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Online Appendix:
Rethinking Optimal Currency Areas

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ABSTRACT

This Appendix contains related literature, an expanded version of the body of the paper, and a Supplement that contains detailed derivations. It is written as a stand-alone document. Here we have more propositions and lemmas than in the body so the numbering systems in the body and this Appendix are different.

Contents

1. Other Related Literature

Here we discuss related theoretical and empirical literature.

A. Theoretical Literature

The idea that delegating policy to other agents can help solve time inconsistency problems dates back at least to the work of Rogoff (1985a). Anchor-client unions are a vivid example of this type of delegation: the client simply delegates policy to the anchor. Forming cooperative or majority rule unions can also be interpreted as a type of delegation. The key difference between our work and Rogoff's is that in our work the delegated agent's objective function is endogenously pinned down by the composition of the union rather than being exogenously given.

Aguiar et al. (2014) also analyze endogenous policy response to the composition of the union. In their model, countries are asymmetric in their initial level of debt. To facilitate comparison with our work, we think of their low debt countries as the North and their high debt countries as the South. In their model the North is always weakly worse off by allowing Southerners into their union: either policy on the equilibrium path does not respond to the composition of the union and the North is indifferent or this policy does respond and the North is strictly worse off. In sharp contrast, in our analysis of asymmetric unions, the less-distorted North strictly benefits from allowing a mass of more distorted Southerners to join because doing so makes the resulting policy more attractive to the North.

A separate literature on policy coordination studies the gains (or losses) from moving from a noncooperative flexible exchange rate regime to a cooperative regime. See, for example, the work of Rogoff (1985b), Cooper and Kempf (2001, 2004), Cooley and Quadrini (2003), and Fuchs and Lippi (2006). None of the gains from forming a union in our paper come from gains in policy coordination because we assume that policy under flexible exchange rates is set cooperatively to begin with. We have also abstracted from externalities arising from the interactions of monetary and fiscal policies in unions. See, for example, the work of Beetsma and Uhlig (1999), Canzoneri, Cumby, and Diba (2006) and Chari and Kehoe (2007).

Devereux and Engel (2003) show that if prices are set in the currency of the importing country, referred to as local currency pricing, the Mundellian gains to flexible exchange rates disappear. Their paper can be interpreted as an argument for forming a union if monetary authorities can commit to their policies. Our argument for forming a union, in contrast, depends critically on how forming a union can help improve credibility.

In our work, the credibility gains from forming a union arise from the defining feature of a monetary union: there is only one currency, and hence monetary policy cannot react to country-specific shocks. In a different literature on fixed exchange rate systems, Giavazzi and Pagano (1988) and others argue that fixed exchange rate systems can also generate credibility gains relative to flexible exchange rates because, even though every country has a separate monetary authority, under fixed exchange rates inflation is more costly for each country's monetary authority than under flexible exchange rates. Clearly, the source of credibility gains in our work is not connected to that in this literature.

In our model, we assume that countries that form a union cannot leave it until the end of the current period. For analyses with endogenous exit, see the work of Fuchs and Lippi (2006) and Alvarez and Dixit (2014).

B. Empirical Literature using OCA Criterion on Aggregates

In the body of the paper we referenced the work of Alesina, Barro, and Tenreyro 2003, Bayoumi and Eichengreen 1993, and De Grauwe 2018. In addition to these contributions to the literature include Cohen and Wyplosz (1989), Alogoskoufis, and Portes (1990), De Grauwe and Vanhaverbeke (1993), Ghosh and Wolf (1994), Eichengreen and Bayoumi (1996), Bayoumi and Eichengreen (1997 and 1999), Frankel and Rose (1998), Frankel (1999), Boone, Laurence, and Maurel (1999), Buiter (2000a and b), Mongelli (2002), Devereux and Lane (2003), Asongu (2014).

2. A Monetary Economy

Our monetary economy builds on the work of Obstfeld and Rogoff (1995), Galí and Monacelli (2005), Kehoe and Pastorino (2014), and especially Farhi and Werning (2013). The economy consists of a continuum of countries, each of which produces traded and nontraded goods and in which consumers use currency to purchase goods. The traded goods sector in each country is perfectly competitive. The nontraded goods sector consists of imperfectly competitive firms with sticky prices and fluctuating markups. Both the productivities and the markups of these firms are subject to aggregate and country-specific shocks. Traded goods have flexible prices and are bought with previously acquired cash, whereas nontraded goods have sticky prices and are bought with credit. We have purposely chosen the ingredients of our model so that it captures key forces and is otherwise as simple as possible.

The assumption that nontraded goods prices are sticky and traded goods prices are flexible captures the key features, outlined by Friedman (1953), that make flexible exchange rates desirable under commitment. Friedman considers an environment in which because of shocks it is desirable to have the relative price of traded and nontraded goods to vary but the prices of nontraded goods are sticky. He argues that since traded goods prices are flexible, a movement in the exchange rate can allow this relative price to move in the same way that it would if nontraded goods prices were flexible.

Consider next the means of payment assumptions. The assumption that goods must be purchased with previous acquired cash implies that surprise inflation is costly, so that without commitment an equilibrium exists. The assumption that only traded are bought with cash is for simplicity.

In the Supplement we also work out a linear-quadratic version of this economy. This version is essentially the reduced-form model in Alesina and Barro (2002). We do so for three reasons. First, doing so helps to highlight how our analysis is complementary to that in Alesina and Barro. Second, this model yields simple closed-form expressions for some cases in which no such expressions exist in the nonlinear model. Third, as Alesina and Barro emphasize, their model, which is built on the Barro and Gordon (1983) model, captures the key ingredients of a large class of monetary models with time inconsistency problems.

A. Environment

In each period t, an i.i.d. aggregate shock $z_t = (z_{1t}, z_{2t}) \in Z$ is drawn, and each of a continuum of countries draws a vector of country-specific shocks $v_t = (v_{1t}, v_{2t}) \in V$ that are i.i.d. both over time and across countries. The probability of aggregate shocks is $f(z_{1t}, z_{2t}) = f^1(z_{1t})f^2(z_{2t})$, and the probability of the country-specific shocks is $g(v_{1t}, v_{2t}) = g^1(v_{1t})g^2(v_{2t})$. Here, Z and V are finite sets. We let $s_t = (s_{1t}, s_{2t})$ with $s_{it} = (z_{it}, v_{it})$ and let $h(s_t) = h^1(s_{1t})h^2(s_{2t})$ with $h^i(s_{it}) = f^i(z_{it})g^i(s_{it})$. These aggregate and country-specific shocks are to the nontraded goods sector. The shock $\theta(s_{1t})$, referred to as a *markup shock,* affects the extent to which the economy is distorted. The shock $A(s_{2t})$, referred to as a *productivity shock,* affects productivity in this sector. We let s^t denote the history of these shocks and $h_t(s^t)$ the corresponding probability, and we use similar notation for any components of these shocks. We will use the notation

$$E_v(\theta|z) = \sum_{v_1} g^1(v_1)\theta(z_1, v_1) \text{ and } E_v(A|z) = \sum_{v_2} g^2(v_2)A(z_2, v_2)$$

to denote the means of θ and A conditional on the aggregate shocks and use similar notation for other random variables.

The timing of events within a period is that the markup shocks are realized, the sticky price firms make their decisions, the productivity shocks are realized, the monetary authority chooses its policy, and finally the consumers and flexible price firms make their decisions. Here we assume that markup shocks are realized before the sticky price setters set their prices and that productivity shocks are set after they do. In our extension section, we also allow a component of markup shocks to be realized after sticky price setters set their prices and a component of productivity shocks are realized before sticky price setters set their prices and show that both are irrelevant to our main results. Thus, if we had allowed both types of shocks to be realized before and after prices are set we would have reached the same conclusions.

In all that follows, we will identify a country by its history of country-specific shocks $v^t = (v_0, \ldots, v_t)$. This identification imposes symmetry in that all countries with the same history of country-specific shocks receive the same allocations.

Production of Traded and Nontraded Goods

The production function for traded goods in a given country is simply $Y_T(s^t) = L_T(s^t)$, where $Y_T(s^t)$ denotes the output of traded goods and $L_T(s^t)$ the input of labor in the traded goods sector. The problem of traded goods firms is then to solve

$$(1) \quad \max_{L_T(s^t)} P_T(s^t)L_T(s^t) - W(s^t)L_T(s^t)$$

where $P_T(s^t)$ is the nominal price of traded goods and $W(s^t)$ is the nominal wage rate. From the zero profit condition, in equilibrium, $P_T(s^t) = W(s^t)$.

The production function for nontraded goods is given by $Y_N(s^t) = A(s_{2t})L_N(s^t)$ where $Y_N(s^t)$ denotes the output of nontraded goods and $L_N(s^t)$ denotes the input of labor in the nontraded goods sector. We posit that the prices of nontraded goods $P_N(s^{t-1}, s_{1t})$ are set as a time-varying markup over a weighted average of the nominal marginal cost of production in that

$$(2) \quad P_N(s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \sum_{s_{2t}} \left(\frac{Q(s^t)Y_N(s^t)}{\sum_{\tilde{s}_{2t}} Q(\tilde{s}^t)Y_N(\tilde{s}^t)} \right) \frac{W(s^t)}{A(s_t)},$$

where $1/\theta(s_{1t}) > 1$ is the *markup* in period t and $Q(s^t)$ is the nominal pricing kernel. To emphasize that such a time-varying markup can arise from many models, we provide three alternative microfoundations for it in the Supplement.

Consumers and the Government

The consumers in any given country have preferences given by

$$(3) \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t h_t(s^t) U\left(C_T(s^t), C_N(s^t), L(s^t)\right),$$

where $C_T(s^t)$ is the consumption of traded goods, $C_N(s^t)$ is the consumption of nontraded goods, and $L(s^t)$ is labor supply. In most of our analysis, we will specialize preferences to be

$$(4) \quad U(C_T, C_N, L) = \alpha \log C_T + (1-\alpha) \log C_N - \psi L$$

and refer to them as *our preferences.* The critical feature of these preferences is their quasi-linearity in labor, which allows us to obtain useful aggregation results along the lines of Lagos and Wright (2005).

The budget constraint of the consumer is given by

$$(5) \quad \begin{aligned} & P_T(s^t)C_T(s^t) + P_N(s^{t-1}, s_{1t})C_N(s^t) + M_H(s^t) + \bar{Q}\left(s^t\right) B(s^t) \\ \leq\ & W(s^t)L(s^t) + M_H(s^{t-1}) + B(s^{t-1}) + T(s^t) + \Pi(s^t), \end{aligned}$$

where $T(s^t)$ are nominal transfers, $\Pi(s^t) = P_N(s^{t-1}, s_{1t})Y_N(s^t) - W(s^t)L_N(s^t)$ are the profits from the nontraded goods firms, $\bar{Q}\left(s^t\right)$ is the price of the noncontingent nominal bond in the domestic currency, and $B(s^t)$ are nominal bonds. Here, for simplicity, we abstract from international capital mobility, so the consumers in a given country hold only domestic nominal bonds. With our preference and shock structure it turns out that consumers have no incentive to borrow and lend across countries.

Consumers are also subject to a cash-in-advance constraint that requires them to buy traded goods at t using money brought in from period $t-1$, namely $M_H(s^{t-1})$, so that

$$(6) \quad P_T(s^t)C_T(s^t) \leq M_H(s^{t-1}).$$

Under flexible exchange rates, consumers use local currency to purchase traded goods so that $M_H(s^{t-1})$ is local currency holdings. In a union, consumers use the common currency of the union so that $M_H(s^{t-1})$ is holdings of the common currency. The subscript H denotes an individual household's holdings of money.

Notice that with our cash-in-advance constraint, traded goods can be bought only with money acquired in the previous period. In particular, money injections in the current period cannot be used to purchase current traded goods. The role of this assumption is to generate a cost of surprise inflation when the monetary authority lacks commitment. To better understand this idea, suppose the cash-in-advance constraint holds with equality $P_T(s^t)C_T(s^t) = M_H(s^{t-1})$. Then, a surprise money injection at t that increases $P_T(s^t)$ necessarily reduces $C_T(s^t)$.[1]

The first order conditions for the consumer are summarized by

$$(7) \quad \frac{U_N(s^t)}{P_N(s^{t-1}, s_{1t})} = -\frac{U_L(s^t)}{W(s^t)}$$

[1] Of course, if we had allowed current money injections to be used for current purchases of traded goods, then a surprise money injection would proportionately increase both the price of traded goods and the stock of money available to purchase those goods. Thus, such an injection would not affect the consumption of traded goods. Hence, there would be no cost of surprise inflation, and in our environment without commitment no equilibrium would exist.

$$(8)\quad \frac{U_T(s^t)}{P_T(s^t)} = -\frac{U_L(s^t)}{W(s^t)} + \phi(s^t)$$

$$(9)\quad -\frac{U_L(s^t)}{W(s^t)} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{U_T(s^{t+1})}{P_T(s^{t+1})}$$

$$(10)\quad \bar{Q}\left(s^t\right) = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1}, s_{1t})}{U_N(s^t)},$$

where $\phi(s^t) \geq 0$ is the (normalized) multiplier on the cash-in-advance constraint. Notice also that the nominal stochastic discount factor for the country is

$$(11)\quad Q(s^{t+1}) = \beta h(s^{t+1}|s^t) \frac{U_N(s^{t+1})}{P_N(s^t, s_{1t+1})} \frac{P_N(s^{t-1}, s_{1t})}{U_N(s^t)},$$

where $Q(s^t)$ is the price of a state-contingent claim to local currency units at s^t in units of local currency at s^t. Firms use this price to discount nominal marginal costs in (2).

The government budget constraint for each country under flexible exchange rates is

$$(12)\quad \bar{Q}\left(s^t\right) B\left(s^t\right) = B\left(s^{t-1}\right) + T(s^t) - \left(M(s^t) - M(s^{t-1})\right),$$

where $M(s^t)$ denotes the money supply in local currency units. In a union, the government budget of a given country is

$$(13)\quad \bar{Q}\left(s^t\right) B\left(s^t\right) = B\left(s^{t-1}\right) + T(s^t) - \left(\bar{M}(z^t) - \bar{M}(z^{t-1})\right),$$

where $\bar{M}(z^t)$ is the *unionwide money supply* where we have assumed that the seignorage revenues are distributed evenly across countries. For initial conditions, we assume that the initial money holdings of consumers in each country $M_{H,-1}$ are equal and these initial money holdings equal the initial money supply in each country M_{-1} and that the initial holdings of bonds B_{-1} are zero in all countries.

With flexible exchange rates, the policy of the monetary authority consists of a specification for each country of prices for debt (or equivalently nominal interest rates), the quantity of money, the quantity of debt, and taxes which satisfies (12). In a union, the policy consists of similar objects with the restriction that they satisfy (13). In both regimes, the policy is set cooperatively or by majority rule. Note, in particular, under flexible exchange rates we do not think of policy as being set noncooperatively. By adopting the same institutional framework for both regimes we ensure that the changes in welfare across regimes do not arise from changes in the extent of cooperation.

An *equilibrium with flexible exchange rates* is a set of allocations, prices, and policy in each of the continuum of countries such that i) the decisions of consumers are optimal, ii) the decisions of firms are optimal, iii) the labor market clears in each country, $L_N(s^t) + L_T(s^t) = L(s^t)$, iv) the traded and nontraded goods markets clear, $C_T(s^t) = Y_T(s^t)$, $C_N(s^t) = A(s^t)Y_N(s^t)$, v) the monetary authority's budget constraint holds, vi) the money market clears $M_H(s^t) = M(s^t)$.

So far we have expressed each country's prices in units of its own currency. Since the law of one price holds for traded goods, we can write the (multilateral) nominal exchange rate between a particular country and all others as

$$(14) \quad e(s^t) = \frac{P_T(s^t)}{\sum_{v^t} P_T(z^t, v^t) g^t(v^t)},$$

where $g^t(v^t) = g(v_0) \ldots g(v_t)$ and the term on the denominator is the simple average over all countries, where countries are identified by the history of their country-specific shocks v^t. Because of the law of large numbers, the exchange rate for a given country does not depend on the realization of country-specific shocks for any countries other than the given country.

In a monetary union, the unionwide money supply, $\bar{M}(z^t)$, is chosen by a single authority. The nominal exchange rate $e(s^t) = 1$ for all s^t and the price of traded goods, expressed in units of the common currency, is equal in all countries. Thus, the price of traded goods only depends on the aggregate shock history z^t and cannot vary with country-specific shocks. We can write this restriction as follows: if one country has a history $s^t = (z^t, v^t)$ and another has history $\tilde{s}^t = (z^t, \tilde{v}^t)$, then for any s^t and $\tilde{s}^t$,

$$(15) \quad P_T(s^t) = P_T(\tilde{s}^t).$$

An *equilibrium in a monetary union* is defined analogously to an equilibrium with flexible exchange rates with several differences. First, there is a unionwide money supply $\bar{M}(z^t)$, the nominal exchange rate $e(s^t) = 1$ for all s^t, and money market clearing requires that the total money held by all the consumers in the union add up to the total money supply in the union, in that

$$\sum_{v^t} M_H(z^t, v^t) g_t(v^t) = \bar{M}(z^t).$$

Under either regime, fluctuations in markups lead to fluctuations in the degree of distortions. To see this point in the simplest way, suppose that productivity is constant. We can then combine the first order condition of the nontraded goods firm with that of private agents to see that

$$(16) \quad -\frac{U_L}{U_N} = A\theta(s_t) < A$$

so that $1 - \theta$ is the *wedge* between the marginal rate of substitution between labor and nontraded goods and the corresponding marginal rate of transformation. Clearly, the higher is the markup $1/\theta(s_t)$, the greater is the distortion from imperfect competition. Clearly, the lower is $\theta(s_t)$, the higher is the markup, the greater is the resulting wedge, and thus, the greater is the distortion from imperfect competition. As we will see in the environment without commitment, higher markups pose a relatively higher temptation for the government to create surprise inflation ex post.

