, k|\pi, \Gamma)$, a transfer strategy for the (optimizing) government $T(b, k, \theta|\pi, \Gamma, s)$, and a law of motion for beliefs, π', such that (i) the debt and investment strategy solves (1); (ii) $b(\pi) = B(\pi)$ and $k(\pi) = K(\pi)$; (iii) the pricing schedule satisfies (2); and (iv) the transfer rule solves (5), where the continuation value W is defined by (7) and the law of motion for beliefs is (6).

3 Bailouts and Spreads Dynamics

In this section, we characterize a class of Markov equilibria and show that the equilibrium outcomes are consistent with the experiences of recent financial and debt crises. In this class of equilibria, investment, debt issuances, the price of debt, and bailout probability are decreasing in the government's reputation. Moreover, the optimizing type government does not bail out in normal times, mixes during a mild crisis, and bails out for sure in a severe crisis. Thus, if private agents observe no bailout in a mild crisis, their beliefs of facing the commitment type increases. Consequently, interest rates go up while debt issuances and investment decrease. If the state of the economy worsens and the economy transits to a severe crisis, then the optimizing type bails out for sure and its reputation collapses; therefore, interest rates decline despite the worse economic fundamentals. The equilibrium outcome can then rationalize the hump-shape of interest rate spreads and why bailouts are often delayed in a crisis. Moreover, we show in an extension that spreads and debt issuances are less responsive to the state when reputation is low; therefore, the cross-sectional volatility of spreads is low when the government has low reputation.

Bailout Decision We begin by characterizing the decision of the government in sub-period 2. One issue that arises is that in a symmetric equilibrium where the distribution of debt holdings is degenerate, the transfer to a borrower that chooses $(b, k) \neq (B, K)$ and deviates from the equilibrium path is not pinned down. This is because each borrower is measure zero; therefore, allowing a single (measure zero) borrower to default does not affect the utility of the government. In principle, it is possible to construct equilibria where transfers off the equilibrium path impose some discipline even absent reputational gains. See Chari et al. (2016), Farhi and Tirole (2012), and Davila and Walther (2017) for related discussions.

Here we select the transfer scheme by considering the limit of the finite borrower case as the number of borrowers converges to infinity. The details of this construction are provided in the Appendix B.1. In this case, either the optimizing type government mimics the strategy of the commitment type or it chooses the statically optimal transfer scheme that transfers the minimal amount required to avoid default by all borrowers who would have done so absent the transfer. That is, for all (π, Γ, s), the bailout transfers $T(b, k, \theta|\pi, \Gamma, s)$ are either zero for all (b, k, θ) or $T^*(b, k, \theta) = \max\{b - \theta k^\alpha, 0\}$. In the context of our example,

$$T^*(b, k, \theta_L) = b \quad \text{and} \quad T^*(b, k, \theta_H) = 0$$

for all s. Therefore,

$$T \in \{\mathbf{0}, T^*\}, \tag{8}$$

where $\mathbf{0}$ is the function identically equal to zero. From a static perspective, it is always optimal for the government to intervene and avoid default. This is because if the government transfers T^*, it can avoid the social cost of default while the sum of utilities of taxpayers and lenders is unchanged.

Notice further that this transfer scheme implies that in case of a bailout, the borrower receives its outside option of defaulting and nothing more. This is because the assumption that the borrower's Pareto weight is lower than that of taxpayers', $\lambda \leqslant 1/2$, implies that any additional transfers make the government strictly worse off. As a result, given such a transfer policy, the borrower's continuation value is independent of the implemented transfers and is given by $U_2(b, k, \theta) = \max\{\theta k^{\alpha} - b, 0\}$. Hence, the bailout authority's decisions only affect the borrower through their effect on the interest rates the borrower faces.

Lemma 1. *Given the transfer scheme in (8), the borrower's continuation value is independent of whether the government chooses the statically optimal transfers,* T^**, or it mimics the commitment type and chooses* **0**.

Given our selection of off-equilibrium transfers, we can summarize the bailout authority's strategy by the probability that it will implement the statically optimal transfer scheme, $\sigma(\pi, B, K, s)$. We will call σ the *bailout policy*. Bailouts generate static benefits and impose dynamic costs on the optimizing type government. Using T^* in the expression for the government's static payoff (4), the static value of bailing out is

$$\omega^{\text{bailout}}(B, K, s) = \lambda h(\theta_H \mid s)(\theta_H K^{\alpha} - B) + (1 - \lambda) h(\theta_H \mid s) B$$

where $h(\theta_H \mid s)$ denotes the fraction of debt that is paid back absent a bailout.[7] The static value of not bailing out (and allowing default) is

$$\omega^{\text{no-bailout}}(B, K, s) = \lambda h(\theta_H \mid s)(\theta_H K^{\alpha} - B) + (1 - \lambda) h(\theta_H \mid s) B - \psi h(\theta_L | s) B.$$

Thus, the *static incentives* to bail out, defined as

$$\Delta\omega(B, K, s) \equiv \omega^{\text{bailout}}(B, K, s) - \omega^{\text{default}}(B, K, s) = \psi h(\theta_L | s) B, \tag{9}$$

are increasing in the debt level B and the fraction of borrowers with θ_L, $h(\theta_L|s)$.

As a consequence of bailing out, there is a loss in reputation of the government, as described in (6). We will later show that the equilibrium value for the government, $W(\cdot)$, is increasing in the government's reputation, π; hence, the loss of reputation is costly for the government.

We now characterize the bailout policy. The state variables in sub-period 2 are (B, s, π). Let $\zeta = 1$ denote the event that a bailout occurs and $\zeta = 0$ denote the event of no bailout.

[7]Recall that since the government cares about taxpayers who have linear utility, implementing a bailout lowers welfare by $\Delta(B, s) B$, which nets out the additional transfer to the lenders.

Given strategy σ, the law of motion for beliefs satisfies

$$\pi' = \begin{cases} p_{nc} + \frac{\pi}{\pi+(1-\pi)(1-\sigma)}\left(p_c - p_{nc}\right) & \text{if } \zeta = 0 \\ p_{nc} & \text{if } \zeta = 1 \end{cases}. \tag{10}$$

If the government chooses to bail out, its value is

$$\Omega^{\text{bailout}}\left(\pi, B, K, s\right) = \omega^{\text{bailout}}\left(B, K, s\right) + \beta W\left(p_{nc}\right), \tag{11}$$

and if it chooses not to bail out, its value is

$$\Omega^{\text{no-bailout}}\left(\pi, B, K, s\right) = \omega^{\text{no-bailout}}\left(B, K, s\right) + \beta W\left(p_{nc} + \frac{\pi\left(p_c - p_{nc}\right)}{\pi + \left(1-\pi\right)\left(1-\sigma\right)}\right). \tag{12}$$

Thus, the value in equilibrium is

$$\Omega\left(\pi, B, K, s\right) = \max\left\{\Omega^{\text{bailout}}\left(\pi, B, K, s\right), \Omega^{\text{no-bailout}}\left(\pi, B, K, s\right)\right\}, \tag{13}$$

where the continuation value $W(\pi)$ is defined by

$$W\left(\pi\right) = \left(1-\lambda\right)\left(e - Q\left(\pi\right)B\left(\pi\right)\right) + q\sum_s p_s \Omega\left(\pi, B\left(\pi\right), K\left(\pi\right), s\right), \tag{14}$$

where $B(\pi)$ and $K(\pi)$ are the allocation rules for aggregate debt and capital along the equilibrium path given prior π, respectively, and $Q(\pi) = Q\left(B(\pi), K(\pi) | \pi, B(\pi), K(\pi)\right)$ is the price of debt in the (symmetric) equilibrium outcome. The optimal strategy for the optimizing type is then

$$\sigma\left(\pi, s\right) = \begin{cases} 0, & \text{if } \psi h\left(\theta_L|s\right)B\left(\pi\right) \leqslant \beta\left[W\left(p_{nc} + \pi\Delta p\right) - W\left(p_{nc}\right)\right] \\ \tilde{\sigma}, & \text{if } \psi h\left(\theta_L|s\right)B\left(\pi\right) = \beta\left[W\left(p_{nc} + \frac{\pi\Delta p}{\pi+(1-\pi)(1-\tilde{\sigma})}\right) - W\left(p_{nc}\right)\right] \\ 1, & \text{if } \psi h\left(\theta_L|s\right)B\left(\pi\right) \geqslant \beta\left[W\left(p_{nc} + \Delta p\right) - W\left(p_{nc}\right)\right] \end{cases}, \tag{15}$$

where $\Delta p \equiv p_c - p_{nc}$. Recall that $\psi h\left(\theta_L|s\right)B\left(\pi\right)$ is the static benefit of bailing out. The dynamic costs of bailing out depend on private agents beliefs that the optimizing type will bail out. Consequently, the optimizing type will choose not to bailout for sure ($\sigma = 0$) if the static benefits are smaller than the dynamic benefits when private agents hold beliefs consistent with this strategy (case 1 in (15)). Similarly, the optimizing type will choose to bailout for sure ($\sigma = 1$) if the static benefits are larger than the dynamic benefits when private agents hold beliefs consistent with this strategy (case 3 in (15)). In the case in which neither $\sigma = 1$ nor $\sigma = 0$ are optimal, the best response of the optimizing type is to mix between bailing out and not with an interior probability $\tilde{\sigma}$ (case 3 in (15). The dynamic benefits of such a strategy are exactly equal to the static benefits when private agents hold beliefs consistent with it.

Debt Issuances and Prices We now characterize the decisions for private agents given a bailout policy σ. Given this policy, the no-arbitrage condition for the lenders (2) can be written as

$$Q(\pi) = q\{p_H + p_M(1-\mu) + p_M\mu(1-\pi)\sigma(\pi, s_M) + p_L(1-\pi)\sigma(\pi, s_L)\}. \tag{16}$$

Note that $Q(\cdot)$ does not depend on the individual debt level and investment, b and k, as long as $b \leqslant \theta_H k^\alpha$.[8] (In Section B.5, we study a more general model in which Q depends on b and k.) We further characterize the private equilibrium by defining a new variable, $\bar{\gamma}$, equal to the probability that an individual borrower will be bailed out conditional on drawing θ_L:

$$\bar{\gamma}(\pi) \equiv \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L}, \tag{17}$$

where $P_L \equiv p_L + p_M\mu$ is the probability that a project fails, i.e., $\theta = \theta_L$. The debt prices can then be written as:

$$Q(\pi) = qP_H + qP_L\bar{\gamma}(\pi)$$

where $P_H \equiv p_H + p_M(1-\mu)$ is the probability that $\theta = \theta_H$. That is, the price of debt is the discounted value of two components: the probability of repayment absent government intervention, P_H, and the probability of receiving a bailout when there would be a default absent government intervention, $P_L\bar{\gamma}(\pi)$.

Using the fact that Q is independent of b and the result in Lemma 1 that the borrower's continuation value is independent of the bailout policy, $U_2(b, k, \theta) = \max\{\theta k^\alpha - b, 0\}$, the problem for the borrower in period 1 (1) can be written as

$$\max_k P_H\left(\theta_H k^\alpha - \frac{k}{Q(\pi)}\right). \tag{18}$$

Using the first order condition and assuming representativeness we have

$$K(\pi) = (\alpha\theta_H Q(\pi))^{\frac{1}{1-\alpha}} \tag{19}$$

and

$$B(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q(\pi)^{\frac{\alpha}{1-\alpha}}. \tag{20}$$

[8]It is easy to show that in equilibrium $b \leqslant \theta_H k^\alpha$.

since $b = k/Q(\pi)$. Clearly, individual debt issuances and investment are increasing in $Q(\pi)$, and since a decrease in π increases Q, we have the following result:

Lemma 2. *If the function $\sigma(\pi, s)$ is decreasing in π, then the price of debt is decreasing in π, i.e., if $\pi_H \geqslant \pi_L$, then $Q(\pi_H) \geqslant Q(\pi_L)$. Furthermore, $B(\pi_L) \geqslant B(\pi_H)$ and $K(\pi_L) \geqslant K(\pi_H)$.*

Continuous Monotone Equilibria

To show that the set of symmetric Markov equilibria is non-empty, we prove the existence of a class of continuous monotone equilibria that have some desirable properties. The equilibrium objects in this class are continuous and monotone in the government's reputation, π.

Let $\Delta(\pi, s) = \Delta(\pi, B(\pi), K(\pi), s)$ and $\sigma(\pi, s) = \sigma(\pi, B(\pi), K(\pi), s)$. We will show that there exist $W(\pi)$, $\sigma(\pi, s)$, $B(\pi)$, $K(\pi)$, and $Q(\pi)$ that jointly satisfy (14), (15), (16), (19), and (20). The next proposition shows that the set of *continuous monotone equilibria* is non-empty.

Proposition 1. *If p_{nc} is sufficiently small, there exists a continuous monotone equilibrium in which debt issuances, investment, and debt prices are decreasing in the level of reputation, the probability of bailout along the equilibrium path, $\sigma(\pi, s)$, is decreasing in the level of reputation with $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H)$, and the value for the government, $W(\pi)$, is increasing in the level of reputation.*

The proof of this proposition is provided in the Appendix. To establish the result, we show that the equilibrium value in (14), the bailout policy *along the equilibrium path*, the equilibrium debt policy rule $B(\pi)$, and the equilibrium pricing schedule $Q(\pi)$ are a fixed point of an operator and then show that the operator admits a fixed point using Tarski's fixed point theorem.[9] Our existence proof also provides a characterization of the equilibrium in this class: all equilibrium objects are monotone in the government's reputation and in the aggregate state.

We now discuss the properties of the equilibria in Proposition 1. First, it is immediate from (16) that if $\sigma(\pi, s)$ is decreasing in the government's reputation, π, then the price of debt, Q, and the amount of debt issued are decreasing in π (see Lemma 2). This is because the probability of a bailout is decreasing in the government's reputation, π. As a result, for low values of π, lenders expect bailouts with high probability, so they are willing to lend at low interest rates. From (19) and (20) it follows that low interest rates

[9]Note, when we refer to the bailout policy along the equilibrium path, we mean that our procedure solves for $\sigma(\pi, s) = \sigma(\pi, B(\pi), K(\pi), s)$ evaluated only at the aggregate amount of debt chosen in equilibrium, and not for arbitrary debt and capital holdings.

(high Q) incentivize borrowers to increase their indebtedness and investment. Thus, debt and capital are decreasing in the government's reputation.

We next discuss why the probability of a bailout, $\sigma(\pi, s)$, is decreasing in the government's reputation. First, note that from Lemma 2 a higher reputation reduces the price of debt and in turn reduces the level of debt outstanding. The reduction in outstanding debt reduces the static incentives to bail out borrowers with low return on investment, $\psi h(\theta_L|s) B(\pi)$. Thus, the static incentives to bail out are low if the government's reputation, π, is sufficiently high. Moreover, since the continuation value for the government, $W(\pi)$, is increasing in the level of reputation, the dynamic gains from not bailing out,

$$\beta\left[W\left(p_{nc}+\frac{\pi}{\pi+(1-\pi)(1-\sigma(\pi,s))}(p_c-p_{nc})\right)-W(p_{nc})\right],$$

are increasing in π for a fixed σ. Thus, higher levels of reputation decrease the static losses of not bailing out and increase the dynamic benefits; therefore, the bailout probability, $\sigma(\pi, s)$, is decreasing in π.

Note that the forces just described can give rise to multiple equilibria and *reputation traps*. This is because there is complementarity between debt issuance and bailout probability. If private agents (lenders and borrowers) expect bailouts with high probability, then the debt price will be high and borrowers will find it optimal to borrow more. As described above, higher borrowing in turn induces the government to bail out with high probability, validating private agents' expectations. At the same time, if private agents expect bailouts with low probability, then less debt will be accumulated, reducing the costs of not bailing out and inducing the government not to bail out.

For any level of reputation, the bailout probability is higher in a severe crisis than in a mild crisis and is zero in normal times: $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H) = 0$. That is, the probability of receiving a bailout is increasing in the share of borrowers with a low productivity shock, $h(\theta_L|s)$. This is because the dynamic benefits of not bailing out are not affected by the current state directly, while the static losses of not bailing out are increasing in the fraction of borrowers with $\theta = \theta_L$. Clearly, if the economy is in normal times, $s = s_H$, there are no static benefits from bailing out, because all borrowers have $\theta = \theta_H$, so $\sigma(\pi, s_L) = 0$ for all levels of reputation.

Next, we provide sufficient conditions such that the monotone continuous equilibrium has mixing in a mild recession and a bailout with probability 1 in a severe recession. This turns out to be useful to connect the equilibrium outcome with our motivating evidence. To do so we first need to define some objects. First, we define the value for a fictitious commitment type who follows a fixed bailout policy summarized by sufficient statistic $\bar{\gamma}$

(defined in 17) when private agents have beliefs $\pi = 1$:[10]

$$W^R(\bar{\gamma}) \equiv \frac{(1-\lambda)\,e - (1-2\lambda)\left[\mathbf{k}(\bar{\gamma}) - qP_H\mathbf{b}(\bar{\gamma})\right] + \lambda\left[qP_H\theta_H\mathbf{k}(\bar{\gamma})^{\alpha} - \mathbf{k}(\bar{\gamma})\right] - \psi(1-\bar{\gamma})\,qP_L\mathbf{b}(\bar{\gamma})}{1-\beta},$$

where $\mathbf{b}(\bar{\gamma}) = (\alpha\theta_H)^{1/(1-\alpha)}\left[q\left(P_H + P_L\bar{\gamma}\right)\right]^{\alpha/(1-\alpha)}$ and $\mathbf{k}(\bar{\gamma}) = \left(\alpha\theta_H q\left(P_H + P_L\bar{\gamma}\right)\right)^{1/(1-\alpha)}$ are the levels of debt and capital that will be issued if borrowers expect to be bailed out with probability $\bar{\gamma}$ conditional on drawing a bad shock. The first term is the lenders' endowment weighted by the lenders' Pareto weight $(1-\lambda)$, the second term is the transfer from tax payers to the borrowers, $[\mathbf{k}(\bar{\gamma}) - qP_H\mathbf{b}(\bar{\gamma})]$, weighted by $-(1-2\lambda)$ which is the Pareto weight of the borrowers (that receive the transfer) minus the Pareto weight of the taxpayers (that pay the transfers), the third term captures the net social value of investing $k(\bar{\gamma})$, and the last term is the social default cost; see Appendix B.2 for a derivation of the expression.

Assumption 1. *Let* $C(x) = \psi x$ *and assume that*

$$\psi\mathbf{b}(0) > \beta\left[W^R(0) - W^R(1)\right] \tag{21}$$

and

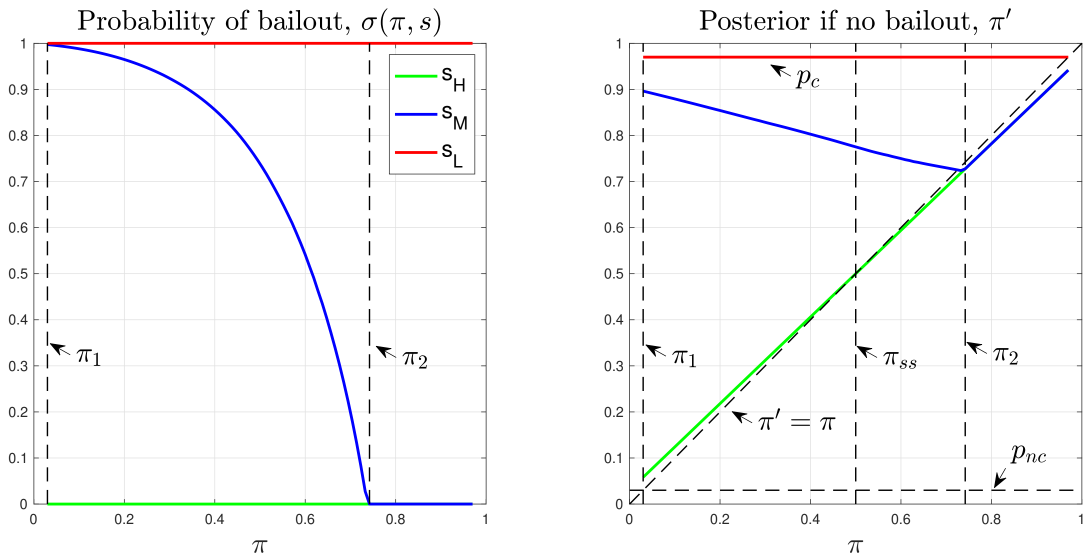
$$\min\left\{\frac{\mathbf{k}(1) - qP_H\mathbf{b}(1)}{1-\beta P_H}, \frac{\{[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)] - [qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)]\}}{1-\beta P_H}\right\} > \frac{\psi\mu\mathbf{b}(1)}{\beta}. \tag{22}$$

Condition (21) guarantees that the static costs of default in a severe crisis – state s_L – are large enough relative to the dynamic benefits so that it is always optimal to bail out the lenders. Since $W^R(0) > W(p_c)$ and $W^R(1) < W(p_{nc})$, the difference $W^R(0) - W^R(1)$ is an upper bound on the dynamic gains from not bailing out. This, along with the fact that $B(\pi) \geqslant \mathbf{b}(0)$, implies that condition (21) ensures that the static gains from bailing out dominate the dynamic costs in state s_L when all borrowers would default absent a bailout.

