Reputation, Bailouts, and Interest Rate Spread Dynamics*

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Abstract

This paper develops a theory of interest rate spread dynamics driven by fundamentals and the endogenous expectations of future bailouts. A government can be either a commitment type that never bails out or a no-commitment type that can sequentially decide whether to bail out lenders or not. Borrowers and lenders do not know the type of the government and learn about it over time by observing the government's actions. We show that there exists a Markov equilibrium in which in normal times, the static costs of not bailing out are small and there is no bailout. Once fundamentals start to deteriorate, it is optimal for the no-commitment type to randomize between bailing out and not. Conditional on not observing a bailout, private agents assign a lower probability to future bailouts, which in turn leads to an increase in spreads and contagion to borrowers not directly hit by the shock. If the crisis becomes more severe, the static incentives to bail out become too large and the no-commitment type bails out. Private agents then anticipate future bailouts with higher probability and spreads for all borrowers decrease despite the crisis being more severe. These dynamics are consistent with the behavior of spreads during the recent European debt crisis and the US financial crisis around the failure of Lehman Brothers.

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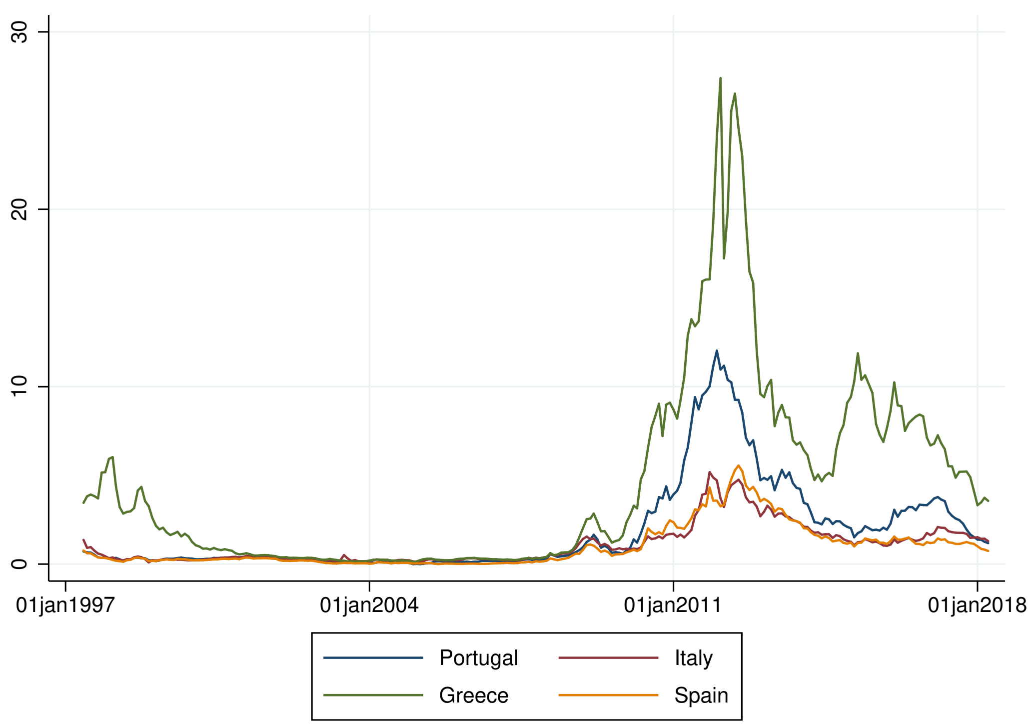
1 Introduction

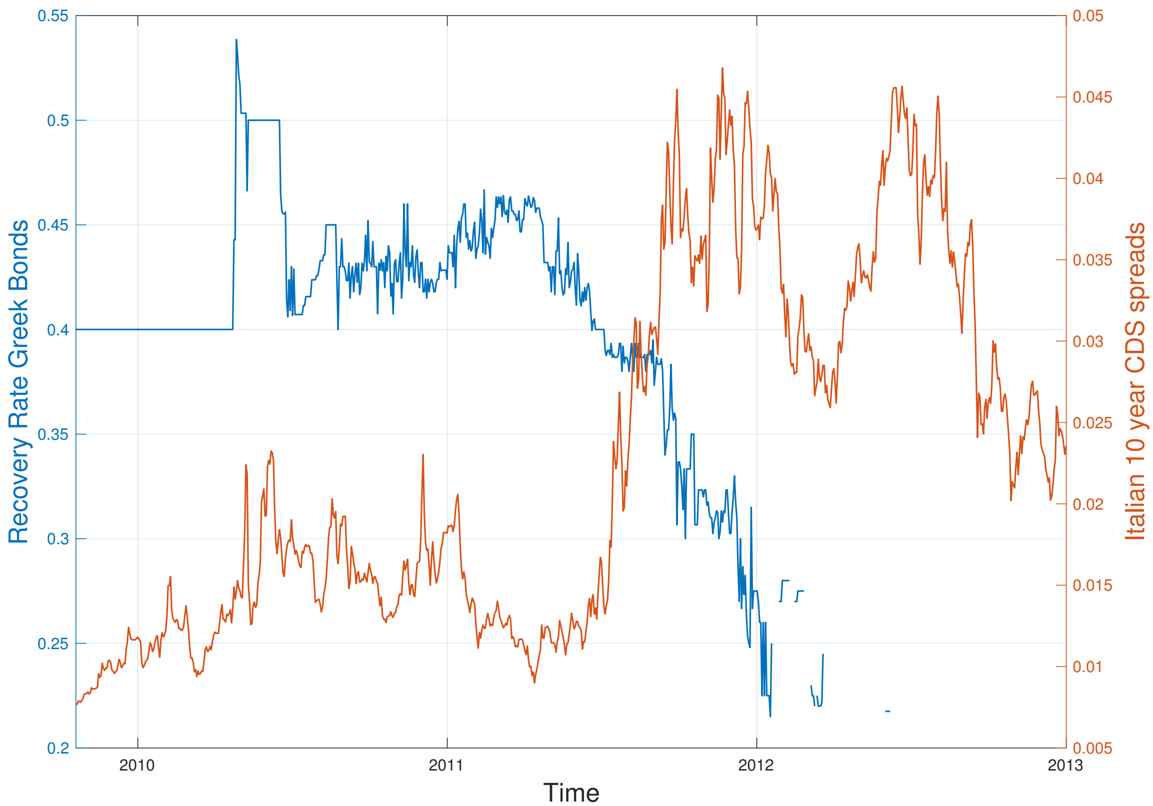
After joining the Euro, Southern European countries were able to borrow at interest rates similar to Germany despite differences in fundamentals, until the beginning of the European debt crisis (see Figure 1). The crisis started in Greece but soon spread to other Southern European countries. News about the willingness of European institutions to bail out Greece impacted the borrowing rates for other EU member countries. Ardagna and Caselli (2014) document that the spike in five- year Italian bond yields coincided with an announcement that the Greek bailout agreement required them to seek a haircut on outstanding debt to private creditors. They argue that this contagion effect might have worsened prospects for peripheral countries. Figure 2 plots the expected recovery rates for Greek bonds on the left axis and the CDS spreads on 10 year Italian bonds on the right axis. The recovery rate is a measure of how much private lenders expect to recover in the event of default. As the figure shows, the sharp increase in Italian CDS spreads coincides the with drop in expected recovery rates for Greek bonds. As the crisis progressed and fundamentals arguably worsened, the interest rates at which Southern European countries continued to rise until the speech by Mario Draghi about the willingness of the ECB to do “whatever it takes”, after which spreads for these countries fell sharply.

Figure 1: Spreads over 10-Year German Bonds



Source: FRED

Figure 2: Recovery Rates and CDS spreads



Source: Markit

These features are not unique to sovereign debt crises. During the U.S. financial crisis, Veronesi and Zingales (2010) document that CDS spreads increased for other banks after Lehman Brothers filed for bankruptcy. After the announcement of the Paulson plan (a $125 billion equity infusion) in October 2008, spreads for many large banks fell. Kelly et al. (2016) document that during the financial crisis, the cost of out-of-the-money (OTM) put options for an index of the financial sector was much cheaper relative to OTM put options on the individual banks comprising the index. They argue that the difference in bailout expectations in case of a single bank failing versus the financial sector collapsing as whole was responsible for this difference. In particular, while the failure of a single bank might not be sufficient to induce a bailout, the failure of a large number of banks might make a bailout more likely.

We take this body of evidence to suggest that news about the generosity of bailout plans is an important driver of spreads. Typically, bailout expectations are high before, lower at the beginning of, and high at the end of the crises, giving rise to "hump-shaped" spreads as in Figure 1. In particular, spreads can suddenly fall even though measured fundamentals might be getting worse, which suggests an alternate factor that drives these spreads. Usually, such crises end after government intervention, as was the case in Europe with OMT or during the U.S. financial crisis with TARP. Our paper is motivated by the following two questions. First, why do interest-rate spreads typically display such patterns around crises, and second, why might governments optimally choose to delay intervention rather than act immediately.

To answer these questions, we study a dynamic model in which spreads are driven by time varying expectations of future bailouts from a common bailout authority. Private agents have beliefs about whether the bailout authority is a commitment or no-commitment type and learn by observing its actions. We jointly characterize the dynamics of spreads and the optimal bailout decision and show that this model can generate spreads which are non-monotonic in fundamentals and also the contagion effect that characterize several debt and financial crises. Moreover, the model provides an explanation for why we often observe the authority in charge of a bailing out (or not) initially refusing to bail out at the beginning of a crisis.

In our model private agents (borrowers) can borrow from risk-neutral lenders who price debt to break even. Borrowers can be interpreted as sovereign governments in the context of the European debt crisis or financial institutions in the context of financial crises.[1] Absent the bailout authority, borrowers default on debt after the realization of a bad idiosyncratic shock. We assume that default imposes both a private and a social cost. We introduce a bailout authority which can be one of two types; a commitment type that never bails out and a no-commitment type which trades off the static benefits of bailing out lenders and avoiding the social default cost with the dynamic costs of losing future reputation. This type is not observed by private agents who learn about it over time by observing its actions. We show that our model can generate an increasing relationship between spreads and reputation. In particular, if the probability of a bailout is low, there is a higher probability of default and so lenders need to increase interest rates in order to break even. The model is subject to an aggregate shock that changes the distribution over idiosyncratic shocks faced by borrowers.

The model can generate equilibrium outcomes that are consistent with the behavior of interest rate spreads around the European debt crisis, for example the contagion effect and the non-monotonicity of spreads. In normal times, when all borrowers receive a high shocks, borrowers do not want to default and so there is no need for a bailout. As a result, there is no learning about the type of the bailout authority and so spreads remain low (and constant) if the initial reputation of the bailout authority is low. If the economy is hit with a shock that affects a small number of borrowers adversely (medium shock), the

[1] While the model in the main text is more applicable to the former, in Appendix C, we describe an environment more suited to studying the latter.

static incentives to bail out increase but by choosing not to bail out the no-commitment type can increase its reputation. If the latter effect is large enough the best response for the no-commitment type is to randomize between bailing out and not. Thus, if there is no bail out, private agents assign a larger probability to no bail outs in the future and thus spreads rise for all borrowers, even those that currently have the high idiosyncratic shock. If the economy is eventually hit by a large negative shock that affects the majority of borrowers, the static bailout benefits dominate the dynamic reputational costs. As a result, the bailout authority chooses to bail out which results in a drop in reputation and hence a sharp reduction in spreads despite the fundamentals being worse.

Along the equilibrium path, spreads and debt issuances are less responsive to the state when reputation is low. That is, if the probability of facing the no-commitment type is high then debt prices are mostly unaffected by the state since lenders expect a bailout in bad states with a high probability. This generates a differential sensitivity of prices to fundamentals that Cole et al. (2016) document in the data.

One message of our paper is that an important driver of spreads are the expectations of future bailouts. In the baseline model, these are driven purely by changes in fundamentals. However, in Section 5 we extend our model to one in which the bailout authority learns about the aggregate state of the world from noisy prices. In this model, we do not need the intermediate (medium) shock in order to generate the hump-shaped spread dynamics typical of crises. The reason is that the government's incentives to bailout are driven by its beliefs about the true state of the world which can be driven by noise rather than changes in fundamentals. At the beginning of a crisis the government is more uncertain about the state of the world and thus it is more likely to find it optimal not to bail out and increase its reputation.

Related Literature

Our paper is related to large literature on repeated games with behavioral types pioneered by Kreps et al. (1982), Kreps and Wilson (1982) and Milgrom and Roberts (1982). Phelan (2006) uses this framework to study a model of government reputation with hidden types which can stochastically evolve. Our model builds on this by embedding this framework in standard sovereign default model. A crucial difference between our models is that in Phelan (2006), the temptation for the government to reveal its type is high when reputation is high. In contrast, in our model, the incentives to reveal are large when reputation is low. This distinction is important for generating the desired movement in spreads. Dovis and Kirpalani (2017) also use a similar framework to study the efficacy of fiscal rules when private agents are strategic. A key difference between these two environments is that they find conditions so that default is never optimal and so that their model is silent on the behavior of spreads.

Nosal and Ordoñez (2013) study a model in which governments learn about the state of the economy through the actions of private agents. In contrast to our model, there is no uncertainty about the type of the government. As mentioned above, uncertainty about the type of the government is crucial to generating the movement in spreads. In particular, their model cannot account for the increase in spreads if there is no bailout observed. In Section 5 we extend our model to allow for the government to learn about the state through the actions of private agents.

Two recent papers, Gourinchas and Martin (2017) and de Ferra (2017) study the effects of bailouts on the debt accumulation decisions in the European Monetary Union. In contrast to these papers, bailout decisions in our model are dynamic due to reputation building incentives. This has important implications for the cost of borrowing and helps explain the behavior of spreads during crises.

Our paper is also related to a literature on reputation and sovereign default, for example Cole et al. (1995), D'Erasmo (2008) and Amador and Phelan (2018). In these models, there is uncertainty about the type of borrower while in ours there is uncertainty about the type of a bailout authority. This allows us to study us an environment in which spreads are driven by bailout expectations.

Finally, our section on two sided learning is related to a literature that studies the link between real activity and the ability to learn about fundamentals. Examples of such papers include Veldkamp (2005) and Fajgelbaum et al. (2017). In our model, when reputation is low, prices are less sensitive to fundamentals which limits the amount the government can learn about the true state . This parallels the idea in these papers that learning is harder in bad economic times when there is low investment.

The rest of paper is organized as follows. In Section 2 we present a simple example economy, and Section 3 characterizes a class of Markov equilibria of this economy. In Section 4 we show how our results apply to a more general model. In Section 5 we extend our model to allow for persistent shocks and in Section 6 we study a model with two sided learning in which the government learns about the true state of the world from prices. Section 7 concludes.

2 Environment

For much of the main text, we will illustrate our results by means of a simple example. We show how these results generalize in Section 4. Time is discrete and is indexed by $\tau = 0, 1, ...$ The economy is populated by a continuum of *borrowers* , $i \in I$, a continuum of *lenders*, a continuum of *tax payers*, and a *government (bailout authority)*. In each period there is a stage game with two sub-periods, $t = 1, 2$.

All borrowers are symmetric ex-ante. In sub-period 1, borrower i has income $Y_{i1} = Y_1$ and can borrow b_i from risk neutral lenders to finance consumption c_{i1}. Lenders have an endowment e that can either be lent to borrowers or invested in a risk free technology that earns a rate of return equal to the inverse of the discount factor.

In sub-period 2, the aggregate state s is realized according to a distribution P. For illustrative purposes assume that the state can take three values: $s \in \{s_L, s_M, s_H\}$ with probabilities p_L, p_M, and p_H respectively. Each borrower receives stochastic income θ drawn from a distribution $H(\cdot|s)$. We assume that θ can take on two values: θ_H and θ_L. In state s_H, all the borrowers receive high endowment so $h(\theta_H|s_H) = 1$ and $h(\theta_L|s_H) = 0$. In state s_M instead, $h(\theta_H|s_M) = 1 - \mu$ and $h(\theta_L|s_M) = \mu$. Finally, in state s_L all borrowers receive the low endowment, $h(\theta_L|s_L) = 1$ and $h(\theta_H|s_L) = 0$. After the realization of θ, each borrower can default on its debt. Default imposes two types of costs: private (imposed on the borrower) and social (imposed on the bailout authority).

Let $c_{i2}(s, \theta)$ denote the consumption of borrower i in sub-period 2 given (s, θ). The preferences of borrower i are given by[2]

$$U = u(c_{i1}) + \delta \sum_s p_s \sum_\theta h(\theta \mid s)\, u(c_{i2}(s, \theta)) \tag{1}$$

where $u(\cdot)$ is increasing, concave, and differentiable, and δ is the borrower's discount factor across sub-periods. The budget constraint of the borrower in sub-period 1 is

$$c_{i1} = Y_1 + Qb_i$$

where b_i is the debt issued by the borrower and Q is the price of the debt. In sub-period 2, if the borrower does not default, its budget constraint is

$$c_{i2}(s, \theta) = \theta - b_i + T_i$$

where T_i are transfers from the bailout authority. We assume that the private cost of default to the borrower defaults, $\underline{u}(s, \theta)$ is given by

$$\underline{u}(s, \theta) = \begin{cases} u(0) & \text{if } \theta = \theta_H \\ u(\theta_L) & \text{if } \theta = \theta_L \end{cases}$$

which implies that these costs exhibit a high degree of convexity. Thus, it is always optimal for borrowers to repay debt if $\theta = \theta_H$ while if $\theta = \theta_L$ there is repayment only if debt

[2] We assume that both borrowers and lenders live for one period.

issued is zero or there is a transfer equal to at least b.