3. Optimal Policy with Commitment

We turn now to analyzing optimal policy under flexible exchange rates and in a monetary union. We will show that the lack of monetary independence in a monetary union imposes a loss on member countries and leads to our modified version of Mundell's optimal currency area criterion: the smaller are the country-specific components of productivity shocks, the smaller are the losses from forming a monetary union.

The Ramsey equilibrium for this economy is the competitive equilibrium that maximizes an equally weighted average of consumer utility across countries. This Ramsey equilibrium naturally corresponds to the cooperative equilibrium with commitment. Corresponding to this equilibrium is a *Ramsey problem* for a country under *flexible exchange rates*. The problem is to choose allocations, prices, and policy given initial conditions to maximize an equally weighted average of consumer utility across countries subject to the consumer and firm first order conditions in each country and the resource constraints in each country as well as the world resource constraint.

Here we assume that both under flexible exchange rates and in a union, monetary policy is set in a cooperative fashion in that it maximizes an equally weighted sum of welfare of utilities of the member countries and show later that doing so is equivalent to majority rule. We do so for a simple reason: by comparing cooperative equilibria under flexible exchange rates with the equilibrium in a union, which is essentially cooperative, we make clear that the differences in welfare between flexible exchange rates and a monetary union arise *solely* because of changes in the ability to use monetary policy to respond to country-specific shocks rather than changes in the degree of cooperation. (It is worth noting that for our particular environment there are no externalities, such as terms of trade externalities, so that the cooperative and noncooperative equilibria coincide and thus there are no gains from cooperation.)

In a monetary union, the price for traded goods cannot vary with country-specific shocks. The *Ramsey problem* in a *monetary union* can thus be written as choosing allocations, prices, and policy to maximize an equally weighted sum of the utilities over all countries subject to the consumer and firm first order conditions and the resource constraints and the additional common price constraint (15).

Since the Ramsey problem under flexible exchange rates is a more relaxed version of the Ramsey problem in a monetary union, we have the following result.

Proposition 1. The Ramsey problem under flexible exchange rates leads to higher welfare than the Ramsey problem in a monetary union.

We turn now to characterizing the Ramsey allocations. The key step in this characterization is to note that the distortions from imperfect competition can be captured by a single constraint on the Ramsey problem. To obtain this constraint, substitute for $W(s^t)$ and $Q(s^t)$ from the consumer first order conditions into (2) to get the *markup condition*:

$$(17)\quad \sum_{s_{2t}} h(s^t|s^{t-1}, s_{1t})C_N(s^t)\left[U_N(s^t) + \frac{1}{\theta(s_{1t})}\frac{U_L(s^t)}{A(s_t)}\right] = 0.$$

Thus, the Ramsey problem under flexible exchange rates reduces to a sequence of static problems of choosing allocations to maximize expected utility in period t subject to the resource constraints and the markup condition (17).

The Ramsey problem in a union reduces to a similar sequence of static problems with the additional constraint that arises from fixed exchange rates. Combining (7), (8), and (15) and comparing two histories $s^t = (z^t, v^{t-1}, v_{1t}, v_{2t})$ and $\tilde{s}^t = (z^t, v^{t-1}, v_{1t}, \tilde{v}_{2t})$ gives the *union constraint*:

$$\frac{U_T(s^t)}{U_N(s^t)} = \frac{U_T(\tilde{s}^t)}{U_N(\tilde{s}^t)} \text{ for all } v_{2t}, \tilde{v}_{2t}.$$

We turn now to comparing the Ramsey allocations and prices under flexible exchange rates with those in a monetary union for our preferences (4). The consumption of traded goods in both regimes is the same and is given by $C_T = \alpha/\psi$. The consumption of nontraded goods under flexible exchange rates and in a union is given by

$$(18)\quad C_N^{flex}(s) = \frac{(1-\alpha)}{\psi} A(s_2)\theta(s_1) \text{ and } C_N^{union}(s) = \frac{1-\alpha}{\psi} \frac{\theta(s_1)}{E_v\left(1/A|z\right)}.$$

Noting that in both regimes the consumption of nontraded goods satisfies $C_N(s) = A(s)L_N(s)$, it follows that the expected value of labor supply is equal across regimes, so that the difference in utility in the regimes solely arises from the differences in the consumption of nontraded goods. Since the utility function over nontraded goods is strictly concave, it follows that whenever the country-specific component of productivity shocks has strictly positive variance, the utility under flexible exchange rates is greater than it is in a monetary union.

Our first main result is that the variability of the country-specific component of productivity shocks plays a key role in determining the costs of forming a union and that markup shocks are irrelevant in determining these costs.

Proposition 2. The difference in expected utility per period between the flexible exchange rate regime and the monetary union is given by

$$(19)\quad (1-\alpha)\, E_z \left[\log E_v \left(\frac{1}{A}|z\right) - E_v \left(\log \frac{1}{A}|z\right)\right] > 0.$$

This proposition follows immediately from substituting (18) into the objective function. The details behind the derivation of (18) as well as the details of most of the subsequent results are in the Supplement.

Clearly, this utility difference is strictly positive, since the log function is a concave function. We find it useful to consider the simple case in which $A(v_2, z_2) = A_v(v_2)A_z(z_2)$ and $A_v(v_2)$ is log normal with mean μ_v and variance σ_v^2. Here, the utility difference reduces to $(1-\alpha)\sigma_v^2/2$ so that the losses in forming the union are increasing in the volatility of the country-specific productivity shocks. Note that markup shocks play no role in determining the utility difference between the two regimes.

One way to gain intuition for Proposition 2 is to recall the classic argument of Friedman (1953) that flexible exchange rate systems are desirable because changes in the exchange rate can be used to mimic the price changes that would have occurred if prices in the economy were flexible rather than sticky. We can apply Friedman's argument by considering a flexible price version of our economy in which imperfectly competitive firms set prices after, rather than before, the productivity shock is realized. With our preferences, it is easy to show that the flexible price allocations under the Friedman rule are also the Ramsey allocations for the sticky price economy.[2] That is, it is indeed desirable to run the flexible exchange rate system to reproduce the flexible price allocations under the Friedman rule. To implement these allocations, the relative price of nontraded to traded goods must move with the productivity shock. Since doing so is not feasible in a monetary union, welfare is lower.

To see how flexible exchange rates allow the relative price of traded to nontraded goods to move with country-specific productivity shocks, note, as shown in the Supplement, that the prices of traded goods under flexible exchange rates and in a union are given by

$$p_T^{flex}(s_{2t}) = \kappa A(s_{2t}) \text{ and } p_T^{union}(z_{2t}) = \frac{\kappa}{E_v(1/A(z_{2t}, v_{2t}))},$$

where κ is a number sufficiently small so that the cash-in-advance constraint is not binding in any state and we have normalized all prices by the relevant beginning-of-period money

[2] Our result that exchange rate policy can be used to implement the Ramsey allocations in an open economy is reminiscent of the work of Correia, Nicolini, and Teles (2008). They show that, in a closed economy model, fiscal policy can be used to implement the Ramsey allocations in a sticky price economy.

stock. These prices imply that, under flexible exchange rates, the exchange rate $e(s) = A(z_2, v_2)/E_v(A|z_2)$ depreciates when the country-specific component of productivity is high.

Proposition 2 implies an optimal currency area criterion expressed in terms of shocks: the smaller are the country-specific productivity shocks, the smaller are the losses from forming a monetary union; the pattern of markup shocks is irrelevant. This criterion represents a refinement of the standard Mundellian criterion. Here, the source of the shocks is critical; some shocks are important, whereas others are irrelevant even though they contribute to aggregate fluctuations.

In empirical work, the optimal currency area criterion is expressed in terms of observables instead of shocks. As we argue later, our refinement implies a very different optimal currency area criterion in terms of observables than does the traditional criterion.

4. Optimal Policy without Commitment

Consider now the same environment except that the monetary authorities cannot commit. We model this lack of commitment by having these authorities choose policies in the standard Markovian fashion. That is, in each period the monetary authority sets its policies as a function of the state and takes as given the evolution of future policy.

Recall that in the environment with commitment, markup shocks play no role in determining the costs or benefits of forming a monetary union. In contrast, in the environment without commitment, markup shocks play a critical role in determining these costs and benefits. In particular, the more variable are markup shocks, the larger are the gains from forming a monetary union. As will become clear later from Proposition 6 and from equations (73) and (74) in the Supplement, productivity shocks play similar roles with and without commitment. To focus on the role of markup shocks, we assume for most of what follows that productivity is constant across countries and time. Under this assumption, there are only first stage shocks and, hence, for simplicity we write (z_1, v_1) as (z, v).

As in the environment with commitment, we assume that the monetary authorities set policy cooperatively with flexible exchange rates. Clearly, policy in a monetary union is made cooperatively. As we have emphasized, we make this assumption to show that our main result that countries can gain by forming a union does not arise because policy is set noncooperatively under flexible exchange rates and cooperatively in the union.

The model features two key frictions. The nontraded goods firms set their prices as a markup over their expected marginal costs and hence distort downward the production of nontraded goods. This distortion gives the monetary authority an incentive to engineer surprise inflation so as to diminish the effective markup and increase the production of nontraded goods. The second friction is that purchases of traded goods must be made with money brought into the period. This feature of the model generates costs for surprise inflation: surprise inflation inefficiently lowers the consumption of traded goods ex post. In equilibrium, the monetary authority balances the benefits of surprise inflation against these costs, and this friction leads to an interior solution for inflation.

The timing is the same as before. Sticky price firms make their decisions after the markup shocks associated with (z, v) have been realized. Then monetary policy is set. Finally, consumer and the flexible price firms make their decisions.

A. Flexible Exchange Rates

In a Markov equilibrium, all choices depend on the state confronting agents at the time they make their decisions. We begin by describing the state variables for the nontraded goods and traded goods firms, the consumers, and the monetary authority. We normalize all nominal variables by the beginning-of-period aggregate stock of money M_{-1} in the given country. With this normalization, the normalized aggregate money stock is 1 in each country. Let b denote the normalized amount of debt in a given country.

Consider a sticky price firm in a given country. The country-specific state is the country-specific shock v. The aggregate state at this stage is the aggregate shock z. Thus, the *nontraded firm state* is (b, v, z), and the nontraded goods firm's normalized decision rule is $p_N(b, v, z)$, where $p_N = P_N/M_{-1}$ denotes the normalized nontraded goods price.

At the time the monetary authority chooses its policy, each country is identified by its country-specific state $x_G = (b, v, p_N)$. The *monetary authority's state* is $S_G = (z, \lambda_G)$, where λ_G is a measure over the states x_G in all countries. The monetary authority's policy rule consists of a choice of bond discount prices, $q\,(x_G, S_G)$, new debt issues, $b'\,(x_G, S_G)$, transfers, $T\,(x_G, S_G)$, and money growth rate, $\mu\,(x_G, S_G)$, for each country. Let $\pi\,(x_G, S_G) =$

$(q\left(x_G,S_G\right), b'\left(x_G,S_G\right), T\left(x_G,S_G\right), \mu\left(x_G,S_G\right))$ denote the policy rule. Next, the country-specific component of the traded goods firm's state is $x_T = (b, v, p_N, \pi)$, where π is the policy in that country and the corresponding aggregate state is $S_T = (z, \lambda_T)$, where λ_T is a measure over country-specific states of traded goods firms in all countries. The traded goods firm's normalized decision rule is $p_T(x_T, S_T)$, where $p_T = P_T/M_{-1}$ denotes the normalized traded goods price. Finally, the *consumer's state* is (b_H, m_H, x_T, S_T), where b_H and m_H denote the amount of debt and money held by an individual in a country divided by the aggregate stock of money in that country, the country-specific state is x_T, and the aggregate state is S_T.

Here we set up the equilibrium recursively, which is easiest to do so by working backward from the end of a period. The consumer's problem is

$$V\left(b_H, m_H, x_T, S_T\right) = \max_{C_T, C_N, L, b'_H, m'_H} U\left(C_T, C_N, L\right) + \beta \sum_s h(s') V\left(b'_H, m'_H, x'_T, S'_T\right)$$

subject to the cash-in-advance constraint in normalized form:

$$p_T(x_T, S_T) C_T \leq m_H$$

and the budget constraint in normalized form:

$$p_T(x_T, S_T) C_T + p_N C_N + \mu m'_H + \mu q b'_H \leq m_H + b_H + w(x_T, S_T) L + T + \Pi(x_T, S_T),$$

where $x'_T = (b', v', p_N(v'), \pi(x'_G, S'_G))$. This problem defines the consumer decision rules. We denote the consumer decision rule for the consumption of the traded good C_T as $C_T(b_H, m_H, x_T, S_T)$ and use similar notation for other consumer choices. For traded goods firms, profit maximization implies

$$p_T(x_T, S_T) = w(x_T, S_T), \tag{20}$$

where p_T and w are normalized by the aggregate stock of money in that country.

The monetary authority acts in a cooperative fashion in that it maximizes an equally weighted sum of utilities across countries. Here the monetary authority chooses a function

$\pi(\cdot, S_G)$ that specifies for any country with country-specific state x_G, the policy $\pi(x_G, S_G)$. This authority internalizes that, in equilibrium, consumers in a given country hold all of that country's money and debt, so that $m_H = M_H/M_{-1} = 1$ and $b_H = b$. The monetary authority's problem is to solve

$$\max_{\{\pi(x_G,S_G)\}} \sum V\left(b, 1, v, p_N, \pi(x_G, S_G), S_T\right) d\lambda_G(x_G),$$

subject to the government budget constraint for each country

$$\text{(21)}\quad \mu q b' = b + T - (\mu - 1)\,,$$

where S_T is induced by the function chosen by the monetary authority.

The pricing rule for nontraded goods is

$$\text{(22)}\quad p_N(b, v, z) = \frac{w(x_T, S_T)}{A\theta(v, z)},$$

where x_T and S_T are induced by the policy rules of other nontraded setting firms and the monetary authority.

A *Markov equilibrium under flexible exchange rates* consists of a pricing rule for nontraded goods $p_N(b, v, z)$, a profit rule $\Pi(x_T, S_T)$, the monetary authority's policy rule $\pi\left(x_G, S_G\right)$, consumer decision rules and value functions, a wage rate rule $w(x_T, S_T)$, and a price rule for traded goods $p_T(x_T, S_T)$, such that *i*) the sticky price firms and the flexible price firms maximize profits, *ii*) the monetary authority maximizes consumer welfare taking as given the decision rule of the consumers and traded goods firms in the current period and the decision rules of the monetary authority and private agents in all future periods, *iii*) consumers maximize welfare, *iv*) the traded goods market, the nontraded goods market, and the labor market clear, the money market clears in that $m'_H(b, 1, x_T, S_T) = 1$, and $b'_H(b, 1, x_T, S_T) = b'$.

To characterize the equilibrium, consider the problem faced by a monetary authority given the nontraded goods prices that have been chosen in each country. We find it convenient to set up this problem in primal form in the sense that we think of this authority as directly choosing prices and allocations subject to the resource constraints and the first order conditions of traded goods firms and consumers. We can summarize these conditions by

$$(23) \quad L = C_T + \frac{C_N}{A}$$

$$(24) \quad \frac{U_N}{p_N} = -\frac{U_L}{p_T}$$

$$(25) \quad \frac{U_T}{p_T} \geq -\frac{U_L}{p_T}$$

$$(26) \quad p_T C_T \leq 1,$$

where if the cash-in-advance constraint (26) is a strict inequality, then (25) holds as an equality, and

$$(27) \quad -\mu \frac{U_L}{p_T} = \beta \sum_{s'} h(s') \frac{U_T(b', 1, x_T', S_T')}{p_T(x_T', S_T')},$$

$$(28) \quad -\mu q \frac{U_L}{p_T} = \beta \sum_{s'} h(s') \frac{U_L(b', 1, x_T', S_T')}{p_T(x_T', S_T')}$$

and (21). Note that in (26), (27) and (28), we have used that money market clearing implies that $m_H = 1$, $m_H' = 1$.

Note first that given p_N, the set of feasible allocations and prices (C_T, C_N, L, p_T, b') is independent of the inherited debt b. Mechanically, any feasible allocation (C_T, C_N, L, p_T, b') for some b is also feasible for any other b because lump sum taxes T can be adjusted appropriately to satisfy (21) and neither b nor T appear in any other equations. This result implies that without loss of generality we can set $b' = 0$, set b to zero as well, and drop (28) from now on.

Thus future states and, therefore, future allocations and continuation utility are unaffected by the current choices of private agents and the monetary authority. The monetary authority's problem reduces to one of choosing allocations and money growth rates for each country to maximize the integral of the country's current utility subject to (23)–(27), that is, to choose the allocations, traded goods prices, and money growth rates in each country to maximize

$$\sum U\left(C_T(x_G, S_G), C_N(x_G, S_G), L(x_G, S_G)\right) d\lambda_G(x_G, S_G)$$

subject to (23)–(27). Since the allocations, traded goods prices, and money growth rates of a given country do not enter into the constraint sets or utility for any other country, we can solve this problem by considering each country in isolation. That is, for any given country indexed by x_G, we can drop the aggregate state S_G and choose that country's allocations and traded goods prices to maximize

$$(29) \quad U\left(C_T(x_G), C_N(x_G), L(x_G)\right)$$

subject to (23)–(27). Note that here the only relevance of x_G is that it records the particular nontraded goods price p_N that has been set in that country. In particular, this feature implies that objects such as the price of traded goods in a country and the monetary policy in a country do not depend on the distribution of states in other countries.

This observation proves that the cooperative solution in which one monetary authority chooses policies for all countries is equivalent to a noncooperative solution in which the monetary authority for each country chooses its own policies to maximize the welfare of its own residents. That is, we have proven the following proposition.

Proposition 3. The cooperative and noncooperative Markov equilibria under flexible exchange rates coincide.

We turn to characterizing the solution to (29). Since μ appears only in (27), we can use this constraint to eliminate μ as a choice variable and solve the *static primal Markov problem* of maximizing current period utility $U(C_T, C_N, L)$ subject to (23)–(26). We can think of this problem as determining the *best response of the monetary authority* $p_T^{flex}(s, p_N)$ to a given choice of nontraded goods price by the nontraded goods firms, and then given this best response, we can determine C_T, C_N, and L from the constraints.