The second part of the assumption, condition (22), provides an upper bound on the static costs of default in a mild crisis so that it is not a best response for the optimizing type to bail out with probability 1 in s_M for all levels of reputation. Moreover, if p_{nc} is sufficiently small, for π close to p_{nc}, a no bailout strategy is not optimal because the dynamic reputation gains, $[W(p_{nc} + \pi\Delta p) - W(p_{nc})]$, are close to zero while the static benefits of bailing out are strictly positive. These two observations imply that the equilibrium bailout strategy calls for randomization in a mild crisis for some level of reputation.

[10]In regard to an earlier discussion on the optimal policy with commitment, we can show that there exists values of α and ψ such that $\frac{\partial W^R(\bar{\gamma})}{\partial \bar{\gamma}} < 0$. Therefore, the optimal policy is to set $\bar{\gamma} = 0$ and never bail out.

Figure 1: Equilibrium objects for computed example



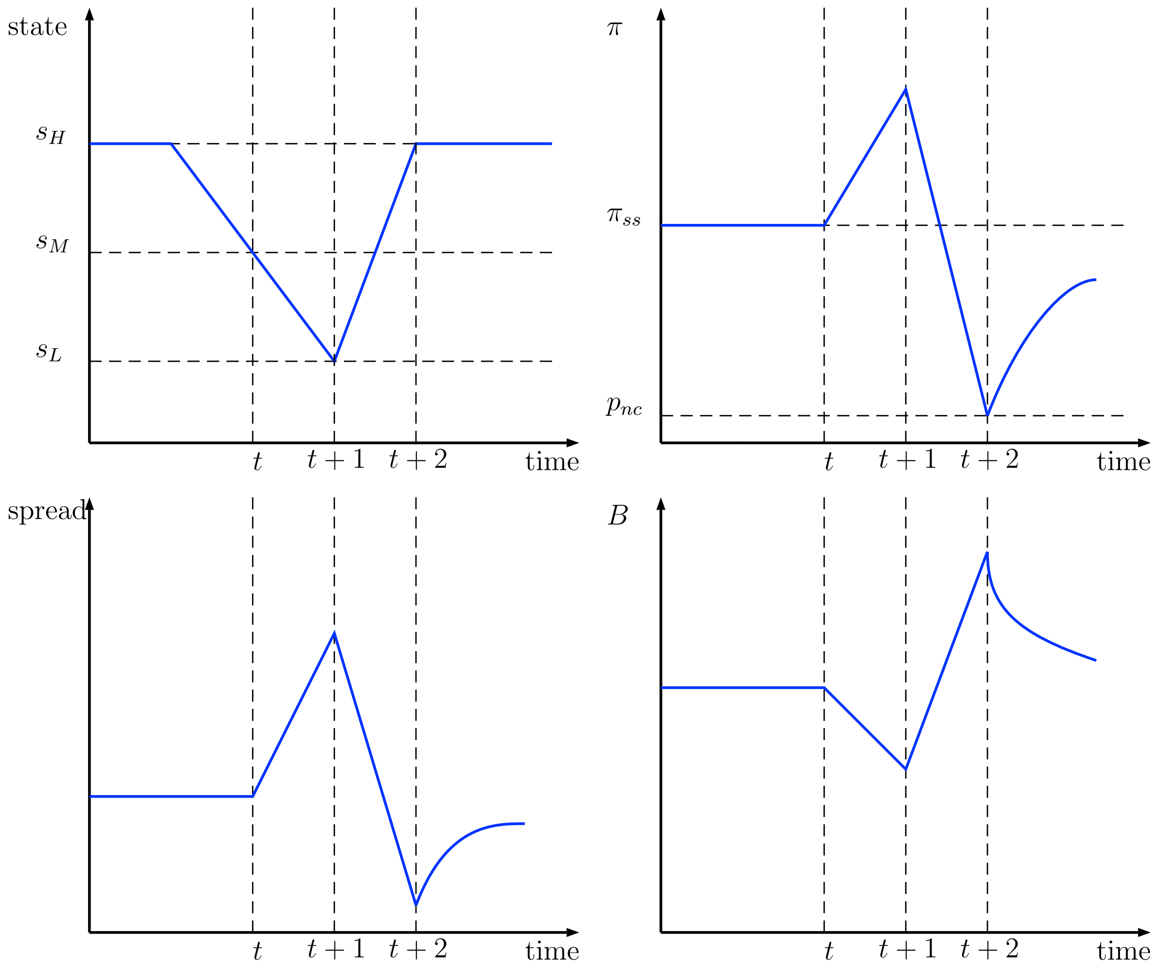
Proposition 2. *Under Assumption 1, if* $p_c \to 1$ *and* $p_{nc} \to 0$*, then in any monotone continuous equilibrium it must be that*

- *it is optimal to bail out with probability 1 in a severe crisis,* $\sigma(\pi, s_L) = 1$ *for all* π*;*
- *it is optimal to mix in a mild crisis, i.e., there exists a set* $[\pi_1, \pi_2]$ *such that* $\sigma(\pi, s_M) \in (0, 1)$ *for all* π *in* $[\pi_1, \pi_2]$*.*

In Figure 1, we use a numerical example to illustrate some of the key properties from the above characterization results. The first panel plots the bailout probability as a function of the government's reputation. Since no borrower defaults in s_H, there are no static benefits of bailing out, so $\sigma(\pi, s_H) = 0$. In state s_L, the static benefits are much larger than the dynamic benefits for all π, so $\sigma(\pi, s_L) = 1$. The plot also shows that there is an interval in which randomizing between bailing out and not is optimal.

The second panel describes how the beliefs about the government type evolve given the equilibrium strategy and conditional on observing no bailout. In state s_H, since private agents believe that the optimizing type will not bail out, the posterior changes because of the exogenous transition from one type to another. In state s_M, since there is a positive probability of bailout, in the event that there is no bailout, the posterior that the government is the commitment type will rise. Therefore, the state s_M offers the government an opportunity to build its reputation. In state s_L, the dynamic gains from not bailing out are the largest, since private agents believe that the optimizing type will bail out with probability 1 and therefore the posterior jumps to p_c in case no bailout is observed. However, as discussed above, the social costs of default are much larger than

Figure 2: Outcome path



these benefits and thus it is not optimal for the optimizing type to abstain from bailing out the borrowers to build up its reputation.

The existence of a region of the state space where the optimizing government mixes between bailing out and not provides one explanation for the heterogeneity in bailout provisions in apparently similar situations. There are however other channels that can account for this. For example, banks could be heterogeneous in their "political connectedness" which might lead to different bailout incentives across borrowers. In fact, Blau et al. (2013) provide evidence that banks with different political connections benefited to different degrees from the TARP rescue program. Our model can incorporate this feature by allowing for heterogeneous (bank specific) default costs.

Equilibrium Outcomes

We now describe a typical equilibrium outcome that generates the dynamics described in the introduction. This outcome is illustrated in Figure 2. Suppose that the optimizing type is in charge and the economy has been in normal times for a sufficiently large number of periods so that the prior has converged to $\pi_{ss} = p_{nc} + \pi_{ss}(p_c - p_{nc})$. Suppose now that the economy suffers a mild crisis in period t, $s_t = s_M$. Assuming that $\pi_{ss} \in [\pi_1, \pi_2]$, where the bounds of the interval are defined in Proposition 2, then the optimizing type mixes between bailing out and not. If it turns out that the outcome of the randomization calls for not bailing out, then the private agents' beliefs of facing the commitment type jump above π_{ss} and consequently the aggregate debt level falls and spreads rise the following period. If the economy is then hit by a severe recession in period $t+1$, $s_{t+1} = s_L$, there is a bailout with probability 1; therefore, private beliefs fall to p_{nc}, the aggregate debt levels rise, and spreads fall in subsequent periods due to the high probability of receiving a bailout in the future. The model can then rationalize the hump-shaped dynamics of spreads often observed during crises where spreads are very high at the beginning of a crisis if there is no bailout right away and then fall after a bailout.

The dynamics in Figure 2 are driven by changes in fundamentals. This is because, in a mild crisis in period t, if $\pi_t \in [\pi_1, \pi_2]$ and the government does not bailout then the posterior increases to $\pi_{t+1} > \pi_2$ as illustrated in the second panel of Figure 1. Therefore, if fundamentals remain constant, it is never optimal to bail out in period $t+1$. Thus, to observe a bailout and the associated fall in spreads a deterioration in fundamentals is necessary. In Section 4, we extend our model to one in which the government learns about the state from noisy prices. This model can generate identical dynamics to the baseline without the true state actually changing.

Reputation and Sensitivity of Spreads to Fundamentals

So far, we have assumed that borrowers are ex-ante identical, so there is no heterogeneity in their borrowing and investment decisions and in the interest rates at which they borrow. We now consider the case with two types of borrowers that differ in their probability of drawing the low productivity shock in sub-period 2. We show that this extension of our baseline model can generate the differential sensitivity effects documented by Acharya et al. (2016) and Cole et al. (2016). The authors document that the sensitivity of bond yields to fundamentals such as GDP growth increased significantly during the US financial crisis and the European debt crisis.

We consider the most parsimonious model to make our point.[11] Assume that in each period there are two types of borrowers indexed by $i \in \{1, 2\}$. Type 1 borrowers (measure μ) draw the idiosyncratic productivity θ_L for sure in aggregate state s_L and s_M so $h_1(\theta_L|s_L) = h_1(\theta_L|s_M) = 1$ and $h_1(\theta_L|s_H) = 0$. Type 2 borrowers (measure $1-\mu$) draw

[11] In Appendix B.6 we show that the conclusion of this section can also be obtained by extending our baseline model to allow for persistence in the borrower's shock.

θ_L only if $s = s_L$. That is, $h_2(\theta_L|s_L) = 1$ and $h_2(\theta_L|s_H) = h_2(\theta_L|s_M) = 0$. The problem of a type i borrower is then

$$\max_k P_{Hi}\left(\theta_H k^\alpha - \frac{k}{Q_i(\pi)}\right),$$

where P_{Hi} is the probability that type i draws θ_H in sub-period 2 and

$$Q_1(\pi) = q\{p_H + p_M(1-\pi)\sigma(\pi, s_M) + p_L(1-\pi)\sigma(\pi, s_L)\} \quad (23)$$

$$Q_2(\pi) = q\{p_H + p_M + p_L(1-\pi)\sigma(\pi, s_L)\}. \quad (24)$$

The next result is immediate:

Proposition 3. *(Sensitivity to fundamentals increasing in reputation) The difference in the price of debt for the low default type,* 2*, and the high default type,* 1*, is increasing in the reputation of the government; that is,* $Q_2(\pi) - Q_1(\pi)$ *is increasing in* π*. Similarly,* $K_2(\pi) - K_1(\pi)$ *is increasing in the government's reputation.*

Debt prices (and debt issuances) are less responsive to the borrower's fundamentals (its type) when the prior is low. If the probability of facing the optimizing type is low, then the lenders are less worried about the type of the borrower since they expect to get bailed out with high probability; therefore, debt prices are not very sensitive to fundamentals.

Proposition 3 has important implications for observed prices after a bailout. The model predicts that once we observe a bailout, the government's reputation falls and the cross-sectional volatility in spreads goes down, as prices are less sensitive to fundamentals. Conversely, after we observe a no-bailout event, the reputation of the government goes up and so does the volatility of spreads.

4 Two-Sided Learning

We now extend the baseline model to allow for uncertainty about the aggregate state and sequential bailout requests within a period. The main result of this section is that we can obtain hump-shaped time series for interest rate spreads without having to rely on a transition through a mild crisis (s_M).

Suppose the aggregate state of the world is $s \in \{s_L, s_H\}$ with probabilities p_L and p_H, respectively. In state s_H, a fraction μ of borrowers draw the low idiosyncratic shock ($h(\theta_L|s_H) = \mu$), while in state s_L all borrowers draw the low shock ($h(\theta_L|s_L) = 1$). We will refer to s_H as normal times and s_L as a systemic crisis. The state of the world s is observed by private agents but is unobservable to the government who must learn from private choices and prices. The timing in the first sub-period of the stage game is identical to the baseline environment. The second sub-period is further divided into two stages.

The timing in the first stage is as follows:

1. Borrowers enter sub-period 2 with debt b and capital k and prior belief π.

2. The state of the world, $s \in \{s_L, s_H\}$, is realized and learned by private agents.

3. Lenders draw the taste shock $\varepsilon \sim G$ and can trade a mutual fund of debt in a secondary market at price $q_2 = Q_2(\pi, B, K, s, \varepsilon|\sigma)$. The taste shock ε is a non-monetary value that lenders attain from holding the portfolio.

4. A fraction μ of borrowers receive the low productivity shock independently of the aggregate state and ask for a bailout.

5. The government bails outs with probability $\sigma_1(\pi, B, K, q_2)$.

Note that in the first stage, since the fraction of borrowers that draw θ_L is independent from the aggregate state, the government does not learn anything about the state from the fraction requesting a bailout. However, it does learn from secondary market prices q_2.

In the second stage, depending on the aggregate state, there is a second wave of bailout requests. If $s = s_H$, no other borrower requests a bailout and the prior is updated to π'. If $s = s_L$, then a fraction $1 - \mu$ of borrowers ask for a bailout and the government bails out with probability $\sigma_2(\pi, B, q_2)$. This implies that the state is perfectly revealed to the government in the second stage.

We now describe the key equilibrium objects. In the second stage, if the state is s_L, the government bails out if and only if

$$\psi(1-\mu)B \geqslant \beta\left[W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)(1-\sigma_2)}\Delta p\right) - W(p_{nc})\right].$$

In what follows we suppose that μ is small enough so that it is always optimal to bail out in the second stage if $s = s_L$.[12] Then we have that $\sigma_2 = 1$. Of course, if the economy is in state s_H, there is no borrower to bail out, and so $\sigma_2 = 0$.

We now consider the bailout decision in the first stage. Here, regardless of the state, a fraction μ of borrowers require a bailout in order to avoid default. The government does not observe the state, but it can observe and learn from prices in the secondary market. The price of a portfolio of debt in the secondary market at the beginning of sub-period 2 is

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\sigma_1(\pi, B, q_2) + \varepsilon, & \text{if } s = s_H \\ (1-\pi)\left[(1-\mu)\sigma_2(\pi, B, q_2) + \mu\sigma_1(\pi, B, q_2)\right] + \varepsilon, & \text{if } s = s_L \end{cases}.$$

[12]Recall that Assumption 1 guaranteed this when $\mu = 0$.

Thus, the bailout authority's beliefs that the true state is s_H conditional on having observed the secondary market price q_2 is

$$\hat{p}\left(s_H|\pi, B, q_2\right) = \frac{p\left(s_H\right) g_H\left(q_2\right)}{p\left(s_L\right) g_L\left(q_2\right) + p\left(s_H\right) g_H\left(q_2\right)},$$

where

$$g_L\left(q_2\right) = g\left(q_2 - (1-\pi)\left[(1-\mu)\,\sigma_2\left(\pi, B, q_2\right) + \mu\sigma_1\left(\pi, B, q_2\right)\right]\right),$$
$$g_H\left(q_2\right) = g\left(q_2 - \left[(1-\mu) + \mu\left(1-\pi\right)\sigma_1\left(\pi, B, q_2\right)\right]\right),$$

and $g(\cdot)$ is the probability density function of ε.

This implies that in the first stage, the optimizing type bails out if and only if

$$\psi\mu B \geqslant \hat{p}\left(s_H|\pi, B, q_2\right)\beta\left[W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\Delta p\right) - W\left(p_{nc}\right)\right].$$

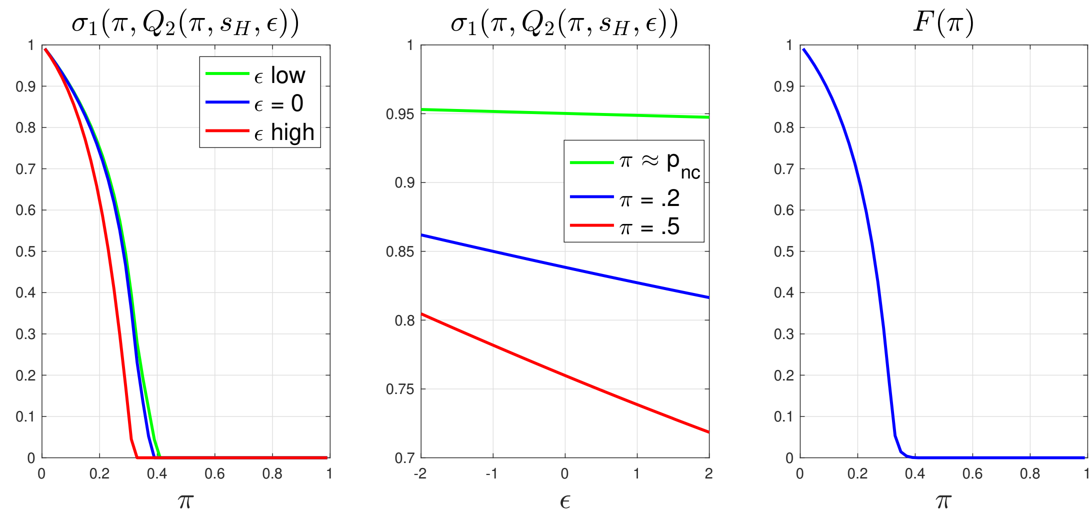
Since the state is unknown in the first stage, the dynamic benefits of not bailing out only accrue with probability $\hat{p}\left(s_H|\pi, B, q_2\right)$, since the optimizing type bails out with probability 1 in the second stage if $s = s_L$. All things being equal, a lower value of q_2 lowers the posterior $\hat{p}\left(s_H|\pi, B, q_2\right)$. As a result, in contrast to the baseline model, the dynamic gains from not bailing out might change for non-fundamental reasons. In particular, all else equal, a lower realization of ε increases the probability of a bailout in the first stage.