Lenders have preferences over consumption in sub-periods 1 and 2, x_1 and $x_2(s)$,

$$V = x_1 + q\sum_s p_s x_2(s) \tag{2}$$

Their budget constraint in sub-period 1 is

$$x_1 + QB = e$$

where B is the average level of debt, $B = \int b_i di$. In sub-period 2, the lender's budget constraint is

$$x_2(s) = (1 - \Delta(b, s, T))\, b$$

where

$$\Delta(b, s, T) = \sum_\theta h(\theta \mid s)\, \mathbb{I}_{\{u(\theta - b + T(B,s)(b,\theta)) < \underline{u}(s,\theta)\}}$$

is the fraction of borrowers that default in state s.

Tax payers have linear utility and an endowment E in the second sub-period, and can be taxed by the government. The bailout authority can be one of two types: commitment or no-commitment. We assume that the true type of the bailout authority evolves according to the transition matrix:

$$\mathbb{P} = \begin{bmatrix} p_c & 1 - p_c \\ p_{nc} & 1 - p_{nc} \end{bmatrix}$$

where p_c is the probability of transiting from the commitment to the commitment type, and p_{nc} is the probability of transiting from the no-commitment to the commitment type. We assume that $p_{nc} > 0$ but sufficiently small and $p_c > p_{nc}$. The type of the bailout authority is not known to private agents (i.e. borrowers and lenders). At the beginning of period 0, private agents share a common prior π_0 that the bailout authority is the commitment type.

The commitment type never bails out the borrowers while in each period the no-commitment type decides whether to bail out or not and the size of the bailout. To implement a bailout it raises lump-sum taxes from tax-payers . The budget constraint for the bailout authority is

$$\int T_i di = T^l(s)$$

where T_i is the transfer to borrower i and T^l is the tax raised from tax-payers. The bailout authority maximizes the preferences of the lenders and the tax payers net of the social default costs. Its preferences are:

$$\tilde{\Omega} = V + \sum p_s \left(E - T^l(s)\right) - \sum p_s C\left(\int_i \Delta(b_i, s, T)\, b_i di\right) \tag{3}$$

where V is the welfare of the lenders and $E - T^l$ is the welfare of the tax payers.[3] The function $C(\cdot)$ is increasing and captures the social cost of default. For the purposes of our example, we assume that $C(x) = \psi x$, with $\psi < 1$, but allow for a more general specification in Section 4. One interpretation of this cost is that the absence of a bailout leads to a reduction in the net-worth of the banking sector. This reduction in net-worth might have real costs associated with it, for example reduced investments, fire sales etc which is represented by the function $C(\cdot)$. The bailout authority discounts utility across periods at rate β.

While we assume that the commitment type never bails out, it is not true in general that the optimal policy with commitment is to never bail out in sub-period 2. For example, if the social default costs are very high, then it might be optimal to bail out, even with commitment. However, if $\psi < 1$, then we can show that the optimal policy with commitment is to never bail out (see Footnote 5).

Finally, it is worth noting that the assumption that borrowing is driven by consumption smoothing motives is not crucial. In Appendix C, we show how our results apply to an investment model in which borrowing is driven by the availability of attractive investment opportunities. One can use this framework to think about the bail out of banks and understand the events surrounding the collapse of Lehman Brothers during the U.S. financial crisis.

3 Markov Equilibria

We now describe the interaction between private agents and the bailout authority. The timing of events in each period is as follows. In the first sub-period, borrowers choose the amount of debt to issue given the bond schedule. In the second sub-period, the state of nature s and the idiosyncratic shocks θ are realized. After observing (s, θ), given the distribution of inherited debt, the no-commitment type bailout authority chooses whether to bail out the borrowers and if so, the level of transfers. The borrower then decides whether to default or not.

We will focus on symmetric Markov equilibria where all histories are summarized by the posterior probability of facing the commitment type, π, and all individual borrowers

[3]Here we assume that the bailout authority cares only about the welfare of lenders and tax-payers, but not the borrowers. In the context of the EU, we can think of the bailout authority as representing Germany who cared about the health of German banks holding Greek bonds.

choose the same level of debt in sub-period 1. Given this restriction and the fact that all borrowers have the same income level in sub-period 1, they will issue the same amount of debt in equilibrium. As a result, in equilibrium, the distribution of debt facing the bailout authority in the second sub-period is degenerate with point mass at B, which corresponds to the average level of debt.

We describe the actions and payoffs of the agents starting from the last sub-period. Given the state (π, B, s) and the distribution of individual debt holdings, the bailout authority chooses the transfers $T(\pi, B, s)(b, \theta)$, which we shall abbreviate as $T(b, \theta)$, to solve

$$\max_{T(b,\theta)} (1 - \Delta(B, s, T)) B - \int \sum_{\theta} h(\theta \mid s) T(b, \theta) \, d\Gamma(b) - \psi(\Delta(B, s, T) B) + \beta W(\pi') \quad (4)$$

where

$$\Delta(B, s, T) \equiv \sum_{\theta} h(\theta \mid s) \mathbb{I}_{\{u(\theta - B + T(b,\theta)) \leqslant \underline{u}(s,\theta)\}} = \Pr(u(\theta - B + T(b, \theta)) \leqslant \underline{u}(s, \theta) | s)$$

and $W(\pi')$ is the continuation value for the bailout authority next period, given by

$$\begin{aligned} W(\pi) = {} & e - Q(\pi, B(\pi)) B(\pi) \quad (5) \\ & + q \sum_{s} p_s \left[(1 - \Delta(B(\pi), s, T)) B(\pi) - \int \sum_{\theta} h(\theta \mid s) T(b, \theta) \, d\Gamma(b) - \psi(\Delta(\pi, B(\pi), T) B(\pi)) \right] \\ & + q\beta \sum_{s} p_s \left[W(\pi'(\pi, B(\pi), s)) \right] \end{aligned}$$

and the distribution Γ is such that $\Gamma(B) = 1$ and $\Gamma(b) = 0$ if $b \neq B$. The new posterior $\pi'(\pi, B(\pi), s)$ is defined using Bayes' rule

$$\pi' = \begin{cases} p_{nc} + \frac{\pi}{\pi + (1-\pi)\Pr(T(\pi, B(\pi), s) = 0)} (p_c - p_{nc}) & \text{if } T(b, \theta) = 0 \, \forall (b, \theta) \\ p_{nc} & \text{if } T \neq 0 \end{cases} \quad (6)$$

The borrower's problem is

$$U_1(\pi) = \max_{b} u(c_1) + \beta \sum_{s} p_s \sum_{\theta} h(\theta \mid s) \max\{u(\theta - b + T(\theta, b)), \underline{u}(s, \theta)\} \quad (7)$$

subject to the budget constraint in the first sub-period

$$c_1 \leqslant Y_1 + Q(\pi, B(\pi))(b) b$$

where the pricing schedule satisfies

$$Q(\pi, B(\pi))(b) = q\left[p_H + p_M(1-\mu) + p_M\mu(1-\pi)\mathbb{I}_{\{T(B,s_M)\geqslant B\}} + p_L(1-\pi)\mathbb{I}_{\{T(B,s_L)\geqslant B\}}\right] \tag{8}$$

Notice that because of the assumption on private default costs, $Q(\cdot)$ does not depend on individual debt level b. In Section 4, we study a more general model in which Q depends on b.

Definition. A *symmetric Markov Equilibrium* is an individual debt strategy $b(\pi)$, aggregate debt $B(\pi)$, a pricing schedule $Q(\pi, B)(b)$, a transfer strategy for the (no-commitment) bailout authority $T(\pi, B, s)(b, \theta)$, and a law of motion for beliefs, π', such that: i) the debt strategy solves (7), ii) $b(\pi) = B(\pi)$, iii) the pricing schedule satisfies (8), iv) the transfer rule solves (4) where the continuation value W is defined by (5), the law of motion for beliefs is (6).

Characterization

Bailout decision We begin by characterizing the decision of the bailout authority in subperiod 2. One issue that arises is that in a symmetric equilibrium where the distribution of debt holdings is degenerate, the transfers to a borrower that chooses $b \neq B$ and deviates from the equilibrium path is not pinned down. This is because each borrower is measure zero and so conditional on bailing out, transfers to a deviating borrower do not affect the utility of the bailout authority. In principle, it is possible to construct equilibria where transfers off the equilibrium path impose some discipline even absent reputational gains. See Chari et al. (2016) for a related discussion.

Here we select the transfer scheme by considering the limit of the finite borrower case as the number of borrowers converges to infinity. The details of this construction is provided in the Appendix B. In this case, for all (π, B, s), the bailout transfers $T(\pi, B, s)(b)$ are either zero for all (b, θ) or

$$T^*(B, s)(b, \theta) = \max\left\{u^{-1}(\underline{u}(s, \theta)) - \theta + b, 0\right\}$$

In the context of our example,

$$T^*(B, s)(b, \theta_L) = b$$

for all $s \in \{s_M, s_L\}$.Therefore,

$$T \in \{\mathbf{0}, T^*\} \tag{9}$$

where $\mathbf{0}$ is the function identically equal to zero. That is, either the no-commitment type bailout authority mimics the strategy of the commitment type, or it chooses the statically optimal transfer scheme that transfers the minimal amount required to avoid default by all borrowers who would have done so absent the transfer. Notice that from a static perspective, it is always optimal for the bailout authority to intervene and avoid default. This is because, if the bailout authority transfers $T^* (B, s) (b, \theta)$ it can avoid the social cost of default. Note that transfers from tax-payers to lenders do not change the utility of the bailout authority.

Notice further that this transfer scheme implies that in case of a bailout, a borrower receives its outside option of defaulting and nothing more. This is because any additional transfers make the tax-payers and consequently the bailout authority strictly worse off. As a result, given such a transfer policy, the borrower's continuation value is independent of the implemented transfers.

Lemma 1. *Given the transfer scheme in (9), the borrower's continuation value is independent of whether the bailout authority chooses the statically optimal transfers*, T^*, *or it mimics the commitment type and chooses* **0**. *In particular,*

$$U_2 (b) = \sum_s p_s \sum_\theta h (\theta \mid s) \max \{u (\theta - b), \underline{u} (s, \theta)\}$$

Hence, the bailout authority's decisions only affect the borrower through its effect on the interest rates they face. Given our selection of off-equilibrium transfers, we can summarize the bailout authority's strategy by the probability that it will implement the statically optimal transfer scheme, $\sigma (\pi, B, s)$. We will call σ the *bailout policy*. Bailouts generate static benefits and impose dynamic costs on the no-commitment type bailout authority. The static value of bailing out is[4]

$$\tilde{\Omega}^{\text{bailout}} (B, s) = (1 - \Delta (B, s)) B$$

where

$$\Delta (B, s) = h (\theta_L \mid s),$$

and we normalize $C (0) = 0$. Here, $(1 - \Delta (B, s)) B$ denotes the fraction of debt that is paid back absent a bailout. The static value of not bailing out (and allowing default) is

$$\tilde{\Omega}^{\text{default}} (B, s) = (1 - \Delta (B, s)) B - \psi (\Delta (B, s) B)$$

where, (with some abuse of notation) we have dropped T from the argument of Δ. The following Lemma is a direct consequence of the above expression and an increasing cost function.

[4]Recall that since the bailout authority also cares about tax-payers who have linear utility, implementing a bailout lowers welfare by $\Delta (B, s) B$ which nets out the additional transfer to the lenders.

Lemma 2. *The static incentive to bailout are increasing in* B*, i.e.*

$$\Delta\tilde{\Omega}(B,s) \equiv \tilde{\Omega}^{bailout}(B,s) - \tilde{\Omega}^{default}(B,s)$$
$$= \psi(\Delta(B,s)B)$$

is increasing in B*.*

As a consequence of bailing out there is the drop in reputation of the bailout authority as described in (6). We will show that the equilibrium value for the bailout authority $W(\cdot)$ is increasing in π which implies that this drop is costly for the bailout authority. We now characterize the problem of the bailout authority. The state variables in sub-period 2 are (B, s, π). Let $\zeta = 1$ denote the event that a bailout occurs and and $\zeta = 0$ denote the event of no bailout. Given strategy σ, the law of of motion for beliefs satisfies

$$\pi' = \begin{cases} p_{nc} + \frac{\pi}{\pi+(1-\pi)(1-\sigma)}(p_c - p_{nc}) & \text{if } \zeta = 0 \\ p_{nc} & \text{if } \zeta = 1 \end{cases} \tag{10}$$

If the bailout authority chooses to bail out, its value is

$$\Omega^{\text{bailout}}(B,s,\pi) = (1-\Delta(B,s))B + \beta W(p_{nc}) \tag{11}$$

and if it chooses not to bail out its value is

$$\Omega^{\text{no-bailout}}(B,s,\pi) = (1-\Delta(B,s))B - \psi(\Delta(B,s)B) + \beta W\left(p_{nc} + \frac{\pi(p_c - p_{nc})}{\pi + (1-\pi)(1-\sigma)}\right) \tag{12}$$

so the value in equilibrium is

$$\Omega(B,s,\pi) = \max\left\{\Omega^{\text{bailout}}(B,s,\pi), \Omega^{\text{no-bailout}}(B,s,\pi)\right\} \tag{13}$$

where the continuation value $W(\pi)$ is defined by

$$W(\pi) = e - Q(\pi)B(\pi) + q\sum p_s \Omega(B(\pi), s, \pi) \tag{14}$$

and $B(\pi)$ is the allocation rule for aggregate debt along the equilibrium path given prior π and $Q(\pi) = Q(B(\pi), \pi)(B(\pi))$ is the price of debt in the (symmetric) equilibrium outcome. Given this, the optimal strategy for the no-commitment type is

$$\sigma(\pi,\varepsilon) = \begin{cases} 0, & \text{if } \psi(\Delta(B,s)B) \leqslant \beta[W(p_{nc} + \pi\Delta p) - W(p_{nc})] \\ \tilde{\sigma}, & \text{if } \psi(\Delta(B,s)B) = W\left(p_{nc} + \frac{\pi\Delta p}{\pi+(1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc}) \\ 1, & \text{if } \psi(\Delta(B,s)B) \geqslant \beta[W(p_{nc} + \Delta p) - W(p_{nc})] \end{cases}$$

where $\Delta p \equiv p_c - p_{nc}$.

Debt issuances and prices We now set up and characterize the decisions for private agents given a bailout policy $\sigma(\pi, B, s)$. The no-arbitrage conditions for the lenders implies that the pricing schedule is given by

$$Q(B,\pi)(b) = q\{p_H + p_M(1-\mu) + p_M\mu(1-\pi)\sigma(\pi, B, s_M) + p_L(1-\pi)\sigma(\pi, B, s_L)\} \quad (15)$$

The problem for the borrower in period 1 is

$$\max_{c,b} u(c) + \beta P_H u(\theta_H - b) + \beta P_L \underline{u}(\theta_L) \quad (16)$$

subject to

$$c \leqslant Y + Q(B,\pi)(b)\, b, \quad b \leqslant \theta_H$$

where $P_H = p_H + p_M(1-\mu)$ and $P_L = p_L + p_M\mu$ are the probabilities that in sub-period 2, the borrower will draw endowments θ_H and θ_L respectively. The optimal debt chosen in period 1 satisfies

$$Q(B,\pi)\, u'(Y + Q(B,\pi)\, b) = \beta P_H u'(\theta_H - b)$$

where we have used the fact that Q is independent of B. Clearly, individual debt issuances are increasing in $Q(B,\pi)$ and since an increase in π increases Q we have the following result:

Lemma 3. *If the function $\sigma(\pi, B, s)$ is decreasing in π for all (B, s), then the price of debt is decreasing in π in that, if $\pi_H \geqslant \pi_L$, then $Q(B,\pi_H) \geqslant Q(B,\pi_L)$. Furthermore $B(\pi_L) \geqslant B(\pi_H)$.*

Proof. To see the second part notice that we can define an interest rate function $R(B,\gamma) = \frac{1}{Q(B,\gamma)}$ and so the first order condition for the borrower becomes

$$u'(Y + b) = \beta R(B,\gamma)\, P_H u'(\theta_H - R(B,\gamma)\, b)$$

Clearly, if R decreases, B must increase. Q.E.D.

Definition 1. A private equilibrium given bailout policy $\sigma(\pi, B, s)$ consists of debt decision $b(B, Q)$, bond price $Q(\pi) = Q(B(\pi), \pi)$, and aggregate debt $B(\pi)$, such that i) b solves (16) given B and Q, ii) Q satisfies (15), and iii) the representativeness condition $b(B(\pi), Q(\pi)) = B(\pi)$.