Consider now the problem of the sticky price producers. Substituting for the wage rate from (20) in the pricing rule for nontraded goods (22) and using that neither traded goods prices nor monetary policy depends on the distribution of states gives $p_N(s) = p_T(x_T)/A\theta(s)$, where $x_T = (0, s, p_N(s), \mu(s, p_N(s)))$. Hence, in any equilibrium, the price of traded goods only varies with s, and the equilibrium *outcome,* which can be written as $\bar{p}_T(s)$ at state s, must be a fixed point of

$$\bar{p}_T(s) = p_T^{flex}\left(s, \frac{\bar{p}_T(s)}{A\theta(s)}\right).$$

Given the fixed point $\bar{p}_T(s)$, the rest of the equilibrium *outcomes* are obtained from the constraints on the monetary authority's problem. (Here bars distinguish outcomes, which vary only with shocks, from decision rules, which also vary with the endogenous state variables.)

In a Markov equilibrium, the cash-in-advance constraint is binding in equilibrium. To understand why, consider the trade-offs confronting the monetary authority in the primal Markov problem. For a given price of nontraded goods p_N, raising the price of traded goods has a marginal benefit because it reduces the markup distortion by moving the marginal rate of substitution closer to the marginal rate of transformation. If the cash-in-advance constraint were not binding, then raising this price has no cost and the solution is to set $p_N = p_T/A$ so that the marginal rate of substitution between traded and nontraded goods equals the marginal rate of transformation between these goods. Such an outcome cannot be an equilibrium because sticky price producers forecasting this policy response will set the price of nontraded goods at a markup over wages, or equivalently, over the price of traded goods, so that $p_N = p_T/A\theta$. Thus, in equilibrium, raising the price of traded goods must have a positive marginal cost, which happens only if it reduces the consumption of traded goods, which, in turn, requires that the cash-in-advance constraint be binding.

Using our preferences and the result that the cash-in-advance constraint binds in a Markov equilibrium, the static primal Markov problem can be written as follows. Given p_N and θ, choose (C_N, C_T, p_T) to solve

$$(30) \quad \max \alpha \log C_T + (1-\alpha) \log C_N - \psi \left[C_T + C_N/A\right]$$

subject to

$$(31) \quad C_T = \frac{1}{p_T} \quad \text{and} \quad C_N = \frac{1-\alpha}{\psi}\frac{p_T}{p_N}.$$

The best response of the monetary authority $p_T^{flex}(s, p_N)$ depends only on p_N and is given by

$$(32) \quad p_T = F\left(\frac{1}{Ap_N}\right)$$

for a quadratic function F defined in the Supplement. This best response function balances off the benefits from lowering the markup distortion against the costs of depressing traded goods consumption by raising the price of traded goods. Since $p_N = p_T/A\theta$, the equilibrium outcome $\bar{p}_T(s)$ solves the fixed point problem $\bar{p}_T(s) = F(\theta(s)/\bar{p}_T(s))$, and given $\bar{p}_T(s)$ the equilibrium outcomes $\bar{C}_T(s)$ and $\bar{C}_N(s)$ are given from the constraints (31).

Here we need to bound the markups from above to guarantee that a Markov equilibrium exists. Briefly, if the benefits of reducing the markup distortion always exceed the costs of depressing traded goods consumption, no equilibrium exists. It turns out that if the markups $1/\theta(s)$ are not too large in that

$$(33) \quad \frac{1}{\theta(s)} < \frac{1-\alpha}{1-2\alpha} \text{ for all } s,$$

then there exists a sufficiently high nontraded goods price such that the benefits equal the costs. In what follows, we will assume without further mention that this bound holds.

Solving the fixed point problem for $\bar{p}_T(s)$, we then have the following lemma.

Lemma 1. The allocations in the Markov equilibrium with flexible exchange rates are given by

$$(34) \quad C_T^{flex}(s) = \frac{\alpha}{\psi} - \frac{1-\alpha}{\psi}(1-\theta(s)) \text{ and } C_N^{flex}(s) = \frac{1-\alpha}{\psi}A\theta(s)$$

and $L(s) = C_T(s) + C_N(s)/A$.

Now compare the commitment outcomes with the no commitment outcomes under flexible exchange rates when productivity is constant in both. From (18) and (34) we see that the consumption of nontraded goods is identical. In contrast, the consumption of traded goods differs: under commitment it is α/ψ, and under no commitment it is given in (34).

We now show that the time inconsistency problem worsens when markup shocks become more volatile. Inspecting the allocations under commitment and no commitment gives that the expected difference in utilities with and without commitment is, up to a constant,

$$-E\log(\alpha-(1-\alpha)(1-\theta(s))).$$

Since the log function is a concave function, we have that a mean-preserving spread in θ increases this expected difference.

Proposition 4. Under flexible exchange rates, a mean-preserving spread in θ worsens the time inconsistency problem in that the differences in welfare between the commitment and the no commitment outcomes increase.

Here, a mean-preserving spread in the markup shock θ makes the consumption of traded goods more volatile and, because preferences are concave, lowers utility. Briefly, as markups become more volatile, the prices set by the nontraded goods firms become more volatile. The monetary authority reacts to this higher volatility by making inflation and, hence, the traded goods prices more volatile. Since the cash-in-advance constraint is binding, this increase in volatility in prices increases the volatility of traded goods. In equilibrium, the attempt by the monetary authority to undo the markup distortions is frustrated, and all the monetary authority accomplishes is an increase in the volatility of traded goods consumption. In short, an increase in the volatility of either aggregate or country-specific markup shocks exacerbates the time inconsistency problem.

B. Monetary Union

To set up the equilibrium in the monetary union recursively, we follow the same procedure as we did with flexible exchange rates: we define the state that confronts each decision maker and then define policies and decision rules as functions of the state. Here, the natural normalization for all nominal variables is the beginning-of-period aggregate money stock for the union as a whole, denoted $\bar{M}_{-1}$. Furthermore, an argument similar to that in the environment with flexible exchange rates implies that the level of debt in any country does not affect the set of feasible allocations. Therefore without loss of generality we set debt to zero in all countries and omit debt from the state.

As under flexible exchange rates, the state at a given stage in the period for a decision maker consist of a complete description of the relevant states of such decision makers in the union, that is, a measure over all such states. Consider, for example, the nontraded goods firm in a given country. The country-specific state x_N consists of the money holdings of consumers in that country relative to the unionwide money stock, $m = M_{-1}/\bar{M}_{-1}$, together with the country-specific shock v. The aggregate state at this stage of the period is $S_N = (z, \lambda_N)$, where z is the aggregate shock and λ_N is a measure over the states of the sticky price firms in the rest of the union. Thus, the *nontraded goods firm state* is (x_N, S_N), and the sticky price firm's normalized decision rule is $p_N(x_N, S_N)$. At the time the monetary authority chooses its policy, each country's state is given by $x_G = (m, p_N, v)$ and the *monetary authority's state* is $S_G = (z, \lambda_G)$, where λ_G is a measure over the states x_G in all countries. The consumer's state and the traded goods firm state are defined in a similar fashion.

We define a Markov equilibrium in a nearly identical fashion to that under flexible exchange rates. Following steps similar to those under flexible exchange rates, we can set up the primal Markov problem in a monetary union. Since countries are indexed by x_G, we think of this authority as choosing the allocations for each country, the unionwide money growth rate γ and the unionwide common price p_T. Thus, suppressing for a moment the dependence of current allocations on the aggregate state S_G, the primal Markov problem is to choose $(C_T(x_G), C_N(x_G), L(x_G))$, γ and p_T to solve

$$W^{union}(S_G) = \max_x \sum U\left(C_T(x_G), C_N(x_G), L(x_G)\right) d\lambda_G + \beta \sum_s h(s') W^{union}\left(S'_G\right)$$

subject to

$$(35) \quad \frac{U_N(x_G)}{p_N} = -\frac{U_L(x_G)}{p_T}$$

$$(36) \quad \frac{U_T(x_G)}{p_T} \geq -\frac{U_L(x_G)}{p_T}$$

$$(37) \quad p_T C_T(x_G) \leq m,$$

where if (37) is a strict inequality, then (36) holds as an equality, and

$$(38) \quad \gamma \frac{-U_L(x_G)}{p_T} = \beta \sum_{s'} h(s') \frac{U_T(m', x'_H, S'_H)}{p_T(x'_H, S'_H)}$$

$$(39) \quad L(x_G) = C_T\,(x_G) + \frac{C_N(x_G)}{A}$$

for all $x_G = (m, p_N, v)$. These constraints capture the market clearing conditions and first order conditions for all the consumers in the union.

Under our preferences, this problem can be simplified because it turns out that the Markov equilibrium has a degenerate distribution for money holdings across countries.

Lemma 2. In any Markov equilibrium in a monetary union, given any initial distribution of money at the beginning of the period, the end-of-period money holdings are concentrated on a single point.

The proof of this lemma has two ideas.

For the first idea, note that the consumer first order condition (38) implies that the marginal cost of earning one unit of money today must be equated to the expected marginal utility that money provides when used to purchase traded goods tomorrow. Since preferences are quasi-linear in labor and nominal wages are equal across countries in the union, this first order condition implies that the expected marginal utility from one unit of money tomorrow must also be equal across countries. If, however, consumers have differing levels of money balances at the end of the period and the cash-in-advance constraint binds in at least one state in the next period, then these consumers have different expected marginal utility from one unit of money tomorrow. This argument yields a contradiction.

The second idea is that combining the incentives of the monetary authority to correct markup distortions, together with the incentives of the nontraded goods firms to set their prices at a markup over expected marginal costs, implies that the cash-in-advance constraint is always binding for reasons similar to those under flexible exchange rates. Combining these two ideas gives Lemma 2.

Lemma 2 implies that regardless of the distribution of money holdings entering period 0, the distribution of money holdings in all future periods is degenerate. In keeping with our assumption that all countries are ex ante identical, we assume that the initial distribution is also degenerate. Then the normalized level of money balances m is one in each country in all periods. (Of course, the absolute level of money balances will typically be changing over time.) Thus, as in the flexible exchange rate case, we can drop m from the individual state, and thus λ_G is a distribution only over (p_N, v). Since $S'_G\ x'_H, S'_H$ are determined by agents in the future and are independent of the choice of current policy, the continuation value and the right side of (38) are also independent of the current money growth rate choice. Since γ appears only in (38), we can use this constraint to eliminate γ as a choice variable and drop this condition also.

Here, the state confronting the monetary authority is a distribution of nontraded goods prices $\{p_N(s)\}$ and an aggregate shock z. Given this state, the primal Markov problem becomes

$$(40)\quad \max_{C_T(s),C_N(s),p_T} \sum_v g(v)\left[\alpha \log C_T(s) + (1-\alpha)\log C_N(s) - \psi\left(C_T(s) + \frac{C_N(s)}{A}\right)\right]$$

subject to

$$C_T(s) = \frac{1}{p_T} \text{ and } C_N(s) = \frac{1-\alpha}{\psi}\frac{p_T}{p_N(s)} \text{ for each } s = (z, v),$$

where we have imposed the result that the cash-in-advance constraint is binding. Let the maximized value of this problem be denoted $U(\{p_N(z,v)\}, z)$. Because policy in the monetary union is chosen to maximize an equally weighted sum of utility of all countries, the weights $g(v)$ in the summation in (40) represent the fraction of all countries with country-specific realization v. Since this fraction also represents the probability that an individual country will experience a country-specific realization v, the maximized value $U(\{p_N(z,v)\}, z)$ is also the expected utility for any individual country.

Solving this problem gives the best response of the monetary authority to any given $\{p_N(s)\}$ and z, which can be written as $p_T = p_T^{Union}(\{p_N(s)\}, z)$. It turns out that this best response only depends on a simple summary statistic of the distribution of nontraded goods prices, namely $E(1/p_N(s)|z)$, the conditional mean of the inverse of these prices. We can then write the best response as

$$(41)\quad p_T^{Union}(\{p_N(s)\}, z) = F\left(E\left(\frac{1}{Ap_N(s)}|z\right)\right)$$

for the same quadratic function F defined under flexible exchange rates in (32). In equilibrium, since nontraded goods prices are set as a markup over marginal cost, the price of traded goods must satisfy the following fixed point equation:

$$(42) \quad \bar{p}_T(z) = F\left(E\left(\frac{\theta(s)}{\bar{p}_T(z)}|z\right)\right).$$

Using this value, it is easy to solve for the rest of the allocations from the constraints.

Lemma 3. The allocations in the Markov equilibrium in a monetary union satisfy

$$(43) \quad C_T^{union}(s) = \frac{\alpha}{\psi} - \frac{1-\alpha}{\psi}\left(1 - E_v(\theta|z)\right) \text{ and } C_N^{union}(s) = \frac{1-\alpha}{\psi}\theta(s)A$$

and $L(s) = C_T(s) + C_N(s)/A$ where $s = (z, v)$.

Note that here the normalized price of traded goods is given by $p_T^{union} = 1/C_T^{union}(s)$ and hence depends on the average of the markup shocks in the union.

The analog of Proposition 4 applies here: a mean-preserving spread in the aggregate component of the markup shock worsens the time inconsistency problem in that the differences in welfare between the commitment and the no commitment outcomes increase. Here, however, a mean-preserving spread to the country-specific component of the markup shock has no effect on the time inconsistency problem because in a union, policy does not react to country-specific shocks.

C. Comparing Welfare

We now compare welfare under flexible exchange rates with that under a monetary union. We show that with only markup shocks, forming a union is beneficial and these benefits are increasing in the variability of country-specific shocks. We then introduce productivity shocks and show that if the country-specific volatility of productivity shocks is sufficiently small relative to that of markup shocks, then a monetary union is preferred to flexible exchange rates.

Only Markup Shocks

Comparing the allocations (34) with those in (43), we see that the allocations under flexible exchange rates differ from those in a monetary union only with respect to the consumption of the traded good and the labor needed to produce it. Using the expressions for tradable and nontradable consumption under the two regimes in the objective function and simplifying, we see that the difference in value for a given initial aggregate state z between the welfare in a union and that under flexible exchange rates is

$$K\left(E\left[\theta|z\right]\right) - E\left[K(\theta)|z\right],$$

where the function $K(\theta) = \alpha \log\left((1-\alpha)(\theta-1)+\alpha\right)$. Since the function $K(\theta)$ is strictly concave in θ, the welfare difference between the regimes is nonnegative and is strictly positive whenever there is variability in the country-specific shock v and is increasing in this variability.

Proposition 5. With only markup shocks, the ex ante utility in the Markov equilibrium for a monetary union is strictly higher than the ex ante utility in the Markov equilibrium with flexible exchange rates. Moreover, a mean-preserving spread in the country-specific component of the markup shock θ increases the gains from forming a union.

The idea behind this proposition is that because of concavity of preferences over traded consumption goods, the ex ante welfare associated with the Markov equilibrium in a monetary union is higher than that under flexible exchange rates.

Interestingly, inflation rates are not only less volatile but also lower on average in a union than they are under flexible exchange rates. To see this result, consider the inflation rates in the tradable and nontradable sectors from state s at one date to state s' at the next. Under flexible exchange rates, these inflation rates are given by

$$\pi_T^{flexible}(s,s') = G(\theta(s)) \text{ and } \pi_N^{flexible}(s,s') = \frac{\theta(s)}{\theta(s')}G(\theta(s)),$$

and in the union they are given by

$$\pi_T^{union}(s,s') = G(E(\theta|z)) \text{ and } \pi_N^{union}(s,s') = \frac{\theta(s)}{\theta(s')}G(E(\theta|z)),$$

where $G(\theta) = \beta\alpha/\left[(1-\alpha)\theta - (1-2\alpha)\right]$. The convexity of G implies that in a monetary union, inflation not only is less volatile than it is under flexible exchange rates but also is lower on average. This lower and less volatile inflation rate is beneficial because it results in distortions in the consumption in the tradable good that are on average lower and less volatile.

Both Shocks

When we allow for both markup shocks and productivity shocks, we have two competing forces. Forming a union has *credibility benefits*: doing so effectively commits the country to not react to the country-specific component of its markup shocks. But forming a union also has *Mundellian losses*: doing so also prevents the country from reacting to the country-specific component of its productivity shocks. Our main result is a new optimal currency area criterion.

Proposition 6. When the volatility of markup shocks is sufficiently high relative to that of productivity shocks, the credibility benefits are higher than the Mundellian losses and forming a union is preferable to flexible exchange rates. In contrast, when the reverse is true, flexible exchange rates are preferred to a union.

The first part of this result immediately follows from Proposition 5 and continuity of the equilibrium values in the parameters of the model. The proof of the second part essentially mimics the argument with commitment.

It is useful to develop a simple approximation that allows us to determine how large markup shocks must be relative to productivity shocks for a union to be beneficial. The approximation is needed because when productivity shocks are stochastic, the Markov equilibrium does not have a closed-form solution. We take a second order Taylor approximation of the objective function and a first order approximation to the price setting rule. This approximation leads to the linear-quadratic policy problem similar to the reduced-form model in Alesina and Barro (2002). Under this approximation, the welfare gains of forming a monetary union are given by

$$(44)\quad W^{union} - W^{flex} = \frac{1}{\kappa} var(\log \theta_v) - \left(\frac{1}{1+\kappa}\right) var(\log A_v),$$

where $\kappa = (1-\alpha)\mu_\theta / \left[(2\alpha - 1) - (1-\alpha)\mu_\theta\right]$ and $var(\log \theta_v)$ and $var(\log A_v)$ are the country-specific variances of $\log \theta$ and $\log A$ and μ_θ is the mean value of θ. (See the Supplement for details.)