As a consequence, one can generate similar dynamics to the baseline model with only two aggregate states. For example, in state s_L, a low realization of ε will induce a very low value of q_2, which will lead to a low value of $\hat{p}$, thus inducing a bailout with probability 1 in the first stage. For the same realization s_L, a higher value of ε will result in a larger value of $\hat{p}\left(s_H\right)$, which in turn can push the government into the randomization region. Similar to state s_M in the baseline model, in the case when a bailout is not observed, the posterior value of the government being the commitment type rises, which in turn decreases the price of debt on the secondary market at the end of the first stage,

$$Q_3\left(\pi, B, s, \varepsilon|\sigma\right) = \begin{cases} 1+\varepsilon & s = s_H \\ (1-\pi)(1-\mu)+\varepsilon & s = s_L \end{cases}.$$

Figure 3 plots the key equilibrium objects for a typical computed numerical example. (In Appendix B.7 we carefully describe all the equilibrium conditions.) As we see in the first panel, for a fixed ε, a higher value of π implies a lower probability of bailout in the first stage. This intuition is similar to our baseline model, in which the state is observable. Similarly, for a fixed π, we see that a lower realization of ε induces a higher probability

Figure 3: Equilibrium objects for computed example with two-sided learning



of a bailout in the first stage since the government's posterior of the state being low rises. The expected probability of a bailout in the first stage,

$$F(\pi) = \int \sigma_1(\pi, \varepsilon)\left[p(s_H)\, g(\varepsilon) + p(s_L)\, g(\varepsilon + (1-\mu)\pi)\right] d\varepsilon,$$

is decreasing in π, which in turn implies the price of debt in sub-period 1, Q, is increasing in π. This generates identical spread dynamics to the baseline environment.

An interesting feature of this model is that information conveyed by secondary market prices depends on the government's reputation, π. For low values of π, since lenders expect a bailout with a high probability, the price q_2 will largely be driven by the taste shock ε, thus conveying little information about the fundamental. In contrast, for high values of π, the price q_2 will be more sensitive to fundamentals. This implies that for low values of π, a much larger value of q_2 is needed in order for the government to increase its posterior belief of s_H.

5 Long Lived Borrowers

In the baseline model, we assumed that borrowers live for one period. This implies that their default decision is static. Here we relax this assumption and allow borrowers to be long lived. Consequently, borrowers' default decisions will be dynamic. This allows us to study two interesting applications of our framework. First, we study why a government would bail out a small bank even though the static benefits of doing so might appear small. Second, we study the value for the optimizing type of a pure announcement of type relative to physical transfers in order to bail out distressed banks.

5.1 Small Bailouts to Avoid Contagion

With static default decisions, it is worthwhile to bail out only if a sufficiently large share of borrowers receives a bailout. This is because the static benefits of a bailout must be large enough to compensate for the dynamic reputation losses. In reality, however, we often observe cases in which governments bail out small banks. One recent example is the case of the Italian government bailing out two mid-sized banks in the Veneto region in 2017. Why would governments bail out small borrowers when the static losses associated with them defaulting is small?

In this section, we show that when the borrowers' default decisions are dynamic – in that they consider future profits in deciding whether to default or not – then the measure of borrowers that end up receiving the bailout is not a sufficient statistic for the static benefits of a bailout. This is because bailouts (or lack thereof) change private beliefs about the future behavior of the government and therefore affect the continuation value of not defaulting for *all* borrowers. In certain circumstances, a bailout to a small number of borrowers is needed in order to avoid contagion to other borrowers. Absent the small bailout, a large number of borrowers might be incentivized to default due to a decrease in future continuation values. By observing a bailout today, these borrowers anticipate higher transfers in the future and therefore choose not to default in order to appropriate these future transfers. Thus, the static benefits of a bailout can be large even if in equilibrium the government ends up bailing out only a small fraction of borrowers.

To illustrate this point, we consider a simple environment. Suppose that there are two periods, $\tau = 1, 2$, and that borrowers also live for two periods. We assume that $\theta \in \{0, \theta_H\}$. Borrowers can be one of two types, $i \in \{T, P\}$, with $\Pr(i = P) = \mu$. Types are perfectly observable. Borrowers of type T have purely transitory productivity shocks and the probability of drawing θ_H is P_H in both periods. Borrowers of type P face persistent productivity shocks. In particular, we assume that $\theta = 0$ is an absorbing state while $\theta = \theta_H$ is transitory. Thus, the probability of drawing θ_H in period 1 is P_H and the probability of drawing θ_H in period 2 conditional on θ_H in period 1 is P_H. We assume that if a borrower defaults in period 1, it is replaced by a type T borrower at the beginning of period 2, which keeps the mass of outstanding borrowers constant. Finally, we also assume that government types are perfectly persistent, i.e., $p_c = 1 - p_{nc} = 1$. To simplify the algebra consider the case in which the government does not value the borrowers' consumption, $\lambda = 0$. Moreover, in period 1 there are only two aggregate states $\{s_L, s_H\}$ with $h_i(\theta_L|s_L) = 1$ and $h_i(\theta_L|s_H) = 0$ so either all borrowers (of all type) draw θ_H or all borrowers draw θ_L.

Consider the second period. Since there are no dynamic gains, the optimizing type government will bail out any borrower with θ_L in period 2 so that $\sigma_2 = 1$. Therefore, the continuation payoff of having access to financial markets in period 2 when the prior is π is given by

$$U_{2i}(\theta_1, \pi) = \max_k P_{Hi}(\theta_1)\left[\theta_H k^\alpha - \frac{k}{Q_{2i}(\pi, \theta_1)}\right], \tag{25}$$

and $Q_{2i} = q\left[P_{Hi} + P_{Li}(1-\pi)\right]$ is the equilibrium price for the debt issued by type i where we use that $\sigma_2 = 1$. The continuation value for the transitory type borrowers, $U_{2T}(\theta_1, \pi)$, is strictly decreasing in π, while the continuation value for the permanent type borrowers is zero if $\theta_1 = 0$, $U_{2P}(0, \pi')$. The value for the optimizing type government at the beginning of period 2 given a prior π is

$$W_2(\pi) = B(\pi)\left(qP_H - Q(\pi)\right)$$

where $B(\pi)$ is given by (20) and $Q(\pi) = q\left(P_H + P_L(1-\pi)\right)$.

Consider the state of the world in which both types of borrowers receive the low endowment in period 1. Absent a bailout, borrowers of type P will default for sure, while borrowers of type T will repay if and only if

$$-B_{1T} + \beta U_{2T}\left(0, \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\right) \geqslant 0,$$

where $\pi / \left[\pi + (1-\pi)(1-\sigma_1)\right]$ is the posterior of facing the commitment type after observing no bailout. Note that here we are assuming that the borrower has access to external resources to repay the debt even if $\theta_L = 0$. We can then be in a situation in which

$$-B_{1T} + \beta U_{2T}\left(0, \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\right) < 0 \tag{26}$$

and

$$-B_{1T} + \beta U_{2T}(0, 0) > 0. \tag{27}$$

That is, if there is no bailout, the borrowers with a transitory bad shock also choose to default. But if the government bails out the permanent type, the drop in reputation increases the continuation value of the transitory type because the borrowers now expect to be able to borrow at low rates in the future. Thus, it is optimal for them to repay their debt even though they do not directly receive transfers in the current period.

Consider now the decision whether to bail out or not for the optimizing type in period 1 when conditions (26) and (27) hold. If the government does not bail out the permanent types, the high reputation of the government will induce the transitory types to default as well. This is true for any μ arbitrarily close to zero. Thus, the static costs of not bailing out are ψB_1 even though only a fraction μ will actually obtain a transfer. Therefore, the government bails out if

$$-\psi B_{1P} + \beta W_2\left(\frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\right) \leqslant \beta W_2(p_{nc}). \tag{28}$$

If ψ is large enough, this is indeed the best response of the optimizing type.

Assumption 2 provides sufficient conditions on parameters so that inequalities (26), (27), and (28) hold so that in equilibrium, the government bails out the permanent type borrowers in period 1 even though their measure is arbitrarily small.

Assumption 2. *Suppose that* $\beta q P_L \leqslant \psi P_H^{1/(1-\alpha)}$ *and* $\frac{\alpha}{\beta(1-\alpha)} \in \left[1, \frac{1}{P_H^{\alpha/(1-\alpha)}}\right]$.

The next proposition follows directly from the above analysis.

Proposition 4. *Suppose that Assumption 2 holds. Then, for any* $\mu > 0$ *and* π*, there exists an equilibrium in which in the state in which all borrowers receive the low endowment, the optimizing type bails out type* P *banks with probability 1.*

5.2 Bailouts vs. Announcements

Next, we study the merits of using a pure announcement versus an actual transfer to help distressed banks. Consider the framework described in the previous subsection but assume that all banks are of the transitory type.[13] We drop the subscript T for convenience. Suppose that

$$-B_1 + \beta U_2(p_{nc}) > 0$$

so that if the government's reputation (p_{nc}) is sufficiently low, expected future rents are high enough to induce the borrower to repay the debt even if current productivity is zero. Now if we assume that the optimizing type can credibly announce its type, the above inequality says that even though banks receive the low endowment in period 1, the continuation payoffs of participating in financial markets are large enough to induce them not to default. As a result, conditional on wanting to bail out the banks, the best response for the optimizing type is to set $T = 0$ and announce its type (or promise future bailouts). The reason for this is that a positive transfer provides a subsidy for the banks and, since the government only cares about the lenders and taxpayers, this makes it strictly worse off.

Proposition 5. *Suppose that*

$$\alpha \leqslant \frac{\beta P_H}{1+\beta P_H}. \tag{29}$$

[13] The analysis would go through as long as types were not perfectly persistent.

Then, the optimizing type either does not bail out or makes an announcement that it will bail out in the next period and chooses $T^* = 0$.

The sufficient condition in the above proposition ensures that just an announcement is sufficient to incentivize the borrowers to repay absent a transfer in period 1.

6 Narrative Analysis

In this section, we show that our model can shed light on three recent events: the US financial crisis, two bailouts within the European banking union, and the European sovereign debt crisis. In order to interpret these events through the lens of our model it is helpful to recall the expression for bond prices,

$$Q(\pi) = qP_H + qP_L\bar{\gamma}(\pi),$$

where P_H is the probability of repayment absent a bailout (a measure of the borrower's fundamentals), P_L is the probability of default absent a bailout, and $\gamma(\bar{\pi})$ is the probability that an individual borrower will be bailed out in the latter case. Consider first an adverse event in which there is no bailout and lenders are not rescued. Our model predicts that $\gamma(\bar{\pi})$ will decrease, which in turn implies that the spreads for borrowers not directly affected by the adverse event will go up. Thus, we should observe average CDS spreads rising after such events.[14] Moreover, as implied by Proposition 3, the sensitivity of interest rates with respect to fundamentals should also rise. Thus, we should observe CDS spreads to be more responsive to an increase in the perceived probability of default. While a direct measure of this sensitivity is hard to obtain, we proxy for this by looking at the cross-sectional standard deviation of CDS spreads under the assumption that the underlying distribution of fundamentals is not affected by news about (future) bailout prospects.

Similarly, our model predicts that the opposite will happen after we observe a bailout or an announcement of future bailouts: $\gamma(\bar{\pi})$ will rise and thus the average level of interest rate spreads will fall and the sensitivity to fundamentals will decrease.

The remainder of this section revisits events in these three crises through the lens of our model and demonstrates that the behavior of interest rates spreads are qualitatively consistent with our theory. Another interesting case for our theory is the Puerto Rican

[14] In the data, movements in counterparty risk in CDS can confound our analysis. Since CDS are traded among financial institutions, a lower probability of bailout could result in higher counterparty risk, which in turn would push CDS spreads down, because agents are less willing to pay for the insurance they provide due to the higher risk not to be repaid. While this concern is valid in theory, studies have shown that counterparty risk is negligible in practice, consistent with the high degree of collateral posted. See Arora et al. (2012).

debt crisis. Chirinko et al. (2019) argue that Puerto Rico has been able to borrow at relatively low costs despite bad fundamentals because investors were anticipating a bailout by the US Treasury. The paper shows that after the decision of the US congress to not bail out the city of Detroit, the Puerto Rican spreads rose dramatically as our theory predicts.

US Financial Crisis We start our analysis by considering the behavior of the CDS spreads of the largest US financial firms around the financial crisis that started in 2007.[15] The first two panels of Figure 4 plot the average and the cross sectional standard deviation of CDS spreads for large financial institutions in the United States. These moments declined – albeit modestly – after the bailout of Bear Stearns, sharply rose after the refusal to bail out Lehman Brothers and its consequent bankruptcy, and sharply declined after the announcement of the Troubled Asset Relief Program (TARP).[16]

These observations are consistent with the narrative suggested by our model if the prior around the Bear Stearns episode was sufficiently low (below π_2 in Proposition 1) so that a bailout was a possible outcome. After the bailout, the reputation of the government fell and so did the spreads and the sensitivity to fundamentals of other financial institutions.[17] The increase in sensitivity to fundamentals here is proxied by the cross-sectional standard deviation of CDS spreads under the assumption that the underlying distribution of fundamentals is not affected by news about (future) bailout prospects. Around the Lehman Brothers bankruptcy, the prior was in the randomization region; thus, after a no-bailout the government's reputation rose, which in turn led to an increase in the mean and standard deviation of CDS spreads for all banks. Finally, we interpret the implementation of TARP as a bailout event resulting in a drop in the reputation of the government and consequently a decrease in the mean and standard deviation of bank CDS spreads.

One potential problem with our interpretation is that spreads for all firms – financial and non-financial – increased during the crisis, as documented by Gilchrist and Zakrajšek (2012). To make sure that the dynamics of spreads for financial firms between the Lehman failure and the announcement of TARP is not driven by either changes in fundamentals or the price of risk, we also plot the same moments for a selection of large non-financial firms, which are arguably less affected by these bailout policies but can be subject to the same movements in volatility and price of risk.[18] From Figure 4, it is clear that the hump-

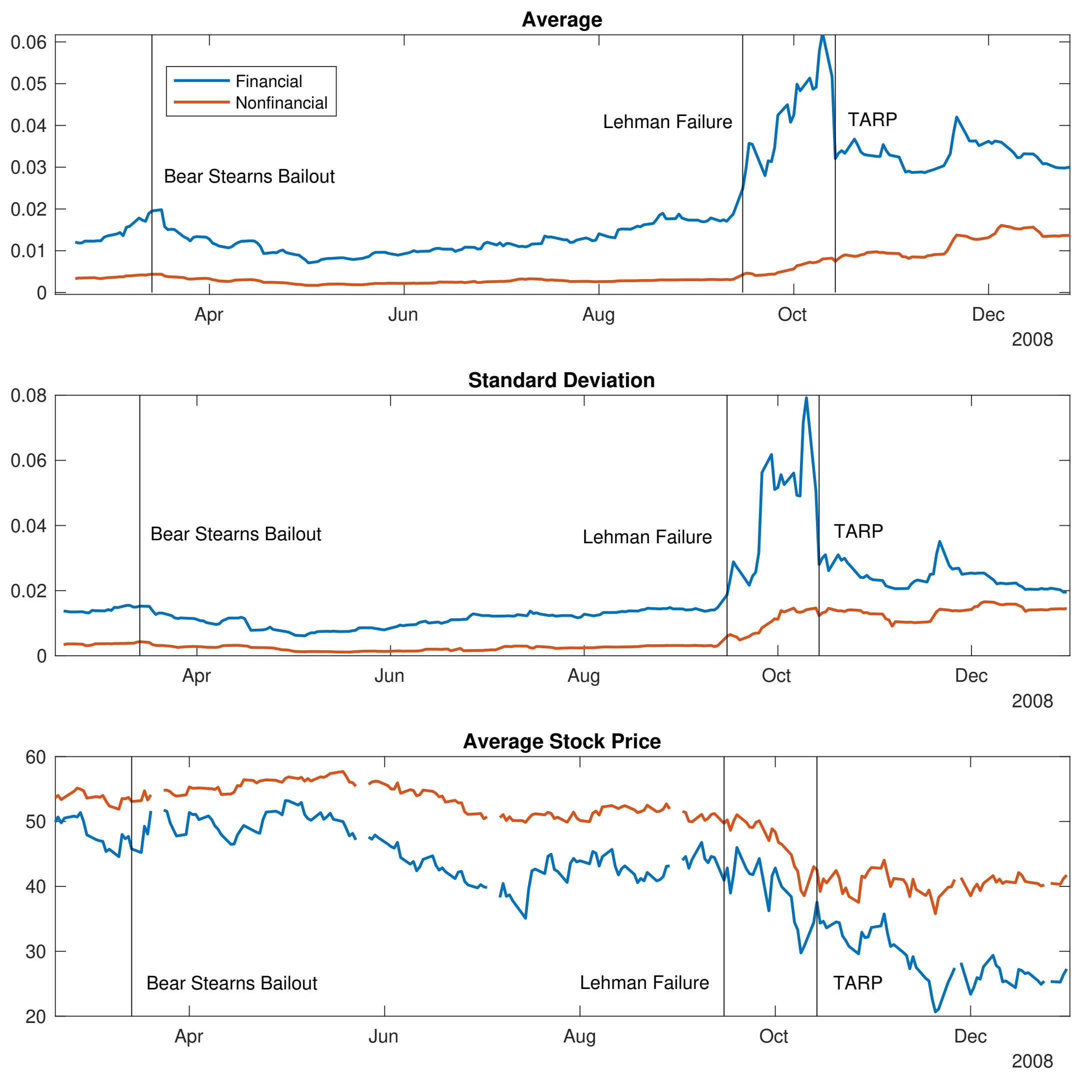
[15]Recall from the discussion in Section 2 that here we interpret the borrowers in the model as financial institutions.

[16]The Federal Reserve Board approved the rescue plan for Bear Stearns on March 14, 2008. Lehman Brothers Holdings Incorporated filed for Chapter 11 bankruptcy protection on September 15, 2008. The US Treasury Department announced TARP on October 14, 2008. See the St. Louis Fed Financial Crisis Timeline at https://www.stlouisfed.org/financial-crisis/full-timeline.

[17] Since the drop was small after the Bear Stearns episode, the model implies that the bailout was expected with high probability.

[18]The comparison builds on the idea that defaults by financial firms have larger social costs and therefore

Figure 4: CDS spreads of large financial and non-financial firms



Source: Markit. The financial firms are American Express, Bank of America, The Bank of New York Mellon, Branch Banking and Trust, Capital One, Citigroup, Fifth Third Bank Corp, Goldman Sachs, JP Morgan Chase, Key Banks, MetLife, Morgan Stanley, PNC, Regions Financial Corp, Suntrust Banks, State Street Boston Corporation, US Bancorps, and Wells Fargo. The non-financial firms are the non-financial companies included in the Dow Jones Industrial Average in February 2008.

shaped dynamics of the mean and standard deviation of CDS spreads for financial institutions is not shared by non-financial firms. If anything, spreads for non-financial firms monotonically increased during this period as the recession deepened. Furthermore, at least for the case of TARP, the reduction in CDS spreads is not associated with an increase in the stock price of financial firms. This is consistent with our channel to the extent that bailouts primarily benefit bond-holders.

The literature has proposed alternative and complementary mechanisms that can also account for the behavior of CDS spreads during the financial crisis. For instance, the increase in the sensitivity of spreads to fundamentals after Lehman can be explained by an increase in the incentive to acquire costly information in periods with high volatility when returns of information are high as in Cole et al. (2016). It is not clear however how this channel can account for the reversal after TARP.[19]

The evidence presented here is consistent with previous studies. Veronesi and Zingales (2010) document that CDS spreads increased for other banks after Lehman Brothers filed for bankruptcy. After the announcement of the Paulson plan (a $125 billion equity infusion) in October 2008, spreads for many large banks fell. Schweikhard and Tsesmelidakis (2011) use the difference in the default risk implied by equity prices and CDS spreads to measure the impact of government guarantees in pricing default risk.[20] They find that this difference increases sharply for banks during bailout events, for example TARP and the Bank of America rescue package. See Figure 3 in their paper. Moreover, they show that this difference is relatively unchanged for non-financial firms.