We can further characterize the private equilibrium by defining a new variable, $\bar{\gamma}$, equal to probability that an individual borrower will be bailed out conditional on drawing θ_L.

$$\bar{\gamma} \equiv \frac{p_L (1-\pi)\, \sigma(\pi, s_L) + p_M \mu (1-\pi)\, \sigma(\pi, s_M)}{P_L}$$

We can then define bond prices

$$\mathbb{Q}(\bar{\gamma}) = qP_H + qP_L\bar{\gamma}$$

and the unique level of debt $\mathbb{B}(\bar{\gamma})$ solves

$$\mathbb{Q}(\bar{\gamma})\, u'(Y + \mathbb{Q}(\bar{\gamma})\, \mathbb{B}(\bar{\gamma})) = \beta P_H u'(\theta_H - \mathbb{B}(\bar{\gamma})) \tag{17}$$

For example, if $u(c) = c^{1-1/\eta}/(1-1/\eta)$ then

$$\mathbb{Q}(\bar{\gamma})\,(Y + \mathbb{Q}(\bar{\gamma})\, \mathbb{B}(\bar{\gamma}))^{-1/\eta} = \beta P_H (\theta_H - \mathbb{B}(\bar{\gamma}))^{-1/\eta}$$

so

$$\mathbb{B}(\bar{\gamma}) = \frac{\left(\frac{\beta P_H}{\mathbb{Q}(\bar{\gamma})}\right)^{-\eta} \theta_H - Y}{\mathbb{Q}(\bar{\gamma}) + \left(\frac{\beta P_H}{\mathbb{Q}(\bar{\gamma})}\right)^{-\eta}}$$

Equilibrium Dynamics

To show that the set of symmetric Markov equilibria is non-empty we show the existence of a class of continuous monotone equilibria which have some desirable properties. The equilibrium objects in this class are continuous and monotone in the prior π.

Existence of a continuous monotone equilibrium Let $\Delta(\pi, s) = \Delta(\pi, B(\pi), s)$ and $\sigma(\pi, s) = \sigma(\pi, B(\pi), s)$. The next Proposition shows that the set of *continuous monotone equilibria* is non-empty.

Proposition 1. *If* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B(\pi) : [0,1] \to \mathbb{R}$ *is decreasing,* $\sigma(\pi, s) : [0,1] \times S \to [0,1]$ *is decreasing,* $W(\pi) : [0,1] \to \mathbb{R}$ *is increasing.*

This result holds for more general environments, as shown in Section 4 and the proof for this more general result is provided in Appendix A. To do this, we show that the equilibrium value in (14), the bailout policy *along the equilibrium path*, the equilibrium debt policy rule $B(\pi)$, and equilibrium pricing schedule $Q(\pi)$ are a fixed point of an operator and then show that the operator admits a fixed point using Tarski's fixed point theorem.

(Note, when we refer to the bailout policy along the equilibrium path we mean that our procedure solves for $\sigma(\pi, s) = \sigma(B(\pi), \pi, s)$ evaluated only at the aggregate amount of debt chosen in equilibrium, and not for arbitrary debt holding B.) One nice consequence of this fixed point theorem is that the set of fixed points are ordered. As a result, we can order equilibria by the probability of bailouts. Moreover, all the equilibrium properties described below hold for every equilibrium in this set.

Properties of the continuous monotone equilibrium Next, we show that in any continuous monotone equilibrium, the bailout policy satisfies a cutoff property: If reputation is high, $B(\pi)$ is low which in turn implies that default incentives will be low and so it is optimal to not bail out ex-post. If instead reputation is low, $B(\pi)$ and default incentives will be high and so the government is incentivized to bail out. For intermediate levels of reputation, the bailout authority finds it optimal to randomize between bailing out and not. The cutoff depends on the realized state of the economy in the second sub-period: if the recession is severe in that there are more defaults on path absent intervention, a higher level of initial reputation is needed for the bailout authority to not bail out. Define $\mathcal{S}^* \equiv \{s : \Delta(B(1), s) > 0\}$ to be the set of states in which it is statically optimal to bail out. Here $B(1)$ is the level of debt issued if private agents expect to face the commitment type with probability one in the current period and so expect no bail out. In our simple example, $\mathcal{S}^* = \{s_M, s_L\}$.

Proposition 2. *In any continuous monotone equilibrium, if* $s \in \mathcal{S}^*$, *there exist cutoffs* $0 \leqslant \pi_1(s) \leqslant \pi_2(s) \leqslant 1$ *such that:*

- *if* $\pi \leqslant \pi_1(s)$ *then* $\sigma(\pi, s) = 1$*;*
- *if* $\pi_1(s) < 1$ *and* $\pi \in (\pi_1(s), \pi_2(s))$ *then* $\sigma(\pi, s) \in (0, 1)$ *and is strictly increasing over the interval;*
- *if* $\pi_2(s_2) < 1$ *and* $\pi > \pi_2(s_2)$ *then* $\sigma(\pi, s_2) = 0$.

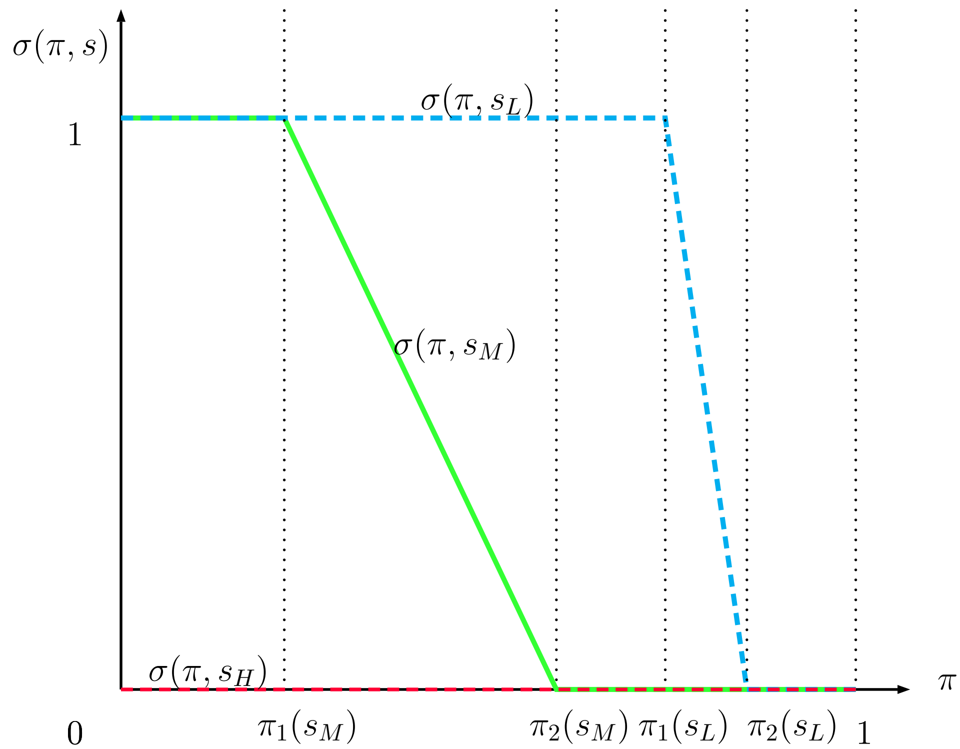
The cutoffs are such that

$$\pi_1(s_H) \leqslant \pi_1(s_L)$$
$$\pi_2(s_H) \leqslant \pi_2(s_L)$$

If instead $s \notin \mathcal{S}^*$ *then there are no static gains from bailout and* $\sigma(\pi, s) = 0$ *for all* π.

This Proposition describes the key property of the equilibria that generates outcomes consistent with the evidence motivating the paper. Proposition 1 implies that for low values of π, the aggregate debt level $B(\pi)$ will be high. As a result, from Lemma 2 we

Figure 3: Probability of a bailout along the equilibrium path in the discrete example



know that the static benefits of bailing out will be very high. Moreover, since $W(\cdot)$ is increasing in π, the dynamic costs of bailing will be low. Therefore, the no-commitment type will bail out with probability one. In contrast, for high values of π, the incentives are reversed: now the static costs of bailing out low and the dynamic costs are high. As a result, the no-commitment type chooses to not bail out. For intermediate values of π for which neither bailing out nor not bailing out with probability one is optimal, the equilibrium strategy for the no-commitment type is to randomize. This probability is chosen so that the bailout authority is indifferent between bailing out and not. From Proposition 2, private agents beliefs of facing the commitment type, if $s \in \mathcal{S}^*$, satisfies

$$\pi'(\pi|\zeta = 0, s) = \begin{cases} p_c & \text{if } \pi \leqslant \pi_1(s) \\ p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma(\pi,s))}(p_c - p_{nc}) & \text{if } \pi \in (\pi_1(s), \pi_2(s)] \\ p_{nc} + \pi(p_c - p_{nc}) & \text{if } \pi \in (\pi_2(s), 1] \end{cases}$$

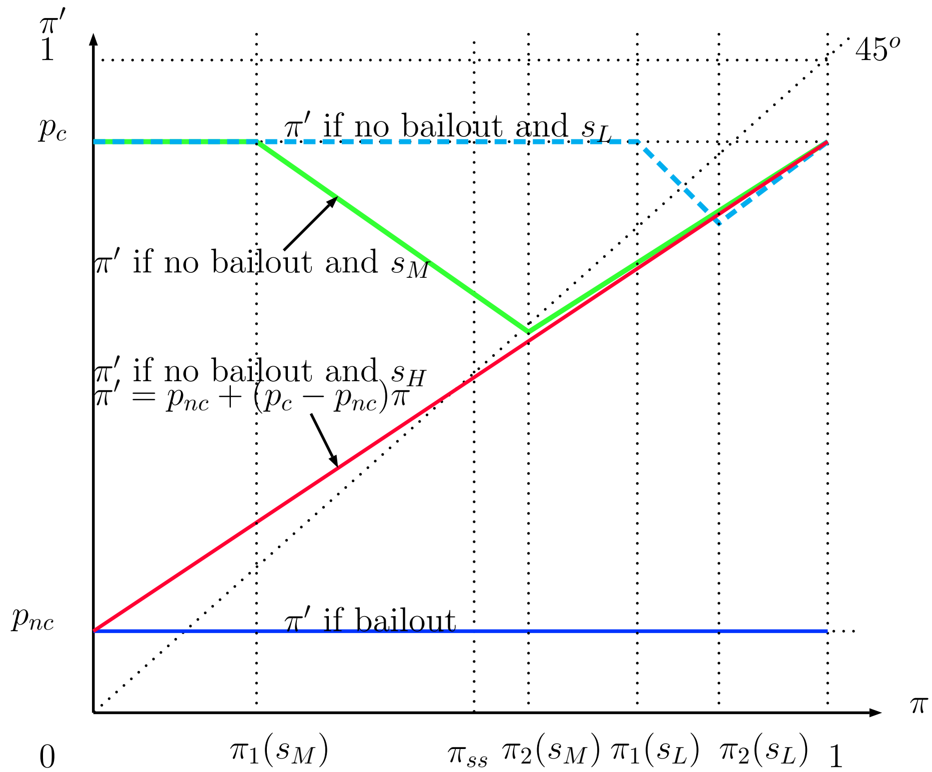
If $s \notin \mathcal{S}^*$ then for all π

$$\pi'(\pi|\zeta = 0, s) = p_{nc} + \pi(p_c - p_{nc})$$

and $\pi'(\pi|\zeta = 1, s_2) = p_{nc}$. The equilibrium bailout policy and the law of motion for beliefs are illustrated in Figures 3 and 4 respectively.

The next proposition characterizes the behavior of interest-rate spreads as a function

Figure 4: Law of motion of beliefs in the discrete example



of π. We define the spread to be the difference between the equilibrium and the risk-free interest rate.

Proposition 3. *In any continuous monotone equilibrium the price of debt,* $Q(\pi, B(\pi))$, *is strictly decreasing in the prior* π. *Thus the spread,* $1/Q(\pi, B(\pi)) - 1/q$ *is increasing in the prior.*

Proof. The results follow from Lemma 3 and the characterization of $\sigma(\pi, s)$ in Proposition 2.Q.E.D.

The intuition for this follows directly from the fact that the probability of a bailout is decreasing in π. As a result, for high values of π, lenders need to increase interest rates in order to break even. It is worth noting that in our simple example, Q is independent of B and so spreads are driven exclusively by bailout expectations. We discuss a more general model in Section 4.

Next, we show that under some sufficient conditions, the monotone continuous equilibrium has mixing in the mild recession and a bailout with probability one in the severe recession.

Assumption 1. *Let* $C(x) = \psi x$ *and define*

$$W^R(\bar{\gamma}) \equiv \frac{e - [\psi(1-\bar{\gamma}) + \bar{\gamma}]\, q P_L \mathbb{B}(\bar{\gamma})}{1-\beta}$$

where $\mathbb{B}(\bar{\gamma})$ *is defined in (17). Assume that,*

$$\psi \mathbb{B}(0) > \beta \left[W^R(0) - W^R(1) \right] \tag{18}$$

and

$$\frac{q\beta P_L}{1 - q\beta p_H} > \psi\mu \frac{\mathbb{B}(0)}{\mathbb{B}(1)} \tag{19}$$

Here, $W^R(\gamma)$ is the value for a fictitious commitment type who follows a fixed bailout policy summarized by sufficient statistic $\bar{\gamma}$ when private agents have beliefs $\pi = 1$. For the commitment type assumed in our model, $\bar{\gamma} = 0$ and so $W^R(0)$ is the value for the commitment type in our model when $\pi = 1$.[5] Since $W^R(0) > W(p_c)$ and $W^R(1) < W(p_{nc})$, the difference $W^R(0) - W^R(1)$ is an upper bound on the dynamic gains from not bailing out. This along with the fact that $B(\pi) \geqslant \mathbb{B}(0)$, implies that (18) ensures that the static gains from bailing out dominate the dynamic costs in state s_L when all borrowers would default absent a bail out. The second part of the assumption, (19) provides an upper bound on μ (the fraction receiving θ_L in s_M) so that it is not a best response for the no-commitment type to bailout with probability one in s_M.

Proposition 4. *Under Assumption 1, if* $p_c \to 1$ *and* $p_{nc} \to 0$ *then in any monotone continuous equilibrium it must be that:*

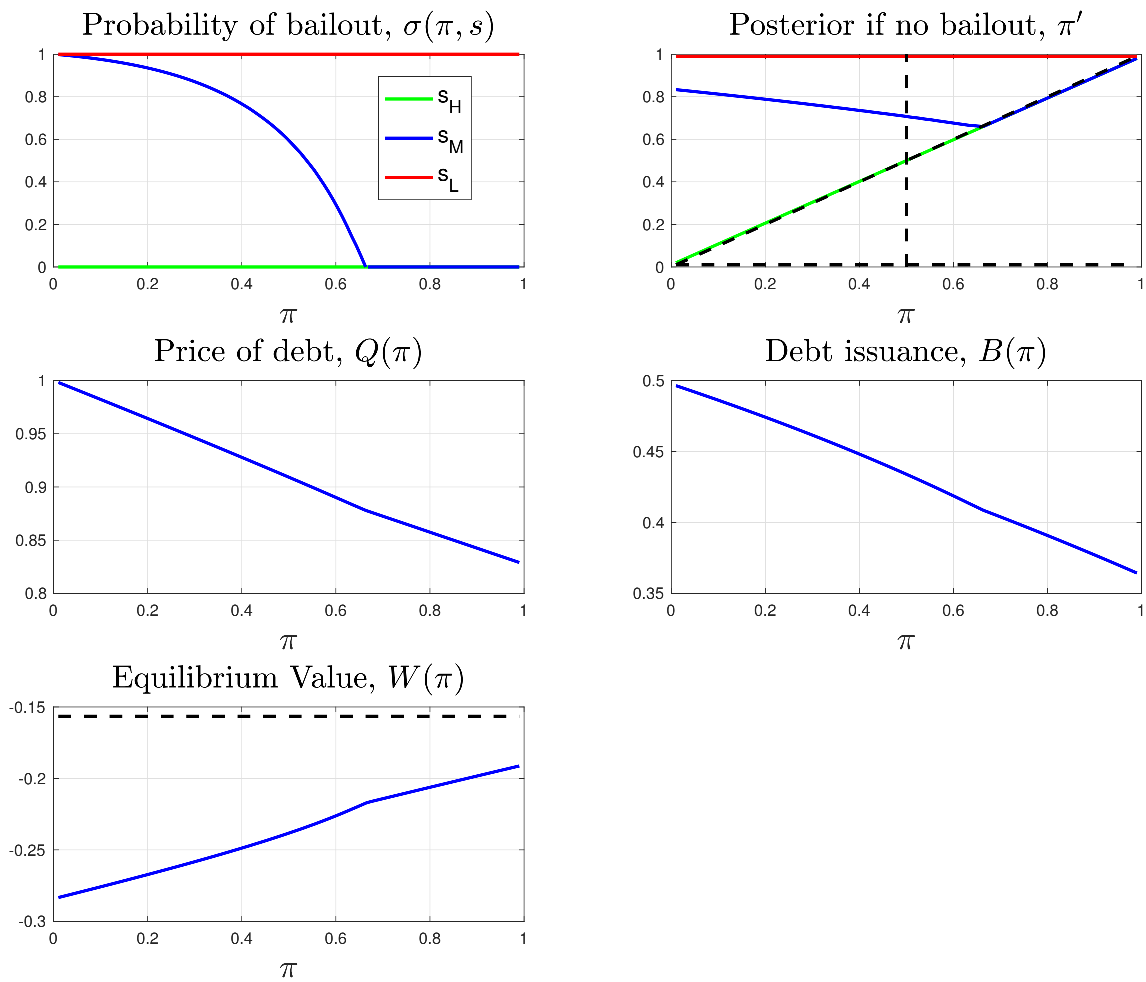
- *It is optimal to bail out with probability one in a severe recession,* $\sigma(\pi, s_L) = 1$ *for all* π
- *It is optimal to mix, i.e.* $\sigma(\pi, s_M) \in (0, 1)$, *in state* s_M *for some values of* π

Proof: See Appendix.