Consider now the welfare gains that result from forming a union. The first term in (44) represents the credibility gains of a monetary union: entering a union allows the country to avoid reacting to country-specific markup shocks, which simply add unwanted volatility to the consumption of traded goods. The second term in (44) represents the standard Mundellian losses associated with the inability to respond to productivity shocks. Thus, there is a cutoff value of the relative variances such that forming a union is preferable to staying with flexible exchange rates if and only if

$$\frac{var(\log A_v)}{var(\log \theta_v)} < \frac{1+\kappa}{\kappa}.$$

We complement this expression with Figure 1, which gives the exact solution for the value of utility in a Markov equilibrium under the two regimes as we vary the relative volatility of the country-specific component of the productivity shock in the nontradable sector.[3] The figure illustrates that there is a cutoff level on the relative variances of these shocks, $var(\log A_v)/var(\log \theta_v)$, such that it is preferable to form the union if and only if these relative variances are below this level.

5. Majority Voting

So far we have considered a monetary union in which monetary policy is set cooperatively to maximize the sum of member countries' utility. Here we show that our results continue to hold when monetary policy is set by majority vote. To do so we use the Median Voter Theorem which states that if preferences are single-peaked over a policy variable, voters are heterogenous with respect to a single-dimensional characteristic, and the most preferred outcomes for each voter are monotone in the characteristic, then the equilibrium outcome with majority voting coincides with the most preferred choice of the voter with the median characteristic.

To begin we suppose that countries are affected only by markup shocks. In this case the policy variable for the union can be thought of as a choice of the price of traded goods $p_T(z)$. We first show that the preferences of the representative consumer in each country are

[3] We parameterize the model by considering a simple case with no aggregate shocks: $\theta(\nu_1) \in \{1.1, 1.2\}$ and $A(v_2) \in \{1-\varepsilon, 1+\varepsilon\}$, where $g^1(\nu_1)$ and $g^2(\nu_2)$ are uniform and we vary $\varepsilon \geq 0$.

single-peaked in $p_T(z)$. Each country is characterized by a price of nontraded goods, p_N. As with a cooperative monetary union, it is straightforward to show that the cash-in-advance constraint is always binding. For any given choice of p_T, the allocations for a country with price of nontraded goods p_N are given by

$$C_T = \frac{1}{p_T} \quad \text{and} \quad C_N = \frac{1-\alpha}{\psi}\frac{p_T}{p_N}.$$

Substituting these allocations into the utility function, we have that the value of utility at the price vector (p_N, p_T) is

$$u\left(p_T | p_N\right) = \alpha \log \frac{1}{p_T} + (1-\alpha) \log\left(\frac{1-\alpha}{\psi}\frac{p_T}{p_N}\right) - \psi\left[\frac{1}{p_T} + \frac{1-\alpha}{\psi}\frac{p_T}{p_N}\frac{1}{A}\right]$$

Clearly, this value is strictly concave and, hence, single-peaked in p_T. The most preferred outcome for each country is given by $p_T = F\left(1/Ap_N\right)$, where the function F is the same quadratic function defined above. It is straightforward to show that this most preferred outcome is monotone in p_N. The Median Voter Theorem implies the policy p_T chosen in the majority voting game coincides with the most preferred policy for the country with the median value of p_N. Thus, letting p_N^{median} denote the median of the prices of nontraded goods, in any equilibrium,

$$(45) \quad p_T^{majority}(\{p_N(s)\}) = F\left(\frac{1}{Ap_N^{median}}\right)$$

Next, optimal price setting behavior by nontraded goods producers implies that in any equilibrium

$$(46) \quad p_N(s) = \frac{p_T^{majority}(\{p_N(s)\})}{A\theta(s)}.$$

Taking medians on both sides of (46) and substituting it into (45) gives that the equilibrium price of traded goods must satisfy

$$(47) \quad p_T^{majority}(\{p_N(s)\}) = F\left(\frac{\theta^{median}}{p_T^{majority}(\{p_N(s)\})}\right).$$

Comparing (42) with (47) we see that the traded goods price under majority voting coincides with that in a cooperative monetary union if $\theta^{median}(z) = E(\theta|z)$ which will occur if θ has a symmetric distribution. Clearly, all the other allocations are identical as well.

With flexible exchange rates clearly the cooperative outcome is an equilibrium outcome with majority voting. Since it also the most preferred outcome for every country, it is the unique equilibrium outcome under a wide variety of refinements of the majority voting game.

This result immediately implies that the analog of one of our main proposition, namely Proposition 5, applies with majority voting.

Proposition 7. With only markup shocks with a symmetric distribution, the ex ante utility in the Markov equilibrium for a monetary union with majority voting is strictly higher than the ex ante utility in the Markov equilibrium with flexible exchange rates. Moreover, a mean-preserving spread in the country-specific component of the markup shock θ increases the gains from forming a union.

With both types of shocks, it turns out that each country can be characterized by a summary statistic, $1/(Ap_N)$ so that the median voter theorem applies and we have the analog of the other main propositions, Proposition 6.

Proposition 8. If markup shocks have a symmetric distribution, then when the volatility of markup shocks is sufficiently high relative to that of productivity shocks, the commitment benefits are higher than the Mundellian losses and forming a union is preferable to flexible exchange rates. In contrast, when the reverse is true, flexible exchange rates are preferred to a union.

6. Criteria in Terms of Macroeconomic Aggregates

So far we have stated our criterion in terms of properties of the stochastic processes for productivity and markups. A large empirical literature has examined whether countries are good candidates for forming a union by looking at the behavior of simple functions of standard macroeconomic aggregates such as the country-specific components of output and real exchange rates. The standard view in the literature is that countries are poor candidates for forming a monetary union if the variances of the country-specific components of output and real exchange rates are large. (See, for example, Alesina, Barro, and Tenreyro (2003) and the references therein.)

Viewed through the lens of our model, this standard view can be misleading: both with and without commitment, even when the variances of the country-specific components of output and real exchange rates are both high, forming a union may be desirable. The key reason our model gives a different prediction from the standard view is that our model implies that the desirability of forming a union depends critically on the source of the shocks, even if these shocks induce similar volatilities in real exchange rates and outputs.

For example, if under commitment a group of countries have large country-specific movements in real exchange rates and output, it is less costly for these countries to form a union if these movements are driven mostly by markup shocks and more costly if they are driven by productivity shocks. Thus, one subtlety is that we need a criterion that is based on observables but can differentiate between these two scenarios. The added subtlety is that the map between observables and shocks is itself a function of the stand we take on commitment: under commitment, policy does not react to markup shocks, whereas under no commitment, it does.

To translate our criterion on shocks into a criterion on macroeconomic aggregates, we use our model to express output ($C_T(s^t)^\alpha C_N(s^t)^{1-\alpha}$) and real exchange rates as functions of shocks and use these functions to rewrite our criterion in terms of observables. We begin by relating output and real exchange rates to the consumption and prices of traded and nontraded goods. To do so, note that we can write output and real exchange rates relative to their world averages in log deviation form as

$$(48)\quad \log y\left(s\right) = \alpha \log C_T\left(s\right) + (1-\alpha)\log C_N\left(s\right) - E_v\left[\alpha \log C_T\left(z,v\right) + (1-\alpha) C_N\left(z,v\right)\right]$$

$$(49)\quad \log q(s) = (1-\alpha)\log p_N(s)/p_T(s) - (1-\alpha)E_v\left[\log p_N(z,v)/p_T(z,v)\right],$$

where the second equation is derived in the Supplement. In order to make clear the role of the country-specific components of the shocks, we assume in what follows that the productivity shocks and the markup shocks can be expressed as multiplicative functions of an aggregate component and a country-specific component and that the two components are independent of each other. Specifically, we assume that $A\left(z,v\right) = A_z\left(z\right) A_v\left(v\right)$ and $\theta\left(z,v\right) = \theta_z\left(z\right)\theta_v\left(v\right).$

Next, suppose that the countries are contemplating forming a union with commitment and currently are in a flexible exchange rate regime pursuing Ramsey policies. Thus, $p_N/p_T = 1/(\theta(s)A(s))$, $C_T = \alpha/\psi$, C_N is given by (18), and since shocks have a multiplicative form,

$$\log y\left(s\right) = (1-\alpha)\left[\log A_v(v) + \log \theta(v)\right] \quad (50)$$

$$\log q(s) = -(1-\alpha)\left[\log A_v(v) + \log \theta(v)\right]. \quad (51)$$

Thus, given any observed volatility of the country-specific components of output and real exchange rates, Proposition 2 makes clear that large welfare losses are associated with forming a union only if most of the volatility in these variables is arising from the productivity shocks. Clearly, since only the sum of the country-specific shocks enters these two expressions, we cannot disentangle the separate roles of each shock from output and real exchange rates alone.

Interestingly, under commitment we can use a simple statistic to infer the volatility of productivity shocks: the volatility of the growth rate of the nominal exchange rate. To see why, note that in log deviation form, we have

$$\log e(s')/e(s) = \left[\log P_T\left(s'\right) - E_v \log P_T(z', v')\right] - \left[\log P_T\left(s\right) - E_v \log P_T(z, v)\right]. \quad (52)$$

Under a regime of flexible exchange rates in which countries are pursuing Ramsey policies, we show in the Supplement that can rewrite (52) as

$$\log e(s')/e(s) = \log A_v(v'). \quad (53)$$

Hence, under commitment, the Mundellian costs associated with moving from a regime of flexible exchange rates to a union are proportional to the country-specific variance of the country's nominal exchange rates before it enters the union.

Next, suppose that the countries are contemplating forming a union without commitment and currently are in a flexible exchange rate regime pursuing Markov policies. Without commitment, the nominal exchange rate is no longer particularly useful. Instead, we use a log-linear approximation of the Markov outcomes for output and real exchange rates and show that in log deviation form, they are given by

$$\text{(54)}\quad \log y(v) = \frac{1-2\alpha}{1+\kappa}\log A_v(v) + \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)\log\theta$$

$$\text{(55)}\quad \log q(v) = (1-\alpha)\log\theta(v) - \left(\frac{1-\alpha}{1+\kappa}\right)\log A(v),$$

where κ is given in (44). Clearly, these two expressions can be solved to express the variances of the country-specific shocks in terms of the variances of the endogenous variables. Doing so and using (44) gives that forming a union is optimal if and only if the relative volatility of output to real exchange rates is sufficiently high in that

$$\text{(56)}\quad var\left(\log y\right)/var\left(\log q\right) > \omega_q/\omega_y,$$

where the constants ω_q and ω_y are given in the Supplement. Here, of course, these volatilities must be calculated from a regime of flexible exchange rates in which countries are following their Markov policies.

Note that the criteria for forming a union differ greatly depending on the extent of commitment. These differences arise both because the criteria in terms of shocks differ and because the map between observables and shocks differs. To see the former, compare (19) and (44). To see the latter, compare (50) and (51) with (54) and (55).

The criterion developed in (56) is novel and stands in contrast to all of the criteria developed in the literature on optimal currency unions. Note that we have derived this criterion from first principles using the workhorse model in international macroeconomics.

7. The Stable Configuration of Unions

So far we have assumed that all countries are symmetric and studied their incentives to form a monetary union rather than stay under a regime of flexible exchange rates. We begin with an analysis with two groups of countries and then briefly analyze the stable configuration of unions with two or more groups.

We introduce asymmetry by assuming that one group of countries (the *North*) is less distorted than another group of countries (the *South*) in that the South's distortions are both larger on average and more variable than those in the North. We imagine that the countries in the North have already formed a union and are choosing the number of countries from the South to let in.

Our first result is that any Southern country gains from joining the Northern union. The reason is that since the North is less distorted on average, it has greater credibility in policy, and the South enjoys increased credibility by joining the union.

Our second result is that if the distortions in the South are not perfectly correlated with those in the North, then as long as the average distortions in the South are not too large, the North will find it optimal to admit some Southerners into their union. The key idea here is that even if each country in the South has a worse time inconsistency problem than each country in the North, admitting some Southerners into the union may be beneficial for the North because the imperfect correlation of distortions leads monetary policy to be less sensitive to fluctuations in the aggregate distortions in the North.

Our model captures the idea that in the European Monetary Union many of the Southern European countries gained credibility by joining a union populated mainly by Northern European countries and hence reduced the inflation bias in the Southern countries. It also provides a motivation for why the Northern European countries might want to admit countries with historically lower credibility in monetary policy.

Next we show that the North will admit fewer countries from the South the greater are the South's mean distortions, the greater is the variance of these distortions, and the greater is the correlation of their distortions with those in the North. We then show that the optimal configuration of unions takes a hierarchical form.

A. Unions with Two Groups of Countries

More formally, we imagine there are two groups of countries, North, N, and South, S, with a measure $\bar{n}^N$ of Northern countries and a measure $\bar{n}^S$ of Southern countries. Here, we focus on an economy with only markup shocks, and we let the markup shocks in the North be $\theta^N(s_t)$ and those in the South be $\theta^S(s_t)$. These shocks are realized at the beginning of the period (and, as before, we drop the subscript 1 denoting the beginning of the period for simplicity). Throughout, we assume that the Southern countries are *more distorted* than the

North in that

(57) $E\theta^S \leq E\theta^N$ and $var(\theta^S) \geq var(\theta^N)$.

Given (16), we see that our condition that Southern countries are more distorted implies that the South has wedges that are both larger on average and more volatile than those in the North. We assume that (57) holds in our later comparisons.

We turn to asking whether a union of Northern countries should admit Southern countries. The Northern countries understand that if they let in a measure n^S of Southern countries, then the policy followed in the mixed union will be one that maximizes a weighted average of the utility of the Northern and Southern countries, where the weights are proportional to group size in that

$$\lambda^N = \frac{\bar{n}^N}{\bar{n}^N + n^S},\ \lambda^S = \frac{n^S}{\bar{n}^N + n^S},$$

so that the resulting vector $\lambda = (\lambda^N, \lambda^S)$ satisfies $\lambda^i \in [0, 1]$ and $\lambda^N + \lambda^S = 1$. For now, we assume that the Southern countries are originally under flexible exchange rates and will join the union only if they receive higher utility in the union than under a regime with flexible exchange rates.

To determine the size of the union, we begin by solving for the Markov equilibrium and the welfare of the Northern and Southern countries for any given composition of the union. We then ask what composition maximizes the welfare of the Northern countries given that the Southern countries that join the union must be made better off by doing so.

Consider the Markov equilibrium for a particular composition of the union (λ^N, λ^S). Note that here the distribution of money holdings is degenerate, since the analog of Lemma 2 applies and the cash-in-advance constraint binds in both the North and the South. The resulting allocations are summarized in the next lemma.

Lemma 4. With constant productivity A^N, A^S, the allocations in the Markov equilibrium in a monetary union with composition λ are

$$(58)\quad C_T^i(s,\lambda) = \frac{\alpha}{\psi} - \frac{1-\alpha}{\psi}\left(1 - \sum_{i=N,S} \lambda^i E_v\left(\theta^i|z\right)\right),\ C_N^i\left(s,\lambda\right) = \frac{1-\alpha}{\psi} A^i \theta^i\left(s\right),$$

and $L^i(s,\lambda) = C_T^i(s,\lambda) + C_N^i(s,\lambda)/A^i$ for $i = N, S$ where $s = (z, v)$.

The expected welfare of both Southern and Northern countries for a given composition is then given by

$$(59)\quad W^i\left(\lambda\right) = \alpha E \log C_T^i(s,\lambda) + (1-\alpha) E \log C_N^i\left(s,\lambda\right) - \psi E L^i(s,\lambda).$$

Note that the allocations imply that Northern and Southern countries rank different compositions the same way: if the North prefers composition $\hat{\lambda}$ to λ, then so does the South. The reason is simply that the North and the South have the same stochastic process for traded goods consumption and have stochastic processes for nontraded goods consumption that are independent of the composition of the union.

We then turn to asking what is the optimal measure of Southern countries that the North finds optimal to admit to the union. Formally, this problem is to solve

$$(60)\quad \max_{\lambda} W^N(\lambda)$$

subject to the feasibility constraint

$$(61)\quad \lambda^N \geq \frac{\bar{n}^N}{\bar{n}^N + \bar{n}^S}$$

and the participation constraint of Southern countries $W^S(\lambda) \geq W^S_{flex}$, where W^S_{flex} is defined from the allocations under flexible exchange rates given in Lemma 1. We will assume that $\bar{n}^S$ is sufficiently large compared with $\bar{n}^N$ so that the feasibility constraint (61) does not bind. It is straightforward to prove that if the Southern countries are more distorted in the sense of (57), then they always prefer joining the union with the North to staying on their own. Hence, we drop the participation constraints in all that follows.

In the Supplement, we show that the solution to this problem is approximately given by

$$(62)\quad \lambda^S = (1-\rho\frac{\sigma_S}{\sigma_N})\frac{var(\theta^N)}{var\left(\theta^N-\theta^S\right)} - \frac{\left(\bar{\theta}^N-\bar{\theta}^S\right)}{var\left(\theta^N-\theta^S\right)}\bar{C}_T$$

whenever this expression is positive and zero otherwise. This expression shows how the measure of Southern countries varies as the stochastic process for distortions in the South varies. From this expression we immediately have the following proposition.

Proposition 9. An increase in the correlation of the distortions in the North and the South decreases the measure of Southerners. Likewise, an increase in the mean distortions in the South decreases the measure of Southerners. When distortions are uncorrelated and have equal means, then $\lambda^S = var(\theta^N)/\left[var\left(\theta^N\right)+var\left(\theta^S\right)\right]$, so that an increase in the volatility of distortions in the South decreases the measure of Southerners.

