

Imperfect Risk-Sharing and the Business Cycle

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March 2019

Imperfect risk-sharing and business cycles

- Does households' heterogeneity matter for business cycle analysis?
- New class of models (HANK): answer is “yes”
 - Amplifies/dampens effects of aggregate shocks
 - Transmission mechanisms of fiscal and monetary policy
- Challenging to assess these channels quantitatively
 - Answers depend on set of financial assets and risk-sharing mechanisms
 - Hard to combine realistic asset markets with standard business cycle models
- We develop a framework robust to these considerations
 - 1 Measure degree of imperfect risk-sharing from households' choices
 - 2 Provide framework to assess its macroeconomic implications

What we do

Our method has two steps

1 Accounting procedure for micro data

- *Prototype* model: households' decision problem under complete markets
- “Wedges” distort risk-sharing and optimal labor supply
- Measure individual wedges that account for micro data (CEX, PSID)

2 Combine micro wedges with a *class* of HANK models

- **Equivalent representation:** RA economy with preference “shocks”
 - State-dependent discount rate
 - State-dependent disutility of labor
- Preference “shocks” are simple statistics of micro wedges

Counterfactuals: what would have happened with perfect risk-sharing?

What we find

Imperfect risk-sharing → drop in aggregate demand in Great Recession

- Mostly due to increase in discount rate in equivalent RA representation
- Higher discount rate increases propensity to save of RA and reduce aggregate demand. At the ZLB, effects sizable

What in the micro data is suggesting higher propensity to save?

- Substantial variation in consumption shares during Great Recession
- Consumption share of income rich/asset poor hh's decreased the most
 - Increase in saving rates for this group in 2008
 - Indication of heightened saving motives

Literature

1 Accounting procedures in macro

- Chari, Kehoe and McGrattan (2008), Hsieh and Klenow (2009), Boerma and Karabarbounis (2017), Ohanian et al. (2018)

2 Aggregation results for models with incomplete markets

- Nakajima (2005), Krueger and Lustig (2010), Werning (2016)

3 New Keynesian models with incomplete markets

- Guerrieri and Lorenzoni (2017), Auclert (2016), Kaplan, Moll and Violante (2017), McKay, Nakamura and Steinsson (2016), Auclert et al. (2018)...
- Precautionary savings and aggregate demand: Heathcote and Perri (2018), Ravn and Sterk (2017), Bayer et al. (2019)

Outline

- 1 Prototype model and accounting procedure
- 2 Measuring the wedges
- 3 A class of New Keynesian model with heterogeneous agents
- 4 An application to the US economy

Prototype model

- z_t and v_t are aggregate and idiosyncratic states. Let $z^t = (z_0, z_1, \dots, z_t)$, $v^t = (v_0, v_1, \dots, v_t)$, $s^t = (z^t, v^t)$, w/ $\Pr(s^t | s^{t-1}) = \Pr(v^t | z^t, v^{t-1}) \Pr(z^t | z^{t-1})$
- Decision problem of an household
 - Takes as given wages $W(s^t)$ and the price of financial assets $Q(s^t, s_{t+1})$
 - Chooses consumption, labor and financial positions
- Individual specific “wedges”
 - Idiosyncratic wage (*efficiency wedge*), $W(s^t) = \theta(v^t)W(z^t)$
 - Tax on labor (*labor wedge*), $\tau_l(s^t)$
 - Tax on financial assets (*risk sharing wedge*), $\tau_a(s^t, s_{t+1})$

Households' problem

$$\max_{\{c(s^t), l(s^t), a(s^t, s_{t+1})\}} \sum_{t=0}^{\infty} \sum_{s^t} \Pr(s^t) \beta^t \left[\frac{c(s^t)^{1-\sigma}}{1-\sigma} - \chi \frac{l(s^t)^{1+\nu}}{1+\nu} \right]$$

subject to

$$\begin{aligned} c(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1}) a(s^t, s_{t+1}) [1 + \tau_a(s^t, s_{t+1})] &\leq \\ &\leq \theta(v^t) W(z^t) l(s^t) [1 - \tau_l(s^t)] + a(s^t) + T(s^t) \end{aligned}$$