In Figure 5 we use a numerical example to illustrate some of the key properties from the above characterization results. The first plot highlights Proposition 4. Since no borrower defaults in s_H, there are no static benefits of bailing out and so $\sigma(\pi, s_H) = 0$. In state s_L, the static benefits are much larger than the dynamic benefits for all π and so $\sigma(\pi, s_L) = 1$. The proposition also shows that interval $(\pi_1(s), \pi_2(s))$ defined in Proposition 2 is non-empty in state s_M and so there is an interval in which randomizing between bailing out and not is optimal. The second plot describes how private beliefs evolve given this strategy. In state s_H, since private agents believe that the no-commitment type will not bailout, the posterior does not change very much if there is no bail out. As a result, the gain in reputation from not bailing out in equilibrium is small. In state s_M, since there is a positive probability of a bail out, in the event that there is no bail out, the posterior that bailout authority is the commitment type will rise. Therefore, the state s_M offers

[5] In regards to an earlier discussion on the optimal policy with commitment, it is easy to see that $\frac{\partial W^R(\bar{\gamma})}{\partial \bar{\gamma}} < 0$ if $\psi < 0$. Therefore, the optimal policy is to set $\bar{\gamma} = 0$ and never bail out.

Figure 5: Equilibrium objects for computed discrete example

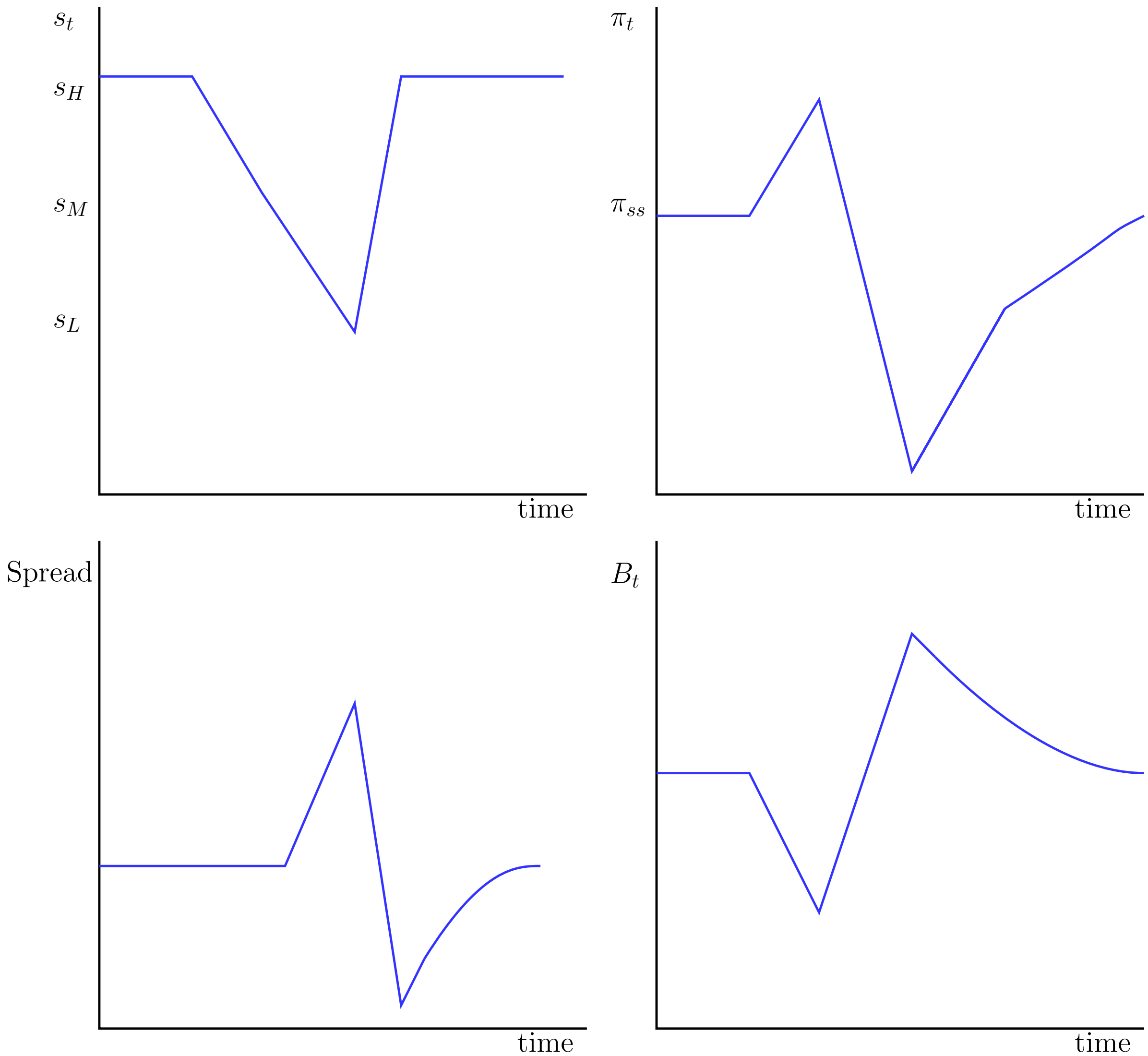


the bailout authority an opportunity to build reputation since the social costs of default are not very large. In state s_L, the dynamic gains from not bailing out are the largest since private agents believe that the no-commitment type will bail out with probability one. However, as discussed above, the social costs of default are much larger than these benefits.

Equilibrium outcome

We now describe an equilibrium outcome that generates the dynamics described in the introduction and in particular Figure 1. This outcome is illustrated in Figure 6. Suppose the economy has been in normal times for a sufficiently large number of periods so that the prior has converged to $\pi_{ss} = p_{nc} + \pi_{ss}(p_c - p_{nc})$. Suppose now that the economy

Figure 6: Outcome Path



suffers a mild recession, i.e. $s = s_M$. Assuming that

$$\pi_{ss} \in (\pi_1(s_M), \pi_2(s_M))$$

where the bounds of the interval are defined in Proposition 2, then the no-commitment type mixes between bailing out and not. If the no-commitment type does not bail out, then the private beliefs of facing the commitment type jumps above π_{ss} and consequently, the aggregate debt level falls and spreads rise the following period. If the economy is then hit by a severe recession, i.e. $s = s_L$, there is a bailout with probability one and so private beliefs fall to p_{nc}, and so the aggregate debt levels rise and spreads fall in subsequent periods due to the high probability of receiving a bail out in the future.

The dynamics in Figure 6 are driven by changes in fundamentals. However, they only require that the no-commitment type's *beliefs* about the true state changes. In Section 6, we extend our model to one in which the bailout authority learns about the state from noisy prices. This model can generate identical dynamics to the baseline without the true state actually changing.

4 Generalization

We now show that the previous results hold in more general environments. We allow both s, θ to be drawn from continuous distributions with densities $P(s)$ and $H(\theta \mid s)$ respectively. Next, we allow private default costs to take the form $\underline{u}(s,\theta) = u(\theta - \chi(s,\theta))$ for some function $\chi(s,\theta)$. Finally, we generalize the social cost function to allow for any increasing function $C(\cdot)$. The details are provided in Appendix A. We show that both the existence and characterization results hold in this environment if the private equilibrium of the stage game satisfies the following condition:

Assumption 2. *For any bailout policy* $\sigma(\pi, B, s)$ *which decreasing in* π *for all* (B, s)*, the private equilibrium outcome is such* $B(\pi)$ *is a decreasing function.*

The assumption requires the debt issued to be decreasing in π which implies that the equilibrium default probabilities in each state s, to be decreasing in π,. In general, as π decreases there are two effects on the equilibrium price of debt Q. First, since the probability of a bail out is higher, Q increases. However, the resulting increase in borrowing increases the probability of default which might lower Q. The assumption requires the first force to dominate so that in equilibrium the price of issuing debt decreases and thus the debt issued increases. It is easy to see that the example described in the previous section satisfies this assumption.

5 Persistent shocks: Contagion and Shock Sensitivity

In the model with iid shocks, there is no heterogeneity among borrowers at the beginning of any period. As a result the model cannot generate the contagion effects described in the introduction. By the contagion effect, we mean the increase in the price of debt for a country not directly affected by an adverse fundamental shock. Moreover, since the price of debt depends only on π and not the state, it is not possible to generate the differential effect of reputation on the sensitivity of prices to fundamentals. To show that our framework can generate such features we extend the baseline model to allow for persistent of aggregate and idiosyncratic states. As a result, the distribution functions of idiosyncratic and aggregate shocks are now $h(\theta' | s', s, \theta)$ and $P(s'|s)$.

Let's consider our simple example. The aggregate state s follows a Markov chain

$$P\left(s'|s\right) = \begin{bmatrix} p_{HH} & p_{HM} & p_{HL} \\ p_{MH} & p_{MM} & p_{ML} \\ p_{LH} & p_{LM} & p_{LL} \end{bmatrix}$$

As before, in state s_H, all borrowers draw θ_H and in state s_L all borrowers draw θ_L, i.e. $h\left(\theta_H|s_H,\theta\right) = 1$ and $h\left(\theta_L|s_L,\theta\right) = 1$ for all θ. We assume that in the medium state, a fraction μ of borrowers have the low output θ_L and

$$h\left(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_L\right) = \rho_L$$
$$h\left(\theta_L|s = s_M, s_- = s_M, \theta_- = \theta_H\right) = \rho_H$$

with $\rho_L\mu + \rho_H\left(1-\mu\right) = \mu$. Thus endowments are persistent in the medium state. Let $\mathbf{z}_- = \left(s_-,\theta_-\right)$, $\mathbf{z} = \left(s,\theta\right)$ and $\nu\left(\mathbf{z}_-\right)$ denote the fraction of type $\mathbf{z}_-$. Next, let $P_H\left(\mathbf{z}_-\right)$ and $P_L\left(\mathbf{z}_-\right)$ be probabilities of a high and low idiosyncratic endowment respectively, conditional on history $\mathbf{z}_-$. Therefore,

$$P_H\left(\mathbf{z}_-\right) = p_{s_-H} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\left(1-\rho_{\theta_-}\right) + \left(1-\mathbb{I}_{s_-=s_M}\right)\left(1-\mu\right)\right]$$

$$P_L\left(\mathbf{z}_-\right) = p_{s_-L} + p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right]$$

Next, define

$$\bar{\gamma}\left(\mathbf{z}_-\right) \equiv \frac{p_{s_-L}\left(1-\pi\right)\sigma\left(\pi, s_-, s_L\right) + p_{s_-M}p_{s_-M}\left[\mathbb{I}_{s_-=s_M}\rho_{\theta_-} + \left(1-\mathbb{I}_{s_-=s_M}\right)\mu\right]\left(1-\pi\right)\sigma\left(\pi, s_-, s_M\right)}{P_L\left(\mathbf{z}_-\right)}$$

to be the probability that an individual borrower with history $\mathbf{z}_-$ will be bailed out conditional on drawing θ_L. As in the i.i.d case this serves as a useful sufficient statistic to characterize private decisions. The price of debt in this environment is

$$\mathbb{Q}\left(\mathbf{z}_-,\bar{\gamma}\right) = qP_H\left(\mathbf{z}_-\right) + qP_L\left(\mathbf{z}_-\right)\bar{\gamma}\left(\mathbf{z}_-\right)$$

and the optimal debt level $\mathbb{B}\left(\mathbf{z}_-,\bar{\gamma}\right)$ solves

$$\mathbb{Q}\left(\mathbf{z}_-,\bar{\gamma}\right)u'\left(Y + \mathbb{Q}\left(\mathbf{z}_-,\bar{\gamma}\right)\mathbb{B}\left(\mathbf{z}_-,\bar{\gamma}\right)\right) = \beta P_H\left(\mathbf{z}_-\right)u'\left(\theta_H - \mathbb{B}\left(\mathbf{z}_-,\bar{\gamma}\right)\right) \tag{20}$$

Define the $\bar{\mathbb{B}}(s_-, \bar{\gamma})$ to be aggregate level of debt where

$$\begin{aligned}
\bar{\mathbb{B}}(s_L, \bar{\gamma}) &\equiv \mathbb{B}((s_L, \theta_H), \bar{\gamma}) = \mathbb{B}((s_L, \theta_L), \bar{\gamma}) \\
\bar{\mathbb{B}}(s_M, \bar{\gamma}) &\equiv \mu \mathbb{B}((s_M, \theta_L), \bar{\gamma}) + (1-\mu)\mathbb{B}((s_M, \theta_H), \bar{\gamma}) \\
\bar{\mathbb{B}}(s_H, \bar{\gamma}) &\equiv \mathbb{B}((s_H, \theta_H), \bar{\gamma}) = \mathbb{B}((s_H, \theta_L), \bar{\gamma})
\end{aligned}$$

Next, we characterize a set of continuous monotone equilibria for the economy for an arbitrary transition matrix P and provide sufficient conditions so that the characterization results for the iid case extend to this more general environment. Assumption 3 is the analog for Assumption 1 for the case with persistent endowments.

Assumption 3. *Let* $C(x) = \psi x$. *Define* $W^R(s, \bar{\gamma})$ as be the solution to

$$\begin{aligned}
W^R(s_-, \bar{\gamma}) = e - \sum_{\theta} \nu(s_-, \theta)\left[\bar{\gamma}(s_-, \theta) + \psi(1-\bar{\gamma}(s_-, \theta))\right] q P_L(s_-, \theta)\mathbb{B}((s_-, \theta), \bar{\gamma}(s_-, \theta)) \\
+ \beta \sum_{s} p_{s_- s} W^R(s, \bar{\gamma})
\end{aligned}$$

Assume that

$$\psi \bar{\mathbb{B}}(s_-, 0) > \beta\left[W^R(s, 0) - W^R(s, 1)\right] \text{ for all } s \tag{21}$$

and

$$\mathbf{A}^{-1} \cdot \mathbf{x} > \mathbf{G} \tag{22}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1-p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1-p_{HH} \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} q(p_{LM}\mu + p_{LL})\bar{\mathbb{B}}(s_L, 1) \\ q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M, \theta_L), 1) + q\mu(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M, \theta_H), 1) \\ q(p_{HM}\mu + p_{HL})\bar{\mathbb{B}}(s_H, 1) \end{bmatrix}$$

and

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}(s_L, 0) \\ \psi\left[\mu\rho_L\mathbb{B}((s_M, \theta_L), 1) + (1-\mu)\rho_H\mathbb{B}((s_M, \theta_H), 1)\right] \\ \psi\mu\bar{\mathbb{B}}(s_H, 0) \end{bmatrix}$$

Proposition 5. *For an arbitrary transition matrix* P, *if* p_{nc} *is sufficiently small, there exists a continuous monotone equilibrium in which* $B(s_-, \pi) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π, $\sigma(s_-, \pi, s) : S \times [0,1] \times S \to [0,1]$ *is decreasing in* π, $W(s_-, \pi) : S \times [0,1] \to \mathbb{R}$ *is increasing* *in* π, *debt price* $Q(s_-, \pi) : S \times [0,1] \to \mathbb{R}$ *is decreasing in* π *for all* s_-, *and*

$$W(s_L, \pi) < W(s_M, \pi) < W(s_H, \pi)$$

Furthermore, under Assumption 3, if $p_c \to 1$ *and* $p_{nc} \to 0$ *then it must be that:*

- *It is optimal to bailout with probability one in a severe recession,* $\sigma(s_-, \pi, s_L) = 1$ *for all* π *and* s_-
- *It is optimal to mix in a mild recession for some values of* π *for all* s_-

Proof. See Appendix.