Acharya et al. (2016) find that the risk sensitivity of spreads for large financial firms is substantially weaker than for small and medium financial firms, while there is no such difference for non-financial firms of different sizes. They also find that "following the collapse of Lehman Brothers in 2008, larger financial institutions experienced greater increases in their spreads than smaller institutions. In contrast, the spreads of large financial institutions also became more risk sensitive after the collapse of Lehman. Following the government's rescue of Bear Stearns in 2008 and the adoption of the TARP and other liquidity and equity support programs, we find that larger financial institutions experienced greater reductions in credit spreads than smaller institutions; the spreads of large financial institutions also became less risk sensitive." They interpret these results as evidence of "too big to fail" status for large financial institutions. This evidence suggests that our

bailouts are more likely for them. Clearly, there are some non-financial firms that impose social costs on society. In fact, during the recession that followed the financial crisis, General Motors and Chrysler ended up receiving government assistance.

[19]The information acquisition channel is complementary with our theory as the incentives to acquire information are higher if the expectations of future bailouts are low.

[20]Atkeson et al. (2018) use the differences in book and equity values of banks to measure the size of the government guarantees.

theoretical analysis is more applicable to this subset of firms.

Relatedly, Kelly et al. (2016) provide evidence that during the US financial crisis the expectation of a system-wide bailout to rescue bank equity holders increased. They document that during the financial crisis, the cost of out-of-the-money (OTM) put options for an index of the financial sector was much cheaper relative to OTM put options on the individual banks comprising the index. They argue that the reason for this was the difference in bailout expectations in the case of a single bank failing versus the financial sector collapsing as a whole. Our model can replicate these dynamics if we assume that it is costly for the optimizing type government to allow banks' equity to be wiped out.

European Banking Union In 2014, member countries of the European Union transferred banks' responsibilities for regulation and resolution to a European authority, the Single Resolution Mechanism (SRM), with the goal of harmonizing the regulations and minimizing the cost to taxpayers from bailing out failing banks. The SRM became fully operative in 2016. The governing principle of the SRM is that bailouts should be avoided and replaced by *bail-ins:* shareholders and debt holders should realize losses before any public funds are used.[21] The credibility of the SRM was tested in 2017 with the prospect of bankruptcy for Monte dei Paschi di Siena, Banca Popolare di Vicenza, and Veneto Banca in Italy. The Italian government used taxpayer money to rescue Monte dei Paschi di Siena. For the two banks in the Veneto region, the Single Resolution Board did not perceive them to be a risk to financial stability, but the Italian government intervened. The two banks closed, and the good assets were acquired by Intesa SanPaolo with a government guarantee. Senior debt holders were bailed out and retail investors were compensated for their losses of junior debt. Thus, the Italian government avoided applying fully the bail-in provision of the new European banking regulations.[22]

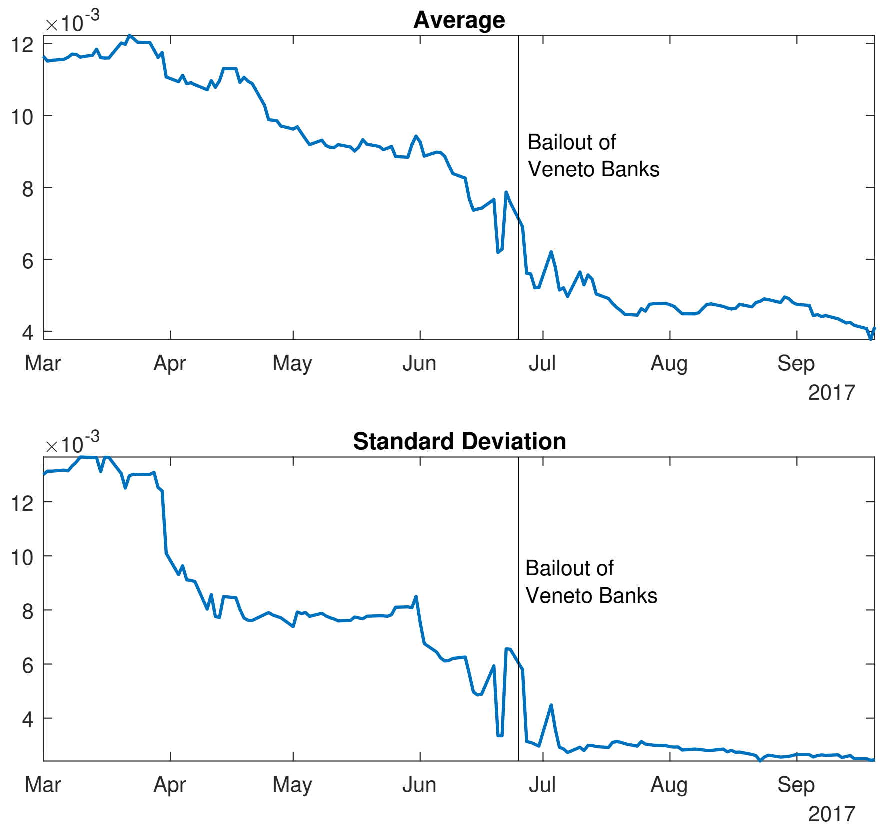
Through the lens of our model, after the Italian government's use of public funds, private agents were less likely to believe that the SRM is credible and thus expected greater government support of banks in the future. Figure 5 plots the mean and standard deviation of CDS spreads for large Italian banks for the period surrounding the bailout of Banca Popolare di Vicenza and Veneto Banca.[23] Consistent with our model, both moments fell right after the announcement.

[21] The same policy was pursued in the US with Title II of Dodd-Frank, which calls for companies in danger of default to be placed in receivership rather than bailed out. In the 2018 David K. Backus Memorial Lecture, Darrell Duffie argues that much of the reason CDS spreads continue to be high compared to pre-crisis levels is due to private beliefs that banks might not be bailed out in the event of a future crisis. He argues that these beliefs are primarily driven by the Lehman Brothers episode as well as the regulation in Dodd-Frank. The resolve of US regulators has not been tested yet.

[22] See Cooley (2017) and https://www.economist.com/finance-and-economics/2017/07/01/the-complicated-failure-of-two-italian-lenders

[23] The two Veneto banks were bailed out on 25 June, 2017.

Figure 5: CDS spreads of Italian banks



Source: Markit. The banks in the sample are Assicurazioni Generali, Banca Carige, Banca Monte dei Paschi di Siena, Banca Nazionale del Lavoro, Banca Popolare di Milano, Banca delle Marche, Banco BPM, Banco Popolare, Beni Stabili, Intesa Sanpaolo, Mediobanca, UniCredit, UBI Banca, and Unipol Gruppo.

A recent paper by Neuberg et al. (2018) provides additional empirical evidence for our mechanism. They exploit a change in the contract terms of credit default swaps to measure the market-perceived credibility of the recent financial reforms. In 2014 the terms of CDS contracts were changed to allow for a government intervention event in which bondholders would be compensated if there was a bail-in. Contracts traded prior to 2014 had no such provision and thus bondholders were subject to losses in case of such events. Since contracts under the old terms continued to trade after 2014, the authors use the difference in CDS spreads of the two types of contracts to measure the cost of protecting bondholders against bail-in events. They find that market implied likelihood of a government intervention decreased a little after the announcement of the SRM but increased after it seemed likely that the Italian banks would be bailed out, consistent with our theory.

One can also wonder why the Italian government chose to bail out these two mid-sized banks, thereby damaging the credibility of the SRM guidelines. Our theory suggests that this bailout might have been driven by the fear of contagion to other banks in bad condition, such as Monte Paschi and Carige, as suggested by Proposition 4.

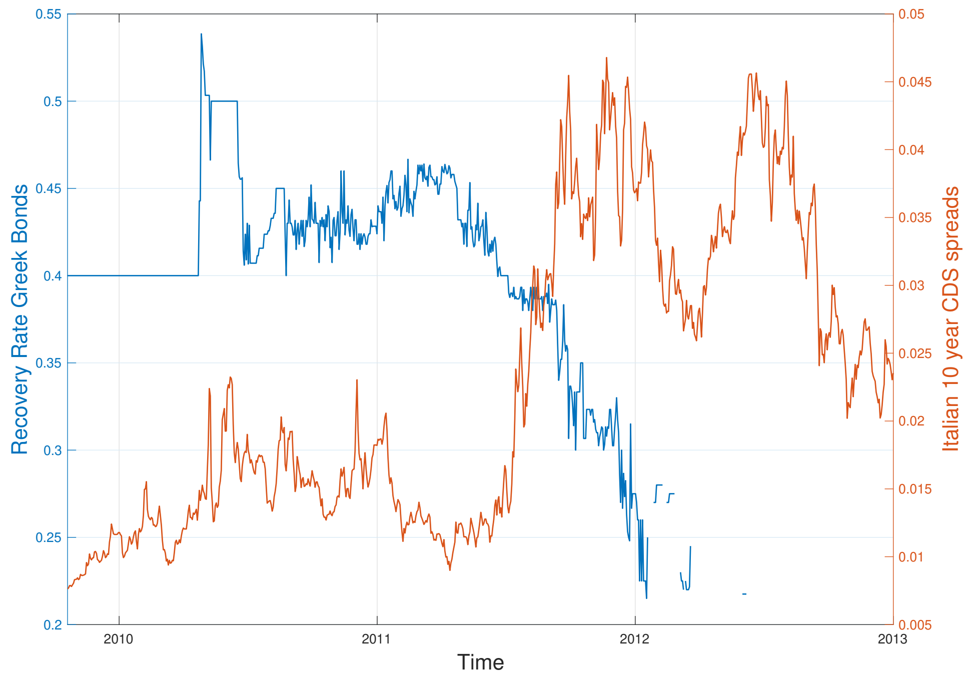
European Debt Crisis After joining the euro area, southern European governments were able to borrow at interest rates similar to Germany, despite differences in fundamentals, until the beginning of the European debt crisis at the end of 2009. The crisis started in Greece but soon spread to other southern European countries. News about the willingness of European institutions to bail out Greece impacted the borrowing rates for other EU member countries. Ardagna and Caselli (2014) document that the spike in five-year Italian bond yields coincided with an announcement that the Greek bailout agreement required them to seek a haircut on outstanding debt to private creditors. They argue that this contagion effect might have worsened prospects for peripheral countries. Figure 6 plots the expected recovery rates for Greek bonds on the left axis and the CDS spreads on 10-year Italian bonds on the right axis. The recovery rate is a measure of how much private lenders expect to recover in the event of default. As the figure shows, the sharp increase in Italian CDS spreads coincides with the drop in expected recovery rates for Greek bonds.

As the crisis progressed and fundamentals arguably worsened, the interest rates at which southern European countries continued to rise until the speech by Mario Draghi about the willingness of the ECB to do "whatever it takes" and the institution of the OMT program, after which spreads for these countries fell sharply. We interpret the institution of OMT as an announcement in the spirit of Proposition 5. Figure 7 plots the mean and standard deviation for CDS spreads of EU countries (not including Greece) for the period around the announcement of the unlimited bond-buying program by the ECB. Consistent with our model, both the mean and standard deviation fell after the announcement.

We provide further evidence that the sensitivity of interest rate spreads to fundamentals increased from the start of Greek debt crisis (2009Q4) to the announcement of OMT (2012Q3) relative to the period 2000Q1-2009Q3 and then reverted back to the pre-crisis level after OMT, 2012Q4-2020Q1. To this end, we regress the 10-year spreads of Eurozone countries relative to Germany on fundamentals like the government debt to GDP ratio and real GDP growth that are informative for spreads according to standard theory, see for instance Arellano (2008) and Aguiar and Gopinath (2006). We allow for the sensitivity of spreads to fundamentals to differ between the initial period (2000Q1–2009Q3), the debt crisis period (T_1 =2009Q4–2012Q3), and the post-OMT period (T_2 =2012Q4–2020Q1). We estimate the following equation

$$spread_{it} = \beta_{0i} + \beta_1 x_{it} + \beta_2 x_{it}\mathbb{I}_{\{t\in T_1\}} + \beta_3 x_{it}\mathbb{I}_{\{t\in T_2\}} + \beta_4\mathbb{I}_{\{t\in t\in T_1\}} + \beta_5\mathbb{I}_{\{t\in T_2\}} + \varepsilon_{it}$$

Figure 6: Recovery rates and CDS spreads



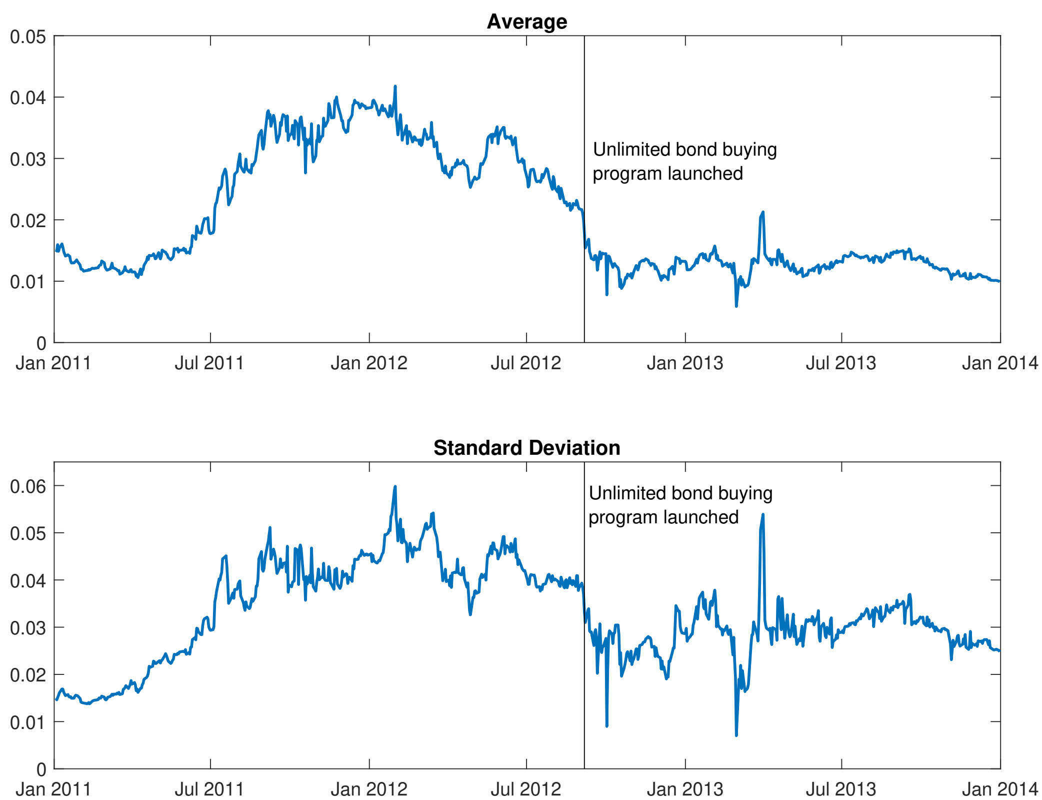
Source: Markit

where $spread_{it}$ is the 10 year yield spread, x_{it} is either debt to GDP ratio, the growth rate of GDP, or the square of the debt to GDP ratio. Table 1 reports the results for these regressions. In all our specifications, the sensitivity of spreads to fundamentals increases during the crisis and then reverts back to pre-crisis levels after OMT.

Since the literature has emphasized the non-linear effects of these fundamentals on spreads, we also include a specification that allows for a squared term (column 3). For this specification, we see that during the crisis period (T_1), the coefficient on the square term increases but the coefficient on the linear term decreases. However, the overall effect is consistent with our theory as seen in Figure 8. This figure plots the relationship between the debt to GDP ratio and interest rate spreads predicted by the regression in the three subperiods together with the data. During the crisis, we see that the sensitivity of the spreads to the debt to GDP ratio rises as indicated by the steeper slope of the line.

A common alternative explanation for the behavior of spreads in the European debt crisis is that high spreads were driven by a self-fulfilling switch to a bad equilibrium – as in Cole and Kehoe (2000) or Calvo (1988) – and the ECB intervention steered the equilibrium back to one with low interest rates. For instance, De Grauwe and Ji (2013) argue this is the case from a regression analysis similar to the one in Table 1. While this is possible in theory, our narrative suggests that in order for this to be true, lenders would

Figure 7: 10 year yield spreads for Eurozone countries



Source: Markit. The countries in the sample are the original Eurozone countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal, and Spain.

need to coordinate on news about the generosity of (future) bailouts.[24]

For this episode, we choose not to contrast the behavior of the spreads for government bonds with the spreads of corporate bonds that are less impacted by the expectations of bailouts by the European institutions (ECB and EU). We do so because there are several channels that transmit sovereign risk to corporate bonds so that similar dynamics of corporate spreads does not by itself invalidate our channel. For example, if EU governments are responsible for corporate bailouts then shocks to their fiscal capacity can affect their ability to bail out firms and thus affect firms' CDS spreads.[25] Moreover, financial institutions hold government debt and thus an increase in spreads reduces their net-worth and can in turn increase corporate spreads (both directly and indirectly through an increase in the price of risk). These channels suggest that corporate spreads might mimic sovereign risk even if not affected by news about bailouts from European institutions. In fact, Gilchrist and Mojon (2018) document that during the Eurozone crisis, the spreads of non-financial firms reflected country specific sovereign risk. In contrast, during the U.S. financial crisis, the spreads of non-financial firms did not reflect individual country risk.

7 Ex-ante Policies

Here we briefly discuss how the implementation of desirable ex-ante policies interacts with our mechanism. A critical ingredient of our theory is that default generates a social default cost, $\psi\Delta B$, so the government has an ex-post incentive to intervene and bail out the borrowers. Thus, borrowing generates negative externalities on society whenever the probability of default is not zero. Consequently, even absent a bailout, the equilibrium borrowing level is not optimal. To see this, consider the objective function in the case in which all agents are equally weighted by the government ($\lambda = 1/2$)[26]

$$\frac{1}{2}\left[qP_H k^{\alpha} - k\right] - q\psi\Delta b.$$

Assume that there are no bailouts so $\Delta = P_L$ and $qP_H b = k$. The socially optimal investment is

$$k = \left(\frac{qP_H\alpha}{1+2\psi P_L/P_H}\right)^{1/(1-\alpha)} < \left(qP_H\alpha\right)^{1/(1-\alpha)}.$$

[24] Bocola and Dovis (2019) use the behavior of the maturity structure of government debt to discipline the component of spreads due to rollover risk and find that it is relatively small.

[25] In our model we abstract from fiscal considerations and assume that the government can always finance the bailout.

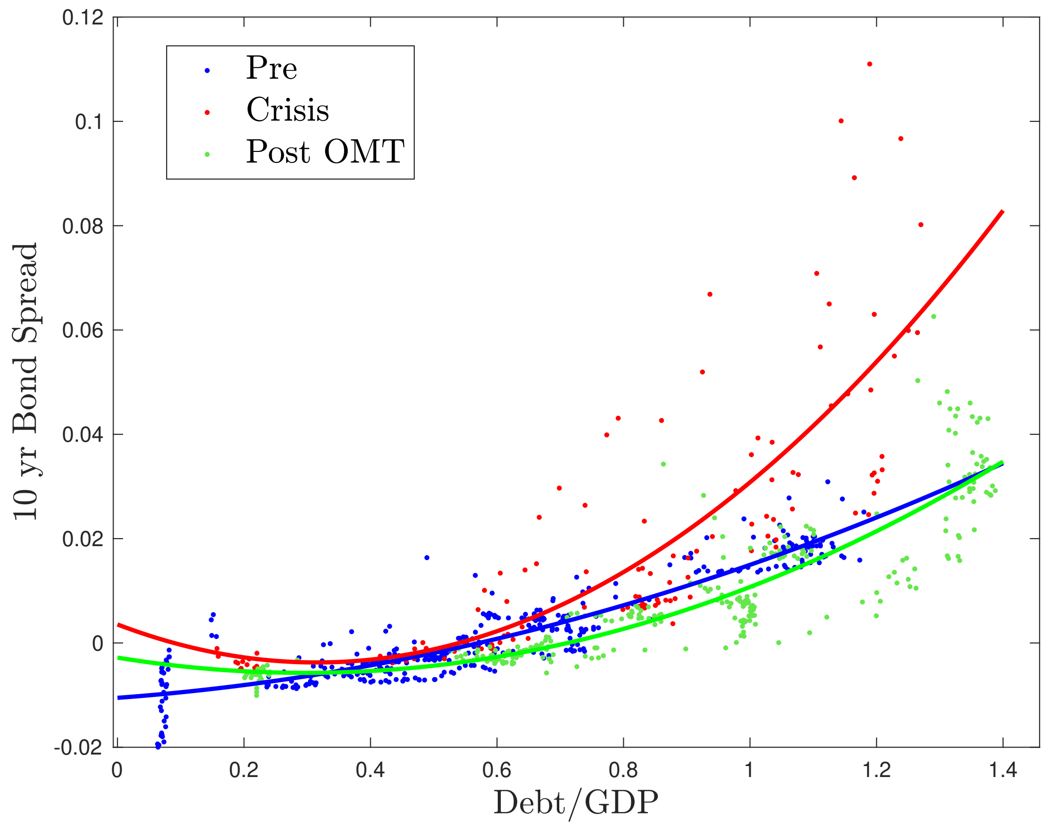
[26] This is just for simplicity. In general, the government's objective in sub-period 1 is $(1-2\lambda)\left[qP_H b - k\right] + \lambda\left[qP_H k^{\alpha} - k\right] - q\psi\Delta b$.