Optimality

$$\begin{aligned} l(s^t)^\nu &= \frac{\theta(v^t) W(z^t) [1 - \tau_l(s^t, s_{t+1})]}{\chi c(s^t)^\sigma} \\ \Pr(s^{t+1} | s^t) \beta \left(\frac{c(s^t, s_{t+1})}{c(s^t)} \right)^{-\sigma} &= Q(s^t, s_{t+1}) [1 + \tau_a(s^t, s_{t+1})] \end{aligned}$$

The procedure in one slide

- We have panel data on $\{c_{it}, w_{it}, l_{it}\}$
- We assume agents face the following prices for Arrow securities

$$Q(s^t, s_{t+1}) = \Pr(s_{t+1}|s^t)\beta \left(\frac{C(z^t, z_{t+1})}{C(z^t)} \right)^{-\sigma}$$

- We recover wedges from the data using the optimality conditions

$$\begin{aligned}\theta_{it} &= \frac{w_{it}}{W_t} \\ \tau_{a,it+1} &= \left[\frac{C_{t+1}/C_t}{c_{it+1}/c_{it}} \right]^\sigma - 1 \\ \tau_{l,it} &= 1 - \chi l_{i,t}^\nu \frac{c_{it}^\sigma}{w_{it}}\end{aligned}$$

The “no-tax” allocation

Suppose that $\tau_a(s^t, s_{t+1}) = 0$ and $\tau_l(s^t) = 0$ for all (s^t, s_{t+1}) . Then

- 1 Individual consumption constant fraction of aggregate consumption

$$c_{it} = \varphi_i C_t$$

for some weight φ_i

- 2 Individual hours given by

$$l_{it} = \frac{(\theta_{it}/\varphi_i)^{1/\nu}}{\mathbb{E}_i [(\theta_{it}/\varphi_i)^{1/\nu}]} L_t$$

Deviations from this allocation require non-zero wedges:

- Risk sharing wedge allows for time-varying consumption shares φ_{it}

$$\varphi_{it} = \prod_{j=0}^t (1 + \tau_{a,ij})^{-\frac{1}{\sigma}} \varphi_{i0}.$$

- Labor wedge allows for deviations from frictionless labor supply

“Detailed” economies impose restrictions on wedges

Detailed economy = structural model w/ given market structure, frictions, ...

- Predictions of detailed economy for $\{c_{it}, l_{it}, W_{it}\}$ can be replicated in prototype model with appropriate sequence of wedges
- Some examples
 - Huggett (1993) and the risk sharing wedge [▶ Details](#)
 - Preference heterogeneity (σ and β) and the risk sharing wedge
 - Sticky wages and idiosyncratic labor wedges