Now we ask what configurations of unions will form in this model. We will focus on configurations that are *stable* in the sense that there is no deviation by a group of countries to form their own union that makes all of the members of the deviating group weakly better off and at least one type of them strictly better off.[4]

In developing our analysis, we will use the result that all countries rank unions with different compositions in the same way. Hence, our economy has a hierarchy of unions: a most preferred union, a second most preferred union, and so on. When there are two types of countries, say North and South as above, then there is a unique stable configuration of unions: a preferred union, which is a mixed North-South union with the mixture chosen as above, say at $\hat{\lambda}$, and a less preferred union. If there are sufficiently many Southern countries so that the feasibility constraint (61) holds, then the less preferred union consists purely of Southern countries. If this constraint is violated, then there are not enough Southerners to reach this optimal mix when all the Northerners are included in the preferred union and the less preferred union consists purely of Northern countries. In either case, since the mixed

[4]More formally, let $\{n_i\}_{i=1}^I$ with $n_i = (n_i^N, n_i^S)$ with $\sum_i n_i^N = \bar{n}^N$ and $\sum_i n_i^S = \bar{n}^S$ and $n_i^N + n_i^S > 0$ for each i be a *partition* of the union, and let $V_i = (V^N(n_i), V^S(n_i))$ be the associated welfare. A configuration $\{n_i\}_{i=1}^I$is *stable* if there does not exist a deviating group of countries $\{\hat{m}_i\}_{i=1}^I$ with $\hat{n}_i \leq n_i$ such that $V^N(\sum \hat{n}_i) \geq V_i^N$ for all i such that $\hat{n}_i^N > 0$ and $V^S(\sum_i \hat{n}_i) \geq V_i^S$ for all i such that $\hat{n}_i^S > 0$, where at least one of these previous inequalities is strict.

union maximizes the welfare of both types of countries, neither type has an incentive to defect. We summarize this discussion as follows.

Proposition 10. Under (57) and (61), a mixed North-South union with λ chosen to solve (60) and a pure Southern union consisting of the remaining Southern countries is the unique stable configuration of unions. The mixed union has higher welfare for both countries than the pure Southern union.

B. Unions with Many Groups of Countries

We briefly consider a more general case with three groups of countries: North (N), Middle (M), and South (S). Let these groups be ranked in a pecking order in that mean distortions and volatilities are increasing from North to South.

The unique stable configuration of unions has a simple hierarchy form and can be constructed as follows. In the highest-ranked union, the weights $\lambda_1 = (\lambda_1^N, \lambda_1^M, \lambda_1^S)$ maximize $W^N(\lambda)$, whereas in the second-ranked union, the weights $\lambda_2 = (0, \lambda_2^M, \lambda_2^S)$ maximize $W^M(\lambda)$ subject to the restriction that $\lambda^N = 0$. In the third-ranked union, the weights are $\lambda_3 = (0, 0, 1)$. It is straightforward to construct the masses of countries (n_i^k) in each of these groups. In the construction we assume that the measure of countries is such that $\bar{n}^M/\bar{n}^N$ and $\bar{n}^S/\bar{n}^M$ are sufficiently large so that the configuration we construct is feasible. In the Supplement we discuss a more general case.

Proposition 11. If $\bar{n}^M/\bar{n}^N$ and $\bar{n}^S/\bar{n}^M$ are sufficiently large, the configuration $\lambda_1, \lambda_2, \lambda_3$ given above is the unique stable configuration of unions. Furthermore, at the stable configuration, $W^i(\lambda_1) > W^i(\lambda_2) > W^i(\lambda_3)$ for $i = N, M, S$.

If one is willing to think of the countries of Southern Europe as relatively more distorted than the countries of Northern Europe, then this proposition provides some perspective on the idea of splitting the current European Monetary Union into two unions: one consisting of Northern countries and one consisting of Southern countries. Our theory suggests that such a split is not desirable unless the distortions in the Southern countries are sufficiently severe.

8. Anchor-Client Unions

So far we have considered environments in which the defining feature of a monetary union is that monetary decisions are made jointly by members of the union. To relate our analysis to some of the existing literature, we turn now to analyzing a very different kind of monetary union that we call an *anchor-client union.* The anchor chooses monetary policy solely to maximize its residents' welfare, and the client maintains a fixed exchange rate with the anchor. Such a union is nearly identical to one in which the client country *dollarizes.*[5]

Here, we find that the similarity of markup shocks is irrelevant. This finding is in sharp contrast with our finding that in cooperative or majority rule unions, countries with dissimilar markup shocks have stronger incentives to form a union. A key distinction between this institutional arrangement and our cooperative or majority rule union setup is that here there is no connection between the composition of the union and the policy followed by it. In contrast, when we considered a Northern union that admitted a positive measure of Southern countries, the union's policy endogenously changed as the composition of the union changed. This lack of endogenous feedback turns out to imply that in an anchor-client union, the correlation of the markup shocks between the anchor and client is irrelevant.

Here, we imagine that the client is contemplating adopting the currency of one of a set of potential anchors. We characterize the optimal anchor from a client's perspective. Throughout, we assume that the distortions in the client country are sufficiently large so that adopting the currency of any of these anchors is welfare improving for the client.

Proposition 12. The ranking of potential anchors by the client is independent of the correlation of the markup shocks of the client and the potential anchor.

We then turn to the characterization of the ideal anchor for a given client. The answer is immediate: the ideal anchor is the country that follows the policies that the client country would follow if it had commitment. Obviously, when the client adopts the policy of such an ideal anchor, it achieves its own Ramsey welfare level and cannot do better. The anchor that achieves the Ramsey outcomes for the client is one that has productivity shocks that are identical to those of the client and either has commitment or follows Markov policies and has no distortions, in that $\theta^i \equiv 1$.

Next, suppose that such an ideal anchor is not available but instead there are I potential anchors, all of which have commitment (or follow Markov policies and have $\theta^i \equiv 1$).

[5]The only distinction is that in an anchor-client union, the client gets to keep the seignorage; under dollarization, it does not.

Let $\{\theta(s_{1t}), A(s_{2t})\}$ denote the stochastic processes of the client, and let $\left\{\theta^i(s_{1t}), A^i(s_{2t})\right\}$ for $i = 1, \ldots, I$ denote the processes for the potential anchors. Within this class, the best anchor for the client is the one with a stochastic process for productivity shocks that is closest to that of the client, in the sense made precise in the following proposition.

Proposition 13. Consider a given client country with stochastic process $\{\theta(s_{1t}), A(s_{2t})\}$. The optimal anchor country i^* for the client from a set of potential anchors that have commitment is the one that solves

$$(63) \quad \min_i \log\left(E\left[\frac{A^i(s_{2t})}{A(s_{2t})}\right]\right) - E\left[\log\frac{A^i(s_{2t})}{A(s_{2t})}\right],$$

which in the log-normal case minimizes the variance of the ratio $A^i(s_{2t})/A(s_{2t})$.

Notice that (63) holds for general specifications of the stochastic processes for the client and the anchor. If we assume that the processes for productivity shocks have the form $A_i(s_{2t}) = A_z(z_{2t})A_{vi}(v_{2t})$ and $A(s_{2t}) = A_z(z_{2t})A_v(v_{2t})$ so that the anchors and the client have a common aggregate component to productivity shocks, then the optimal anchor i^* solves

$$\min_i \log\left(E\left[A_{vi}(v_{2t})\right]\right) - E\left[\log A_{vi}(s_{2t})\right],$$

which in the log-normal case implies that it is optimal to pick the anchor with the lowest variance of country-specific shocks.

We turn now to the optimal choice of an anchor by the client when the anchor follows Markov policies and the set of potential anchors does not include one with no distortions. For simplicity, assume that the set of potential anchors all have the same mean distortions $E\theta^i$ but have different variances. Using a second order approximation for welfare gives the following proposition.

Proposition 14. Consider a given client country with stochastic process $\{\theta(s_{1t})\}$. The optimal anchor country i^* for the client from the set of potential anchors with $E\theta^i$ at the same level is the one that has the smallest value of

$$(64) \quad \frac{var(\log A^i)}{1+\kappa^i} - 2\frac{\rho\sigma_{A^i}\sigma_A}{1+\kappa^i} + \frac{var(\log\theta^i)}{\kappa^i},$$

where $\kappa^i = (1-\alpha)\mu_{\theta^i} / [(2\alpha - 1) - (1-\alpha)\mu_{\theta^i}]$ and μ_{θ^i} is the mean θ^i.

Here, the client prefers an anchor with correlated productivity shocks for usual Mundellian reasons. The client also prefers an anchor with low variability of markup shocks because such shocks only introduce undesirable fluctuations in inflation.

In sum, in an anchor-client union, the lack of feedback between the composition of the union and the policies pursued by the union makes the selection of the best anchor by a client simple: find a country with small and stable distortions that has highly correlated productivity shocks. In contrast, our criterion for forming cooperative or majority rule unions is very different because of the endogenous feedback from the composition of the union to its policies.

9. Extensions

Our model has been purposely set up to have the minimal forces needed to make our points. Here, we discuss alternative assumptions about the timing of shocks and more general stochastic processes for these shocks.

A. Timing of Shocks

Consider the timing of shocks. We assumed that the markup shocks are realized before the nontraded goods prices are set and that productivity shocks are realized after these prices are set.

Consider first the markup shocks. In the Supplement we have three microfoundations for them. In each of them, the whole point of the markup shocks is to affect the incentive of the nontraded goods firms to change their prices and hence for the monetary authority to inflate. In each of these three settings, if we made the shocks realized after the nontraded goods prices are set, they clearly are irrelevant because by the time they are realized, it is too late for the nontraded goods firm to react to them.

Consider next the productivity shocks. For simplicity, suppose that total productivity is the product of two components $A_1(s_{1t})$, and $A_2(s_{2t})$, where the first component is realized before the nontraded goods prices are set and the second component is realized after these prices are set. Clearly, the nontraded goods prices can adjust to the realization of the first component; thus, in this sense, nontraded goods prices are not sticky with respect to this component. Hence, there is neither a need for nor an advantage to having the exchange rate move when this component is realized, since prices play their usual allocative role. Thus, this component will play no role in our comparison of flexible exchange rates and a monetary union. We formally demonstrate this result in the Supplement in our version of the Alesina-Barro model.

B. Serial Correlation of Shocks

Consider next the serial correlation of the shocks. Under commitment, our formula immediately extends to an arbitrary specification of uncertainty.

Proposition 2'. Under commitment, the utility difference between the flexible exchange rate regime and the monetary union is given by

$$(65)\quad (1-\alpha)\sum_{t}\sum_{h_{1t}}\beta^{t}\Pr\left(h_{1t}\right)\left[\log\left(\sum_{v_{2t}}\Pr\left(v_{2t}|h_{1t}\right)\frac{1}{A(s_t)}\right)-\sum_{v_{2t}}\Pr\left(v_{2t}|h_{1t}\right)\log\left(\frac{1}{A\left(s_t\right)}\right)\right].$$

where $h_{1t}=(s^{t-1},z_t,v_{1t})$ is a history of shocks. Here, as before, nontraded goods prices in period t are sticky only with respect to the period t innovations in the productivity shock.

Consider next our results without commitment. Proposition 6 is now modified as follows.

Proposition 6'. When the volatility of markup shocks is sufficiently high relative to that of the innovation in productivity shocks, the credibility benefits are higher than the Mundellian losses and forming a union is preferable to flexible exchange rates. In contrast, when the reverse is true, flexible exchange rates are preferred to a union.

Note that in this proposition, the relevant comparison is between the unconditional variance of the country-specific markup shock and innovation variance of the productivity shock. The reason is that nontraded goods prices are effectively flexible with respect to all shocks in the information set of nontraded goods producers and are sticky only with respect to innovations in the productivity shocks.

Next, we modify our criterion in terms of observables to allow both markup shocks and productivity shocks to follow autoregressive processes. In the Supplement, we show that with serially correlated shocks for forming a union the criterion has the identical form as with independent shocks: forming a union is optimal if and only if the relative volatility of output to real exchange rates is sufficiently high in that

(66) $var\left(\log y\right)/var\left(\log q\right) > \omega_q'/\omega_y'$,

where the constants ω_q' and ω_y' are modified versions of ω_q and ω_y.

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10. Supplement

A. An Alesina-Barro model

One illustration of our message is in a reduced-form model in the spirit of Barro and Gordon (1983) and Alesina and Barro (2002). Barro and Gordon (1983) emphasize two benefits of surprise inflation that give rise to two types of temptation shocks: the benefits from exploiting a short-run expectational Phillips curve and the benefits from inflating away nominal government liabilities. Shocks to these benefits are examples of temptation shocks. Thus, interpreted in the light of this model, our criterion becomes: in environments without commitment, countries with dissimilar shocks to the Phillips curve or dissimilar shocks to government revenue or expenditure are good candidates for forming a union.

Here, we show that an approximation to our general equilibrium model is essentially identical to the reduced-form model in Alesina and Barro (2002). For notational simplicity only we abstract from aggregate shocks. In each period t, each of a continuum of countries draws a vector of country-specific shocks $v_t = (v_{1t}, v_{2t})$, which are i.i.d. both over time and across countries. The probability of the country-specific shocks is $g(v_{1t}, v_{2t}) = g^1(v_{1t})g^2(v_{2t})$. There are three shocks: a *markup* shock, $\theta(v_{1t})$, an *ex ante productivity* shock, $A_1(v_{1t})$, and an *ex post productivity* shock, $A_2(v_{2t})$. We normalize the unconditional mean of the both productivity shocks to be 1 and let $A(v_{1t}, v_{2t}) = A_1(v_{1t})A_2(v_{2t})$.

The timing within the period is as follows: the markup shock and the ex ante productivity shocks are realized, the sticky price P_N is chosen by private agents, the ex post productivity shock is realized, and then the policy P_T and the allocations (C_T, C_N, L) are chosen. The utility of the monetary authority in period t is

(67) $\alpha \log C_T + (1-\alpha)\log C_N - \psi L,$

where $C_T = min\,[\alpha/\psi, M/P_T]$, $C_N = (1-\alpha)P_T/\psi P_N$ and $L = (C_T + C_N/A)$. Note that the expression for C_N comes from the first order condition (7), our functional form, and that $P_T = W$. With our functional form, the rule for the price setters is given by

(68) $$P_N(v_1) = \frac{1}{\theta(v_1)A_1(v_1)} \sum_{v_2} g^2(v_2)\frac{P_T(v_1,v_2)}{A_2(v_2)}.$$

Commitment

The utility of the monetary authority is

$$\sum_v g(v)\left[\alpha \log C_T(v) + (1-\alpha)\log\frac{1-\alpha}{\psi}\frac{P_T(v)}{P_N(v_1)} - \psi\left(C_T(v) + \frac{1-\alpha}{\psi}\frac{P_T(v)}{A(v)P_N(v)}\right)\right].$$

Now adding and subtracting $\log A(v)$ in each state and dropping state-specific constants, we can rewrite this objective function as $\sum_v g(v)U(C_T(v), P_T(v)/\left(A(v)P_N(v)\right))$, where

$$U(C_T(v), \frac{P_T(v)}{A(v)P_N(v)}) = \left[\alpha \log C_T(v) + (1-\alpha)\log\frac{P_T(v)}{A(v)P_N(v)} - \psi\left(C_T(v) + \frac{1-\alpha}{\psi}\frac{P_T(v)}{A(v)P_N(v)}\right)\right].$$

Flexible Exchange Rates. Here the choice variables of the monetary authority can be thought of as $P_T(v)$ and $X(v) = P_T(v)/(A(v)P_N(v))$. Let $\bar{P}_T, \bar{P}_N$ be the commitment outcomes in a deterministic version of the model in which $A_i = 1$ and $\theta = \bar{\theta}$. Here and throughout, we let lower case letters denote log deviations, so for example, $p_T(v) = \log(P_T(v)) - \log(\bar{P}_T)$. The exception to this notation is that η is the log deviation of the markup $1/\theta$ from its mean, that is,

$$\eta = \log(1/\theta(v_1)) - \log(1/\bar{\theta}).$$

We can approximate the objective function with a second order Taylor series expansion around a deterministic steady state. Dropping additive and multiplicative constants gives the objective function

$$(69) \quad -\frac{1}{2}\sum_{v} g(v)\left[(p_T - (a_1 + a_2) - p_N)^2 + \kappa p_T^2\right].$$

The price setting rule is

$$(70) \quad p_N(v_1) = \sum_{v_2} g_2(v_2)(p_T(v) + \eta(v_1) - (a_1(v_1) + a_2(v_2))).$$

The optimal plan determines both the constants $\bar{P}_T$ and $\bar{P}_N$ and the responsiveness of these prices to shocks. Substituting the price setting rule into the objective function gives that (69) becomes

$$-\frac{1}{2}E\left[(p_T - E(p_T|v_1) - a_2 - \eta))^2 + \kappa p_T^2\right],$$

where E denotes the expected value with respect to v and we have used that $Ea_2 = 0$. Note that the ex ante productivity shock does not enter this objective function, so that it is optimal not to respond to it. The reason is simply that prices of nontraded goods are flexible with respect to that shock, so prices perform their usual allocative role. Clearly, the optimal rule is linear in the shocks so that $p_T(a_2, \eta) = Ba_2 + C\eta$ and so that with this form the objective function becomes

$$= -\frac{1}{2}[(B-1)^2\, var(a_2) + var(\eta) + \kappa B^2 var(a_2) + \kappa C^2 var(\eta)].$$

The optimal policy has $C = 0$ because reacting to the markup shock only adds unnecessary variability to inflation. From the first order condition for B, we find that the optimal response to the ex post productivity shock is $B = 1/(1+\kappa)$. Here, simple algebra shows that because the cash-in-advance constraint never binds in the steady state, locally there is no cost to inflation and $\kappa = 0$. To sum up, optimal policy under commitment has no response to ex ante productivity shocks or to the markup shocks and thus

$$(71) \quad p_T^{flex} = a_2.$$

Substituting this response into the objective function and simplifying gives that, up to a constant, welfare under flexible exchange rates is given by

$$(72) \quad W^{flexible} = -\frac{1}{2}var(\eta).$$

Union. Here the choice variables of the monetary authority can be thought of as the union-wide price of traded goods P_T and $X(v) = P_T/(A(v)P_N(v))$. Since there are no aggregate shocks, P_T is a constant so that the $p_T = 0$. Here $p_N(v_1)$ is pinned down by (70). Welfare is now given by $W^{union} = -\frac{1}{2}[var(a_2) + var(\eta)]$. Hence, the welfare gains to flexible exchange rates is given by

$$(73)\quad W^{flex} - W^{union} = \frac{var(a_2)}{2}.$$

Clearly, the gains from flexible exchange rates are increasing in the variance of the country-specific component of ex post productivity shocks and markup shocks are irrelevant.