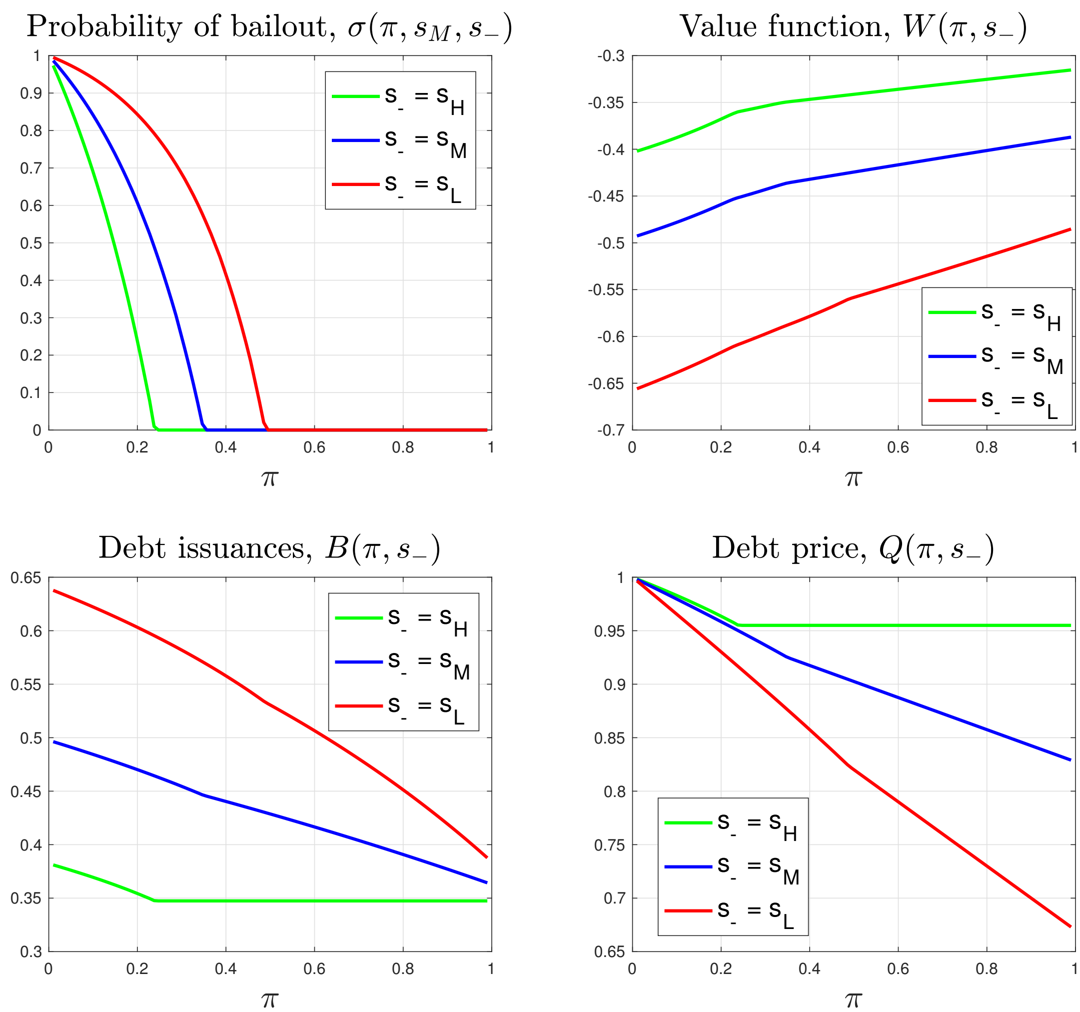
The proposition shows that under some assumptions, a continuous monotone equilibrium exists when shocks are persistent and the economy displays similar dynamics to the i.i.d case. We now show that the introduction of persistence can generate the contagion effects described previously. In the i.i.d case, there is only a single type of borrower in each period. However, with persistent shocks, if $s_- = s_M$, then in the following period there are two types of borrowers: (s_M, θ_L) and (s_M, θ_H). If there is no bail out and a subsequent rise in reputation, the interest rates faced by both types rise due to the presence of a common bailout authority. This provides an explanation as to why the CDS spreads for Italy rose after the the perceived recovery rates for Greek bonds declined. The announcement that private creditors were expected to receive haircuts on Greek bonds signaled that EU countries were less likely to receive the benefit of a full bail out in case of default in the future. As a result, the cost of borrowing for other countries that might have been considered at risk of default rose as well.

Proposition 6. *(Contagion) If the reputation of the government increases after observing no bail out in state* s_M, *then the price of debt for types* (s_M, θ_H) *decreases.*

The proofs follows from the observation that the pricing function Q depends on π. We next show that this model is capable of generating the differential sensitivity effects documented by Cole et al. (2016). The authors document that the sensitivity of bond yields to fundamentals such as GDP growth increased significantly during the course of the European debt crisis.

Proposition 7. *(Sensitivity) For any* s_-, *the difference in the price of debt for a* $\theta_- = \theta_H$ *borrower and a* $\theta_- = \theta_L$ *borrower is increasing in the reputation of the government. That is,* $Q((s_-, \theta_{H-}), \pi) - Q((s_-, \theta_{L-}), \pi)$ *is increasing in* π. *Similarly, for any* θ_-, *the differences* $Q((s_H, \theta_-), \pi) - Q((s_M, \theta_-), \pi)$ *and* $Q((s_M, \theta_-), \pi) - Q((s_L, \theta_-), \pi)$ *are increasing in* π *for* π *large enough.*

Figure 7: Equilibrium objects for computed discrete example with persistent shocks



Debt prices (and debt issuances) are less responsive to the state s_- when the prior is low. That is, if the probability of facing the no-commitment type is low then lenders are less worried about the state of the world since they expect to get bailed out with high probability and therefore, debt prices are not sensitive to the state. These effects are illustrated in Figure 7. As the fourth plot illustrates, the difference between the price of debt across the different states is increasing in π. At $\pi = 0$, the prices are identical and equal to the risk-free rate since lenders expect to be bailed out with probability one. At $\pi = 1$, prices are driven exclusively by the probability of default and since the states are persistent, the difference in prices is large.

6 Two Sided Learning

We now extend the baseline model to allow for uncertainty about the aggregate state and sequential bailouts request within a period. The main result of this section is that we can obtain hump-shaped interest rate spreads without having to rely on a transition through a mild crisis (s_M)..

Suppose the aggregate state of the world is $s \in \{s_L, s_H\}$ with probabilities p_L and p_H respectively. In state s_H, a fraction μ of borrowers draw the low idiosyncratic shock ($h(\theta_L|s_H) = \mu$) while in state s_L all borrowers draw the low shock ($h(\theta_L|s_L) = 1$). We will refer to s_H as normal times and s_L as a systemic crisis. As in the baseline model we assume that the social default costs are linear, $C(x) = \psi x$. The state of the world s is observed by private agents but is unobservable to the government. The timing in the first sub-period of the stage game is identical to the baseline environment. The second sub-period is further divided into two stages. The timing in the first stage is as follows:

1. Borrowers enter the sub-period period 2 with aggregate debt B and prior belief π
2. $s \in \{s_L, s_H\}$ is realized and learned by private agents
3. Lenders draw taste shock $\varepsilon \sim G$ and can trade a mutual fund of debt in a secondary market at price $q_2 = Q_2(\pi, B, s, \varepsilon|\sigma)$
4. A measure μ of borrowers ask for a bailout and the government bails outs with probability $\sigma_1(\pi, B, q_2)$

The realization of endowments in the second sub-period is staggered over the two stages. In the first stage, a fraction μ of borrowers receive the low endowment shock independently of the state. This implies that the bailout authority does not learn anything about the state from the fraction requesting a bail out. In the second stage, depending on the state, there is a second wave of bailout requests. If $s = s_H$ no other borrower requests a bailout and the prior is updated to π' . If $s = s_L$ then a fraction $1 - \mu$ of borrowers ask for a bailout and the government bails out with probability $\sigma_2(\pi, B, q_2)$. This implies that the state is perfectly revealed to the bailout authority in the second stage.

We now characterize the equilibrium behavior starting from the end of the period. In the second stage, if the state is s_L, the government bails out iff

$$\psi(1-\mu) B \geqslant \beta \left[W_1 \left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)(1-\sigma_2)} \Delta p \right) - W_1(p_{nc}) \right]$$

In what follows we suppose that μ is small enough so that it is always optimal to bail out in the second stage if $s = s_L$.[6] Then we have that $\sigma_2 = 1$. Of course, if the economy is in

[6] Recall that Assumption 1 guaranteed this when $\mu = 0$.

state s_H there is no borrower to bail out and trivially $\sigma_2 = 0$ in normal times.

We now consider the bailout decision in the first stage. Here, regardless of the state, a fraction μ of borrowers require a bailout in order to avoid default. The bailout authority does not observe the state but it can observe and learn from prices in the secondary market. The price of a portfolio of debt in the secondary market at the beginning of the second sub-period is

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\,\sigma_1(\pi, B, q_2) + \varepsilon, & \text{if } s = s_H \\ (1-\pi)\left[(1-\mu)\,\sigma_2(\pi, B, q_2) + \mu\sigma_1(\pi, B, q_2)\right] + \varepsilon, & \text{if } s = s_L \end{cases}$$

Thus, the bailout authority's beliefs that the true state is s_H conditional on have observed the secondary market price q_2 is

$$\hat{p}(s_H|\pi, B, q_2) = \frac{p(s_H)\,g_H(q_2)}{p(s_L)\,g_L(q_2) + p(s_H)\,g_H(q_2)}$$

where

$$\begin{aligned} g_L(q_2) &= g\left(q_2 - (1-\pi)\left[(1-\mu)\,\sigma_2(\pi, B, q_2) + \mu\sigma_1(\pi, B, q_2)\right]\right), \\ g_H(q_2) &= g\left(q_2 - \left[(1-\mu) + \mu(1-\pi)\,\sigma_1(\pi, B, q_2)\right]\right), \end{aligned}$$

and $g(\cdot)$ is the probability density function of ε.

This implies that in the first stage, the no-commitment type bails out iff

$$\psi\mu B \geqslant \hat{p}(s_H|\pi, B, q_2)\,\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1)}\Delta p\right) - W_1(p_{nc})\right]$$

where Since the state is unknown in the first stage, the dynamic benefits of not bailing out only accrue with probability $\hat{p}(s_H|\pi, B, q_2)$ since the no-commitment type bails out with probability one in the second stage if $s = s_L$. All things being equal, a lower value of q_2 lowers the posterior $\hat{p}(s_H|\pi, B, q_2)$. As a result, in contrast the baseline model, the dynamic gains from not bailing out might change for non-fundamental reasons. In particular, all else equal, a lower realization of ε increases the probability of a bailout in stage one. As we will see, this feature allows the model to generate similar dynamics to the baseline without changes in true fundamentals.

We can simplify the analysis by noting that since $\sigma_2 = 1$, the price in the secondary market simplifies to

$$q_2 = Q_2(\pi, B, s, \varepsilon|\sigma) = \begin{cases} (1-\mu) + \mu(1-\pi)\,\sigma_1(\pi, B, q_2) + \varepsilon & s = s_H \\ (1-\pi)\left[(1-\mu) + \mu\sigma_1(\pi, B, q_2)\right] + \varepsilon & s = s_L \end{cases}$$

It follows that if $Q_2(\pi, s_H, \varepsilon_H) = Q_2(\pi, s_L, \varepsilon_L)$ then

$$\varepsilon_L = \varepsilon_H + (1-\mu)\pi$$

If we assume that $\text{supp}(g) = (-\infty, +\infty)$ we can then make a change of variable and express all the equilibrium objects as a function of the realization of ε in state s_H. Define

$$F(\pi) \equiv \int \sigma_1(\pi, \varepsilon)\left[p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon + (1-\mu)\pi)\right] d\varepsilon \tag{23}$$

to be ex-ante probability of a bailout in the first stage of the sub-period two given prior π. Then, the price of issuing debt in the first sub-period is

$$Q(\pi) = q\left[p(s_H)(1-\mu) + p(s_L)(1-\mu)(1-\pi) + \mu(1-\pi)F(\pi)\right] \tag{24}$$

and so the optimal choice of debt satisfies

$$Q(\pi)u'(y + Q(\pi)B(\pi)) = \beta p(s_H)(1-\mu)u'(y_H - B(\pi)) \tag{25}$$

The value for the government is given by

$$\begin{aligned} W(\pi) = &-Q(\pi)B(\pi) + \\ &+ qp(s_H)\{(1-\mu)B(\pi) \\ &+ \int \sigma_1(\pi,\varepsilon)\beta W(p_{nc}) g(\varepsilon) d\varepsilon \\ &+ \int [1-\sigma_1(\pi,\varepsilon)]\left[-c\mu B(\pi) + \beta W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\sigma_1(\pi,\varepsilon))}\Delta p\right)\right] g(\varepsilon) d\varepsilon\Big\} \\ &+ qp(s_L)\left\{\int [1-\sigma_1(\pi, \varepsilon + (1-\mu)\pi)][-c\mu B(\pi)] g(\varepsilon + (1-\mu)\pi) d\varepsilon + \beta W(p_{nc})\right\} \end{aligned} \tag{26}$$

Finally, the probability of a bailout in the first stage $\sigma_1(\pi, \varepsilon)$ is given by

$$\sigma_1(\pi,\varepsilon) = \begin{cases} 0, & \text{if } c\mu B(\pi) \leqslant \beta \hat{p}_H(\pi,\varepsilon)\left[W(p_{nc} + \pi\Delta p) - W(p_{nc})\right] \\ \tilde{\sigma}, & \text{if } c\mu B(\pi) = \beta \hat{p}_H(\pi,\varepsilon)\left[W\left(p_{nc} + \frac{\pi\Delta p}{\pi+(1-\pi)(1-\tilde{\sigma})}\right) - W(p_{nc})\right] \\ 1, & \text{if } c\mu B(\pi) \geqslant \beta \hat{p}_H(\pi,\varepsilon)\left[W(p_{nc} + \Delta p) - W(p_{nc})\right] \end{cases} \tag{27}$$

where

$$\hat{p}_H(\pi,\varepsilon) = \frac{p(s_H)}{p(s_H) g(\varepsilon) + p(s_L) g(\varepsilon + (1-\mu)\pi)}$$

Thus, (23)–(27) define a set of functional equations that can be solved for the equilibrium objects $F(\pi)$, $Q(\pi)$, $B(\pi)$, $W(\pi)$, and $\sigma_1(\pi, \varepsilon)$.

As a consequence of two sided learning, one can generate similar dynamics to the baseline model with only two aggregate states. For example, in state s_L, a low realization of ε will induce a very low value of q_2 which will lead to a low value of $\hat{p}$ and so inducing a bailout with probability 1 in the first stage. For the same realization s_L, a higher value of ε will result in a larger value of $\hat{p}(s_H)$ which in turn can push the government into the randomization region. Similar to state s_M in the baseline model, in the case when a bailout is not observed, the posterior value of the government being the commitment type rises which in turn decreases the price of debt on the secondary market at the end of the first stage,

$$Q_3(\pi, B, s, \varepsilon|\sigma) = \begin{cases} 1+\varepsilon & s = s_H \\ (1-\pi)(1-\mu)+\varepsilon & s = s_L \end{cases}.$$

Figure 8 plots the key equilibrium objects for a typical computed numerical example. As we see in the first panel, for a fixed ε, a higher value of π implies a lower probability of bailout in the first stage. This intuition is similar to our baseline model in which the state is observable. Similarly, for a fixed π, we see that a lower realization of ε induces a higher probability of a bailout in the first stage since the government's posterior of the state being low rises. The expected probability of a bailout in stage 1, $F(\pi)$ is decreasing in π which in turn implies the price of debt in the first sub-period, Q is increasing in π. This generates identical spread dynamics to baseline environment.

An interesting feature of this model is that information conveyed by secondary market prices depends on government reputation π. For low values of π, since lenders expect a bailout with a high probability, the price Q_2 will largely be driven by the taste shock ε, thus conveying little information about the fundamental. In contrast, for high values of π, the price Q_2 will be more sensitive to fundamentals. This implies that for low values of π, a much larger value of Q_2 is needed in order for the government to increase its posterior belief of s_H.

Unlike the baseline model, for a fixed ε, the probability of a bailout in the first stage σ_1 need not be monotone in π. The reason for this is that the right hand side of (27) need not be monotone increasing in π since the posterior is more sensitive to ε for larger values of π. As a result, for low values of ε, as π increases, the reduction in the posterior might be larger than the increase in reputation gains leading to a larger bailout probability. Figure 9 plots an example of when this can occur. Note however, that the expected bailout probability $F(\pi)$ is still decreasing in π which implies the behavior of spreads is identical to the previous cases.