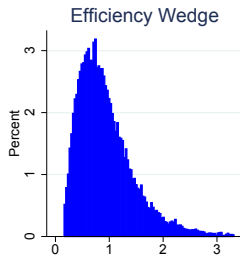
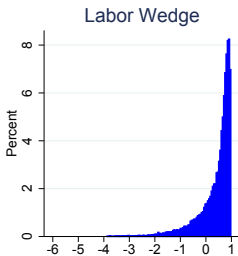
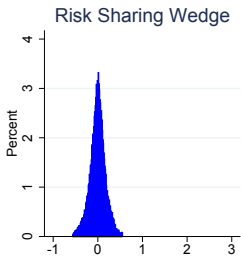
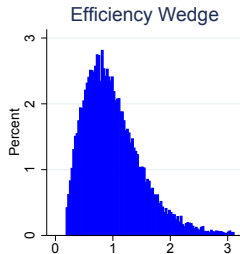
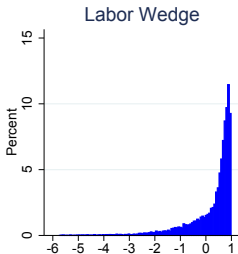
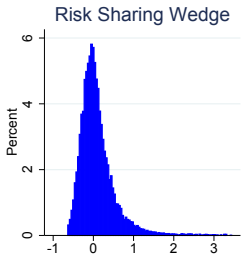
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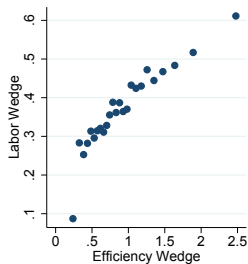
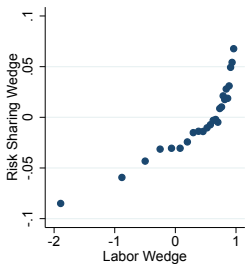
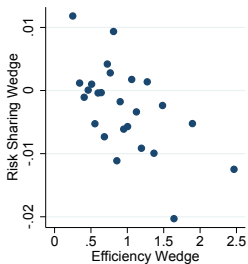
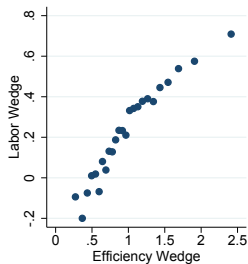
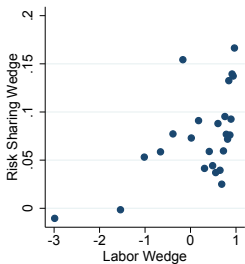
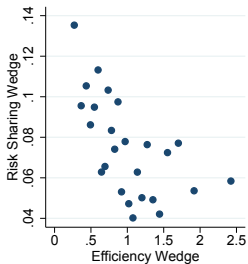
Measuring idiosyncratic wedges

- Need panel on consumption expenditures, wages and hours worked. We use the CEX (1996-2012) and the PSID (1999-2015)
- Data definitions
 - Consumption: Dollar spending in non-durables and services
 - Earnings: Labor + business income
 - Hours: Total hours worked per year
- Mapping between model and data
 - Measure at household level and adjust for number of members
 - Control for characteristics that are typically not included in macro models: education, age, sex, race, state, and family size
- Set $\sigma = 1$, $\nu = 1$ and χ such that labor wedge is on average 0.3 in 2006

Marginal distribution of the wedges



Cross-sectional patterns



Time-series patterns



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Taking stock

- We have measured micro wedges in the data. We now put them into use
- We consider a *class* of New Keynesian models with heterogeneous agents
 - **Macro block:** Standard 3 equations NK model (Woodford, 2002)
 - **Micro block:** unrestricted, allow for a wide range of asset structures
- **Key result:** Micro block summarized by few statistics of micro wedges
 - Law of motion for aggregates as in RA economy with “taste shocks”
 - Taste shocks simple functions of micro wedges
- Use framework to perform counterfactuals
 - What would happen if risk sharing wedges were set to zero?

Preferences, technology, and monetary policy

- Households' preferences, $u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{l^{1+\nu}}{1+\nu}$
- Competitive final good firms use intermediates to produce final good

$$Y(z^t) = \left(\int_0^1 y_i(z^t)^{1/\mu} di \right)^\mu$$

- Intermediate good firms are monopolistic competitive, face quadratic adjustment costs á la Rotemberg, production function

$$y_i(z^t) = \exp\{A(z_t)\} n_i(z^t)$$

- Monetary policy described by a Taylor rule

$$i(z^t) = \max \left\{ \bar{i}^{1-\rho_i} i(z^{t-1})^{\rho_i} \left(\frac{\Pi(z^t)}{\bar{\Pi}} \right)^{\gamma_\pi} \left(\frac{Y(z^t)}{Y(z^{t-1})} \right)^{\gamma_y} \exp\{\epsilon_m(z_t)\}, 1 \right\}$$