Without Commitment

Here the monetary authority chooses policy after price setters have made their decisions and after all shocks have been realized. The key difference from commitment is that this authority takes the price of nontraded goods as given.

Flexible Exchange Rates. Maximizing (69) with respect to p_T and taking as given p_N gives the policy rule

$$p_T = \frac{p_N + a_1 + a_2}{1+\kappa}.$$

To find the equilibrium, we substitute this policy rule into the price setter's equation (70) and solve to get the policy rule

$$p_T^{flex} = \frac{1}{1+\kappa} a_2 + \frac{1}{\kappa}\eta$$

and the price setting rule

$$p_N = \frac{1+\kappa}{\kappa}\eta - a_1.$$

The resulting welfare turns out to be

$$W^{flex} = -\frac{1}{2}\left(\frac{\kappa}{1+\kappa}\right) var(a_2) - \frac{1}{2}\left(1 + \frac{1}{\kappa}\right) var(\eta).$$

Union. Here again since there are no aggregate shocks, $p_T = 0$ and $p_N(v_1)$ is pinned down by (70). Hence, welfare is

$$W^{union} = -\frac{1}{2}[var(a_2) + var(\eta)].$$

Thus, without commitment the difference in welfare in the two regimes is proportional to

$$(74)\quad W^{union} - W^{flex} = \frac{1}{\kappa} var(\eta) - \left(\frac{1}{1+\kappa}\right) var(a_2)$$

so that the welfare in the union is higher than under flexible exchange rates if the variance of country-specific markup shocks are sufficiently high relative to that of productivity shocks. Here the cash-in-advance binds in the steady state, so that locally there is a cost of inflation. Simple algebra shows that

$$(75)\quad \kappa = \frac{(1-\alpha)}{(2\alpha - 1)\mu_\eta - (1-\alpha)},$$

where μ_η is the mean of $1/\theta$.

Macroeconomic Aggregates without Commitment

Here we compute the macroeconomic aggregates without commitment. We then use these variables to express our criterion for forming a union in terms of standard macroeconomic aggregates rather than shocks. The allocations are given by

$$c_T(v) = -\frac{1}{1+\kappa}a_2 - \frac{1}{\kappa}\eta,\; c_N(v) = \frac{1}{1+\kappa}a_2 - \eta,$$

where we have suppressed the ex ante productivity shock a_1. Output $y(v) = \alpha c_T(v) + (1-\alpha)c_N(v)$ is given by

$$y(v) = (1-2\alpha)\frac{1}{1+\kappa}a_2 - \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)\eta.$$

The real exchange rate in levels with the rest of the world is proportional to $(P_N(v)/P_T(v))^{1-\alpha}$ so that the real exchange rate in log-deviation form is

$$q(v) = (1-\alpha)\left[\eta - \frac{1}{1+\kappa}a_2\right].$$

The expressions for the variances output and real exchange rates are thus related to the variances of the shocks according to

$$var\,(y) = \left[\frac{1-2\alpha}{1+\kappa}\right]^2 var(a_2) + \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)^2 var(\eta)$$

$$var(q) = (1-\alpha)^2\left[\left(\frac{1}{1+\kappa}\right)^2 var(a_2) + var(\eta)\right].$$

Inverting these two equations gives

$$var(a_2) = \frac{1}{\Delta}\left[(1-\alpha)^2 var(y) - \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)^2 var(q)\right]$$

$$var(\eta) = \frac{1}{\Delta}\left[-\left(\frac{1-\alpha}{1+\kappa}\right)^2 var(y) + \left[\frac{1-2\alpha}{1+\kappa}\right]^2 var(q)\right],$$

where

$$\Delta = \left(\frac{1-\alpha}{1+\kappa}\right)^2\left[(1-2\alpha)^2 - \left(1 - \left(\frac{1+\kappa}{\kappa}\right)\alpha\right)^2\right] < 0.$$

Substituting the expressions for $var(a_2)$ and $var(\eta)$ into (74) gives that forming a union is preferable to a regime with flexible exchange rates only if

$$\frac{var(y)}{var(q)} > \frac{\omega_q}{\omega_y},$$

where

$$(76) \quad \omega_q = \left[\frac{(1-2\alpha)^2}{1+\kappa} + \kappa\left(\frac{\alpha}{\kappa} + 1 - \alpha\right)^2\right] \text{ and } \omega_y = (1-\alpha)^2\left[\frac{1}{1+\kappa} + \kappa\right].$$

B. Microfoundations for the markup equation

In the text we simply posited that the nontraded goods firms set their prices as a stochastically fluctuating markup over the discounted value of the nominal marginal cost of production. Here we show that our model is robust to the details of how this markup arises and why it fluctuates. We do so by providing three different microfoundations for this equation. Since in all three scenarios $Y_N(s^t) = A(s_t)L_N(s^t)$ and the prices $P_N(s^t)$ satisfy (2), the results we derive are identical under all three foundations.

The first two foundations are simple ways to make the imperfectly competitive price setting firms have time-varying market power. The third foundation shows that other forces such as time-varying tax policy will lead to identical results.

Time-varying market power from time-varying elasticities of substitution

The well-cited paper by Smets and Wouters (2007) posits a technology for differentiated products that has time-varying elasticities of substitution between differentiated products. This paper also provides evidence for the quantitative importance of the influence of this time-varying elasticity on aggregates.

Here we follow Smets and Wouters (2007) by making the elasticity of substitution between differentiated traded goods time-varying. Specifically, we assume that the nontraded good in any given country is produced by a competitive final consumption firm using $j \in [0,1]$ intermediates according to

$$Y_N(s^t) = \left[\int y_N(j,s^t)^{\theta(s_{1t})} dj\right]^{1/\theta(s_{1t})}.$$

This firm maximizes

$$P_N(s^{t-1}, s_{1t})Y_N(s^t) - \int P_N(j, s^{t-1}, s_{1t})y_N(j,s^t)dj,$$

where the notation makes clear that, consistent with our timing assumption, the prices of nontraded goods cannot vary with s_{2t}. The demand for an intermediate of type j is thus given by

$$y_N(j,s^t) = \left(\frac{P_N(s^{t-1}, s_{1t})}{P_N(j, s^{t-1}, s_{1t})}\right)^{\frac{1}{1-\theta(s_{1t})}} Y_N(s^t).$$

The intermediate goods are produced by monopolistic competitive firms using a linear technology $y_N(j,s^t) = A(s_{2t})L_N(j,s^t)$. The problem of an intermediate good firm of type j is to choose $P = P(j, s^{t-1}, s_{1t})$ to solve

$$(77) \quad \max_P \sum_{s_{2t}} Q(s^t)\left[P - \frac{W(s^t)}{A(s_{2t})}\right]\left(\frac{P_N(s^{t-1}, s_{1t})}{P}\right)^{\frac{1}{1-\theta(s_{1t})}} Y_N(s^t),$$

where $Q(s^t)$ is the nominal stochastic discount factor. Throughout we will assume that $\theta(s_{1t}) \in (0,1)$, so that the induced demands are elastic and that the optimal price for the monopolist is finite. The solution to this problem gives that all intermediate goods producers j set their prices according to

$$(78)\quad P_N(j,s^{t-1},s_{1t}) = \frac{1}{\theta(s_{1t})}\frac{\sum_{s_{2t}} Q(s^t)Y_N(s^t)\frac{W(s^t)}{A(s_t)}}{\sum_{s_{2t}} Q(s^t)Y_N(s^t)},$$

where $1/\theta(s_{1t})$ is the markup in period t. Since this price does not depend on j, we can write $P_N(j,s^{t-1},s_{1t}) = P_N(s^{t-1},s_{1t})$ so that (78) reduces to the markup equation (2) in the body. This result also implies that the labor hired by each intermediate goods firm within a country is the same, so that $L_N(j,s^t)$ can be written as $L_N(s^t)$ and the final output of nontraded goods is simply $Y_N(s^t) = A(s_{2t})L_N(s^t)$. Thus, this economy provides a microfoundation for the time-varying markup formulation in the text.

Bertrand competition

Here the nontraded good in any given country is produced by a competitive final consumption firm using inputs from a continuum of intermediate goods sectors $j \in [0,1]$ according to

$$(79)\quad Y_N(s^t) = \left[\int y_N(j,s^t)^{\varepsilon}dj\right]^{1/\varepsilon},$$

where ε is a constant. The demand for an intermediate of type j is thus given by

$$y_N(j,s^t) = \left(\frac{P_N(s^{t-1},s_{1t})}{P_N(j,s^{t-1},s_{1t})}\right)^{\frac{1}{1-\varepsilon}} Y_N(s^t).$$

We assume that each sector has a large number of potential firms that have the ability to produce intermediate good j. Each sector has a single leader who has the lowest costs of production. The technology of the leader in sector j is

$$y_N(j,s^t) = A(s_{2t})L_N(j,s^t).$$

The technology of the next most productive entrant (the "follower") is

$$y_N^f(j,s^t) = A(s_{2t})\theta(s_{1t})L_N(j,s^t),$$

where $\theta(s_{1t}) < 1$, which means that the follower needs $1/\theta(s_{1t})$ times as much labor as the leader does to produce one unit of intermediate good j. The price charged is determined by Betrand competition between the leader and potential entrants. If $1/\varepsilon < 1/\theta(s_{1t})$, the leader sets the markup over the weighted marginal cost to be $1/\varepsilon$ and serves the whole market, whereas if $1/\varepsilon \geq 1/\theta(s_{1t})$ the leader sets the markup to be (just under) $1/\theta(s_{1t})$ over the weighted marginal cost and serves the whole market. We assume that the latter case always prevails, so that

$$(80)\quad P_N(j,s^{t-1},s_{1t}) = \frac{1}{\theta(s_{1t})}\sum_{s_{2t}}\left(\frac{Q(s^t)Y_N(s^t)}{\sum_{\tilde{s}_{2t}} Q(\tilde{s}^t)Y_N(\tilde{s}^t)}\right)\frac{W(s^t)}{A(s_t)}.$$

Since the right side of (80) does not depend on j, the leading firms in each sector set their prices equal to the right side of (80). Thus, a situation with constant elasticity of demand and time-varying relative productivity of the leader and potential entrants is a second microfoundation for (2).

Time-varying taxes

Here we again assume that the production function for final goods is given by (79). But now the fluctuations in the markup arise from time-varying taxes. In particular, we assume that the sales tax rate on all intermediate goods firms is $\tau(s_{1t})$. The government rebates the revenues from these taxes to consumers in a lump-sum fashion. The problem of an intermediate good firm of type j is to choose $P = P(j, s^{t-1}, s_{1t})$ to solve

$$\max_{P} \sum_{s_{2t}} Q(s^t) \left[(1 - \tau(s_{1t}))P - \frac{W(s^t)}{A(s_{2t})} \right] \left(\frac{P_N(s^t)}{P} \right)^{\frac{1}{1-\varepsilon}} Y_N(s^t).$$

The solution is that

$$(81) \quad P_N(j, s^{t-1}, s_{1t}) = \frac{1}{\varepsilon(1 - \tau(s_{1t}))} \sum_{s_{2t}} \left(\frac{Q(s^t) Y_N(s^t)}{\sum_{\tilde{s}_{2t}} Q(\tilde{s}^t) Y_N(\tilde{s}^t)} \right) \frac{W(s^t)}{A(s_t)}.$$

Since the right side of (81) does not depend on j, $P_N(j, s^{t-1}, s_{1t}) = P_N(s^{t-1}, s_{1t})$. Defining $\theta(s_{1t}) = \varepsilon(1 - \tau(s_{1t}))$ this model also gives rise to (2).

C. Derivation of the Ramsey Outcome

The equilibrium allocations both under flexible exchange rates and in a monetary union satisfy the markup condition (17) and the resource constraints. Consider a relaxed version of the Ramsey problem: choose allocations to maximize utility subject to the markup condition and the resource constraints. Clearly, the consumption of traded goods is given by

$$(82) \quad C_T(s^t) = \frac{\alpha}{\psi}.$$

Letting $\chi(s^{t-1}, s_{1t})$ be the multiplier associated with (17) and dividing the first order condition for $C_N(s^t)$ by that for $L(s^t)$ gives

$$(83) \quad C_N(s^t) = \frac{1}{\psi} \frac{A(s_{2t})(1 - \alpha)}{1 + \chi(s^{t-1}, s_{1t})\theta(s_{1t})}.$$

Then, substituting this expression for $C_N(s^t)$ into (17) and solving for $\chi(s^{t-1}, s_{1t})$, we get

$$1 + \chi(s^{t-1}, s_{1t})\theta(s_{1t}) = \frac{1}{\theta(s_{1t})},$$

which when substituted back into (83) gives that the expression for nontraded consumption in (18) and labor is clearly given by

$$L(s^t) = C_T(s^t) + \frac{C_N(s^t)}{A(s_{2t})}.$$

We next show that this allocation can be implemented as a competitive equilibrium and therefore solves the original Ramsey problem under flexible exchange rates. We construct prices so that the multiplier on the cash-in-advance constraint is zero in all states. To do so, we construct the prices so that the cash-in-advance constraint holds with equality at the highest level of productivity of the nontraded goods and is a strict inequality at all other shocks. (A moment's reflection makes clear that there is a one-dimensional degree of indeterminacy in the price level. Here we have resolved this indeterminacy in one particular way, but we could support the same allocations with prices such that the cash-in-advance constraint holds as a strict inequality at all shocks.)

For all t, s^t, recursively construct prices normalized by the beginning-of-the-period money holdings, $p_T(s^t) = P_T(s^t)/M(s^{t-1})$ and $p_N(s^{t-1}, s_{1t}) = P_N(s^{t-1}, s_{1t})/M(s^{t-1})$ and the money growth rate as

$$(84) \quad p_N(s^{t-1}, s_{1t}) = \frac{1}{\theta(s_{1t})} \min_{s_2} \left\{ \frac{\psi}{\alpha} \frac{1}{A(s_{2t})} \right\} = \frac{1}{\theta(s_{1t})} \frac{\psi}{\alpha} \frac{1}{\max A(s_{2t})}$$

$$(85) \quad p_T(s^t) = A(s_{2t})\theta(s_{1t})p_N(s^{t-1}, s_{1t})$$

$$(86) \quad \frac{M(s^t)}{M(s^{t-1})} = \beta \sum_{s^{t+1}} h(s^{t+1}|s^t) \frac{A(s_{2t})}{A(s_{2t+1})}.$$

Let $W(s^t) = P_T(s^t)$, and let the nominal interest rates $\{r(s^t)\}$ and state prices $\{Q(s^t)\}$ be given by (10) and (11) at these allocations and prices. We claim that our constructed allocations, prices, and money supplies are a competitive equilibrium outcome. First notice that the necessary conditions for consumers' optimality are satisfied: since $W(s^t) = P_T(s^t)$ and (82) holds, then (8) holds; combining (85), (84), and (83) and using $W(s^t) = P_T(s^t)$ gives (7); next (86), (82), (83), (85), and (84) imply (9); (10) and (11) hold by construction; finally, notice that (6) is satisfied by substituting (85) and (82) in the cash-in-advance constraint. The constructed prices satisfy (2) because the allocations satisfy (17). Finally, market clearing follows from the feasibility of the allocations.

We now turn to the Ramsey problem for a monetary union. We begin by showing that in a union, nontraded goods consumption cannot vary with country-specific productivity shocks. To see how this arises, note that the restriction that the price of traded goods is equal in all countries, (15), when combined with the consumer first order condition (8) and our preferences, implies that

$$\frac{C_N(s^t)\,\psi}{1-\alpha} = \frac{P_T(z^t)}{P_N(s^{t-1}, s_{1t})},$$

which in turn implies that in the union $C_N(s^t)$ cannot vary with v_{2t}, that is,

$$(87) \quad C_N(s^t) = C_N(s^{t-1}, s_{1t}, z_{2t}) \quad \text{for all } v_{2t}.$$

Consider the following relaxed problem:

$$\max_{\{C_T(s^t), C_N(s^t), L(s^t)\}} \sum_t \sum_{s^t} \beta^t h(s^t) \left[\alpha \log\left(C_T(s^t)\right) + (1-\alpha)\log\left(C_N\left(s^t\right)\right) - \psi L(s^t)\right]$$

subject to the resource constraints (17) and (87). The first order condition for $C_T(s^t)$ gives

$$(88) \quad C_T(s^t) = \frac{\alpha}{\psi}.$$

After substituting the restriction on nontraded goods consumption into the objective function, the first order condition for the consumption of nontraded goods can be written as

$$(89) \quad C_N(s^{t-1}, s_{1t}, z_{2t}) = \frac{(1-\alpha)\,\theta(s_{1t})}{(1+\chi(s^{t-1}, s_{1t}))\,\psi X(z_{2t})},$$

where $\chi(s^{t-1}, s_{1t})$ is the multiplier on (17) where $X(z_2) = \sum_{v_2} g^2(v_2)/A(s_2)$. Substituting back into

(17), we can solve for the multiplier

$$1+\chi(s^{t-1},s_{1t})=\sum_{s_2}h^2(s_{2t})\frac{1}{A(s_{2t})X(z_{2t})}.$$

Substituting this expression for $\chi(s^{t-1},s_{1t})$ into (89) gives

$$\text{(90)}\quad C_N(s^{t-1},s_{1t},z_{2t})=\theta(s_{1t})\frac{1-\alpha}{\psi}\frac{1}{X(z_{2t})\sum_{\tilde{s}_2}h^2(\tilde{s}_2)/\left(A(\tilde{s}_2)X(\tilde{z}_2)\right)},$$

and obviously

$$\text{(91)}\quad L(s^t)=C_T(s^t)+\frac{C_N(s^t)}{A(s_{2t})}.$$

We now show that the allocations in (88), (90)–(91) can be implemented as a competitive equilibrium under a monetary union. Here also there is a one-dimensional degree of indeterminacy in the price level, and we resolve it by having the cash-in-advance constraint hold with equality in the aggregate state at which a unionwide average of the inverse of productivity, namely $X(z_2)=\sum_{v_2}g^2(v_2)/A(s_2)$, is at its lowest value (but with a multiplier of zero) and hold as an inequality at all other values.