Table 1: Sensitivity of spreads to fundamentals

	(1)	(2)	(3)
debt/GDP	0.0345***	0.0342***	0.0089
	(0.0027)	(0.0025)	(0.0106)
debt/GDP$\times \mathbb{I}_{\{t\in T_1\}}$	0.0320***	0.0264***	-0.0552***
	(0.0028)	(0.0026)	(0.0116)
debt/GDP$\times \mathbb{I}_{\{t\in T_2\}}$	0.0053**	0.00443**	-0.0287***
	(0.0021)	(0.0019)	(0.0081)
real GDP growth		-0.206***	
		(0.0485)	
real GDP growth $\times \mathbb{I}_{\{t\in T_1\}}$		-1.085***	
		(0.1310)	
real GDP growth$\times \mathbb{I}_{\{t\in T_2\}}$		-0.2460***	
		(0.0723)	
(debt/GDP)^2			0.0166**
			(0.0075)
(debt/GDP)$^2 \times \mathbb{I}_{\{t\in T_1\}}$			0.0570***
			(0.0079)
(debt/GDP)$^2 \times \mathbb{I}_{\{t\in T_2\}}$			0.0167***
			(0.0057)
$\mathbb{I}_{\{t\in t\in T_1\}}$	-0.0215***	-0.0089***	0.0141***
	(0.0030)	(0.0021)	(0.0041)
$\mathbb{I}_{\{t\in T_2\}}$	-0.0074***	-0.0051***	0.0077**
	(0.0022)	(0.0017)	(0.0031)
Country fixed effects	Yes	Yes	Yes
Observations	790	790	790
R^2	0.553	0.644	0.594

Note: Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1. The sample is 2000Q1–2020Q1, T_1 =2009Q4–2012Q3, and T_2 =2012Q4–2020Q1. The countries in the sample are the original Eurozone countries as in Figure 7. The data is from Eurostat.

Figure 8: Sensitivity of spreads to debt to GDP ratio



This figure reports the relationship between the 10-year spreads and the debt to GDP ratio estimated in column 3 of Table 1 together with the data.

This socially optimal level of investment can be implemented by imposing a debt tax equal to $1/\left(1+2\psi\frac{P_L}{P_H}\right)^{1/(1-\alpha)}$.

In the spirit of Kareken and Wallace (1978), the combination of ex-ante policies and bailouts can lead to an allocation that attains higher welfare than the Ramsey outcome when these ex-ante policies are not available. Trivially, if the government promises to bail out the lenders for sure and has the power to limit the investment or borrowing through a tax or a quantity limit, then the government can attain the efficient level of capital without incurring the default costs.[27]

Our model is too stylized to think about the optimal design of this policy mix. Here we provide a brief discussion of the interaction between these ex-ante policies and the bailout incentives ex-post. Suppose we have a fixed ex-ante tax on debt (or on capital). To justify the no bailout policy of the commitment type, assume that the tax is sufficiently small

[27]In our setup with full information these policies are trivial but can be more complicated in practice if the government has an informational disadvantage.

so that not bailing out is still optimal from an ex-ante perspective. In this setting all our results go through. Comparative statics about the frequency of bailouts with respect to this tax are ambiguous because the tax reduces both the static benefits of bailing out, due to a reduction in the amount of debt, as well as the the dynamic benefits since reputation is less valuable. The latter is true because the ex-ante tax reduces overborrowing which is more valuable when reputation is low. Therefore, it is unclear if the introduction of a fixed sub-optimal tax leads to more or less bailouts in equilibrium.

A key issue when thinking about such ex-ante policies is whether they can be credibly enforced. For instance, suppose that the government in our economy imposes a cap on investment. The government can verify ex-post if the borrower adhered to the cap. To incentivize borrowers to respect the cap, assume that the government can impose a fine if it is violated. As we show in Dovis and Kirpalani (2020), in the context of fiscal rules, if the government cannot commit to imposing these fines (in addition to not being able to commit to not bail out), then using this additional tool can have negative consequences. In particular, we show that the imposition of such fines can lead to more borrowing and subsequently more bailouts in equilibrium because they increase the cost for the government of maintaining a good reputation. In concurrent work, Dovis and Kirpalani (2019), we study the optimal policy in an environment without commitment but with incentives for building reputation. One application of this framework is the optimal design of bailout policies.

8 Conclusion

In this paper we study a model in which the expectation of future bailouts is an important determinant of interest rate spreads. We jointly characterize these spreads and the optimal bailout decisions of a government that lacks commitment but has incentives to build reputation. The model can help to account for several features of both the behavior of spreads around crises and the delay in intervention we often observe from governments once the crisis has started. Moreover, it can account for the observation that the correlation of spreads for certain borrowers (such as financial firms or governments in the European Monetary Union) is higher than the correlation of their fundamentals because they are linked by a common bailout probability.

Our narrative analysis demonstrates qualitatively that certain patterns in the data are consistent with our theory. More work is needed to differentiate alternative theories that have been proposed in the literature. On the model side, we purposely build a parsimonious environment to highlight one novel channel. It would be worthwhile to quantify how much of the movement in spreads can be accounted for by a combination of fundamentals and reputation in richer models.

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Appendix

A Omitted Proofs

A.1 Proof of Proposition 1

We prove that a monotone continuous equilibrium exists by applying Tarski's fixed point theorem. Let

$$X = \{\Delta W : [0,1] \to \mathbb{R} \text{ and } \Delta W(\cdot) \text{ is increasing and continuous and } \Delta W(p_{nc}) = 0\}$$

and

$$\Sigma = \{\sigma : [0,1] \times S \to [0,1]\}$$

where S is the set of states. Define $f : X \to \Sigma$ as $f(\Delta W) = \sigma$ to be the *smallest*[28] solution to the functional equation

$$\sigma(\pi, s) = \begin{cases} 0 & \text{if } \beta \Delta W(p_{nc} + \pi \Delta p) \geqslant \Delta \omega(Q(\sigma, \pi), s) \\ \tilde{\sigma} & : \beta \Delta W\left(p_{nc} + \frac{\pi \Delta p}{\pi + (1-\pi)(1-\tilde{\sigma})}\right) = \Delta \omega(Q(\sigma, \pi), s) \\ 1 & \text{if } \beta \Delta W(p_c) < \Delta \omega(Q(\sigma, \pi), s) \end{cases} \tag{30}$$

where $Q(\sigma, \pi) = qP_H + q(1-\pi)\sum_s p_s h(\theta_L|s)\sigma(\pi, s)$,

$$\begin{aligned} \omega^*(Q, s) &= \lambda h(\theta_H|s)(\theta_H k(Q)^\alpha - b(Q)) + (1-\lambda) h(\theta_H|s) b(Q) \\ \omega(Q, s) &= \lambda h(\theta_H|s)(\theta_H k(Q)^\alpha - b(Q)) + (1-\lambda) h(\theta_H|s) b(Q) - \psi h(\theta_L|s) b(Q) \\ \Delta \omega(Q, s) &= \psi h(\theta_L|s) b(Q) \end{aligned}$$

with $k(Q) = (\alpha \theta_H Q)^{1/(1-\alpha)}$, $b(Q) = (\alpha \theta_H)^{1/(1-\alpha)} Q^{\alpha/(1-\alpha)}$.

Define the operator $\mathbb{T}_\Delta : X \to X$ as

$$\Delta W'(\pi) = \mathbb{T}_\Delta(\Delta W)(\pi) = W'(\pi) - W'(p_{nc})$$

[28] By doing so we are selecting the best equilibrium. We could alternatively look for the worst by choosing the largest solution.

where

$$W'(\pi) = \sum_s p_s \sigma(\pi, s)\, \omega^*(Q(\sigma, \pi), s)$$
$$+ \sum_s p_s [1 - \sigma(\pi, s)] \left[\omega(Q(\sigma, \pi), s) + \beta \Delta W \left(p_{nc} + \frac{\pi \Delta p}{\pi + (1-\pi)(1 - \sigma(\pi, s))} \right) \right].$$

and $\sigma = f(\Delta W)$. Note that if $\Delta W = \mathbb{T}_{\Delta} \Delta W$ then $\sigma = f(\Delta W)$ is an equilibrium for the model. In fact, if we have a fixed point then we can compute

$$W(p_{nc}) = \sum_s p_s \sigma(p_{nc}, s) \frac{\omega^*(Q(\sigma, p_{nc}), s)}{(1-\beta)}$$
$$+ \sum_s p_s (1 - \sigma(p_{nc}, s)) \frac{\left[\omega(Q(\sigma, p_{nc}), s) + \beta \Delta W \left(p_{nc} + \frac{\pi \Delta p}{p_{nc} + (1-p_{nc})(1-\sigma(p_{nc}, s))} \right) \right]}{1-\beta}$$

and

$$W(\pi) = W(p_{nc}) + \Delta W(\pi)$$

and so all equilibrium conditions are satisfied.

We first establish a preliminary result that characterizes $f(\Delta W)$:

Claim. Given $\Delta W \in X$, $\sigma(\pi, s) = f(\Delta W)$ is decreasing in π. Moreover, if $\Delta W_H \geqslant \Delta W_L$ then $f(\Delta W_H) \leqslant f(\Delta W_L)$.

Proof. Note that the smallest solution of (30) is the smallest fixed point of the following operator: $\mathbb{T}_\sigma : \Sigma \to \Sigma$ defined as

$$\mathbb{T}_\sigma \sigma(\pi, s) = \begin{cases} 0 & \text{if } \beta \Delta W(p_{nc} + \pi \Delta p) \geqslant \Delta \omega(Q(\sigma, \pi), s) \\ \tilde{\sigma} & : \beta \Delta W \left(p_{nc} + \frac{\pi \Delta p}{\pi + (1-\pi)(1-\tilde{\sigma})} \right) = \Delta \omega(Q(\sigma, \pi), s) \\ 1 & \text{if } \beta \Delta W(p_c) < \Delta \omega(Q(\sigma, \pi), s) \end{cases}$$

First we show that for any $\sigma \in \Sigma$ and $\Delta W \in X$, $\sigma'(\pi, s) = (\mathbb{T}_\sigma \sigma)(\pi, s)$ is decreasing in π. Suppose by way of contradiction that there exists $\pi_L < \pi_H$ and $\sigma'(\pi_L, s) < \sigma'(\pi_H, s)$ for some s so that the bailout probability is larger if we start from a higher prior.

Suppose first that $\sigma'(\pi_L, s) = 0$ then

$$\Delta \omega(Q(\sigma, \pi_L), s) \leqslant \beta [W(p_{nc} + \pi_L \Delta p) - W_1(p_{nc})].$$

We also have

$$\Delta \omega(Q(\sigma, \pi_H), s) < \Delta \omega(Q(\sigma, \pi_L), s)$$

and

$$\beta\left[W\left(p_{nc}+\pi_L\Delta p\right)-W\left(p_{nc}\right)\right]\leqslant\beta\left[W\left(p_{nc}+\pi_H\Delta p\right)-W\left(p_{nc}\right)\right]$$

where the first inequality follows from $Q\left(\sigma,\pi_H\right)<Q\left(\sigma,\pi_L\right)$ and the fact that $B\left(Q\right)$ is increasing, and the second inequality from $W\left(\pi\right)$ being an increasing function. Therefore,

$$\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right)\leqslant\beta\left[W\left(p_{nc}+\pi_H\Delta p\right)-W\left(p_{nc}\right)\right]$$

and $\sigma'\left(\pi_H,s\right)=0$, yielding a contradiction.

Next, suppose that $0<\sigma'\left(\pi_L,s\right)<\sigma'\left(\pi_H,s\right)<1$. Then,

$$\begin{aligned}\beta\left[W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma'\left(\pi_H,s\right)\right)}\right)-W\left(p_{nc}\right)\right]&=\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right),\\ \beta\left[W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma'\left(\pi_L,s\right)\right)}\right)-W\left(p_{nc}\right)\right]&=\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right).\end{aligned}$$

Therefore, since $\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right)\leqslant\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right)$ and W is increasing, it must be that

$$\begin{aligned}&p_{nc}+\frac{\pi_L\Delta p}{\pi_L+(1-\pi_L)\left(1-\sigma'\left(\pi_L,s\right)\right)}\geqslant p_{nc}+\frac{\pi_H\Delta p}{\pi_H+(1-\pi_H)\left(1-\sigma'\left(\pi_H,s\right)\right)}\\ &\iff\frac{(1-\pi_H)}{\pi_H}\left(1-\sigma'\left(\pi_H,s\right)\right)\geqslant\frac{(1-\pi_L)}{\pi_L}\left(1-\sigma'\left(\pi_L,s\right)\right)\\ &\iff 1-\sigma'\left(\pi_H,s\right)\geqslant\frac{(1-\pi_L)}{\pi_L}/\frac{(1-\pi_H)}{\pi_H}\left(1-\sigma'\left(\pi_L,s\right)\right)>1-\sigma'\left(\pi_L,s\right)\\ &\iff\sigma'\left(\pi_L,s\right)>\sigma'\left(\pi_H,s\right)\end{aligned}$$

obtaining a contradiction.

Finally, if $0<\sigma'\left(\pi_L,s\right)<\sigma'\left(\pi_H,s\right)=1$ then

$$\beta\left[W\left(p_c\right)-W\left(p_{nc}\right)\right]<\Delta\omega\left(Q\left(\sigma,\pi_H\right),s\right)<\Delta\omega\left(Q\left(\sigma,\pi_L\right),s\right)$$

implying $\sigma'\left(\pi_L,s\right)=1$, which is also a contradiction. Thus $\sigma'\left(\pi,s\right)=\left(\mathbb{T}_\sigma\sigma\right)\left(\pi,s\right)$ is decreasing in π.

Next, we show that $\mathbb{T}_\sigma$ is monotone in σ. That is, if $\sigma_H\geqslant\sigma_L$ then $\sigma_H'\geqslant\sigma_L'$. Suppose by way of contradiction that for some π and s, $\sigma_H'\left(\pi,s\right)<\sigma_L'\left(\pi,s\right)$. Then, it must be that $\sigma_L'\left(\pi,s\right)>0$ so

$$\beta\Delta W\left(p_{nc}+\frac{\pi\Delta p}{\pi+(1-\pi)\left(1-\sigma_L'\left(\pi,s\right)\right)}\right)\geqslant\Delta\omega\left(Q\left(\sigma_L,\pi\right),s\right).$$

Since $\sigma_H \geqslant \sigma_L$ then $Q(\sigma_H, \pi) \geqslant Q(\sigma_L, \pi)$ and

$$\Delta\omega\left(Q\left(\sigma_H, \pi\right), s\right) \geqslant \Delta\omega\left(Q\left(\sigma_L, \pi\right), s\right).$$

Thus,

$$\begin{aligned}\beta\Delta W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)\left(1-\sigma_H'(\pi, s)\right)}\right) &> \beta\Delta W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)\left(1-\sigma_L'(\pi, s)\right)}\right) \\ &\geqslant \Delta\omega\left(Q\left(\sigma_L, \pi\right), s\right) \\ &\geqslant \Delta\omega\left(Q\left(\sigma_H, \pi\right), s\right)\end{aligned}$$

which implies that $\sigma_H'(\pi, s) = 1$, a contradiction. Therefore the operator $\mathbb{T}_\sigma$ is monotone and we can find f as

$$f(\Delta W) = \lim_{n\to\infty} \mathbb{T}_\sigma^n \mathbf{0} \quad \mathbf{0}(\pi, s) = 0 \quad \forall (\pi, s)$$

where $\mathbf{0}$ is some initial feasible value.

Finally, we show that if $\Delta W_H \geqslant \Delta W_L$ then $f(\Delta W_H) \leqslant f(\Delta W_L)$. We know that

$$f(\Delta W_i) = \lim_{n\to\infty} \mathbb{T}_\sigma^n(\mathbf{0}; \Delta W_i)$$

so it suffices to show that for all n

$$\sigma_H^n(\pi, s) = \mathbb{T}_\sigma^n(\mathbf{0}; \Delta W_H) \leqslant \mathbb{T}_\sigma^n(\mathbf{0}; \Delta W_L) = \sigma_H^n(\pi, s)$$

which is true since $\mathbb{T}_\sigma$ is monotone increasing in σ and monotone decreasing in ΔW. This must also be true in the limit so that $f(\Delta W_H) \leqslant f(\Delta W_L)$. □

We now show that $\mathbb{T}_\Delta \Delta W \in X$. Notice that the definition of $\mathbb{T}_\Delta$ implies that $\mathbb{T}_\Delta(\Delta W)(p_{nc}) = 0$. Thus, what is left is to show that the operator $\mathbb{T}_\Delta$ maps increasing functions into increasing functions.

Claim. $\Delta W'(\pi) = \mathbb{T}_\Delta(\Delta W)(\pi)$ is increasing in π.

Proof. Let $\sigma = f(\Delta W)$. Consider $\pi_L < \pi_H$ and define $\mathcal{S}_1 = \{s : \sigma(\pi_L, s) = \sigma(\pi_H, s) = 0\}$, $\mathcal{S}_2 = \{s : \sigma(\pi_L, s) > \sigma(\pi_H, s) = 0\}$, and $\mathcal{S}_3 = \{s : \sigma(\pi_L, s) \geqslant \sigma'(\pi_H, s) > 0\}$ so that in $\mathcal{S}_1$ there are no bailouts under both π_L and π_H, in $\mathcal{S}_2$ there is a positive probability of bailouts under π_L but not under π_H, and in $\mathcal{S}_3$ bailouts happen with positive probability under both π_L and π_H. Then

$$\begin{aligned} W'(\pi_L) = & \sum_{s \in \mathcal{S}_1} p_s \left[\omega\left(Q\left(\pi_L, \sigma\right), s\right) + \beta \Delta W\left(p_{nc} + \pi_L \Delta p\right)\right] \\ & + \sum_{s \in \mathcal{S}_2} p_s \left[\omega^*\left(Q\left(\pi_L, \sigma\right), s\right) + \beta \Delta W\left(p_{nc}\right)\right] \\ & + \sum_{s \in \mathcal{S}_3} p_s \left[q\omega^*\left(Q\left(\pi_L, \sigma\right), s\right) + \beta \Delta W\left(p_{nc}\right)\right] \end{aligned}$$

and

$$\begin{aligned} W'(\pi_H) = & \sum_{s \in \mathcal{S}_1} p_s \left[\omega\left(Q\left(\pi_H, \sigma\right), s\right) + \beta \Delta W\left(p_{nc} + \pi_H \Delta p\right)\right] \\ & + \sum_{s \in \mathcal{S}_2} p_s \left[\omega\left(Q\left(\pi_H, \sigma\right), s\right) + \beta \Delta W\left(p_{nc} + \pi_H \Delta p\right)\right] \\ & + \sum_{s \in \mathcal{S}_3} p_s \left[q\omega^*\left(Q\left(\pi_H, \sigma\right), s\right) + \beta \Delta W\left(p_{nc}\right)\right]. \end{aligned}$$

Since $Q(\pi_H, \sigma) < Q(\pi_H, \sigma)$ and $\omega(Q, s)$ is decreasing in Q we have that for all s

$$\begin{aligned} \omega\left(Q\left(\pi_H, \sigma\right), s\right) - \omega\left(Q\left(\pi_L, \sigma\right), s\right) > 0, \\ \omega^*\left(Q\left(\pi_H, \sigma\right), s\right) - \omega^*\left(Q\left(\pi_L, \sigma\right), s\right) > 0. \end{aligned}$$

Moreover, since ΔW is increasing we have

$$W\left(p_{nc} + \pi_H \Delta p\right) \geqslant W\left(p_{nc} + \pi_L \Delta p\right).$$

Thus, for all $s \in \mathcal{S}_1 \cup \mathcal{S}_3$ we have that the value at π_H is higher than at π_L. Finally, for $s \in \mathcal{S}_2$ we have that

$$\begin{aligned} \omega\left(Q\left(\pi_H, \sigma\right), s\right) + \beta \Delta W\left(p_{nc} + \pi_H \Delta p\right) & \geqslant \omega^*\left(Q\left(\pi_H, \sigma\right), s\right) + \beta \Delta W\left(p_{nc}\right) \\ & \geqslant \omega^*\left(Q\left(\pi_L, \sigma\right), s\right) + \beta \Delta W\left(p_{nc}\right) \end{aligned}$$

and so it follows that

$$W'(\pi_H) - W'(\pi_L) > 0 \Rightarrow \Delta W'(\pi_H) - \Delta W'(\pi_L) > 0.$$

□

Finally, we show that the operator $\mathbb{T}_\Delta$ is monotone.