The problem of the households

Have access to \mathcal{J} assets and risk-free nominal bond

$$\max_{c,l,b,\{a_j\}} \sum_t \sum_{s^t} \beta^t \Pr(s^t|s_0) \left[\frac{c(s^t)^{1-\sigma}}{1-\sigma} - \chi \frac{l(s^t)^{1+\nu}}{1+\nu} \right]$$

subject to

$$\begin{aligned} P(z^t) c(s^t) + \sum_{j \in J} a_j(s^t) + \frac{b(s^t)}{i(z^t)} &\leq (1 - \tau_l(s^t)) W(z^t) \theta(v_t) l(s^t) + T(s^t) \\ &+ b(s^{t-1}) + \sum_{j \in J} R_j(s^t) a_j(s^{t-1}) \\ H(b(s^t), \{a_j(s^t)\}_{j \in J}, s^t) &\geq 0 \quad H_b(\cdot) \geq 0 \end{aligned}$$

Remark: Nests large class of incomplete market models. Key restriction is that agents with highest marginal valuation for b are on their Euler equation

Heterogeneity and the Euler equation

Euler equation holds for household(s) with highest marginal valuation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{s_{t+1}} \Pr(s^{t+1}|s^t) \left\{ \frac{\beta}{\Pi(z^{t+1})} \left(\frac{c(s^t, s_{t+1})}{c(s^t)} \right)^{-\sigma} \right\}$$

Heterogeneity and the Euler equation

Divide and multiply by $[C(z^{t+1})/C(z^t)]^{-\sigma}$

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{s_{t+1}} \Pr(s^{t+1}|s^t) \left\{ \frac{\beta}{\Pi(z^{t+1})} \underbrace{\left(\frac{C(z^{t+1})/C(z^t)}{c(s^{t+1})/c(s^t)} \right)^\sigma}_{[1+\tau_a(s^t, s_{t+1})]} \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma} \right\}$$

Heterogeneity and the Euler equation

Aggregate C , Π and i satisfy the Euler equation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{z^{t+1}} \Pr(z^{t+1}|z^t) \left\{ \frac{\beta(v^t, z^{t+1})}{\Pi(z^{t+1})} \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma} \right\}$$

where

$$\beta(v^t, z^{t+1}) = \beta \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t) [1 + \tau_a(s^t, s_{t+1})]$$

Heterogeneity manifests itself as a “shock” to discount factor (Krueger and Lustig, 2009; Werning, 2016)

- Agents on Euler equation discount more aggregate states characterize by higher average taxes on Arrow securities

Heterogeneity and the Euler equation: complete markets

Aggregate C , Π and i satisfy the Euler equation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{z^{t+1}} \Pr(z^{t+1}|z^t) \left\{ \frac{\beta(v^t, z^{t+1})}{\Pi(z^{t+1})} \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma} \right\}$$

where

$$\beta(v^t, z^{t+1}) = \beta \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t) [1 + \tau_a(s^t, s_{t+1})]$$

With **complete markets**, $\tau_a(s^t, s_{t+1}) = 0 \forall s_{t+1}$ and

$$\beta^c(v^t, z^{t+1}) = \beta$$

- Euler equation as in RA economy

Heterogeneity and labor supply

Optimal labor supply

$$\chi l(s^t)^\nu = (1 - \tau_l(s^t))w(z^t)\theta(v_t)c(s^t)^{-\sigma}$$

Heterogeneity and labor supply

Divide both sides by $\theta(v_t)/C(z^t)^{-\frac{\sigma}{\psi}}$ and aggregate across households

$$\chi^{\frac{1}{\nu}} \underbrace{\left[\sum_{v^t} \Pr(v^t|z^t) \theta(v_t) l(s^t) \right]}_{L_e(z^t)} C(z^t)^{\frac{\sigma}{\nu}} = w(z^t)^{\frac{1}{\nu}} \left\{ \sum_{v^t} \Pr(v^t|z^t) (1 - \tau_l(s^t))^{\frac{1}{\nu}} \theta(v_t)^{\frac{1+\nu}{\nu}} \left[\frac{c(s^t)}{C(z^t)} \right]^{-\frac{\sigma}{\nu}} \right\}$$