For all t,s^t, construct prices normalized by the beginning-of-the-period money holdings, $p_T(s^t)=P_T(s^t)/M(s^{t-1})$ and $p_N(s^{t-1},s_{1t})=P_N(s^{t-1},s_{1t})/M(s^{t-1})$ and the money growth rate as follows:

$$\text{(92)}\quad p_N(s^{t-1},s_{1t})=\frac{1}{\theta(s_{1t})}\frac{\psi}{\alpha}\min_{z_2}\{X(z_2)\}\sum_{s_2}h^2(s_2)\frac{1/A(s_{2t})}{X(z_{2t})}$$

$$\text{(93)}\quad p_T(s^t)=A(s_{2t})\theta(s_{1t})p_N(s^{t-1},s_{1t})=\frac{\psi}{\alpha}\frac{\min_{z_2}\{X(z_{2t})\}}{X(z_{2t})}$$

$$\text{(94)}\quad \frac{M(s^t)}{M(s^{t-1})}=\beta\sum_{s^{t+1}}h(s^{t+1}|s^t)X(z_{2t+1})/X(z_{2t}).$$

We also let $W(s^t)=P_T(s^t)$ and let the nominal interest rates $\{r(s^t)\}$ and state prices $\{Q(s^t)\}$ be given by (10) and (11) at these allocations and prices.

We claim that our constructed allocations, prices, and policies are a competitive equilibrium outcome in a monetary union. First notice that the sufficient conditions for consumers' optimality are satisfied. Here $W(s^t)=P_T(s^t)$ and (88) gives (8); combining (93), (92), and (90) and using $W(s^t)=P_T(s^t)$ gives (7); (94), (88), (90), (93), and (92) imply (9); finally, notice that (6) is satisfied by substituting (93) and (88) in the cash-in-advance constraint. The constructed prices satisfy (2) because the allocations satisfy (17). Finally, market clearing follows from the feasibility of the allocations.

D. Markov Equilibrium Outcomes under Flexible Exchange Rates and Lemma 1

We start with the characterization of the Markov equilibrium under flexible exchange rates. Here we derive the outcomes allowing for both productivity shocks and markup shocks, since we will use this more general formulation in Proposition 5.

Allowing for productivity to be stochastic makes the analysis a bit more subtle than the main case of the text when productivity is constant. The reason is that in the Markov equilibrium, the cash-in-advance constraint may be slack when the realization of productivity shocks is sufficiently below its average level. To see why, suppose the realized productivity is sufficiently low that it is possible to completely offset the markup distortion and not have the cash-in-advance constraint bind. This occurs when at $p_T = p_N A$, the cash-in-advance constraint is slack so that $C_T = \alpha/\psi < 1/p_T = 1/Ap_N$. Hence, when $Ap_N \leq \psi/\alpha$, this is the outcome; otherwise it is the typical case in which the cash-in-advance constraint binds. Of course, for such an outcome to be part of an equilibrium, such a setting for p_N must be optimal for the nontraded goods firms. From (2) it is clear that the nontraded goods firms set their prices, in part, based roughly on the average productivity shock. When the realization of productivity is sufficiently low relative to this average, then p_N can be such that $Ap_N \leq \psi/\alpha$ and this scenario can occur. Of course, when productivity is constant, $p_N = p_T/\theta A$ and it cannot.

We formalize this logic in the following lemma.

Lemma A1. In the Markov equilibrium outcome with flexible exchange rates, the price of traded goods $\bar{p}_T(s^t)$ only depends on the current shock s_t and if $Ap_N \leq \psi/\alpha$ satisfies $\bar{p}_T(s_t) = A(s_{2t})\bar{p}_N(s_{1t})$ and otherwise satisfies

$$(95) \quad \bar{p}_T(s_t) = \frac{\bar{p}_N(s_{1t})A(s_{2t})}{2(1-\alpha)}\left[(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_{2t})}\frac{\psi}{\bar{p}_N(s_{1t})}}\right],$$

and the normalized nontraded good price $\bar{p}_N(s^{t-1}, s_{1t})$ only depends on s_{1t} and solves

$$(96) \quad \bar{p}_N(s_{1t}) = \frac{1}{\theta(s_{1t})}\sum_{s_{2t}} h^2(s_{2t})\frac{\bar{p}_T(s_t)}{A(s_{2t})}.$$

Furthermore, $C_T(s_t) = \min\{1/\bar{p}_T(s_t), \alpha/\psi\}$, and $C_N(s_t) = (1-\alpha)\bar{p}_T(s_t)/\psi\bar{p}_N(s_{1t})$. Finally, the money growth rate is $\mu(s_t) = \beta\alpha\bar{p}_T(s_t)/\psi$ and the inflation rate in sector $i = T, N$, defined as $\pi_i(z_{t-1}, z_t) = P_i(s_t)/P_i(s_{t-1})$, is $\pi_i(s_{t-1}, s_t) = \mu(s_{t-1})\bar{p}_i(z_t)/\bar{p}_i(z_{t-1})$.

Proof. Suppose first that the realization of A is such that the cash-in-advance constraint is binding. Then using an argument similar to that in the text, the primal problem for the monetary authority is (30). From the first order conditions to that problem, it is easy to show that the optimal price for traded goods satisfies

$$(97) \quad \frac{1-2\alpha}{p_T/p_N} = (1-\alpha)\frac{1}{A(s_2)} + \frac{\psi}{p_N}\frac{1}{(p_T/p_N)^2}.$$

If this constraint is slack, the optimal price clearly satisfies $p_T = Ap_N$. Thus, the monetary authority's best response has two parts: if $Ap_N \leq \psi/\alpha$ then $p_T(p_N, s) = A(s_2)p_N$; otherwise it equals the p_T that solves (97), namely

$$(98) \quad p_T(p_N, s) = \frac{p_N A(s_2)}{2(1-\alpha)}\left[(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_2)}\frac{\psi}{p_N}}\right],$$

where the right-hand side of this equation defines the function F in the text. Substituting into the pricing rule for nontraded goods (22) for the wage rate from (20) gives

$$(99) \quad p_N(s_1) = \frac{1}{\theta(s_1)}\sum_{s_2} h^2(s_2)\frac{p_T(p_N(s_1), s)}{A(s_2)}.$$

The equilibrium outcome for nontraded goods is a fixed point of these equations, and hence, combining the two-part best response of the monetary authority $p_T(p_N, s)$ and the pricing rule for nontraded goods $p_N(s_1)$ in (99) gives

$$1 = \frac{1}{\theta(s_1)} \sum_{s_2} h^2(s_2) \max\left\{ \frac{(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s_2)}\frac{\psi}{p_N(s_1)}}}{2(1-\alpha)}, 1 \right\},$$

which implicitly defines $p_N(s_1)$. Using such a $p_N(s_1)$, we then have that (98) implies the equilibrium outcome $p_T(s)$. The other relevant equilibrium objects can be recovered by substituting for $p_N(s_1)$ and $p_T(s)$ into the constraints (31). *Q.E.D.*

If A is not stochastic, the cash-in-advance constraint always binds and we can solve

$$1 = \frac{1}{\theta(s_1)} \frac{(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A}\frac{\psi}{p_N(s_1)}}}{2(1-\alpha)}$$

and (98) to get the expressions for prices and then use the constraints on the primal problem to construct the consumption and labor allocations of Lemma 1.

E. Proof of Lemma 2

We start by showing that with our preferences, the primal Markov problem can be split into a static part and a dynamic part. The static part is to solve

$$(100) \quad \max_{p_T, C_T(x_G), C_N(x_G)} \sum \left[U\left(C_T(x_G), C_N(x_G), C_T(x_G) + \frac{C_N(x_G)}{A(x_G)} \right) \right] d\lambda_G$$

subject to

$$(101) \quad C_N(x_G) = \frac{1-\alpha}{\psi} \frac{p_T}{p_N(x_G)}$$

$$(102) \quad C_T(x_G) = \min\left\{ \frac{m(x_G)}{p_T}, \frac{\alpha}{\psi} \right\}.$$

For any given p_T, the dynamic part is to solve

$$(103) \quad \max_{\mu(x_G)} \sum_s h(s') W^{union}\left(S'_G\right)$$

$$(104) \quad \gamma \frac{\psi}{p_T} = \beta \sum_{s'} h(s') \frac{\alpha}{p_T(x'_H, S'_H) C_T(m\mu(x_G)/\gamma, x'_H, S'_H)}$$

$$\gamma = \sum_{x_G} \left[\mu(x_G, S_G) m \right] d\lambda_G.$$

We can separate these problems because the value of the dynamic part is independent of p_T. To see why, note that the aggregate growth rate of money is homogeneous of degree 1 in $\mu(x_G)$, whereas the value $W^{union}(S'_G)$ and the right-hand side of the constraint (104) are homogeneous of degree 0 in $\mu(x_G)$. Hence the value in (103) does not depend on p_T.

We prove a preliminary lemma that immediately implies Lemma 2.

Lemma A2. *i*) Under our preferences (4), if at the end of any period there is a nondegenerate money holding distribution, then the cash-in-advance constraint in the next period has a zero multiplier for all m and all z, and ii) in any Markov equilibrium, the multiplier on the cash-in-advance constraint is binding for at least one level of aggregate shocks z and for a positive of measure of relative money holdings m in the support of λ_m.

Proof of part i. Suppose by way of contradiction that the end-of-period money holding distribution across countries is not degenerate so that there are two countries, say country 1 and country 2, whose consumers have money holdings at the beginning of the next period that satisfy $m_1 < m_2$ and the cash-in-advance constraint in the next period binds for country 1 for some realization of the shocks.

From (102) we see that the value of consumption of the traded good, $p_T C_{Ti} = \min[m_i, \alpha p_T/\psi]$ for $i = 1, 2$ does not vary with the country-specific shock. It follows that $p_T C_{T1} \leq p_T C_{T2}$ with strict inequality for at least one aggregate state. It follows that

$$(105)\quad \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_1, x_{H1}, S_H)} > \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_2, x_{H2}, S_H)}.$$

But the first order condition for money holdings from period $t-1$ to t implies that for both countries $i = 1, 2$,

$$(106)\quad \frac{\psi}{\alpha p_T} = \beta \sum_s h(s) \frac{1}{p_T(S_H) C_T(m_i, x_{Hi}, S_H)},$$

where p_T is the price of traded goods in period $t-1$. Clearly, (105) contradicts (106).

Proof of part ii. Suppose by way of contradiction that the cash-in-advance constraint is slack for all countries for all realizations of the aggregate shock. Consider the static problem (100). The first order conditions with respect to p_T evaluated with the equilibrium rule imply that

$$(107)\quad 1 = \sum_{x_G} \frac{p_T(z, \lambda_G)}{A(x_G) p_N(x_F, S_F)} d\lambda_G(x_G).$$

Now the sticky price first order condition evaluated in equilibrium is

$$p_N(x_F, S_F) = \frac{1}{\theta(s_1)} \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2)},$$

which since $1/\theta(s_1) > 1$ for all s_1 implies that

$$(108)\quad \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2) p_N(x_F, S_F)} < 1.$$

Integrating (108) over the state x_F with respect to the measure λ_F implies that

$$(109)\quad \sum_{x_F} \sum_{s_2} h^2(s_2) \frac{p_T(z, \lambda_G)}{A(s_2) p_N(x_F, S_F)} d\lambda_F(x_F) = \sum_{x_G} \frac{p_T(z, \lambda_G)}{A(x_G) p_N(x_G)} d\lambda_G(x_G) < 1,$$

where in the first equality we have used the property that the marginal measure of λ_G over x_F is λ_F. The inequality in (109) contradicts (107). *Q.E.D.*

Note that the intuition for the second part of the lemma is similar to why the cash-in-advance constraint must be binding under flexible exchange rates. If the cash-in-advance constraint were slack in all states, then the monetary authority would eliminate the markup distortion on average in the sense of (107). But the nontraded goods producers always set their price as a markup over the average value of the price of traded goods in the sense of (108). These two conditions are incompatible if the markup is always positive. Thus, in equilibrium the cash-in-advance constraint must bind for enough countries so that the benefits of raising the price of traded goods to correct the distortions from imperfect competition just balance the costs of lowering the consumption of traded goods.

Combining parts *i*) and *ii*) of Lemma A2 immediately implies Lemma 2.

F. Markov Equilibrium Outcome for a Monetary Union and Lemma 3

It turns out that it is particularly simple to characterize the Markov equilibrium with fixed exchange rates when the cash-in-advance constraint always holds with equality. It follows from the proof of Lemma 2 that a sufficient condition for this result to be true is that productivity shocks in the nontraded goods sector have no aggregate component or more generally that the fluctuations in this aggregate component are not too large.

Lemma A3. Assume that all agents begin with the same initial holdings of money, (4) holds, and the cash-in-advance constraint holds with equality in all states. Then the Markov equilibrium outcome in a monetary union is such that the prices and consumption of nontraded and traded goods can be written as $p_N(s_{1t}), C_N(s_{1t}, z_{2t})$, $p_T(z_t)$, and $C_T(z_t)$ and solve

$$(110)\quad p_N(s_{1t}) = \frac{1}{\theta(s_{1t})} \sum h^2(s_{2t}) \frac{p_T(z_t)}{A(s_{2t})},$$

where

$$(111)\quad p_T(z_t) = \frac{(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4\sum_v g\left(v\right) \frac{(1-\alpha)}{A(z_{2t},v_2)} \frac{\psi}{p_N(z_{1t},v_1)}}}{\sum_v g\left(v\right) \frac{2(1-\alpha)}{A(z_{2t},v_2)} \frac{1}{p_N(z_{1t},v_1)}}.$$

Furthermore, $C_T(z_t) = 1/p_T(z_t)$ and $C_N(s_{1t}, z_{2t}) = (1-\alpha)p_T(z_t)/\psi p_N(s_{1t})$. Finally, the aggregate money growth rate is $\gamma(z_t) = \beta\alpha p_T(z_t)/\psi$, and the inflation rate in sector $i = T, N$, defined as $\pi_i(z_{t-1}, z_t) = P_i(z_t)/P_i(z_{t-1})$, is $\pi_i(z_{t-1}, z_t) = \gamma(z_{t-1})p_i(z_t)/p_i(z_{t-1})$.

Proof. Under our assumptions, the problem for the unionwide monetary authority is (40). The solution to the problem above satisfies

$$0 = \frac{1-2\alpha}{p_T} + \psi \left(\frac{1}{p_T}\right)^2 - \sum_v g\left(v\right) \frac{(1-\alpha)}{A(z_2,v_2)} \frac{1}{p_N\left(z_1,v_1\right)}.$$

Solving this expression gives the monetary authority's best response:

$$(112)\quad p_T(z, \{p_N\left(z_1, v_1\right)\}) = \frac{(1-2\alpha) + \sqrt{(1-2\alpha)^2 + 4\sum_v g\left(v\right) \frac{(1-\alpha)}{A(z_2,v_2)} \frac{\psi}{p_N(z_1,v_1)}}}{\sum_v g\left(v\right) \frac{2(1-\alpha)}{A(z_2,v_2)} \frac{1}{p_N(z_1,v_1)}},$$

where in the text we write the right-hand side at $F(E(1/Ap_N))$.

In equilibrium we must impose that the sticky price firm's first order condition (110) is satisfied. Hence, (110) and (112) give a system of equations in $\bar{p}_N(z_1, v_1)$ and $\bar{p}_T\left(z\right)$ that can be solved, yielding the price of the nontraded and traded goods on the equilibrium path. Finally, $C_T(s)$ and $C_N(s)$ can be recovered using (110) and (111) in (101), (102) with a cash-in-advance constraint holding with equality.

When A is not stochastic, we can solve for the equilibrium outcomes obtaining the expressions in Lemma 3.

G. Proof of Proposition 6

The first part of this result immediately follows from Proposition 5 and continuity of the equilibrium values in the parameters of the model.

The proof of the second part has two parts. The easy part mimics the logic with the commitment case in that for any given price of nontraded goods, under flexible exchange rates the monetary authority is better able to adjust the price of traded goods to country-specific shocks. The more subtle part shows that, in equilibrium, the price of nontraded goods that confronts the monetary authority under flexible exchange rates is actually lower than it is under a monetary union. A lower price of nontraded goods means that the economy is less distorted in terms of market power, and this feature tends to reinforce the benefits of flexible exchange rates.

We begin with the more subtle part by showing that, in equilibrium, the price of nontraded goods that confronts the monetary authority under flexible exchange rates is lower than it is under a monetary union, that is, $p_N^{flex} < p_N^{Union}$. Combining the expressions for p_T and p_N from Lemma A1, namely (95) and (96), and assuming the cash-in-advance constraint binds in all states, p_N^{flex}, is defined by

$$\sum_z f(z) \sum_v g(v) H\left(\frac{1}{A(z,v)} \frac{1}{p_N^{flex}}\right) = B(\theta),$$

where the function H is defined by

$$H\left(\frac{1}{A(s)} \frac{1}{p_N}\right) \equiv \left[(1-2\alpha)^2 + 4(1-\alpha)\frac{1}{A(s)} \frac{\psi}{p_N}\right]^{1/2}$$

and $B(\theta) \equiv 2(1-\alpha)\theta - (1-2\alpha)$. For the union, a similar analysis gives that $p_N^{union}(z)$ solves

$$\sum_z f(z) H\left(\sum_v g(v) \frac{1}{A(z,v)} \frac{1}{p_N^{union}}\right) = B(\theta)$$

for the same functions H and B. Since the function H is concave in $1/A$ for a given p_N, $p_N^{flex} < p_N^{union}$.