Claim. If p_{nc} is sufficiently small and $\Delta W_H \geqslant \Delta W_L$ then

$$\Delta W'_H = \mathbb{T}_\Delta\left(\Delta W_H; p_{nc}\right) \geqslant \mathbb{T}_\Delta\left(W_L; p_{nc}\right) = \Delta W'_L.$$

Proof. Let $\mathcal{S}^*_i(\pi) \equiv \{s : f(\Delta W_i)(\pi, s) > 0\}$ and $\mathcal{S}^0_i(\pi) \equiv \{s : f(\Delta W_i)(\pi, s) = 0\}$. We can then write

$$W'_H(\pi) = \sum_{s\in\mathcal{S}^*_H} p_s \omega^*\left(Q_H(\pi), s\right) + \sum_{s\in\mathcal{S}^0_H} p_s\left[\omega\left(Q_H(\pi), s\right) + \beta\Delta W_H\left(p_{nc} + \pi\Delta p\right)\right] dP(s)$$
$$W'_L(\pi) = \sum_{s\in\mathcal{S}^*_L} p_s \omega^*\left(Q_L(\pi), s\right) dP(s) + \sum_{s\in\mathcal{S}^0_L} p_s\left[\omega\left(Q_L(\pi), s\right) + \beta\Delta W_L\left(p_{nc} + \pi\Delta p\right)\right] dP(s).$$

We know that

$$\mathcal{S}^*_H \subset \mathcal{S}^*_L$$

and for $s \in \mathcal{S}^0_L$,

$$\omega\left(Q_L(\pi), s\right) + \beta\Delta W_L\left(p_{nc} + \pi\Delta p\right) \leqslant \omega\left(Q_H(\pi), s\right) + \beta\Delta W_H\left(p_{nc} + \pi\Delta p\right)$$

and for $s \in \mathcal{S}^*_H$,

$$\omega^*\left(Q_L(\pi), s\right) + \beta\Delta W_L\left(p_{nc}\right) \leqslant \omega^*\left(Q_H(\pi), s\right) + \beta\Delta W_H\left(p_{nc}\right)$$

and for $s \in \mathcal{S}^0_H \cap \mathcal{S}^*_L$,

$$\omega\left(Q_H(\pi), s\right) + \beta\Delta W_H\left(p_{nc} + \pi\Delta p\right) \geqslant \omega^*\left(Q_H(\pi), s\right) \geqslant \omega^*\left(Q_L(\pi), s\right).$$

Thus,

$$W'_H(\pi) - W'_L(\pi) > \sum_{s\in\mathcal{S}^*_L} p_s\left[\omega^*\left(Q_H(\pi), s\right) - \omega^*\left(Q_L(\pi), s\right)\right] \geqslant 0.$$

If $p_{nc} \to 0$ then $f(\Delta W, p_{nc})(p_{nc}, s) = 1$ for all ΔW and so

$$W'_H(p_{nc}) = \sum_{s\in S} p_s \omega^*\left(Q(1,0), s\right) dP(s),$$
$$W'_L(p_{nc}) = \sum_{s\in S} p_s \omega^*\left(Q(1,0), s\right) dP(s).$$

So for all $\pi > p_{nc}$

$$\lim_{p_{nc}\to 0}\left[\mathbb{T}_\Delta\left(\Delta W_H; p_{nc}\right) - \mathbb{T}_\Delta\left(W_L; p_{nc}\right)\right] > \lim_{p_{nc}\to 0}\sum_{s\in\mathcal{S}^*_L} p_s\left[\omega^*\left(Q_H(\pi), s\right) - \omega^*\left(Q_L(\pi), s\right)\right] \geqslant 0$$

and thus

$$\lim_{p_{nc}\to 0} \mathbb{T}_{\Delta}\left(\Delta W_H; p_{nc}\right)(\pi) > \lim_{p_{nc}\to 0} \mathbb{T}_{\Delta}\left(W_L; p_{nc}\right)(\pi).$$

□

Claim. X is a complete lattice.

Proof. For any $F, G \in X$ we define a binary relation $\succeq$ where $F \succeq G$ iff $\forall s \in S$, $F(\pi, s) \geqslant G(\pi, s)$ for all π . We want to argue that $(X, \succeq)$ is a complete lattice. That is, for any arbitrary subset $\tilde{X}$ of X: 1) there exists $\underline{x} \in \tilde{X}$ such that i) for all $x \in \tilde{X}$, $x \succeq \underline{x}$ and ii) for all $x' \in X$ such that $x \succeq x'$ for all $x \in \tilde{X} \Rightarrow \underline{x} \succeq x$ ($\underline{x}$ is the greatest lower bound); 2) there exists $\bar{x} \in \tilde{x}$ such that i) for all $x \in \tilde{X}$, $\bar{x} \succeq x$ and ii) for all $x' \in X$ such that $x' \succeq x \Rightarrow x' \succeq \bar{x}$ ($\bar{x}$ is the least upper bound). Clearly for any subset $X' \subset X$, for each s these correspond to the lower and upper envelopes of functions in the set. In particular, for each s and each π define

$$\bar{x}(\pi, s) = \max_{x(\pi,s)\in X'} x(\pi, s)$$

and

$$\underline{x}(\pi, s) = \min_{x(\pi,s)\in X'} x(\pi, s).$$

Notice that both $\bar{x}(\pi, s)$ and $\underline{x}(\pi, s)$ are continuous, decreasing, and satisfy $\bar{x}(p_{nc}, s) = \underline{x}(p_{nc}, s) = 0$. Therefore, $\bar{x}$ and $\underline{x}$ belong to X. □

Thus, we have verified all the conditions needed to apply Tarski's fixed point theorem to establish that the set of fixed points of $\mathbb{T}$ is in X and is non-empty. Consider a fixed point $\Delta W = \mathbb{T}_{\Delta} W$ in X. Since $\Delta W \in X$ then ΔW is increasing in π. Since we showed that $\sigma = f(\Delta W)$ is decreasing in π for all $\Delta W \in X$, the equilibrium bailout probability on path, σ, is decreasing in π. Finally, $\sigma(\pi, s_L) \geqslant \sigma(\pi, s_M) \geqslant \sigma(\pi, s_H)$ follows from the fact that the static costs of not bailing out are are increasing in s. Q.E.D.

A.2 Proof of Proposition 2

We first show that under condition (21) in Assumption 1 we have $\sigma(\pi, s_L) = 1$ for all π. To this end, note that in any equilibrium $B(\pi) = \mathbf{b}(\bar{\gamma}(\pi)) \geqslant \mathbf{b}(0)$. Moreover, note that the dynamic gains from bailing out, $W(p_c) - W(p_{nc})$, are bounded by $W^R(0) - W^R(1)$, i.e.,

$$W(p_c) - W(p_{nc}) \leqslant W^R(0) - W^R(1).$$

This is because $W^R(0) = W^R \geqslant W(p_c)$ since the value of the Markov equilibrium is lower than the value of the Ramsey plan, and $W(p_{nc}) \geqslant W^R(1)$ because along the equilibrium path private agents believe that with some probability they are facing the commitment type. Hence we have that

$$\psi B(\pi) \geqslant \psi \mathbf{b}(0) > \beta \left[W^R(0) - W^R(1)\right] \geqslant \beta \left[W(p_c) - W(p_{nc})\right]$$

and so it is optimal to bail out with probability one if $s = s_L$.

Next, we show that for some π it is optimal to mix in a mild crisis under assumption (22). Since we know that $\sigma(\pi, s_L) = 1$ for all π then

$$\bar{\gamma}(\pi) = \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M \mu (1-\pi)\sigma(\pi, s_M)}{P_L} \in \left[(1-\pi)\frac{p_L}{P_L}, (1-\pi)\right]$$

so

$$B(\pi) \in \left[\mathbf{b}(1-\pi), \mathbf{b}\left((1-\pi)\frac{p_L}{P_L}\right)\right].$$

First, suppose by way of contradiction that $\sigma(\pi, s_M) = 0$ for all π. Then it must be that

$$0 < \psi\mu B(\pi) \leqslant \beta \left[W(p_{nc} + \pi\Delta p) - W(p_{nc})\right]$$

but as $p_{nc} \to 0$, for $\pi = p_{nc} = 0$ we have

$$0 < \psi\mu B(p_{nc}) \leqslant \beta \left[W(p_{nc}) - W(p_{nc})\right] = 0$$

which is a contradiction. Thus for π low enough we have $\sigma(\pi, s_M) > 0$.

We now show that it is not optimal to bail out for sure in state s_M. Suppose by way of contradiction that $\sigma(\pi, s_M) = 1$ for all π. Thus, we have that $\bar{\gamma} = 1$ so it must be that for all π

$$\psi\mu \mathbf{b}(1-\pi) \leqslant \beta \left[W(p_c) - W(p_{nc})\right].$$

Under the contradiction hypothesis, $\sigma(\pi, s_i) = 1$ for all π and $s_i = s_M, s_L$ so the continuation value for the optimizing type is

$$\begin{aligned} W(\pi) = {} & (1-2\lambda)\left[qP_H \mathbf{b}(1-\pi) - \mathbf{k}(1-\pi)\right] + \lambda\left[qP_H \mathbf{k}(1-\pi)^{\alpha} - \mathbf{k}(1-\pi)\right] \\ & + \beta P_H W(p_{nc} + \pi\Delta p) + \beta P_L W(p_{nc}) \end{aligned}$$

which evaluated at $\pi = p_c$ and p_{nc} reduces to

$$W(p_{nc}) = (1-2\lambda)\left[qP_H\mathbf{b}(1-p_{nc}) - \mathbf{k}(1-p_{nc})\right] + \lambda\left[qP_H\mathbf{k}(1-p_{nc})^{\alpha} - \mathbf{k}(1-p_{nc})\right]$$
$$+ \beta P_H W(p_{nc} + p_{nc}\Delta p) + \beta P_L W(p_{nc})$$
$$W(p_c) = (1-2\lambda)\left[qP_H\mathbf{b}(1-p_c) - \mathbf{k}(1-p_c)\right] + \lambda\left[qP_H\mathbf{k}(1-p_c)^{\alpha} - \mathbf{k}(1-p_c)\right]$$
$$+ \beta P_H W(p_{nc} + p_c\Delta p) + \beta P_L W(p_{nc})$$

and as $p_{nc} \to 0$ and $p_c \to 1$ we have

$$W(p_{nc}) = (1-2\lambda)\left[qP_H\mathbf{b}(1) - \mathbf{k}(1)\right] + \lambda\left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right]$$
$$+ \beta P_H W(p_{nc}) + \beta P_L W(p_{nc})$$
$$W(p_c) = (1-2\lambda)\left[qP_H\mathbf{b}(0) - \mathbf{k}(0)\right] + \lambda\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right]$$
$$+ \beta P_H W(p_c) + \beta P_L W(p_{nc}).$$

Thus, using $\left[qP_H\mathbf{b}(0) - \mathbf{k}(0)\right] = 0$ and subtracting the two expressions above we obtain

$$\Delta W(p_c) = \frac{(1-2\lambda)}{1-\beta P_H}\left\{\mathbf{k}(1) - qP_H\mathbf{b}(1)\right\}$$
$$+ \frac{\lambda}{1-\beta P_H}\left\{\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right] - \left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right]\right\}.$$

Since for all $\lambda \in [0, 1/2]$ we have that $(1-2\lambda) + \lambda = 1 - \lambda \leqslant 1$, condition (22) ensures that

$$\beta\frac{\mathbf{k}(1) - qP_H\mathbf{b}(1)}{1-\beta P_H} > \psi\mu\mathbf{b}(1),$$
$$\beta\frac{\left\{\left[qP_H\mathbf{k}(0)^{\alpha} - \mathbf{k}(0)\right] - \left[qP_H\mathbf{k}(1)^{\alpha} - \mathbf{k}(1)\right]\right\}}{1-\beta P_H} > \psi\mu\mathbf{b}(1),$$

which in turn imply that

$$\beta\Delta W(p_c) > \psi\mu(1)$$

so that it is not optimal to have $\sigma(\pi, s_M) = 1$ because the static costs of not bailing out are smaller than the dynamic benefits. This is a contradiction. Q.E.D.

A.3 Proof of Proposition 4

To prove that the conjectured equilibrium with $\sigma_1 = 1$ exists, we need to show that (26), (27), and (28) hold at $\pi = p_c = 1$ and $\sigma_1 = 1$. From (25), a simple computation gives us the transitory type's continuation value:

$$U_{2T}(\theta_1, \pi) = P_H\left(q\left(P_H + (1-\pi)P_L\right)\right)^{\alpha/(1-\alpha)}\theta_H^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}(1-\alpha). \quad (31)$$

Recall that under the conjectured equilibrium, the transitory types do not receive a bailout but repay in both states.Their problem in period 1 is

$$\max_{k} P_H \theta_H k^{\alpha} - k/Q$$

with $Q = q$. Thus,

$$k_{1T} = (\alpha P_H \theta_H q)^{1/(1-\alpha)}$$

and

$$B_{1T} = (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} < B_{1P}. \tag{32}$$

Using (31) and (32), conditions (26), (27), and (28) can be written as (recall that $\pi = 1$)

$$-(\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) > 0,$$

$$-(\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H (q P_H)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) < 0,$$

$$\beta [W_2(1) - W_2(0)] < \psi [\mu B_{1P} + (1-\mu) B_{1T}].$$

Using the expression for $W_2(\pi)$ we have

$$\begin{aligned} W_2(1) - W_2(0) &= 0 - (\alpha \theta_H)^{\frac{1}{1-\alpha}} Q(0)^{\frac{\alpha}{1-\alpha}} [q P_H - Q(0)] \\ &= (\alpha \theta_H q)^{\frac{1}{1-\alpha}} P_L. \end{aligned}$$

Moreover, since $B_{1P} > B_{1T}$, to show that (26), (27), and (28) are satisfied it is sufficient to show that

$$-(\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) > 0,$$

$$-(\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H (q P_H)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) < 0,$$

$$\beta [W(1) - W(0)] < \psi (\alpha P_H \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)}.$$

It is straightforward to verify that these three inequalities are satisfied under Assumption 2. Q.E.D.

A.4 Proof of Proposition 5

We now want to show that

$$-b_1 + \beta U_2(p_{nc}) \geqslant 0.$$

Note that

$$b_1 \leqslant b_2(0) = (\alpha \theta_H)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)}.$$

Thus it is sufficient to show that

$$-\left(\alpha\theta_H\right)^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} + \beta P_H q^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\right] \geqslant 0$$

or

$$\theta_H^{1/(1-\alpha)} q^{\alpha/(1-\alpha)} \left\{\beta P_H \left[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\right] - \alpha^{1/(1-\alpha)}\right\} \geqslant 0$$

or

$$\beta P_H - \left(1 + \beta P_H\right) \alpha \geqslant 0.$$

But this follows directly from (29). Q.E.D.

B Extensions

B.1 Model with N borrowers

Consider an environment with N borrowers. Let $\mathbf{B} = (b_1, ..., b_N)$, $\mathbf{K} = (k_1, ..., k_N)$ and $\Theta = (\theta_1, ..., \theta_N)$. We show that as $N \to \infty$, the equilibrium outcome is the one in the main text.

Consider first the incentives for the government in sub-period 2. If the government bails out, its static value is

$$\omega^*\left(\mathbf{B}, \mathbf{K}, \Theta\right) = \max_{T_i} \lambda \left[\sum_i \frac{1}{N}\left[\max\left\{\theta_i k_i^\alpha + T_i - b_i; 0\right\}\right]\right] + \\ (1-\lambda)\left[\sum_i \frac{1}{N} b_i \mathbb{I}_{\left\{\theta_i k_i^\alpha + T_i - b_i \geqslant 0\right\}} - \sum_i \frac{1}{N} T_i\right].$$

The value with no transfers is

$$\omega\left(\mathbf{B}, \mathbf{K}, \Theta\right) = \lambda \left[\sum_i \frac{1}{N}\left[\max\left\{\theta_i k_i^\alpha - b_i; 0\right\}\right]\right] + (1-\lambda) \sum_{i:\theta_i = \theta_H} \frac{1}{N} b_i - \psi \sum_{i:\theta_i = 0} \frac{1}{N} b_i.$$

Notice that since N is finite and $\lambda \leqslant 1/2$, the optimal transfers will satisfy $T \in \{\mathbf{0}, T^*\}$ since choosing any other level only imposes costs on the government. Thus, we can summarize the government's decision is sub-period 2 by the bailout policy $\sigma\left(\pi, \mathbf{B}, \mathbf{K}, \Theta\right)$. The static benefits of bailing out are

$$\Delta\omega\left(\mathbf{B}, \mathbf{K}, \Theta\right) = \psi \sum_{i:\theta_i = 0} \frac{1}{N} b_i. \tag{33}$$

Consider now the problem for a borrower. This problem is identical to the one in the main text, except that now they internalize the effect of their choices on the government's equilibrium bailout policy $\sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta)$. The first order condition that characterizes debt issuance is

$$\alpha\theta_H (Q_i b_i)^{\alpha-1} \left(Q_i + Q_{b_i} b_i\right) - 1 = 0 \tag{34}$$

where $Q_i = Q_i(\mathbf{B}, \mathbf{K}, \pi, \sigma)$ is the pricing schedule for borrower i which can be written as

$$Q_i(\mathbf{B}, \mathbf{K}, \pi, \sigma) = qP_H + q(1-\pi)\sum_s p_s \sum_\Theta \Pr(\Theta|s)\,\sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta)$$

where

$$\Pr(\Theta|s) = \prod_{i=1}^{N} h(\theta_i|s)$$

and

$$Q_{bi} = \frac{\partial Q_i}{\partial b_i} = q(1-\pi)\sum_s p_s \sum_\Theta \Pr(\Theta|s)\,\frac{\partial\sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta)}{\partial b_i}.$$

This is identical to the characterization with a continuum of borrowers if $Q_{bi} = 0$. We next show this is the case in the limit as $N \to \infty$.