Heterogeneity and labor supply

So, in the aggregate we must have

$$\omega(z^t) \chi L_e(z^t)^\nu = \frac{w(z^t)}{C(z^t)^\sigma}$$

where

$$\omega(z^t) = \left\{ \sum_{v^t} \Pr(v^t | z^t) \varphi(z^t, v^t)^{-\frac{\sigma}{\nu}} \theta(v_t)^{\frac{1+\nu}{\nu}} (1 - \tau_l(s^t))^{\frac{1}{\nu}} \right\}^{-\nu}$$

Same FOC of RA agent economy with state-dependent disutility of labor

Heterogeneity and the Phillips curve

Aggregate $\tilde{\Pi} \equiv \Pi(1 + \Pi)$, C and Y must satisfy the Phillips curve

$$\tilde{\Pi}(z^t) = \frac{Y(z^t)}{\kappa(\mu - 1)} \left[\mu \chi \frac{Y(z^t)^\nu C(z^t)^\sigma \omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}} - 1 \right] + \sum_{s^t} Q(z^{t+1}|z^t) \tilde{\Pi}(z^{t+1})$$

where

$$\omega(z^t) = \left\{ \sum_{v^t} \Pr(v^t|z^t) \varphi(z^t, v^t)^{-\frac{\sigma}{\nu}} \theta(v_t)^{\frac{1+\nu}{\nu}} (1 - \tau_l(s^t))^{\frac{1}{\nu}} \right\}^{-\nu}$$

Heterogeneity manifests itself as a shock to the disutility of labor

Suppose high $\theta(v^t)$ also have high consumption shares

- If consumption share of rich decreases \Rightarrow High θ agents work more, low θ agents work less
- Equivalent to positive labor supply shock \rightarrow decrease in marginal cost

Heterogeneity and the Phillips curve: complete markets

Aggregate $\tilde{\Pi} \equiv \Pi(1 + \Pi)$, C and Y must satisfy the Phillips curve

$$\tilde{\Pi}(z^t) = \frac{Y(z^t)}{\kappa(\mu - 1)} \left[\mu \chi \frac{Y(z^t)^\nu C(z^t)^\sigma \omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}} - 1 \right] + \sum_{s^t} Q(z^{t+1}|z^t) \tilde{\Pi}(z^{t+1})$$

where

$$\omega(z^t) = \left\{ \sum_{v^t} \Pr(v^t|z^t) \varphi(z^t, v^t)^{-\frac{\sigma}{\nu}} \theta(v_t)^{\frac{1+\nu}{\nu}} (1 - \tau_l(s^t))^{\frac{1}{\nu}} \right\}^{-\nu}$$

With **complete markets**, $\tau_a(s^t, s_{t+1}) = 0 \forall s_{t+1}$ and

$$\omega^c(z^t) = \left\{ \sum_{v^t} \Pr(v^t|z^t) \varphi(v_0)^{-\frac{\sigma}{\nu}} \theta(v_t)^{\frac{1+\nu}{\nu}} (1 - \tau_l(s^t))^{\frac{1}{\nu}} \right\}^{-\nu}$$

An equivalent representative-agent economy

Suppose that C, Y, Π, i are part of an equilibrium. Then they satisfy

$$\begin{aligned}\Pi(z^t) [1 + \Pi(z^t)] &= \frac{Y(z^t)}{\kappa(\mu - 1)} \left[\mu \chi \frac{Y(z^t)^\nu C(z^t)^\sigma \omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}} - 1 \right] + \\ &+ \sum_{s'} Q(z^{t+1}|z^t) \Pi(z^{t+1}) [1 + \Pi(z^{t+1})] \\ \frac{1}{i(z^t)} &= \max_{v^t} \sum_{z^{t+1}} \Pr(z^{t+1}|z^t) \left\{ \frac{\beta(v^t, z^{t+1})}{\Pi(z^{t+1})} \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma} \right\} \\ i(z^t) &= \max \left\{ \bar{i}^{1-\rho_i} i(z^{t-1})^{\rho_i} \left(\frac{\Pi(z^t)}{\bar{\Pi}} \right)^{\gamma_\pi} \left(\frac{Y(z^t)}{Y(z^{t-1})} \right)^{\gamma_y} \exp\{\epsilon_m(z_t)\}, 1 \right\} \\ Y(z^t) &= C(z^t) + \frac{\kappa}{2} [\Pi(z^t) - 1]^2\end{aligned}$$