For the rest of the proof, note first that for the same p_N, the value of the Markov primal problem under flexible exchange rates is greater than that under a union, $U^{flex}(p_N) \geq U^{union}(p_N)$, simply because the problem under flexible exchange rates is less constrained. Note next that the $U^{flex}(p_N)$ is decreasing in p_N. Intuitively, the higher is p_N, the higher are the implied distortions for the traded good. Then since $p_N^{flex} < p_N^{union}$, we have that

$$U^{flex}(p_N^{flex}) > U^{flex}(p_N^{union}) \geq U^{union}(p_N^{union}).$$

H. Derivation of Expressions for the Real and Nominal Exchange Rates

We start by deriving our expression for real exchange rates (49). To do so, start with the definition of the multilateral real exchange rate of a country with country-specific shock history v^t, namely

$$q(s^t) = \frac{e(s^t)P_T(s^t)^\alpha P_N(s^t)^{1-\alpha}}{\sum_{v^t} e(s^t)P_T(s^t)^\alpha P_N(s^t)^{1-\alpha}},$$

where $P_T(s^t)^\alpha P_N(s^t)^{1-\alpha}$ is the consumer price index for a country with country-specific shock history v^t. Hence,

$$q(s^t) = \frac{(P_N(s^t)/P_T(s^t))^{1-\alpha}}{\sum_{v^t}(P_N(s^t)/P_T(s^t))^{1-\alpha}} = \frac{(p_N(s_t)/p_T(s_t))^{1-\alpha}}{\sum_{v^t}(p_N(s_t)/p_T(s_t))^{1-\alpha}},$$

where the first equality follows from using our expression for the multilateral nominal exchange (14). The second equality, which gives (49), follows by definition of the normalized prices. Note that the second equality implies that the real exchange rate depend only on the current shocks.

We now derive equation (53) assuming $A(s) = A_z(z)A_v(v)$ and $E_vA_v(v) = 1$. Under the Ramsey policy, letting $\bar{p}_T(z^t) = \sum_{v^t} P_T(z^t, v^t)g^t(v^t)/\bar{M}(z^{t-1})$ we can write the growth in nominal exchange rate as

$$\frac{e(s^{t+1})}{e(s^t)} = \frac{p_T(s^{t+1})}{p_T(s^t)} \frac{\bar{p}_T(z^t)}{\bar{p}_T(z^{t+1})} \frac{M(s^t)}{M(s^{t-1})} \frac{\bar{M}(z^{t-1})}{\bar{M}(z^t)},$$

which using (86) to express the money growth rate and $A(s) = A_z(z)A_v(v)$, we have

$$\frac{e(s^{t+1})}{e(s^t)} = \frac{p_T(s^{t+1})}{p_T(s^t)} \frac{\bar{p}_T(z^t)}{\bar{p}_T(z^{t+1})} \frac{A_v(v_t)E\left(1/A_v\left(v_{t+1}\right)A_z(z_{t+1})\right)}{E\left(1/A_z\left(z_{t+1}\right)\right)}.$$

Using $p_T(s^t) = \psi A(s_t)/\left(\alpha \max A(s_t)\right)$ and $\bar{p}_T(z^t) = \psi A_z(z_t)/\left(\alpha \max A_z(z_t)\right)$ straightforward algebra yields

$$\frac{e(s^{t+1})}{e(s^t)} = A_v(v^t),$$

which is (53) in the text.

I. Proof of Lemma 4

For a given state $\left(z, \left\{p_N^i\left(z,v\right)\right\}\right)$, the Markov primal problem reduces to

$$\max_{p_T^i} \sum_{i=N,S} \lambda^i \sum_v g(v)\left[-\alpha \log p_T + (1-\alpha)\log \frac{1-\alpha}{\psi}\frac{p_T}{p_N^i(s_1)} - \psi\left(\frac{1}{p_T} + \frac{1-\alpha}{\psi}\frac{p_T}{p_N^i(z,v)}\frac{1}{A^i}\right)\right].$$

The first order condition for this problem, namely,

$$0 = (1-2\alpha)\frac{1}{p_T} + \psi\frac{1}{p_T^2} - (1-\alpha)\sum_{i=N,S}\lambda^i\sum_v g(v)\frac{1}{p_N^i(z,v)A^i},$$

defines the best response of the monetary authority in state $\left(z, \left\{p_N^i\left(z,v\right)\right\}\right)$. From the sticky price first order condition we have

$$p_N^i\left(z,v\right) = \theta^i\left(s_1\right)\frac{p_T\left(z\right)}{A^i}.$$

The equilibrium is a fixed point of these two equations: combining them and using the result that the cash-in-advance constraint is binding so $C_T^i(z) = 1/p_T(z)$, we can solve for $C_T^i(s)$ obtaining (58).

J. Proof of Proposition 9

The problem (60) reduces to one of maximizing the expected value of $W(x(s,\lambda)) = \log(\alpha - (1-\alpha)(1-x(s,\lambda)))$ where $x(s,\lambda) = \left(1-\lambda^S\right)\theta^N(s) + \lambda^S\theta^S(s)$. Up to a second order approximation around $\bar{x} = E\left(x\left(s,\bar{\lambda}\right)\right)$ where $\bar{\lambda} = (0,1)$ we have that

$$\begin{aligned} EW(x(s,\lambda)) &= \alpha \log\left(\frac{\alpha}{\psi} - \frac{1-\alpha}{\psi}\left[1 - \sum_i \lambda^i \bar{\theta}^i\right]\right) \\ &\quad - \frac{1}{2}\frac{\alpha(1-\alpha)^2}{\bar{C}_T^2}\left[(1-\lambda^S)^2 var\left(\theta^N\right) + \left(\lambda^S\right)^2 var\left(\theta^S\right) + 2\left(1-\lambda^S\right)\lambda^S cov\left(\theta^N,\theta^S\right)\right], \end{aligned}$$

where we treat $\bar{C}_T^2$ as a constant that does not vary with λ^S.

Taking first order conditions and solving for λ^S we get

$$\begin{aligned} (113) \quad \lambda^S &= \frac{var\left(\theta^N\right) - cov\left(\theta^N,\theta^S\right) - \bar{C}_T\left(\bar{\theta}^N - \bar{\theta}^S\right)/(1-\alpha)^2}{var\left(\theta^N\right) + var\left(\theta^S\right) - 2cov\left(\theta^N,\theta^S\right)} \\ &= \left(1-\rho\frac{\sigma_S}{\sigma_N}\right)\frac{var(\theta^N)}{var\left(\theta^N-\theta^S\right)} - \frac{\left(\bar{\theta}^N - \bar{\theta}^S\right)}{var\left(\theta^N-\theta^S\right)}\frac{\bar{C}_T}{(1-\alpha)^2} \end{aligned}$$

if the expression is positive or zero otherwise. Notice that if $\rho < \sigma_N/\sigma_S$, both the numerator and the denominator in the first term of (113) are positive. This is a necessary condition for λ^S to be interior as the second term is positive, $\bar{\theta}^N - \bar{\theta}^S > 0$. *Q.E.D.*

K. Remarks on Proposition 11

Consider the general case with arbitrary measures of $\bar{n}^N, \bar{n}^M, \bar{n}^S$ and let $\left(n_i^N, n_i^M, n_i^S\right)$ for $i = 1,2,3$ be the composition of the three unions. Suppose first that

$$(114) \quad \lambda_1^M \bar{n}^N \leq \bar{n}^M \text{ and } \lambda_1^S \bar{n}^N \leq \bar{n}^S,$$

then $n_1^N = \bar{n}^N$. Here there are sufficiently many Middle and Southern countries to achieve the optimal mixture in the most preferred union even when all of the Northerners are in this union. Now if

$$(115) \quad \lambda_2^S(\bar{n}^M - \lambda_1^M \bar{n}^N) \leq \bar{n}^S - \lambda_1^S \bar{n}^N$$

holds, then there are enough Southern countries left over to achieve the optimal mixture in the second most preferred union even when all of the remaining Southerners are in this union. Under (114) and (115), the construction in Proposition 10 is feasible. Clearly, a sufficient condition for (114) (115) to hold is that $\bar{n}^M/\bar{n}^N$ and $\bar{n}^S/\bar{n}^M$ are sufficiently large.

Now if the measures are such that (114) holds but (115) fails, then the second most preferred union has all the Southerners that are left over from the first union in that $n_2^S = \bar{n}^S - \lambda_1^S \bar{n}^N$ and

$$n_2^M = \frac{\lambda_2^M}{\lambda_2^S}(\bar{n}^S - \lambda_1^S \bar{n}^N),$$

whereas the third most preferred union consists solely of middle countries.

Many more cases work similarly. In all of them, the most preferred union has the weights as constructed in the text, and that union has all the members of at least one of the three groups, that is the group that is, in the relevant sense, the most scarce. The second most preferred union has the optimal mixture subject to the constraint that it contain no members of this most scarce group. This union has the remaining members of the second most scarce group. The third union is composed solely of the members of one group that is the most abundant. Notice that if $\bar{n}^N$ is sufficiently large relative to both $\bar{n}^M$ and $\bar{n}^S$ and the Middle and Southern countries are not too distorted in that λ_1^M and λ_1^S are both positive, then the least preferred group is composed solely of Northern countries.

L. Proofs of Propositions 12, 13, and 14

To prove all three propositions, we first characterize the equilibrium outcomes for a client given that the anchor follows an arbitrary policy. The anchor's policy matters for the client only to the extent that it influences the stochastic process for the price of traded goods denoted $p_T^i(s)$, where i denotes the anchor and prices are normalized by the anchor's money supply.

Following logic very similar to that in the proof of the first part of Lemma 2, it follows that we can set the money supply of the client equal to that of the anchor, $M\left(s^t\right) = M^i\left(s^t\right)$ for all $t \geq 0$. When the anchor is not following the Friedman rule, this is a necessary condition (as it was in Lemma 2), whereas if the anchor is following the Friedman rule, it is without loss of generality. Thus, the price of traded goods normalized by the anchor's money supply equals the price of traded goods normalized by the client's money supply.

The prices and allocations in the client country are then given by

$$\begin{aligned}
p_N(s) &= \frac{1}{\theta(s)} E\left(p_T^i(s)/A(s)\right) \\
C_N(s) &= \frac{1-\alpha}{\psi}\frac{p_T^i(s)}{p_N(s)} = \frac{1-\alpha}{\psi}\theta(s)\frac{p_T^i(s)}{E\left(p_T^i(s)/A(s)\right)} \\
C_T(s) &= \min\left\{1/p_T^i(s), \alpha/\psi\right\},
\end{aligned}$$

and, ignoring constants, the utility is given by

$$\begin{aligned}
(116) \qquad & \left[\alpha E \log\left(\min\left\{\frac{1}{p_T^i(s)}, \frac{\alpha}{\psi}\right\}\right) + (1-\alpha)\left[E\log p_T^i(s) - \log E_A\left(\frac{p_T^i(s)}{A(s)}\right)\right]\right] \\
& + (1-\alpha)E\log\theta(s) - \psi E \min\left\{\frac{1}{p_T^i(s)}, \frac{\alpha}{\psi}\right\} - (1-\alpha)E\theta(s).
\end{aligned}$$

Proof of Proposition 12. Clearly the value of (116) does not depend on the correlation of the markup shocks of the client and the anchor.

Proof of Proposition 13. Under commitment, the cash-in-advance constraint does not bind for either the anchor or the client, so that $C_T(s) = \alpha/\psi$ and the price of traded goods has the form $p_T^i(s) = \kappa A^i(s)$ for a constant κ chosen so that the cash-in-advance constraint never binds. Substituting into (116) and ignoring terms that do not vary with the anchor gives that maximizing the welfare of the client is equivalent to maximizing

$$\log\left(E\left[\frac{A^i(s)}{A(s)}\right]\right) - E\left[\log\frac{A^i(s)}{A(s)}\right].$$

Proof of Proposition 14. Here the price of traded goods for the anchor is the Markov equilibrium price

$$p_T^i(s) = \frac{\psi}{c_0 + (1-\alpha)\,\theta^i\,(s)},$$

where $c_0 = 2\alpha - 1$ and we have used Lemma 1 and $p_T^i(s) = 1/C_T^i(s)$. Substituting this price into (116) and suppressing constants gives

$$\alpha E \log\left(c_0 + (1-\alpha)\,\theta^i\,(s)\right) - E\left[c_0 + (1-\alpha)\,\theta^i\,(s)\right].$$

Taking a second order approximation of this expression gives that the utility of the client is

$$\alpha \log\left(c_0 + (1-\alpha)\,E\theta^i\right) - \left[c_0 + (1-\alpha)\,E\theta^i\right] - \frac{(1-\alpha)}{\left[c_0 + (1-\alpha)\,E\theta^i\right]^2} var\left(\theta^i\right).$$

Thus, if the set of potential anchors have the same expected distortions $E\theta^i$, then the best anchor is the one that minimizes $var\left(\theta^i\right)$.

M. Anchor-Client Unions

Consider now an anchor-client union in which the anchor does not have commitment in our approximated model. We consider a set of potential anchors with the same mean distortion. The policy of the anchor is

$$(117)\;\; p_T^{flex} = \frac{1}{1+\kappa^A} a^A + \frac{1}{\kappa^A}\eta^A,$$

where a^A and η^A denote the productivity shocks and markup shocks. Here we drop ex ante productivity shocks and let a^A and a denote the ex post productivity shocks of the anchor and the client. Throughout all variables are in log-deviation form. When confronted with this policy, from (70) we see that price setters in the client country set

$$p_N(v_1) = E(\frac{1}{1+\kappa^A} a^A + \frac{1}{\kappa^A}\eta^A + \eta - a).$$

Substituting these into the objective function (69)

$$W = -\frac{1}{2}\sum_v g(v)\left[(p_T - a - p_N)^2 + \kappa p_T^2\right]$$

and using $Ea^A = Ea = 0$ and simplifying, we can rewrite this as

$$W = -\frac{1}{2}\left[var\left(\frac{a^A}{1+\kappa^A} - a\right) + var(\eta) + \kappa\left[\frac{var(a^A)}{(1+\kappa^A)^2} + \frac{var(\eta^A)}{(\kappa^A)^2}\right]\right].$$

Clearly, the best anchor is the one that has the smallest value of

$$\frac{var(a^A)}{1+\kappa^A} - 2\frac{\rho\sigma_{a^A}\sigma_a}{1+\kappa^A} + \frac{var(\eta^A)}{\kappa^A}.$$

N. Extensions

Serially correlated shocks without commitment. We assume that the log of country-specific productivity follows a first order autoregressive process

$$a_{vt} = (1-\rho)\,\mu_{av} + \rho a_{vt-1} + \varepsilon_{avt}$$

and the processes for the aggregate shocks to productivity and the country-specific and aggregate shocks to markup can be arbitrary stochastic processes. Proceeding as in our version of the Alesina-Barro model, it is straightforward to show that the welfare difference between the union and flexible exchange rates is given by

$$W^{union} - W^{flex} = \frac{1}{\kappa} var(\log\theta_v) - \frac{1}{1+\kappa} var(\varepsilon_{av}). \tag{118}$$

Note that in this expression, the unconditional variance of the country-specific markup shock appears, but only the variance of the innovation of the productivity shock appears. Here, the first term reflects the gains that arise because the monetary authority in a union does not respond to country-specific markups. The second term reflects the losses from being unable to react to the innovations in productivity. Note that price setters react to the predictable components of productivity so that the monetary authority does not have to. Thus, under flexible exchange rates the monetary authority need only react to the innovations in productivity.

Next we compute macroeconomic aggregates with serially correlated shocks. We then use these variables to express our criterion for forming a union in terms of standard macroeconomic aggregates rather than shocks. Given the policy function and the price setting rule, the allocations are given by

$$c_T(s, s_{-1}) = \frac{1}{\kappa}\theta - \frac{1}{1+\kappa}\varepsilon_{av} - \frac{1}{\kappa}\eta,\; c_N(v) = E\left(a_2|s_{-1}\right) + \frac{1}{1+\kappa}\varepsilon_{av} - \eta.$$

Output $y(s, s_{-1}) = \alpha c_T(s, s_{-1}) + (1-\alpha)c_N(s, s_{-1})$ can then be expressed as

$$y(s, s_{-1}) = (1-2\alpha)\frac{1}{1+\kappa}\varepsilon_{av} + (1-\alpha)E\left(a_2|s_{-1}\right) - \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)\eta.$$

The real exchange rate in levels relative to the rest of the world is proportional to $\left(P_N(s, s_{-1})/P_T(s, s_{-1})\right)^{1-\alpha}$ so that the real exchange rate in log-deviation form is

$$q(s, s_{-1}) = (1-\alpha)\left[\eta - E\left(a_2|s_{-1}\right) - \frac{1}{1+\kappa}a_2\right].$$

The expressions for the variances output and real exchange rates are thus related to the variances of the shocks according to

$$var\,(y) = \left[\frac{1-2\alpha}{1+\kappa}\right]^2 var(\varepsilon_{av}) + (1-\alpha)^2 var\left(E\left(a_2|s_{-1}\right)\right) + \left(\frac{\alpha}{\kappa} + 1 - \alpha\right)^2 var(\eta)$$

$$var(q) = (1-\alpha)^2\left[\left(\frac{1}{1+\kappa}\right)^2 var(\varepsilon_{av}) + var\left(E\left(a_2|s_{-1}\right)\right) + var(\eta)\right].$$

Here $varE\left(a_2|s_{-1}\right) = \rho_{a2v}^2 var(\varepsilon_{av})/(1-\rho_{a2v}^2)$. These expressions can be inverted and substituted into (118) so that forming a union is welfare improving only if

$$\omega_q' var(q) < \omega_y' var(y),$$

where

$$\begin{aligned}
\omega_y' &= (1-\alpha)^2 \left[\frac{1}{1+\kappa} + \kappa + \frac{(1+\kappa)\rho_{a2v}^2}{1-\rho_{a2v}^2} \right], \\
\omega_q' &= \left[\frac{(1-2\alpha)^2}{1+\kappa} + \kappa \left(\frac{\alpha}{\kappa} + 1 - \alpha \right)^2 + (1-\alpha)^2 \frac{(1+\kappa)\rho_{a2v}^2}{1-\rho_{a2v}^2} \right].
\end{aligned}$$