First note that the dynamic benefits are independent of b_i and so borrowers can only affect the bailout decision by affecting the static benefits of bailing out. Differentiating (33) we obtain

$$\frac{\partial}{\partial b_i}\Delta\omega(\mathbf{B}, \mathbf{K}, \Theta) = \begin{cases} \frac{\psi}{N} & \text{if } \theta_i = 0 \\ 0 & \text{if } \theta_i = \theta_H \end{cases}$$

which converges to 0 as $N \to \infty$. Therefore Q_{bi} converges to zero and in the limit the first order condition for the borrower is $\alpha\theta_H (Q_i b_i)^{\alpha-1} Q_i - 1 = 0$ so

$$b_i(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q_i(\pi)^{\frac{\alpha}{1-\alpha}}, \quad k_i(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q_i(\pi)^{\frac{1}{1-\alpha}}.$$

Using the law of large numbers we have,

$$Q_i(\pi) = qP_H + q(1-\pi)\sum_s p_s \sigma(\pi, \mathbf{B}, \mathbf{K}, \Theta(s))$$

where $\Theta(s)$ is a sequence $\Theta = \{\theta_n\}_{n=1}^{\infty}$ with a share $h(\theta_L|s)$ of borrowers with realizations equal to θ_L and a share $h(\theta_H|s)$ of borrowers with realizations equal to θ_H. The expression above is the same as the one in Lemma 18 for the case with a continuum of borrowers. Since the static benefits in (33) are the same as the ones in (9) then the limits of W and σ are also equal to that in the continuum case. To see why, notice that we can just apply the same argument as in Proposition 1.

B.2 Ramsey Problem

Here we show that if α is sufficiently high and ψ is sufficiently low, then a government with commitment chooses not to bail out. Thus, bailouts are not optimal ex-ante but are only optimal ex-post.

The objective of the government with commitment is to maximize

$$W^c \equiv (1-\lambda)\left[e-K+qP_H B\right]+\lambda\left[qP_H\left(\theta_H K^{\alpha}-B\right)\right]-\psi P_L\left(1-\bar{\gamma}\right)B \tag{35}$$

where the first term is the consumption of taxpayers and lenders, the second term is the borrowers' consumption, and the last term is the social default cost. Note that we can rewrite the first two terms of (35) as

$$\begin{aligned}
&(1-\lambda)\left[e-K+qP_H B\right]+\lambda\left[qP_H\left(\theta_H K^{\alpha}-B\right)\right]-\lambda K+\lambda K\\
&=(1-\lambda)\,e+(1-\lambda)\left(-K+qP_H B\right)+\lambda\left[qP_H\theta_H K^{\alpha}-K\right]+\lambda\left(K-qP_H B\right)\\
&=(1-\lambda)\,e+(1-2\lambda)\left(-K+qP_H B\right)+\lambda\left[qP_H\theta_H K^{\alpha}-K\right].
\end{aligned}$$

Thus, the Ramsey problem is

$$\max_{B,K,\bar{\gamma}}\,(1-\lambda)\,e-(1-2\lambda)\left(K-qP_H B\right)+\lambda\left[qP_H\theta_H K^{\alpha}-K\right]-\psi P_L\left(1-\bar{\gamma}\right)B$$

subject to

$$\begin{aligned}
B&=\mathbf{b}\left(\bar{\gamma}\right)=\left(\alpha\theta_H\right)^{1/(1-\alpha)}\left[q\left(P_H+P_L\bar{\gamma}\right)\right]^{\alpha/(1-\alpha)}\\
K&=\mathbf{k}\left(\bar{\gamma}\right)=\left(\alpha\theta_H q\left(P_H+P_L\bar{\gamma}\right)\right)^{1/(1-\alpha)}.
\end{aligned}$$

Differentiating with respect to $\bar{\gamma}$ we have:

$$\begin{aligned}
\frac{\partial W^c}{\partial\bar{\gamma}}=&\,\mathbf{k}'\left(\bar{\gamma}\right)\left[-(1-\lambda)+\lambda\alpha qP_H\theta_H\mathbf{k}\left(\bar{\gamma}\right)^{\alpha-1}\right]\\
&+\mathbf{b}'\left(\bar{\gamma}\right)\left[(1-\lambda)\,qP_H-\lambda-\psi P_L\left(1-\bar{\gamma}\right)\right]+\psi P_L\mathbf{b}\left(\bar{\gamma}\right).
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{\partial W^c}{\partial\bar{\gamma}}&\leqslant\lambda\left[\alpha qP_H\theta_H\mathbf{k}\left(\bar{\gamma}\right)^{\alpha-1}-1\right]\mathbf{k}'\left(\bar{\gamma}\right)-\mathbf{b}'\left(\bar{\gamma}\right)\psi P_L\left(1-\bar{\gamma}\right)+\psi P_L\mathbf{b}\left(\bar{\gamma}\right)\\
&\leqslant-\psi P_L\left[\mathbf{b}'\left(\bar{\gamma}\right)\left(1-\bar{\gamma}\right)-\mathbf{b}\left(\bar{\gamma}\right)\right]
\end{aligned}$$

where the first inequality follows from the fact that the term $(1-2\lambda)\left(K-qP_H B\right)$ is increasing in $\bar{\gamma}$, the second from the fact that $\alpha q P_H \theta_H \mathbf{k}(\bar{\gamma})^{\alpha-1} - 1 < 0$ and $\mathbf{k}'(\bar{\gamma}) > 0$. Thus, it suffices to show that $[\mathbf{b}'(\bar{\gamma})(1-\bar{\gamma}) - \mathbf{b}(\bar{\gamma})] > 0$ or

$$\frac{\alpha q P_L}{(1-\alpha)} (\alpha\theta_H)^{1/(1-\alpha)} [q(P_H + P_L\bar{\gamma})]^{\alpha/(1-\alpha)-1} (1-\bar{\gamma}) - (\alpha\theta_H)^{1/(1-\alpha)} [q(P_H + P_L\bar{\gamma})]^{\alpha/(1-\alpha)} > 0$$

or

$$\frac{\alpha q P_L}{(1-\alpha)} \frac{(1-\bar{\gamma})}{[q(P_H + P_L\bar{\gamma})]} - 1 > 0$$

which is true if α is sufficiently close to 1 and $\bar{\gamma} < 1$. For $\bar{\gamma}$ close to one, we have that

$$\lim_{\bar{\gamma}\to 1} \frac{\partial W^c}{\partial \bar{\gamma}} = \mathbf{k}'(1)\left[-(1-\lambda) + \lambda\alpha q P_H \theta_H \mathbf{k}(1)^{\alpha-1}\right] + \mathbf{b}'(1)[(1-\lambda) q P_H - \lambda] + \psi P_L \mathbf{b}(1)$$

which is negative if ψ is small enough. Therefore if α is sufficiently close to one and ψ is sufficiently small then the Ramsey problem has $\bar{\gamma} = 0$.

B.3 A Consumption Smoothing Model

In this section we describe a model in which the desire to borrow arises from consumption smoothing motives. All of our results go through in this economy.

All borrowers are one-period lived and symmetric ex-ante. In sub-period 1, borrower i has income $Y_{i1} = Y_1$ and can borrow b_i from risk neutral lenders to finance consumption c_{i1}. In sub-period 2, the aggregate state s is realized according to a distribution P. As in our baseline model, assume that the state can take three values: $s \in \{s_L, s_M, s_H\}$ with probabilities p_L, p_M, and p_H respectively. Each borrower receives stochastic income θ drawn from a distribution $H(\cdot|s)$. We assume that θ can take on two values: θ_H and θ_L. In state s_H, all the borrowers receive the high endowment so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M instead, $h(\theta_H|s_M) = 1-\mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L all borrowers receive the low endowment, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$. After the realization of θ, each borrower can default on its debt. The rest of the model is unchanged.

Let $c_{i2}(s,\theta)$ denote the consumption of borrower i in sub-period 2 given (s,θ). The preferences of borrower i are given by

$$u(c_{i1}) + \delta \sum_s p_s \sum_\theta h(\theta \mid s) u(c_{i2}(s,\theta)) \tag{36}$$

where $u(\cdot)$ is increasing, concave, and differentiable, and δ is the borrower's discount factor across sub-periods. The budget constraint of the borrower in sub-period 1 is

$$c_{i1} = Y_1 + Qb_i$$

where b_i is the debt issued by the borrower and Q is the price of the debt. In sub-period 2, if the borrower does not default, its budget constraint is

$$c_{i2}(s,\theta) = \theta - b_i + T_i$$

where T_i are transfers from the government. We assume that the private cost of default to the borrower, $\underline{u}(s,\theta)$ is given by

$$\underline{u}(s,\theta) = \begin{cases} u(0) & \text{if } \theta = \theta_H \\ u(\theta_L) & \text{if } \theta = \theta_L \end{cases}$$

which implies that these costs exhibit a high degree of convexity. Thus, it is always optimal for borrowers to repay debt if $\theta = \theta_H$ while if $\theta = \theta_L$ there is repayment only if debt issued is zero or there is a transfer equal to at least b. Therefore, the fraction of borrowers defaulting is given by $\Delta = \sum_\theta h(\theta \mid s)\, \mathbb{I}_{\{u(\theta - b + T(B,s)(b,\theta)) < \underline{u}(s,\theta)\}}$ and the optimal transfer is $T \in \{\mathbf{0}, T^*\}$ where

$$T^*(B,s)(b,\theta_L) = b.$$

Given this setup, it is easy to see that all the results in the baseline model apply here as well.

B.4 Payoff Types

Here we show that the equilibrium outcome in the main text is also the equilibrium outcome of a policy game with payoff types if the cost of default for the low cost type is sufficiently small.

Suppose there are two types of governments: low and high cost types which are indexed by subscripts L and H respectively. In particular, the social default cost for the high cost type is $\psi_H = \psi$ as in the baseline model (i.e. the one in the main text) and the default cost for the low cost type is ψ_L. Here, π is the probability that the government is the low default cost type L. Let $\sigma_i(\pi, s)$ for $i = L, H$ be the probability of a bailout given prior π in state s and $\sigma(\pi, s)$ denote the equilibrium strategy for the optimizing type in the baseline model.

Proposition. *Under the assumptions in Proposition 1 and 2, there exists $\bar{\psi}_L > 0$ such that for* *all* $\psi_L \leqslant \bar{\psi}_L$

$$\sigma_H(\pi, s) = \sigma(\pi, s) \text{ and } \sigma_L(\pi, s) = 0, \; \forall \pi, s$$

is an equilibrium.

Proof. Given the conjectured bailout strategies, the levels of debt and capital $B(\pi)$ and $K(\pi)$ are the same as in the case with the commitment type considered in the text. Given $B(\pi)$, $K(\pi)$, and $\sigma_L(\pi, s) = 0$, the problem of the high cost type is identical to the problem of the optimizing type in the baseline model and thus $\sigma_H(\pi, s) = \sigma(\pi, s)$ is optimal. We are left to check that never bailing out is optimal for the low default cost type. To this end, define the value for the low cost government type of following the conjectured strategy, $W_L(\pi; \psi_L)$, as the unique solution to the following functional equation:

$$W_L(\pi; \psi_L) = w(\pi) + \beta \sum_s p_s W_L\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}; \psi_L\right)$$

where the static value given by

$$w(\pi; \psi_L) = (1-\lambda)e + (1-2\lambda)[qP_H B(\pi) - K(\pi)] + \lambda[qP_H\theta_H K(\pi)^\alpha - K(\pi)] - \psi_L q P_L B(\pi).$$

It is easy to see that since $w(\pi; \psi_L)$ is strictly increasing in reputation π, this property is inherited by $W_L(\pi; \psi_L)$. For $\sigma_L(\pi, s) = 0$ to be an equilibrium, it must be that for all $\pi \in [p_{nc}, p_c]$ and $s \in \{s_M, s_L\}$

$$\beta\left[W_L\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}; \psi_L\right) - W_L(p_{nc}; \psi_L)\right] \geqslant \psi_L h(\theta_L|s) B(\pi). \quad (37)$$

We now show that the posterior after no bailout, $p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi,s))}$, is bounded away from p_{nc}. Note that for π close to p_{nc}, we know from the proof of Proposition 2 that $\sigma(\pi, s) > 0$. Let $0 < \underline{\sigma}(s) = \min_{\pi \in [p_{nc}, p_{nc}+\varepsilon]} \sigma(\pi, s)$ for some $\varepsilon > 0$. Thus, we have

$$p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))} \geqslant \begin{cases} p_{nc} + \frac{p_{nc}(p_c - p_{nc})}{p_{nc} + (1-p_{nc})(1-\underline{\sigma}(s))} > p_{nc} & \text{for } \pi \in [p_{nc}, p_{nc} + \varepsilon] \\ p_{nc} + \varepsilon(p_c - p_{nc}) > p_{nc} & \text{for } \pi > p_{nc} + \varepsilon \end{cases}$$

so we can define

$$\eta \equiv \min_{s \in \{s_M, s_L\}} \min_{\pi \in [p_{nc}, p_c]} \left\{p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma(\pi, s))}\right\} > p_{nc}.$$

Then the dynamic benefits of not bailing out are at least

$$\beta[W_L(\eta; \psi_L) - W_L(p_{nc}; \psi_L)] > 0$$

where the strict inequality follows from $W_L(\pi;\psi_L)$ being strictly increasing in π and $\eta > p_{nc}$. In particular, for $\psi_L = 0$ we have that

$$\beta\left[W_L(\eta;0) - W_L(p_{nc};0)\right] > 0$$

so condition (37) holds for $\psi_L = 0$. Thus, by continuity, there exists $\bar{\psi}_L > 0$ such that for all $\psi_L \leqslant \bar{\psi}_L$ condition (37) holds and so it is optimal for the low default cost government to follow a strategy of never bailing out. Q.E.D.

The proposition implies that any equilibrium outcome in the model with a behavioral commitment type is an equilibrium outcome in this model with payoff types.

B.5 Arbitrary Distributions and Recovery Rates

We now argue that Proposition 1 hold in more general environments with more general distributions and recovery rates. We allow both s, θ to be drawn from continuous distributions $P(s)$ and $H(\theta \mid s)$ respectively. We generalize the social cost function to allow for any increasing function $C(\cdot)$. Finally, in the event of default, we assume that lenders and borrowers can renegotiate the contract so that borrowers make a partial repayment to the lenders and avoid the default cost. Let

$$\Delta(B,K,s) = \int \mathbb{I}_{\{B>\theta K^{\alpha}\}} dH(\theta|s),$$

$$\tilde{\Delta}(B,K,s) = \int \theta K^{\alpha} \mathbb{I}_{\{B>\theta K^{\alpha}\}} dH(\theta|s)$$

where $\tilde{\Delta}(B,s)$ denotes the maximal transfer that can be extracted from the borrower such that it is indifferent between defaulting and not.

The static value of bailing out is

$$\omega^{\text{bailout}}(B,K,s) = \lambda \int \max\{\theta K^{\alpha} - B, 0\}\, dH(\theta|s) + (1-\lambda)\left[(1-\Delta(B,K,s))\,B + \tilde{\Delta}(B,s)\right]$$

and the static value of not bailing out (and allowing default) is

$$\begin{aligned}\omega^{\text{no-bailout}}(B,K,s) = \lambda \int \max\{\theta K^{\alpha} - B, 0\}\, dH(\theta|s) + (1-\lambda)\left[(1-\Delta(B,K,s))\,B + \tilde{\Delta}(B,K,s)\right] \\ - C(\Delta(B,K,s)\,B).\end{aligned}$$

Note that even absent a bailout, since private agents can re-negotiate contracts, lenders will extract $\tilde{\Delta}(B,K,s)$ from borrowers who default. Given this the pricing schedule for debt is

$$Q(b,k|\pi,B,K) = q\left\{\int (1-\Delta(b,k,s))\, dP(s) + \int \tilde{\Delta}(b,k,s)\, dP(s)\right\} \tag{38}$$
$$+ q\left\{(1-\pi)\int \sigma(\pi,B,K,s)\left[\Delta(B,K,s)B - \tilde{\Delta}(B,K,s)\right] dP(s)\right\}.$$

Given that private contracts can be renegotiated, lenders receive at least $\tilde{\Delta}(B,K,s)$ in the event of default. The expression on the second line denotes the additional transfer received in the event of a bailout. From Lemma 18, the problem for the borrower in period 1 is

$$\max_{b,k} \int\int \max\{\theta k^{\alpha} - b, 0\}\, dH(\theta|s)\, dP(s) \tag{39}$$

subject to

$$K \leqslant Q(b,k|\pi,B,K)\, b.$$

Define $\Theta^{s}_{+}(B,K) \equiv \{\theta : \theta K^{\alpha} - B \geqslant 0\}$. The following Lemma characterizes the private outcome in the stage game given the bailout policy σ if the distribution for θ is continuous:

Lemma 3. *Given π and a bailout policy σ, (B,K,Q) is a symmetric equilibrium outcome if*

$$K = (QB)^{\alpha},$$

$$\int_{s}\int_{\theta\in\Theta^{s}_{+}(B,K)} \alpha\theta(QB)^{\alpha-1}\left(Q + Q'B\right) dH(\theta \mid s)\, dP(s) - 1 = 0,$$

and $Q = Q(B,K|\pi,B,K)$ *where*

$$Q' = \left.\frac{dQ(b,k|\pi,B,K)}{db}\right|_{(b,k)=(B,K)}.$$

We show that both the existence and characterization results hold in this environment if the private equilibrium of the stage game satisfies the following condition:

Assumption 3. *For any bailout policy $\sigma(\pi,B,K,s)$ which decreasing in π for all (B,K,s), the private equilibrium outcome is such that $B(\pi)$ is a decreasing function.*

The assumption requires the debt issued to be decreasing in π which implies that the equilibrium default probabilities in each state s, to be decreasing in π. In general, as π decreases there are two effects on the equilibrium price of debt Q. First, since the probability of a bail out is higher, Q increases. However, the resulting increase in borrowing increases the probability of default which might lower Q. The assumption requires the first force to dominate so that in equilibrium the price of issuing debt decreases and thus the debt issued increases. It is easy to see that the example described in the previous section satisfies this assumption. Under this Assumption, the steps in Proposition 1 go through unchanged.

B.6 Persistent Shocks: Contagion and Shock Sensitivity

In the model with iid shocks, there is no heterogeneity among borrowers at the beginning of any period. As a result the model cannot generate the contagion effects described in the introduction. By the contagion effect, we mean the increase in the price of debt for a country not directly affected by an adverse fundamental shock. Moreover, since the price of debt depends only on π and not the state, it is not possible to generate the differential effect of reputation on the sensitivity of prices to fundamentals unless we introduce multiple types of borrowers. To show that our framework can generate such features we extend the baseline model to allow for persistent of aggregate and idiosyncratic states. As a result, the distribution functions of idiosyncratic and aggregate shocks are now $h(\theta'|s', s, \theta)$ and $P(s'|s)$.

Let's consider our simple example. To simplify the algebra we assume that the government cares only about the borrowers ($\lambda = 0$). The aggregate state s follows a Markov chain

$$P(s'|s) = \begin{bmatrix} p_{HH} & p_{HM} & p_{HL} \\ p_{MH} & p_{MM} & p_{ML} \\ p_{LH} & p_{LM} & p_{LL} \end{bmatrix}.$$

As before, in state s_H, all borrowers draw θ_H and in state s_L all borrowers draw θ_L, i.e. $h(\theta_H|s_H, \theta) = 1$ and $h(\theta_L|s_L, \theta) = 1$ for all θ. We assume that in the medium state, a fraction μ of borrowers have the low output θ_L and

$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_L) = \rho_L$$
$$h(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_H) = \rho_H$$

with $\rho_L\mu + \rho_H(1-\mu) = \mu$. Thus, the productivity shock is persistent in the medium state.