Figure 8: Equilibrium objects for computed example with two-sided learning

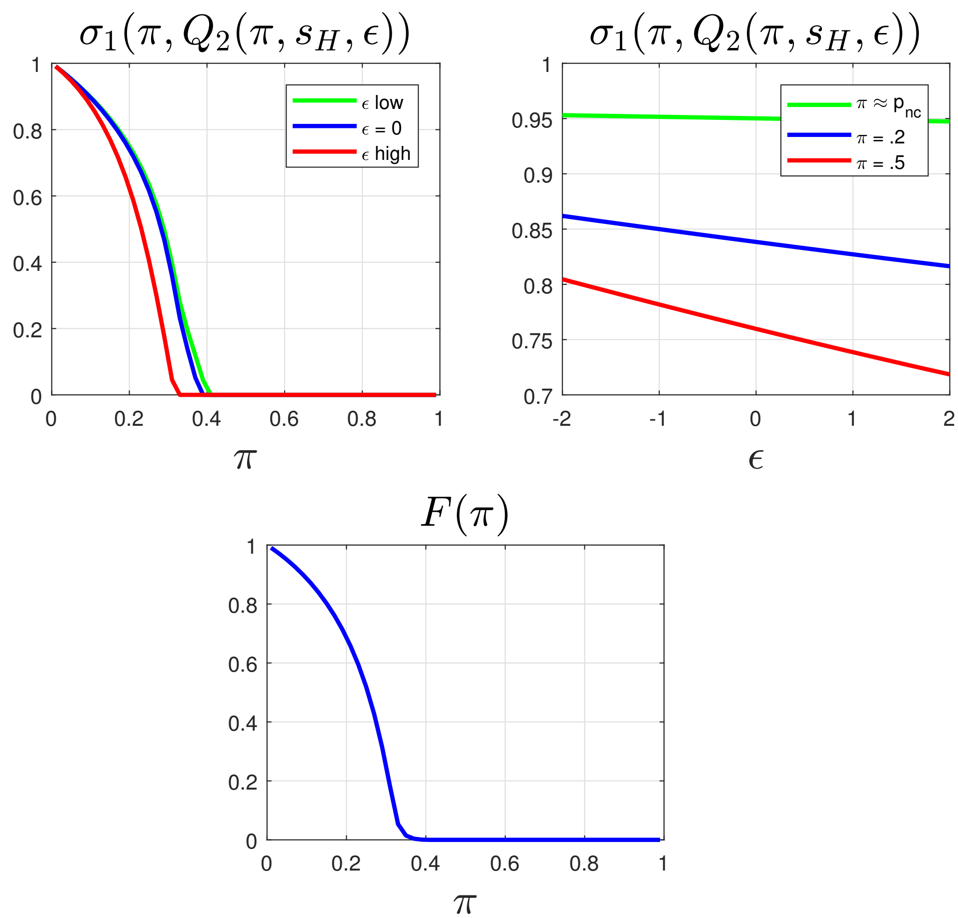
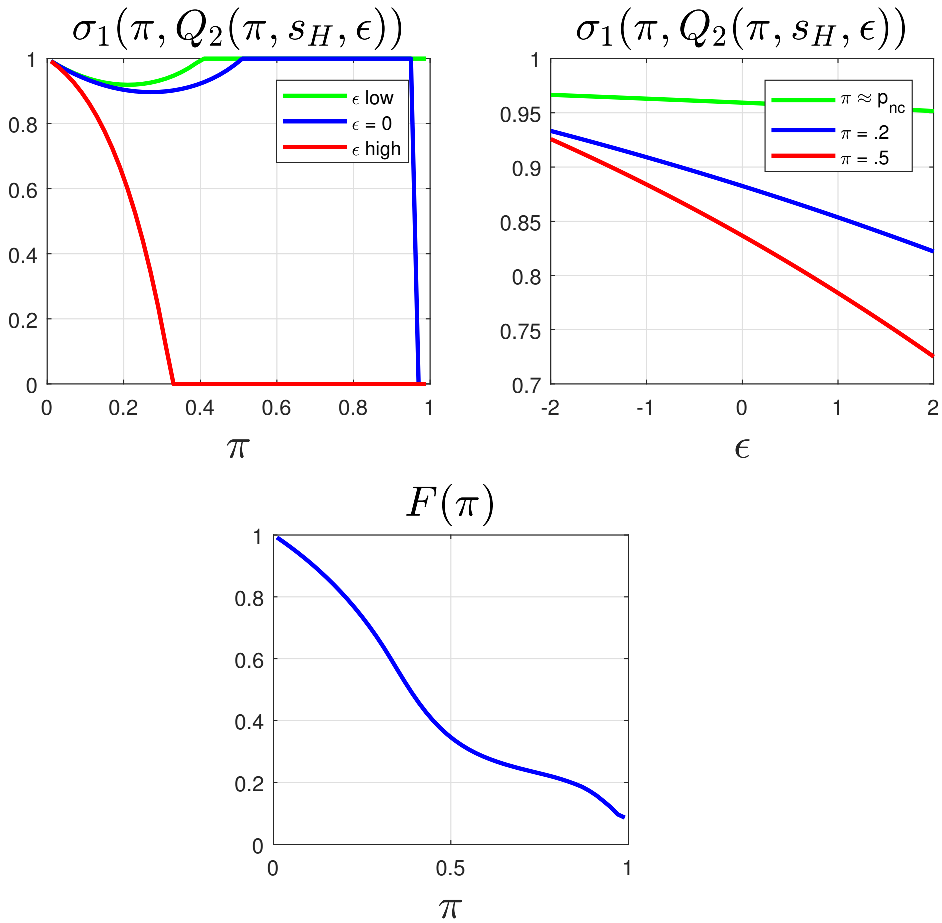


Figure 9: Two-sided learning example with non-monotone σ_1



7 Conclusion

Our paper studies a model in which the expectation of future bailouts are an important determinant of interest rate spreads. We jointly characterize these spreads and the optimal bailout decisions of a government that lacks commitment but has incentives to build reputation. This model can help explain both the behavior of spreads around crises and the delay in intervention we often observe from governments once the crisis has started.

There are lots of interesting extensions worth studying that are outside the scope of this paper. In concurrent work, we study the optimal policy in an environment without commitment but incentives for building reputation. One application of this would be to characterize the optimal bailout policy in this model.

A second extension would be to allow debt holdings to be a state variable across periods. As discussed earlier, higher reputation has two opposing effects on interest rates. First, since the probability of future private default is larger, interest rates need to rise to allow lenders to break even. Second, a higher cost of borrowing discourages debt accumulation which pushes down interest rate. Allowing debt accumulation to be a dynamic choice would likely lead to the first force dominating the other. This is because consumption smoothing motives would make large changes in debt holds very costly. As a result this extension is likely to strengthen our result. Finally, a third interesting extension to see how much of the movement in spreads can be accounted for by a combination of fundamentals and reputation.

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A Omitted Proofs

A.1 Proofs

We now prove proposition 1 for the general environment described in Section 4. In this environment, in the event of default, we assume that lenders and borrowers can renegotiate the contract so that borrowers make a partial repayment to the lenders and avoid the default cost. The static value of bailing out is

$$\Omega^{\text{bailout}}(B,s) = (1-\Delta(B,s))\,B + \tilde{\Delta}(B,s)$$

where

$$\Delta(B,s) = \int \mathbb{I}_{\left\{B>\theta-u^{-1}(\underline{u}(s,\theta))\right\}} dH(\theta|s),$$

$$\tilde{\Delta}(B,s) = \int \left[\theta - u^{-1}(\underline{u}(s,\theta))\right] \mathbb{I}_{\left\{B>\theta-u^{-1}(\underline{u}(s,\theta))\right\}} dH(\theta|s)$$

and as before we normalize $C(0)=0$. Here, $\tilde{\Delta}(B,s)$ denotes the additional transfer extracted from the borrower in case of a bailout. The value of not bailing out (and allowing default) is

$$\Omega^{\text{default}}(B,s) = (1-\Delta(B,s))\,B + \tilde{\Delta}(B,s) - C(\Delta(B,s)\,B)$$

Note that even absent a bailout, since private agents can re-negotiate contracts, lenders will extract $\tilde{\Delta}(B,s)$ from borrowers who default. Given this the pricing schedule for debt is

$$Q(B,\pi,\sigma)(b) = q\left\{\int (1-\Delta(B,s))\,dP(s) + \int \tilde{\Delta}(B,s)\,dP(s)\right\} \\ + q\left\{(1-\pi)\int \sigma(\pi,B,s)\left[\Delta(B,s)\,B - \tilde{\Delta}(B,s)\right] dP(s)\right\} \tag{28}$$

Given that private contracts can be renegotiated, lenders receive at least $\tilde{\Delta}(B,s)$ in the event of default. The expression on the second line denotes the additional transfer received in the event of a bailout. The problem for the borrower in period 1 is

$$\max_{c,b} u(c) + \beta \int\int \max\{u(\theta-b), \underline{u}_i(s,\theta)\}\, dH(\theta|s)\, dP(s) \tag{29}$$

subject to

$$c \leqslant Y + Q(B,\pi)(b)\,b$$

The price schedule depends on borrower's debt choice b, the aggregate level of debt, B, and the prior π. Define $\Theta_{+}^{s}(B) \equiv \{\theta : u(\theta - B) \geqslant \underline{u}(s, \theta)\}$. The following Lemma characterizes the private outcome in the stage game given the bailout policy σ if the distribution for θ is continuous:

Lemma 4. *Given π and a bailout policy σ, (B, Q) is a symmetric equilibrium outcome if*

$$u'(Y_1 + QB)\left(Q + Q'B\right) = \delta \int_s \int_{\theta \in \Theta_{+}^{s}(B)} u'(\theta - B)\, dH(\theta \mid s)\, dP(s) \tag{30}$$

and $Q = Q(B, \gamma)(B)$ *where*

$$Q' = \frac{dQ(B, \pi)}{dB}$$

Given this setup, the general version of Proposition 1 is

Proposition 1 (Generalized). *Under Assumption 2, if p_{nc} is sufficiently small, there exists a continuous monotone equilibrium in which $B(\pi) : [0,1] \to \mathbb{R}$ is decreasing, $\sigma(\pi, s) : [0,1] \times S \to [0,1]$ is decreasing, $W(\pi) : [0,1] \to \mathbb{R}$ is increasing.*

Proof of Proposition 1

Define the following operator: $\mathbb{T} : \Sigma \to \Sigma$ where

$$\Sigma = \{\sigma : [0,1] \times S \to [0,1] : \forall s \in S, \sigma(\cdot, s) \text{ is decreasing, continuous}\}$$

and the operator is defined as follows:

1. Given $\sigma_0 \in \Sigma$, compute $b(\cdot|\sigma_0)$ that solves the borrower's problem in (29) with $Q(\pi, B|\sigma_0)(b)$ from (28) imposing the representativeness condition so $B(\pi|\sigma_0) = b(\pi|\sigma_0)$.

2. Given σ_0 and $B(\cdot|\sigma_0)$ compute $W(\cdot|\sigma_0)$ as the solution to the following fixed point problem

$$\begin{aligned} \mathbb{T}^{W} W(\pi) &= w_i(\pi|\sigma_0) \\ &+ \beta \int [1 - \sigma_0(\pi, s)]\, W\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1 - \sigma_0(\pi, s))}(p_c - p_{nc})\right) dP(s) \\ &+ \beta \int \sigma_0(\pi, s)\, W(p_{nc})\, dP(s) \end{aligned} \tag{31}$$

where

$$\begin{aligned} w_i(\pi \mid \sigma_0) = & [e - Q(\pi|\sigma_0) B(\pi|\sigma_0)] \\ & + q \int [1 - \sigma(\pi, s_2)] \Omega(\pi, s_2 \mid \sigma_0) dP(s_2|s_1) \\ & + q \int \sigma(\pi, s_2) \Omega^*(\pi, s_2 \mid \sigma_0) dP(s_2|s_1) \end{aligned}$$

and

$$\Omega(\pi, s \mid \sigma_0) = (1 - \Delta(\pi, s)) B(\pi) + \tilde{\Delta}(\pi, s) - C(\Delta(\pi, s) B(\pi))$$

where

$$\tilde{\Delta}(\pi, s) = \int \left[\theta - u^{-1}(\underline{u}(s, \theta))\right] \mathbb{I}_{\{B(\pi) \geqslant \theta - u^{-1}(\underline{u}(s,\theta))\}} dH(\theta|s)$$

and

$$\begin{aligned} \Omega^*(\pi, s \mid \sigma_0) & = \int \min\left\{B(\pi), \theta - u^{-1}(\underline{u}(s, \theta))\right\} dH(\theta|s) \\ & = (1 - \Delta(\pi, s)) B(\pi) + \tilde{\Delta}(\pi, s) \end{aligned}$$

Note that here $\Delta(\pi, s) = \Delta(B(\pi), s)$. The operator $\mathbb{T}^W$ maps the space of continuous and bounded functions into itself. Further, it is easy to see that the mapping is a contraction and thus by the Contraction mapping theorem, there exists a unique fixed point.

3. Compute $\sigma_1 = \mathbb{T}\sigma_0$ as

$$\sigma_1(\pi, s_2) = \begin{cases} 0 & \text{if } \beta[W_1(p_{nc} + \pi\Delta p \mid \sigma_0) - W_1(p_{nc} \mid \sigma_0)] \geqslant \Delta\Omega(\pi, s \mid \sigma_0) \\ \tilde{\sigma} & : \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma})}\Delta p \mid \sigma_0\right) - W_1(p_{nc} \mid \sigma_0)\right] = \Delta\Omega(\pi, s \mid \sigma_0) \\ 1 & \text{if } \beta[W_1(p_c \mid \sigma_0) - W_1(p_{nc} \mid \sigma_0)] < \Delta\Omega(\pi, s \mid \sigma_0) \end{cases} \tag{32}$$

with $\Delta p = p_c - p_{nc}$ and

$$\Delta\Omega(\pi, s \mid \sigma_0) = \Omega(\pi, s \mid \sigma_0) - \Omega^*(\pi, s \mid \sigma_0)$$

To prove the proposition, we will establish that $\mathbb{T}$ has a fixed point in $\Sigma^{|S_2|}$ using Tarski's theorem.

Step 1: For any $f, g \in \Sigma$ we define a binary relation $\succeq$ where $f \succeq g$ iff $\forall s_2 \in S_2$, $f(\pi, s_2) \geqslant g(\pi, s_2)$ for all π . We want to argue that $(\Sigma, \succeq)$ is a complete lattice. That is, for any arbitrary subset $\tilde{\Sigma}$ of Σ: 1) there exists $\underline{\sigma} \in \tilde{\Sigma}$ such that i) for all $\sigma \in \tilde{\Sigma}$, $\sigma \succeq \underline{\sigma}$ and ii) for all $\sigma' \in \Sigma$ such that $\sigma \succeq \sigma'$ for all $\sigma \in \tilde{\Sigma} \Rightarrow \underline{\sigma} \succeq \sigma'$ ($\underline{\sigma}$ is the greatest lower bound); 2) there exists $\bar{\sigma} \in \tilde{\Sigma}$ such that i) for all $\sigma \in \tilde{\Sigma}$, $\bar{\sigma} \succeq \sigma$ and ii) for all $\sigma' \in \Sigma$ such that $\sigma' \succeq \sigma$ $\Rightarrow \sigma' \succeq \bar{\sigma}$ ($\bar{\sigma}$ is the least upper bound). Clearly for any subset $\Sigma' \subset \Sigma$, for each s_2 these correspond to the lower and upper envelope of functions in the set. In particular, for each s_2 and each π define

$$\bar{\sigma}(\pi, s_2) = \max_{\sigma(\pi, s_2) \in \Sigma'} \sigma(\pi, s_2)$$

and

$$\underline{\sigma}(\pi, s_2) = \min_{\sigma(\pi, s_2) \in \Sigma'} \sigma(\pi, s_2)$$

Notice that both $\bar{\sigma}(\pi, s_2)$ and $\underline{\sigma}(\pi, s_2)$ are continuous, increasing, and satisfy $\bar{\sigma}(0, s_2) = \underline{\sigma}(0, s_2) = 1$. Therefore, $\bar{\sigma}$ and $\underline{\sigma}$ belong to Σ.

Step 2: The value function $W(\pi|\sigma_0)$ that solves (31) is strictly increasing in π. To prove this we will use the corollary to the Contraction Mapping Theorem. Given a weakly increasing, bounded and continuous function $W_0(\pi)$ let $W_1 = \mathbb{T}^W W_0$. We want to show that W_1 is strictly increasing. Consider $\pi_L < \pi_H$ and associated $\sigma_0(\pi_L, s) \leqslant \sigma_0(\pi_H, s)$. Define $\mathcal{S}_1 = \{s : \sigma_0(\pi_L, s) = \sigma_0(\pi_H, s) = 0\}$, $\mathcal{S}_2 = \{s : \sigma_0(\pi_L, s) > \sigma_0(\pi_H, s) = 0\}$, and $\mathcal{S}_3 = \{s : \sigma_0(\pi_L, s) \geqslant \sigma_0(\pi_H, s) > 0\}$ so that in $\mathcal{S}_1$ there are no bailouts under both π_L and π_H, in $\mathcal{S}_2$ there is a positive probability of bailouts under π_L but not under π_H, finally in $\mathcal{S}_3$ bailouts happen with positive probability under both π_L and π_H. Then

$$\begin{aligned}
W_1(\pi_L) = {} & [e - Q(\pi_L) B(\pi_L)] \\
& + \int_{\mathcal{S}_1} [q\Omega(\pi_L, s) + \beta W_0(p_{nc} + \pi_L \Delta p)] \, dP(s) \\
& + \int_{\mathcal{S}_2} [q\Omega^*(\pi_L, s) + \beta W_0(p_{nc})] \, dP(s) \\
& + \int_{\mathcal{S}_3} [q\Omega^*(\pi_L, s) + \beta W_0(p_{nc})] \, dP(s)
\end{aligned}$$

and

$$\begin{aligned}
W_1(\pi_H) = {} & [e - Q(\pi_H) B(\pi_H)] \\
& + \int_{\mathcal{S}_1} [q\Omega(\pi_H, s) + \beta W_0(p_{nc} + \pi_H \Delta p)] \, dP(s) \\
& + \int_{\mathcal{S}_1} [q\Omega(\pi_H, s) + \beta W_0(p_{nc} + \pi \Delta p)] \, dP(s) \\
& + \int_{\mathcal{S}_3} [q\Omega^*(\pi_H, s) + \beta W_0(p_{nc})] \, dP(s)
\end{aligned}$$

Then we have,

$$\begin{aligned}
&W_1(\pi_L) - W_1(\pi_H) \\
\leqslant &Q(\pi_H) B(\pi_H) - Q(\pi_L) B(\pi_L) \\
&+ q \int_{\mathcal{S}_1} [\Omega(\pi_L, s) - \Omega(\pi_H, s)] \, dP(s) \\
&+ q \int_{\mathcal{S}_2} [\Omega^*(\pi_L, s) - \Omega^*(\pi_H, s)] \, dP(s) \\
&+ q \int_{\mathcal{S}_3} [\Omega^*(\pi_L, s) - \Omega^*(\pi_H, s)] \, dP(s) \\
= &\left\{ -Q(\pi_L) B(\pi_L) + q \int (1 - \Delta(\pi_L, s)) B(\pi_L) \, dP(s) + q \int \tilde{\Delta}(\pi_L, s) \, dP(s) - q \int_{\mathcal{S}_1} C(\Delta(\pi_L, s) B(\pi_L)) \, dP(s) \right\} \\
&- \left\{ -Q(\pi_H) B(\pi_H) + q \int (1 - \Delta(\pi_H, s)) B(\pi_H) \, dP(s) + q \int \tilde{\Delta}(\pi_H, s) \, dP(s) - q \int_{\mathcal{S}_1} C(\Delta(\pi_H, s) B(\pi_H)) \, dP(s) \right\} \\
= &\left\{ -(1 - \pi_L) \int \sigma(\pi_L, s) \mathbb{I}_{B(\pi_L) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi_L) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi_L, s) \, dP(s) \right\} \\
&- \left\{ -(1 - \pi_H) \int \sigma(\pi_H, s) \mathbb{I}_{B(\pi_H) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi_H) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi_H, s) \, dP(s) \right\} \\
< &\, 0
\end{aligned}$$

where the first inequality follows from

$$W_0(p_{nc} + \pi_L \Delta p) \leqslant W_0(p_{nc} + \pi_H \Delta p)$$

and

$$[\Omega(\pi_H, s) + \beta W_0(p_{nc} + \pi_H \Delta p)] \geqslant [\Omega^*(\pi_H, s) + \beta W_0(p_{nc})]$$

for $s \in \mathcal{S}_2$, the second equality is algebra, the third equality uses the definition of $Q(\pi)$, finally the last inequality follows since $\sigma(\pi_L, s) \geqslant \sigma(\pi_H, s)$, Assumption 1, and C is an increasing function.

Step 3: Showing that $\mathbb{T}$ maps Σ into Σ. Now we argue that $\sigma_1 = \mathbb{T}\sigma_0$ is decreasing by showing that $\mathbb{T}\sigma_0(\pi, s)$ is decreasing in π for all s. Suppose by way of contradiction there exists $\pi_L < \pi_H$ and $\sigma_1(\pi_L, s) < \sigma_1(\pi_H, s)$ for some s so that the bailout probability is higher if we start from a higher prior. First, note that if $\sigma_1(\pi_L, s) = 0$ then

$$\Delta\Omega(\pi_L, s) \leqslant \beta [W_1(p_{nc} + \pi_L \Delta p) - W_1(p_{nc})]$$

but

$$\begin{aligned}
\Delta\Omega(\pi_H, s) &\leqslant \Delta\Omega(\pi_L, s) \\
\beta [W_1(p_{nc} + \pi_L \Delta p) - W_1(p_{nc})] &\leqslant \beta [W_1(p_{nc} + \pi_H \Delta p) - W_1(p_{nc})]
\end{aligned}$$

where the first inequality follows from the fact that $B(\pi|\sigma_0)$ is decreasing and the second from W_1 being an increasing function. Therefore

$$\Delta\Omega\left(\pi_H, s\right) \leqslant \beta\left[W_1\left(p_{nc} + \pi_H \Delta p\right) - W_1\left(p_{nc}\right)\right]$$

and $\sigma\left(\pi_H, s\right) = 0$, yielding a contradiction. Second, if $0 < \sigma_1\left(\pi_L, s\right) < \sigma_1\left(\pi_H, s\right) < 1$ then

$$\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)\left(1-\sigma_1\left(\pi_H, s\right)\right)}\Delta p\right) - W_1\left(p_{nc}\right)\right] = \Delta\Omega\left(\pi_H, s\right)$$
$$\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)\left(1-\sigma_1\left(\pi_L, s\right)\right)}\Delta p\right) - W_1\left(p_{nc}\right)\right] = \Delta\Omega\left(\pi_L, s\right)$$

so since $\Delta\Omega\left(\pi_H, s\right) \leqslant \Delta\Omega\left(\pi_L, s\right)$ and W_1 is increasing it must be that

$$\begin{aligned} p_{nc} + \frac{\pi_L}{\pi_L + (1-\pi_L)\left(1-\sigma_1\left(\pi_L, s\right)\right)}\Delta p &> p_{nc} + \frac{\pi_H}{\pi_H + (1-\pi_H)\left(1-\sigma_1\left(\pi_H, s\right)\right)}\Delta p \\ \iff \frac{(1-\pi_H)}{\pi_H}\left(1-\sigma_1\left(\pi_H, s\right)\right) &> \frac{(1-\pi_L)}{\pi_L}\left(1-\sigma_1\left(\pi_L, s\right)\right) \\ \iff 1-\sigma_1\left(\pi_H, s\right) &> \frac{(1-\pi_L)}{\pi_L} \Big/ \frac{(1-\pi_H)}{\pi_H}\left(1-\sigma_1\left(\pi_L, s\right)\right) > 1-\sigma_1\left(\pi_L, s\right) \\ \iff \sigma_1\left(\pi_L, s\right) &> \sigma_1\left(\pi_H, s\right) \end{aligned}$$

obtaining a contradiction. Finally, if $0 < \sigma_1\left(\pi_L, s\right) < \sigma_1\left(\pi_H, s\right) = 1$ then

$$\beta\left[W_1\left(p_c\right) - W_1\left(p_{nc}\right)\right] < \Delta\Omega\left(\pi_H, s\right) < \Delta\Omega\left(\pi_L, s\right)$$

implying $\sigma_1\left(\pi_L, s\right) = 1$, again a contradiction. Hence $\mathbb{T} : \Sigma \to \Sigma$ as desired.

Step 4: As an intermediate step we show that if $\sigma_H \succeq \sigma_L$ then $W_1\left(\pi \mid \sigma_H\right) \leqslant W_1\left(\pi \mid \sigma_L\right)$ for all π. This is because the operator $\mathbb{T}^W$ is decreasing in σ. That is, if $W_{Ln}(\cdot) \geqslant W_{Ln}(\cdot)$ then $W_{Ln+1} = \mathbb{T}^W_L W_{Ln} \geqslant \mathbb{T}^W_H W_{Hn} = W_{Hn+1}$ where $\mathbb{T}^W_i$ is the operator $\mathbb{T}^W$ with $\sigma_0 = \sigma_i$. The proof for this claim is similar to the argument in step 2. Fix π and define $\mathcal{S}_1 = \{s : \sigma_H(\pi, s) = \sigma_L(\pi, s) = 0\}$, $\mathcal{S}_2 = \{s : \sigma_H(\pi, s) > \sigma_L(\pi, s) = 0\}$, and $\mathcal{S}_3 =$

$\{s : \sigma_H(\pi, s) \geqslant \sigma_L(\pi, s) > 0\}$. Then

$$
\begin{aligned}
& W_{Ln+1}(\pi) - W_{Hn+1}(\pi) \\
=& Q(\pi|\sigma_H) B(\pi|\sigma_H) - Q(\pi|\sigma_L) B(\pi|\sigma_L) \\
& + \delta \int_{\mathcal{S}_1} [\Omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc} + \pi_L \Delta p)] \, dP(s) - \delta \int_{\mathcal{S}_1} [\Omega(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc} + \pi_H \Delta p)] \, dP(s) \\
& + \delta \int_{\mathcal{S}_2} [\Omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})] \, dP(s) - \delta \int_{\mathcal{S}_2} [\Omega^*(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc} + \pi_H \Delta p)] \, dP(s) \\
& + \delta \int_{\mathcal{S}_3} [\Omega^*(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})] \, dP(s) - \delta \int_{\mathcal{S}_3} [\Omega^*(\pi, s|\sigma_H) + \beta W_{Hn}(p_{nc})] \, dP(s) \\
\geqslant& Q(\pi|\sigma_H) B(\pi|\sigma_H) - Q(\pi|\sigma_L) B(\pi|\sigma_L) \\
& + \delta \int_{\mathcal{S}_1} [\Omega(\pi, s|\sigma_L) - \Omega(\pi_H, s|\sigma_H)] \, dP(s) \\
& + \delta \int_{\mathcal{S}_2} [\Omega^*(\pi, s|\sigma_L) - \Omega^*(\pi, s|\sigma_H)] \, dP(s) \\
& + \delta \int_{\mathcal{S}_3} [\Omega^*(\pi, s|\sigma_L) - \Omega^*(\pi, s|\sigma_H)] \, dP(s) \\
=& \left\{ -Q(\pi \mid \sigma_H) B(\pi \mid \sigma_H) + q \int (1 - \Delta(\pi, s) \mid \sigma_H) B(\pi \mid \sigma_H) \, dP(s) + q \int \tilde{\Delta}(\pi, s \mid \sigma_H) \, dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_H) \, dP(s) \right\} \\
& - \left\{ -Q(\pi \mid \sigma_L) B(\pi \mid \sigma_L) + q \int (1 - \Delta(\pi, s) \mid \sigma_L) B(\pi \mid \sigma_L) \, dP(s) + q \int \tilde{\Delta}(\pi, s \mid \sigma_L) \, dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_L) \, dP(s) \right\} \\
=& \left\{ -(1-\pi) \int \sigma_H(\pi, s) \mathbb{I}_{B(\pi|\sigma_H) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi \mid \sigma_H) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_H) \, dP(s) \right\} \\
& - \left\{ -(1-\pi) \int \sigma_L(\pi, s) \mathbb{I}_{B(\pi|\sigma_L) > \theta - u^{-1}(\underline{u}(s,\theta))} \left[B(\pi \mid \sigma_L) - \left[\theta - u^{-1}(\underline{u}(s, \theta)) \right] \right] dP(s) - q \int_{\mathcal{S}_1} C(\pi, s \mid \sigma_L) \, dP(s) \right\} \\
\geqslant& 0
\end{aligned}
$$

where the first equality simply follows from a definition of $W_{Ln+1}(\pi)$ and $W_{Hn+1}(\pi)$, the second inequality follows from $W_{Hn}(\cdot) \leqslant W_{Ln}(\cdot)$ and $[\Omega(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc} + \pi \Delta p)] \geqslant [\Omega^*(\pi, s|\sigma_L) + \beta W_{Ln}(p_{nc})]$ for $s \in \mathcal{S}_2$, the third equality uses the definition of Ω^*, and finally the last inequality follows from Assumption 1. The above argument implies that starting from the same guess we have that

$$
W(\pi|\sigma_L) = \lim_{n \to \infty} \left(\mathbb{T}_L^W W_0 \right)^n (\pi) \geqslant \lim_{n \to \infty} \left(\mathbb{T}_H^W W_0 \right)^n (\pi) \geqslant W(\pi|\sigma_H)
$$

as wanted.

Step 5: Showing that $\mathbb{T}$ is monotone increasing. To apply the Tarski theorem we need to show that $\mathbb{T}$ is increasing in that if $\sigma_H \succeq \sigma_L$ then $(\mathbb{T}\sigma_H) \succeq (\mathbb{T}\sigma_L)$. Suppose not. Suppose first that for some s and π we have that $0 < \tilde{\sigma}_H = \mathbb{T}\sigma(\pi, s) < \mathbb{T}\sigma(\pi, s) = \tilde{\sigma}_L < 1$. First, note that $\Delta\Omega(\pi, s \mid \sigma_H) \geqslant \Omega(\pi, s \mid \sigma_L)$. Then from the definition of $\mathbb{T}$ we have that

$$\begin{aligned}
0 =& \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] - \Delta\Omega(\pi, s \mid \sigma_H) \\
=& \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] - \Delta\Omega(\pi, s \mid \sigma_L)
\end{aligned}$$

This can be written as

$$\begin{aligned}
&\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] \\
&- \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] \\
=& \Delta\Omega(\pi, s \mid \sigma_L) - \Delta\Omega(\pi, s \mid \sigma_H) < 0
\end{aligned} \tag{33}$$

where the inequality follows from the fact that $\Delta\Omega(\pi, s \mid \cdot)$ is increasing in the expected probability of a bailout σ. The left side of (33) is

$$\begin{aligned}
&\beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - W_1(p_{nc} \mid \sigma_L)\right] \\
&- \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_H)}\Delta p \mid \sigma_H\right) - W_1(p_{nc} \mid \sigma_H)\right] \\
>& \beta W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_L\right) - \beta W_1(p_{nc} \mid \sigma_L) \\
&- \beta W_1\left(p_{nc} + \frac{\pi}{\pi + (1-\pi)(1-\tilde{\sigma}_L)}\Delta p \mid \sigma_H\right) + \beta W_1(p_{nc} \mid \sigma_H) \\
\geqslant& \beta W_1(p_{nc} \mid \sigma_H) - \beta W_1(p_{nc} \mid \sigma_L)
\end{aligned}$$

where the first inequality follows from the contradiction hypothesis, $\tilde{\sigma}_L > \tilde{\sigma}_H$, and $W_1(\cdot|\sigma)$ being increasing, the second inequality follows from the fact that $W_1(\pi \mid \sigma_H) \leqslant W_1(\pi \mid \sigma_L)$ as shown in step 4. Finally, if we take the limit as $p_{nc} \to 0$ then, the above expression converges to zero but the right side of (33) is strictly less than zero which is a contradiction. Therefore, for p_{nc} small enough, by continuity it must be that $\tilde{\sigma}_H \geqslant \tilde{\sigma}_L$.

Next, suppose that $0 \leqslant \tilde{\sigma}_H = \mathbb{T}\sigma_H(\pi, s) < \mathbb{T}\sigma_L(\pi, s) = 1$. Then we have

$$\begin{aligned}
\Delta\Omega(\pi, s \mid \sigma_H) > \Delta\Omega(\pi, s \mid \sigma_L) &\geqslant \beta\left[W_1(p_c \mid \sigma_L) - W_1(p_{nc} \mid \sigma_L)\right] \\
&\geqslant \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_L)\right]
\end{aligned}$$

where the last inequality follows from $W_1(p_c \mid \sigma_L) \geqslant W_1(p_c \mid \sigma_H)$. Therefore, for p_{nc}

close enough to zero we have that

$$\Delta\Omega(\pi, s \mid \sigma_H) > \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_L)\right] = \beta\left[W_1(p_c \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right]$$

which implies that $\mathbb{T}\sigma_H(\pi, s) = 1$, a contradiction.

Finally, suppose that $0 = \mathbb{T}\sigma_H(\pi, s) < \mathbb{T}\sigma_L(\pi, s) < 1$. Then,

$$\beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right] > \Delta\Omega(\pi, s \mid \sigma_H) \geqslant \Delta\Omega(\pi, s \mid \sigma_L)$$

As in the previous case, we know that $W_1(p_c \mid \sigma_H) > W_1(p_c \mid \sigma_L)$. Therefore, for p_{nc} small enough

$$\beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_L) - W_1(p_{nc} \mid \sigma_L)\right] \geqslant \beta\left[W_1(p_{nc} + \pi\Delta p \mid \sigma_H) - W_1(p_{nc} \mid \sigma_H)\right] > \Delta\Omega(\pi, s \mid \sigma_L)$$

which implies that $\mathbb{T}\sigma_L(\pi, s) = 0$, a contradiction.

We then verified all the conditions needed to apply the Tarski's fixed point theorem to establish that set of fixed points of $\mathbb{T}$ is in Σ and is non-empty. Q.E.D.

Proof of Proposition 2

Clearly if $s \notin \mathcal{S}^*$ then $\Delta\Omega(\pi, s) = 0$ for all π and so it is optimal not to bailout, $\sigma(\pi, s) = 0$ for all $s \notin \mathcal{S}^*$. Now consider $s \in \mathcal{S}^*$. Suppose there exits $\pi_2(s) \in (0, 1)$ implicitly defined as

$$\Delta\Omega(\pi_2(s), s) = \beta\left[W_1\left(p_{nc} + \frac{\pi_2(s)\Delta p}{\pi_2(s) + (1 - \pi_2(s))\,1}\right) - W_1(p_{nc})\right] \tag{34}$$

so that the local government is indifferent between bailout and not if private agents expect the no-commitment type bailout authority to bailout with probability 0. Since the left side is decreasing in π wile the right side is increasing in π, it must be that for $\pi \geqslant \pi_2(s)$ it is optimal not to bailout, $\sigma(\pi, s) = 0$. Instead, to the left of $\pi_2(s)$ it must be that $\sigma(\pi, s) > 0$ and it solves

$$\Delta\Omega(\pi, s) = \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1 - \pi)\,\tilde{\sigma}}\Delta p\right) - W_1(p_{nc})\right]$$

Since at zero it must be that

$$\Delta\Omega(0, s) > \beta\left[W_1\left(p_{nc} + \frac{\pi}{\pi + (1 - \pi)\,0}\Delta p\right) - W_1(p_{nc})\right]$$

then by monotonicity of the left and right side in π there exists $\pi_1(s) \in (0, \pi_2(s))$ implicitly defined as

$$\Delta\Omega\left(\pi_1\left(s\right), s\right) = \beta\left[W_1\left(p_{nc} + \frac{\pi_1\left(s\right)\Delta p}{\pi_1\left(s\right) + \left(1-\pi_1\left(s\right)\right)0}\right) - W_1\left(p_{nc}\right)\right] \tag{35}$$

so that the local government is indifferent between bailout and not if private agents expect the no-commitment type bailout authority to bailout with probability 1. Therefore if there exists a $\pi_2\left(s\right) \in \left(0,1\right)$ defined in (34) then the bailout policy followed by the bailout authority is as described in the proposition.