Let $\mathbf{z}_- = (s_-, \theta_-)$, $\mathbf{z} = (s, \theta)$ and $\nu(\mathbf{z}_-)$ denote the fraction of type $\mathbf{z}_-$. Next, let $P_H(\mathbf{z}_-)$ and $P_L(\mathbf{z}_-)$ be probabilities of a high and low idiosyncratic endowment respectively, conditional on history $\mathbf{z}_-$. Therefore,

$$P_H(\mathbf{z}_-) = p_{s_-H} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\left(1-\rho_{\theta_-}\right) + \left(1-\mathbb{I}_{s_-=s_M}\right)(1-\mu)\right],$$

$$P_L(\mathbf{z}_-) = p_{s_-L} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right].$$

Next, define

$$\bar{\gamma}(\mathbf{z}_-) \equiv \frac{p_{s_-L}(1-\pi)\,\sigma(\pi, s_-, s_L) + p_{s_-M}p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right](1-\pi)\,\sigma(\pi, s_-, s_M)}{P_L(\mathbf{z}_-)}$$

to be the probability that an individual borrower with history $\mathbf{z}_-$ will be bailed out conditional on drawing θ_L. As in the i.i.d case this serves as a useful sufficient statistic to characterize private decisions. The price of debt in this environment is

$$\mathbb{Q}(\mathbf{z}_-, \bar{\gamma}) = qP_H(\mathbf{z}_-) + qP_L(\mathbf{z}_-)\,\bar{\gamma}(\mathbf{z}_-)$$

and the optimal debt level $\mathbb{B}(\mathbf{z}_-, \bar{\gamma})$ is given by

$$\mathbb{B}(\mathbf{z}_-, \bar{\gamma}) = (\alpha\theta_H)^{\frac{1}{1-\alpha}}\,\mathbb{Q}(\mathbf{z}_-, \bar{\gamma})^{\frac{\alpha}{1-\alpha}}. \tag{40}$$

Define the $\bar{\mathbb{B}}(s_-, \bar{\gamma})$ to be aggregate level of debt where

$$\begin{aligned}
\bar{\mathbb{B}}(s_L, \bar{\gamma}) &\equiv \mathbb{B}((s_L, \theta_H), \bar{\gamma}) = \mathbb{B}((s_L, \theta_L), \bar{\gamma}), \\
\bar{\mathbb{B}}(s_M, \bar{\gamma}) &\equiv \mu\mathbb{B}((s_M, \theta_L), \bar{\gamma}) + (1-\mu)\,\mathbb{B}((s_M, \theta_H), \bar{\gamma}), \\
\bar{\mathbb{B}}(s_H, \bar{\gamma}) &\equiv \mathbb{B}((s_H, \theta_H), \bar{\gamma}) = \mathbb{B}((s_H, \theta_L), \bar{\gamma}).
\end{aligned}$$

Next, we characterize a set of continuous monotone equilibria for the economy for an arbitrary transition matrix P and provide sufficient conditions so that the characterization results for the iid case extend to this more general environment. Assumption 4 is the analog for Assumption 1 for the case with persistent endowments.

Assumption 4. *Let* $C(x) = \psi x$. *Define* $W^R(s, \bar{\gamma})$ as be the solution to

$$\begin{aligned}
W^R(s_-, \bar{\gamma}) = e - \sum_{\theta} \nu(s_-, \theta)\left[\bar{\gamma}(s_-, \theta) + \psi\left(1 - \bar{\gamma}(s_-, \theta)\right)\right] qP_L(s_-, \theta)\,\mathbb{B}((s_-, \theta), \bar{\gamma}(s_-, \theta)) \\
+ \beta \sum_{s} p_{s_-s} W^R(s, \bar{\gamma})
\end{aligned}$$

Assume that

$$\psi\bar{\mathbb{B}}(s_-, 0) > \beta\left[W^R(s, 0) - W^R(s, 1)\right] \text{ for all } s \tag{41}$$

and

$$\mathbf{A}^{-1} \cdot \mathbf{x} > \mathbf{G} \tag{42}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1-p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1-p_{HH} \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} q\left(p_{LM}\mu + p_{LL}\right)\bar{\mathbb{B}}\left(s_L, 1\right) \\ q\mu\left(p_{MM}\rho_L + p_{ML}\right)\mathbb{B}\left(\left(s_M, \theta_L\right), 1\right) + q\mu\left(p_{MM}\rho_H + p_{ML}\right)\mathbb{B}\left(\left(s_M, \theta_H\right), 1\right) \\ q\left(p_{HM}\mu + p_{HL}\right)\bar{\mathbb{B}}\left(s_H, 1\right) \end{bmatrix}$$

and

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}\left(s_L, 0\right) \\ \psi\left[\mu\rho_L\mathbb{B}\left(\left(s_M, \theta_L\right), 1\right) + (1-\mu)\rho_H\mathbb{B}\left(\left(s_M, \theta_H\right), 1\right)\right] \\ \psi\mu\bar{\mathbb{B}}\left(s_H, 0\right) \end{bmatrix}.$$

Proposition 6. *For an arbitrary transition matrix* P*, if* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π*,* $\sigma\left(s_-, \pi, s\right) : S \times [0,1] \times S \to [0,1]$ *is decreasing in* π*,* $W\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is increasing in* π*,* $Q\left(s_-, \pi\right) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π *for all* s_-*, and*

$$W\left(s_L, \pi\right) < W\left(s_M, \pi\right) < W\left(s_H, \pi\right).$$

Furthermore, under Assumption 4, if $p_c \to 1$ *and* $p_{nc} \to 0$ *then it must be that:*

- *It is optimal to bailout with probability one in a severe recession,* $\sigma\left(s_-, \pi, s_L\right) = 1$ *for all* π *and* s_-*.*
- *It is optimal to mix in a mild recession for some values of* π *for all* s_-*.*

Proof. The proof of the first part is identical to the i.i.d case. To see the second, we first show that under condition (41) in Assumption 4 we have $\sigma\left(\pi, s_-, s_L\right) = 1$ for all $\left(\pi, s_-\right)$. To this end, note that in any equilibrium $\mathbb{B}\left(\mathbf{z}_-, \bar{\gamma}\right) \geqslant \mathbb{B}\left(\mathbf{z}_-, 0\right)$. Moreover, note that the dynamic gains from bailing out, $W\left(s, p_c\right) - W\left(s, p_{nc}\right)$, are bounded by $W^R\left(s, 0\right) - W^R\left(s, 1\right)$ in that

$$W\left(s, p_c\right) - W\left(s, p_{nc}\right) \leqslant W^R\left(s, 0\right) - W^R\left(s, 1\right)$$

because $W^R\left(s, 0\right) \geqslant W\left(s, p_c\right)$, and $W\left(s, p_{nc}\right) \geqslant W^R\left(s, 1\right)$. Hence we have that

$$\psi B\left(s_-, \pi\right) \geqslant \psi\bar{\mathbb{B}}\left(s_-, 0\right) > \beta\left[W^R\left(s, 0\right) - W^R\left(s, 1\right)\right] \geqslant \beta\left[W\left(s, p_c\right) - W\left(s, p_{nc}\right)\right]$$

and so it is optimal to bail out with probability one if $s = s_L$.

Next we show that it is optimal to mix in a mild recession under assumption (42). Suppose by way of contradiction that $\sigma\left(\pi, s_-, s_M\right) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the optimizing type in state s for $\pi = 1$ is

$$W\left(s, 1\right) = qp_{sH}\left[0 + \beta W\left(s_H, 1\right)\right] + qp_{sM}\left[0 + \beta W\left(s_M, 0\right)\right] + qp_L\left[0 + \beta W\left(s_L, 0\right)\right].$$

For $\pi = 0$, since $\bar{\gamma}(0) = 1$ we have for $s = \{s_H, s_L\}$

$$W(s,0) = -q(p_{sM}\mu + p_{sL})\bar{\mathbb{B}}(s,1) + qp_{sH}\beta W(s_H,0) \\ + qp_{sM}\beta W(s_M,0) + qp_{sL}\beta W(s_L,0)$$

and for $s = s_M$

$$W(s_M,0) = -q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M,\theta_L),1) - q(1-\mu)(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M,\theta_H),1) \\ + qp_{MH}\beta W(s_H,0) + qp_{MM}\beta W(s_M,0) + qp_{ML}\beta W(s_L,0)$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(s,1) - W(s,0) = x_s + q\beta \sum_{s'} p_{ss'}\left[W(s',1) - W(s',0)\right]$$

for some constant x_s. Hence we can write

$$\mathbf{A} \cdot \mathbf{W} = \mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1-p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1-p_{HH} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W(s_L,1) - W(s_L,0) \\ W(s_M,1) - W(s_M,0) \\ W(s_H,1) - W(s_H,0) \end{bmatrix}$$

and

$$\mathbf{x} = \begin{bmatrix} q(p_{LM}\mu + p_{LL})\bar{\mathbb{B}}(s_L,1) \\ q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M,\theta_L),1) + q\mu(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M,\theta_H),1) \\ q(p_{HM}\mu + p_{HL})\bar{\mathbb{B}}(s_H,1) \end{bmatrix}$$

and so

$$\mathbf{W} = \mathbf{A}^{-1} \cdot \mathbf{x}.$$

The static gains of bailing out in a mild recession if $\pi = 1$ is given by

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}(s_L,0) \\ \psi\left[\mu\rho_L\mathbb{B}((s_M,\theta_L),1) + (1-\mu)\rho_H\mathbb{B}((s_M,\theta_H),1)\right] \\ \psi\mu\bar{\mathbb{B}}(s_H,0) \end{bmatrix}.$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the government prefers not to incur the default costs, or

$$\mathbf{G} \geqslant \mathbf{A}^{-1} \cdot \mathbf{x}$$

which contradicts in Assumption 1. Hence it must be that $\sigma(\pi, s_-, s_M) = 1 < 1$ for some π.

We are now left to show that we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_-, s_M) = 0$. Hence, it must be that

$$\bar{\gamma}(\mathbf{z}_-) = \frac{p_L(1-\pi)\,\sigma(\pi, s_L) + p_M \mu (1-\pi)\,\sigma(\pi, s_M)}{P_L(\mathbf{z}_-)} = \frac{p_{s_-L}(1-\pi)}{P_L(\mathbf{z}_-)}.$$

The posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi(p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and for $s_- \in \{s_H, s_L\}$

$$\mu \bar{B}(s_-, \bar{\gamma}) \leqslant \beta\left[W(s, p_{nc}) - W(s, p_{nc})\right].$$

This is a contradiction since

$$0 < \mu \bar{B}(s_-, \bar{\gamma}) \leqslant \beta\left[W(s, p_{nc}) - W(s, p_{nc})\right] = 0.$$

Hence, we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Therefore, there is mixing for some interval of π. A similar argument holds for $s_- = s_M$. Q.E.D.

Thus, a continuous monotone equilibrium exists when shocks are persistent and the economy displays similar dynamics to the i.i.d case. We now show that the introduction of persistence can generate the contagion effects described previously. In the i.i.d case, there is only a single type of borrower in each period. However, with persistent shocks, if $s_- = s_M$, then in the following period there are two types of borrowers: (s_M, θ_L) and (s_M, θ_H). If there is no bail out and a subsequent rise in reputation, the interest rates faced by both types rise due to the presence of a common government. This provides an explanation as to why the CDS spreads for Italy rose after the perceived recovery rates for Greek bonds declined. The announcement that private creditors were expected to receive haircuts on Greek bonds signaled that EU countries were less likely to receive the benefit of a full bail out in case of default in the future. As a result, the cost of borrowing for other countries that might have been considered at risk of default rose as well.

Proposition 7. *(Contagion) If the reputation of the government increases after observing no bail out in state s_M, then the price of debt for types (s_M, θ_H) decreases.*

The proofs follows from the observation that the pricing function Q depends positively on π.
We next show that this model is capable of generating higher sensitivity to fundamentals when reputation is high.

Proposition 8. *(Sensitivity) For any s_-, the difference in the price of debt for a $\theta_- = \theta_H$ borrower and a $\theta_- = \theta_L$ borrower is increasing in the reputation of the government. That is, $Q((s_-, \theta_{H-}), \pi) - Q((s_-, \theta_{L-}), \pi)$ is increasing in π. Similarly, for any θ_-, the differences $Q((s_H, \theta_-), \pi) - Q((s_M, \theta_-), \pi)$ and $Q((s_M, \theta_-), \pi) - Q((s_L, \theta_-), \pi)$ are increasing in π for π large enough.*

Debt prices (and debt issuances) are less responsive to the state s_- when the prior is low. That is, if the probability of facing the optimizing type is low then lenders are less worried about the state of the world since they expect to get bailed out with high probability and therefore, debt prices are not sensitive to the state. These effects are illustrated in Figure 9. As the fourth plot illustrates, the difference between the price of debt across the different states is increasing in π. At $\pi = 0$, the prices are identical and equal to the risk-free rate since lenders expect to be bailed out with probability one. At $\pi = 1$, prices are driven exclusively by the probability of default and since the states are persistent, the difference in prices is large.

B.7 Learning Model

We can simplify the analysis by noting that since $\sigma_2 = 1$, the price in the secondary market simplifies to

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\sigma_1(\pi, B, q_2) + \varepsilon & s = s_H \\ (1-\pi)[(1-\mu) + \mu\sigma_1(\pi, B, q_2)] + \varepsilon & s = s_L \end{cases}.$$

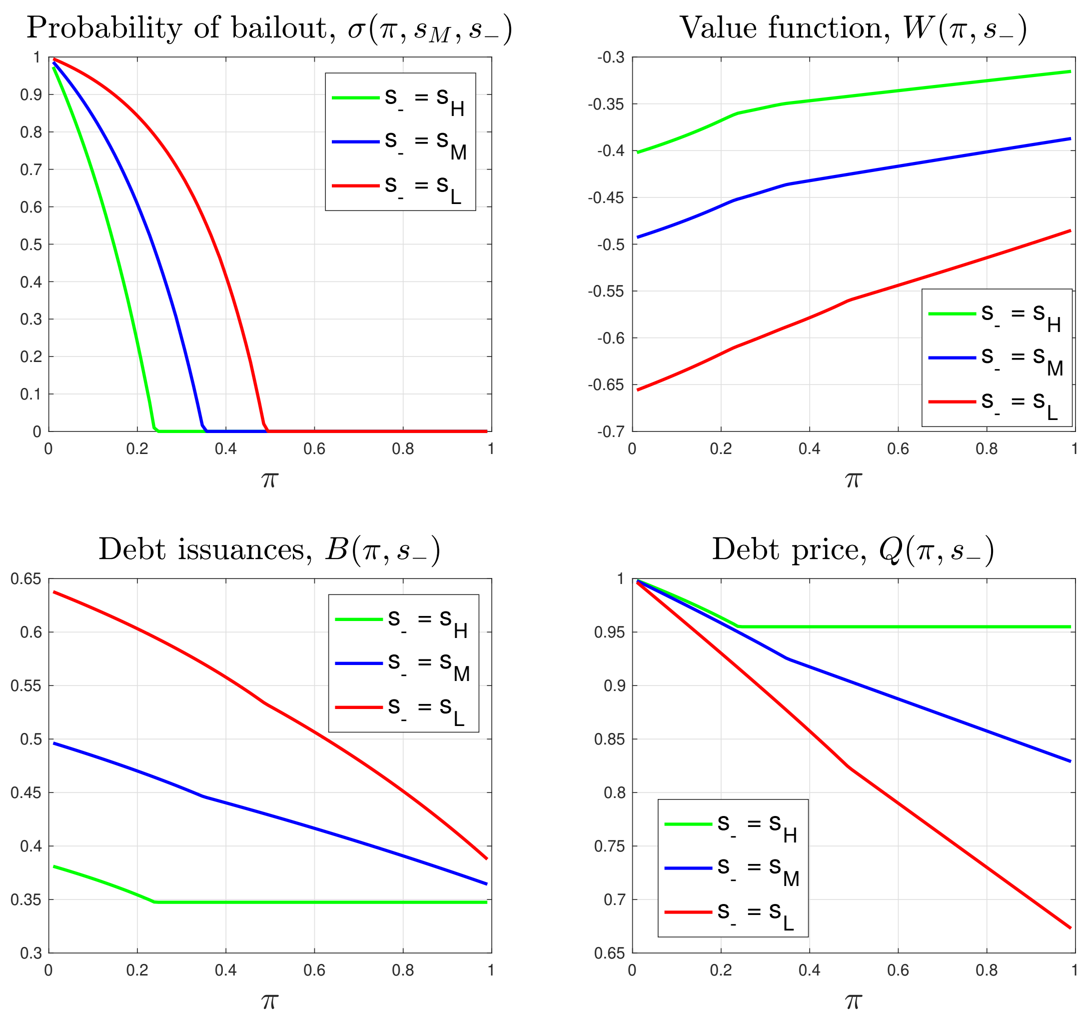
It follows that if $Q_2(\pi, s_H, \varepsilon_H) = Q_2(\pi, s_L, \varepsilon_L)$ then

$$\varepsilon_L = \varepsilon_H + (1-\mu)\pi.$$

If we assume that $\text{supp}(g) = (-\infty, +\infty)$ we can then make a change of variable and express all the equilibrium objects as a function of the realization of ε in state s_H. Define

$$F(\pi) \equiv \int \sigma_1(\pi, \varepsilon)[p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon + (1-\mu)\pi)] d\varepsilon \tag{43}$$

Figure 9: Equilibrium objects for computed discrete example with persistent shocks



to be ex-ante probability of a bailout in the first stage of the sub-period two given prior π. Then, the price of issuing debt in the first sub-period is

$$Q(\pi) = q\left[p(s_H)(1-\mu) + p(s_L)(1-\mu)(1-\pi) + \mu(1-\pi)F(\pi)\right] \tag{44}$$

and so the optimal choice of debt satisfies

$$B(\pi) = (\alpha\theta_H)^{\frac{1}{1-\alpha}} Q(\pi)^{\frac{\alpha}{1-\alpha}}. \tag{45}$$

The value for the government, assuming $\lambda = 0$, is given by

$$\begin{aligned} W(\pi) = & -Q(\pi)B(\pi) + \\ & + qp(s_H)\{(1-\mu)B(\pi) \\ & + \int \sigma_1(\pi,\varepsilon)\beta W(p_{nc})g(\varepsilon)d\varepsilon \\ & + \int [1-\sigma_1(\pi,\varepsilon)]\left[-c\mu B(\pi) + \beta W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi,\varepsilon))}\Delta p\right)\right] g(\varepsilon)d\varepsilon\Big\} \\ & + qp(s_L)\left\{\int [1-\sigma_1(\pi,\varepsilon + (1-\mu)\pi)][-c\mu B(\pi)]g(\varepsilon + (1-\mu)\pi)d\varepsilon + \beta W(p_{nc})\right\}. \end{aligned} \tag{46}$$

Finally, the probability of a bailout in the first stage $\sigma_1(\pi,\varepsilon)$ is given by

$$\sigma_1(\pi,\varepsilon) = \begin{cases} 0, & \text{if } c\mu B(\pi) \leqslant \beta\hat{p}_H(\pi,\varepsilon)\left[W(p_{nc} + \pi\Delta p) - W(p_{nc})\right] \\ \tilde{\sigma}, & \text{if } c\mu B(\pi) = \beta\hat{p}_H(\pi,\varepsilon)\left[W\left(p_{nc} + \frac{\pi\Delta p}{\pi + (1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc})\right] \\ 1, & \text{if } c\mu B(\pi) \geqslant \beta\hat{p}_H(\pi,\varepsilon)\left[W(p_{nc} + \Delta p) - W(p_{nc})\right] \end{cases} \tag{47}$$

where

$$\hat{p}_H(\pi,\varepsilon) = \frac{p(s_H)}{p(s_H)g(\varepsilon) + p(s_L)g(\varepsilon + (1-\mu)\pi)}.$$

Thus, (43)–(47) define a set of functional equations that can be solved for the equilibrium objects $F(\pi)$, $Q(\pi)$, $B(\pi)$, $W(\pi)$, and $\sigma_1(\pi,\varepsilon)$.