Consider now the case in which there does not exist some $\pi_2\left(s\right) \in \left(0,1\right)$ that solves (34). We can have two cases: if there exists $\pi_1\left(s\right) \in \left(0,1\right)$ implicitly defined by (35) then $\sigma\left(\pi, s\right) = 1$ for $\pi \leqslant \pi_1\left(s\right)$ and strictly decreasing over the interval $\left[\pi_1\left(s\right), 1\right]$. If instead there is no $\pi_1\left(s\right) \in \left(0,1\right)$ that solves (35) then $\sigma\left(\pi, s\right) = 1$ for all π. Q.E.D.

Proof of Proposition 4

We first show that under condition (18) in Assumption 1 we have $\sigma\left(\pi, s_L\right) = 1$ for all π. To this end, note that any equilibrium $B\left(\pi\right) = \mathbb{B}\left(\bar{\gamma}\left(\pi\right)\right) \geqslant \mathbb{B}\left(0\right)$. Moreover, note that the dynamic gains from bailing out, $W\left(p_c\right) - W\left(p_{nc}\right)$, is bounded by $W^R\left(0\right) - W^R\left(1\right)$ in that

$$W\left(p_c\right) - W\left(p_{nc}\right) \leqslant W^R\left(0\right) - W^R\left(1\right)$$

because $W^R\left(0\right) = W^R \geqslant W\left(p_c\right)$ since the value of the Markov equilibrium is lower than the value of the Ramsey plan, and $W\left(p_{nc}\right) \geqslant W^R\left(1\right)$ because along the equilibrium path private agents believe that with some probability they are facing the commitment type. Hence we have that

$$\psi B\left(\pi\right) \geqslant \psi\mathbb{B}\left(0\right) > \beta\left[W^R\left(0\right) - W^R\left(1\right)\right] \geqslant \beta\left[W\left(p_c\right) - W\left(p_{nc}\right)\right]$$

and so it is optimal to bailout with probability one if $s = s_L$.

Next, we show that it is optimal to mix in a mild recession under assumption (19). Suppose by way of contradiction that $\sigma\left(\pi, s_M\right) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the no-commitment type for $\pi = 1$ is

$$W\left(1\right) = e + qp_H\left[0 + \beta W\left(1\right)\right] + qp_M\left[0 + \beta W\left(0\right)\right] + qp_L\left[0 + \beta W\left(0\right)\right]$$

and for $\pi = 0$, since $\bar{\gamma}\left(0\right) = 1$ we have

$$W\left(0\right) = e - \mathbb{Q}\left(1\right)\mathbb{B}\left(1\right) + q\int \Delta\left(0, s\right)\mathbb{B}\left(1\right)dP\left(s\right) + qp_H\left[0 + \beta W\left(0\right)\right]$$
$$+ qp_M\left[0 + \beta W\left(0\right)\right] + qp_L\left[0 + \beta W\left(0\right)\right]$$

Moreover,

$$\mathbb{Q}(1)\,\mathbb{B}(1) - q\int \Delta(0,s)\,\mathbb{B}(1)\,dP(s) = q\mathbb{B}(1) - q\left[p_H + p_M(1-\mu)\right]\mathbb{B}(1)$$
$$= q\mathbb{B}(1)\left[1 - \left[p_H + p_M(1-\mu)\right]\right] = q\mathbb{B}(1)\,P_L$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(1) - W(0) = q\mathbb{B}(1)\,P_L + q\beta p_H\left[W(1) - W(0)\right]$$

or

$$W(1) - W(0) = \frac{q\mathbb{B}(1)\,P_L}{1 - q\beta p_H}$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the bailout authority prefers not to incur the default costs

$$\psi\mu B(\pi = 1) = \psi\mu\mathbb{B}(0)$$

than to obtain the dynamic gains $\beta\left[W(1) - W(0)\right]$. Note that we use that inherited debt at $\pi = 1$ under the contradiction hypothesis because private agents expect a bailout with probability zero, $\bar{\gamma} = (1-\pi) = 0$. Thus it must be that

$$\frac{\beta q\mathbb{B}(1)\,P_L}{1 - q\beta p_H} < \psi\mu\mathbb{B}(0)$$

But this contradicts condition (19) in Assumption 1. Hence it must be that $\sigma(\pi, s_M) < 1$ for some π. In particular, $\sigma(1, s_M) < 1$ because of monotonicity of σ.

We are now left to show that we cannot have that $\sigma(\pi, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_M) = 0$. Hence it must be that

$$\bar{\gamma} = \frac{p_L(1-\pi)\,\sigma(\pi, s_L) + p_M\mu(1-\pi)\,\sigma(\pi, s_M)}{P_L} = \frac{p_L(1-\pi)}{p_L + p_M\mu}$$

and the posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi(p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and finally

$$\mu B(\bar{\gamma}) \leqslant \beta\left[W(p_{nc}) - W(p_{nc})\right]$$

but this is a contradiction since

$$0 < \mu \mathbb{B}(\bar{\gamma}) \leqslant \beta [W(p_{nc}) - W(p_{nc})] = 0$$

Hence we cannot have that $\sigma(\pi, s_M) = 0$ for all π. Therefore there is mixing for some interval. Q.E.D.

Proof of Proposition 5

The proof of the first part is identical to the i.i.d case. To see the second, we first show that under condition (21) in Assumption 3 we have $\sigma(\pi, s_-, s_L) = 1$ for all (π, s_-). To this end, note that any equilibrium $\mathbb{B}(\mathbf{z}_-, \bar{\gamma}) \geqslant \mathbb{B}(\mathbf{z}_-, 0)$. Moreover, note that that the dynamic gains from bailing out, $W(s, p_c) - W(s, p_{nc})$, are bounded by $W^R(s, 0) - W^R(s, 1)$ in that

$$W(s, p_c) - W(s, p_{nc}) \leqslant W^R(s, 0) - W^R(s, 1)$$

because $W^R(s, 0) \geqslant W(s, p_c)$, and $W(s, p_{nc}) \geqslant W^R(s, 1)$. Hence we have that

$$\psi B(s_-, \pi) \geqslant \psi \bar{\mathbb{B}}(s_-, 0) > \beta \left[W^R(s, 0) - W^R(s, 1) \right] \geqslant \beta [W(s, p_c) - W(s, p_{nc})]$$

and so it is optimal to bailout with probability one if $s = s_L$.

Next we show that it is optimal to mix in a mild recession under assumption (22). Suppose by way of contradiction that $\sigma(\pi, s_-, s_M) = 1$ for all π. Under the assumption that the government type is absorbing, the value for the no-commitment type in state s for $\pi = 1$ is

$$W(s, 1) = qp_{sH}[0 + \beta W(s_H, 1)] + qp_{sM}[0 + \beta W(s_M, 0)] + qp_L[0 + \beta W(s_L, 0)]$$

and for $\pi = 0$, since $\bar{\gamma}(0) = 1$ we have for $s = \{s_H, s_L\}$

$$\begin{aligned} W(s, 0) = -q(p_{sM}\mu + p_{sL})\bar{\mathbb{B}}(s, 1) + qp_{sH}\beta W(s_H, 0) \\ + qp_{sM}\beta W(s_M, 0) + qp_{sL}\beta W(s_L, 0) \end{aligned}$$

and for $s = s_M$

$$\begin{aligned} W(s_M, 0) = & -q\mu(p_{MM}\rho_L + p_{ML})\mathbb{B}((s_M, \theta_L), 1) - q(1-\mu)(p_{MM}\rho_H + p_{ML})\mathbb{B}((s_M, \theta_H), 1) + qp_{MH}\beta W \\ & + qp_{MM}\beta W(s_M, 0) + qp_{ML}\beta W(s_L, 0) \end{aligned}$$

and so $W(p_c) - W(p_{nc}) = W(1) - W(0)$ equals

$$W(s,1) - W(s,0) = x_s + q\beta \sum_{s'} p_{ss'} \left[W\left(s',1\right) - W\left(s',0\right)\right]$$

for some constant x_s. Hence we can write

$$\mathbf{A} \cdot \mathbf{W} = \mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 - p_{LL} & p_{LM} & p_{LH} \\ p_{ML} & 1 - p_{MM} & p_{MH} \\ p_{HL} & p_{HM} & 1 - p_{HH} \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} W(s_L,1) - W(s_L,0) \\ W(s_M,1) - W(s_M,0) \\ W(s_H,1) - W(s_H,0) \end{bmatrix}$$

and

$$\mathbf{x} = \begin{bmatrix} q\left(p_{LM}\mu + p_{LL}\right)\bar{\mathbb{B}}(s_L,1) \\ q\mu\left(p_{MM}\rho_L + p_{ML}\right)\mathbb{B}\left((s_M,\theta_L),1\right) + q\mu\left(p_{MM}\rho_H + p_{ML}\right)\mathbb{B}\left((s_M,\theta_H),1\right) \\ q\left(p_{HM}\mu + p_{HL}\right)\bar{\mathbb{B}}(s_H,1) \end{bmatrix}$$

and so

$$\mathbf{W} = \mathbf{A}^{-1} \cdot \mathbf{x}$$

The static gains of bailing out in the medium state if $\pi = 1$ is given by

$$\mathbf{G} = \begin{bmatrix} \psi\mu\bar{\mathbb{B}}(s_L,0) \\ \psi\left[\mu\rho_L\mathbb{B}\left((s_M,\theta_L),1\right) + (1-\mu)\rho_H\mathbb{B}\left((s_M,\theta_H),1\right)\right] \\ \psi\mu\bar{\mathbb{B}}(s_H,0) \end{bmatrix}$$

For the contradiction hypothesis to be valid, it must then be that even for $\pi = 1$ the bailout authority prefers not to incur the default costs, or

$$\mathbf{G} \geqslant \mathbf{A}^{-1} \cdot \mathbf{x}$$

which contradicts in Assumption 1. Hence it must be that $\sigma(\pi, s_-, s_M) = 1 < 1$ for some π.

We are now left to show that we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Suppose by way of contradiction this is indeed the case. In particular, we have that $\sigma(0, s_-, s_M) = 0$.

Hence it must be that

$$\bar{\gamma}(\mathbf{z}_-) = \frac{p_L(1-\pi)\sigma(\pi, s_L) + p_M\mu(1-\pi)\sigma(\pi, s_M)}{P_L(\mathbf{z}_-)} = \frac{p_{s_-L}(1-\pi)}{P_L(\mathbf{z}_-)}$$

and the posterior after no-bailout (if $\pi = 0$), is

$$\pi' = p_{nc} + \pi(p_c - p_{nc}) = p_{nc}$$

since a no-bailout is expected under the contradiction hypothesis, and finally for $s_- \in \{s_H, s_L\}$

$$\mu\bar{B}(s_-, \bar{\gamma}) \leqslant \beta[W(s, p_{nc}) - W(s, p_{nc})]$$

but this is a contradiction since

$$0 < \mu\bar{B}(s_-, \bar{\gamma}) \leqslant \beta[W(s, p_{nc}) - W(s, p_{nc})] = 0$$

Hence, we cannot have that $\sigma(\pi, s_-, s_M) = 0$ for all π. Therefore there is mixing for some interval of π. A similar argument holds for $s_- = s_M$. Q.E.D.

B Model with N borrowers

Consider an environment with N borrowers. The problem for each borrower is identical to main text, except that now the internalize the effect of their choices on the policies of the government. The first order condition for the borrower is

$$u'(Y_1 + Qb_i)(Q + Q_b b_i) = \delta \int_s \int_{\theta \in \Theta_+^s(b_i)} u'(Y_2(\theta, s) - b)\, dF(\theta \mid s)\, dP(s) \tag{36}$$

and $Q = Q(\mathbf{B}, \pi, \sigma)(b_i)$ where $\mathbf{B} = (b_1, ..., b_N)$ and

$$Q(\mathbf{B}, \pi, \sigma)(b) = q\left[\int (1 - \Delta(b_i, s))\, dP(s) + \int \tilde{\Delta}(b_i, s)\, dP(s)\right] + q(1-\pi)\int \sigma(\pi, \mathbf{B}, s) \int \left[\Delta(b_i, s)\, b - \tilde{\Delta}(b_i, s)\right] dP(s)$$

$$Q_b = q\frac{\partial \int (1 - \Delta(b_i, s))\, dP(s) + \int \tilde{\Delta}(b_i, s)\, dP(s)}{\partial b_i} + q\frac{(1-\pi)\int \sigma(\pi, \mathbf{B}, s) \int \left[\Delta(b_i, s)\, b - \tilde{\Delta}(b_i, s)\right] dP(s)}{\partial b_i}$$

where each borrower internalizes the effect of b_i on $\sigma(\pi, \mathbf{B}, s)$.

Let's consider now the incentives for the bailout authority in sub-period 2. We will normalize $C(0) = 0$ so if the bailout authority bails out, its static value is

$$\Omega^*(\mathbf{B}, s) = \sum_i \frac{1}{N}\left[(1-\Delta(b_i, s))\, b_i + \tilde{\Delta}(b_i, s)\right]$$

since transfers are a wash. The value if it allows default is

$$\Omega^*(\mathbf{B}, s) = \sum_i \frac{1}{N}\left[(1-\Delta(b_i, s))\, b_i + \tilde{\Delta}(b_i, s)\right] - C\left(\frac{\sum_i \Delta(b_i, s)\, b_i}{N}\right)$$

Consider the derivative

$$\begin{aligned}
&\frac{\partial}{\partial b}\Delta\Omega(\mathbf{B}, s)\\
=&\frac{\partial}{\partial b_i}\frac{1}{N}C\left(\frac{\sum_i \Delta(b_i, s)\, b_i}{N}\right)\\
=&\frac{1}{N^2}C'\left(\frac{\sum_i \Delta(b_i, s)\, b_i}{N}\right)\frac{\partial \Delta(b_i, s)\, b_i}{\partial b_i}
\end{aligned}$$

which converges to 0 as $N \to \infty$. Note the dynamic benefits are independent of b_i. Therefore,

$$\lim_{N\to\infty}(1-\pi)\int\left(\frac{\partial}{\partial b_i}\sigma(\pi, \mathbf{B}, s)\right)\int\left[\Delta(b, s)\, b - \tilde{\Delta}(b, s)\right] dP(s) = 0$$

and so in the limit the first order condition for the borrower is

$$u'(Y_1 + Qb)(Q + Q_b b) = \delta \int_s \int_{\theta\in\Theta_+^s(b)} u'(Y_2(\theta, s) - b)\, dF(\theta \mid s)\, dP(s)$$

where

$$\begin{aligned}
Q_b = q&\frac{\partial \int (1-\Delta(b, s))\, dP(s) + \int \tilde{\Delta}(b, s)\, dP(s)}{\partial b}\\
&+ q(1-\pi)\int \sigma(\pi, \mathbf{B}, s)\int \frac{\partial\left[\Delta(b, s)\, b - \tilde{\Delta}(b, s)\right]}{\partial b} dP(s)
\end{aligned}$$

C An Investment Model

We now consider an alternative model in which the desire to borrow is driven by profitable investment opportunities. Consider a stage game in which investors start with no capital and must borrow to finance investment k. Next, the aggregate state of the world s is realized and an idiosyncratic productivity level θ is realized according to $h(\theta \mid s)$. The output for the investor who invests k is θk^{α}. Finally, at the end of the stage game, borrowers repay (absent transfers) if $\theta k^{\alpha} \geqslant R(k,\pi)k$. Similar to our baseline economy we can consider an example in which $s = \{s_L, s_M, s_H\}$ and $\theta \in \{\theta_H, 0\}$.

The investor's problem is

$$\max_{k} \sum_{s} \sum_{\theta} \max\{\theta k^{\alpha} - R(k,\pi)k, 0\} h(\theta \mid s) p_s$$

The optimal choice of k satisfies

$$P_H \theta_H \alpha k^{\alpha-1} = P_H R$$
$$\Longrightarrow k = \left(\frac{\alpha\theta_H}{R}\right)^{\frac{1}{1-\alpha}}$$

where $P_H = \sum_s p(s) h(\theta_H|s)$is the probability of a high shock. The interest rate is given by

$$1 = q\left(P_H + \sum_{s} p(s) h(\theta_L|s)(1-\pi)\sigma(\pi,s)\right) R$$

or

$$R = \frac{1}{q\left(P_H + \sum_s p(s) h(\theta_L|s)(1-\pi)\sigma(\pi,s)\right)}$$

Therefore, it is easy to see that k is increasing in π and R is decreasing in π. As a result, the environment satisfies Assumption 2 which implies that we can use Proposition 1 to prove the existence of a Markov equilibrium and use the subsequent propositions to show that this economy displays the same properties as the baseline.