Key observation: Knowledge of $\{\beta(v^t, z^{t+1}), \omega(z^t)\}$ is all we need from the “micro block” to characterize law of motion for aggregate variables

As $\{\beta(v^t, z^{t+1}), \omega(z^t)\}$ varies, the aggregate allocation varies with them

Some examples

“ β ” shocks important to explain Great Recession in RA economies

- $\beta \uparrow \rightarrow$ RA wants to save more
- Aggregate demand and interest rates fall. Large effects if ZLB binds

HA economies endogenously induce time-variation in β . What mechanisms?

- 1 Time-varying idiosyncratic risk (Heathcote and Perri, 2018, ...) ▶ Example
 - Increase in idiosyncratic income risk + incomplete markets \rightarrow more precautionary savings \rightarrow as if $\beta \uparrow$
- 2 Tightening of borrowing constraints (Eggertson and Krugman, 2012, ...)
 - Borrowers cannot borrow \rightarrow Savers cannot save \rightarrow as if $\beta \uparrow$

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Counterfactuals: conceptual experiment

- Suppose we know

$$x = \{A(z_t), \epsilon_m(z_t), \beta(v^t, z^{t+1}), \omega(z^t)\}$$

- Use equivalent RA economy and x to solve for

$$y = \{Y(z^t), \Pi(z^t), i(z^t)\}$$

- Use equivalent RA economy and $x^c = \{A(z_t), \epsilon_m(z_t), \beta^c(v^t, z^{t+1}), \omega^c(z^t)\}$ to solve for counterfactual

$$y^c = \{Y^c(z^t), \Pi^c(z^t), i^c(z^t)\}$$

Contribution of imperfect risk-sharing to macroeconomic aggregates

$$y - y^c$$

Counterfactuals in practice

- Use micro wedges to construct time path for $\{\beta_{it}, \omega_t\}$
- Assume Markov process for $\{A_t, \epsilon_{mt}, \beta_{it}, \omega_t\}$
- Estimate structural parameters of the equivalent RA economy using $\{Y_t, \Pi_t, i_t, \beta_{it}, \omega_t\}$ as observables
- Apply particle filter to estimate state vector and $y = \{Y_t, \Pi_t, i_t\}$
- Solve equivalent RA economy under complete markets and compute counterfactual $y^c = \{Y_t^c, \Pi_t^c, i_t^c\}$ by feeding $\{A_t, \epsilon_{mt}, \omega_t^{cm}\}$

Contribution of imperfect risk-sharing to macroeconomic aggregates

$$y - y^c$$

Constructing $\beta(v^t, z^{t+1})$

$$\begin{aligned}\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) [1 + \tau_a(s^t, s_{t+1})] \\ &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^\sigma\end{aligned}$$

Want:

- Measure change in consumption shares for an individual with history v^t in every possible state v_{t+1}

Problem:

- For each individual, v^t , we observe only one realization of v_{t+1}

Constructing $\beta(v^t, z^{t+1})$

$$\begin{aligned}\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) [1 + \tau_a(s^t, s_{t+1})] \\ &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^\sigma\end{aligned}$$

What we do:

- Group individuals with same history v^t
- Compute realized cross-sectional mean of change in consumption shares between z^t and z^{t+1} for individuals in the group
- By law of large numbers, it equals $\beta(v^t, z^{t+1})$

Constructing $\beta(v^t, z^{t+1})$

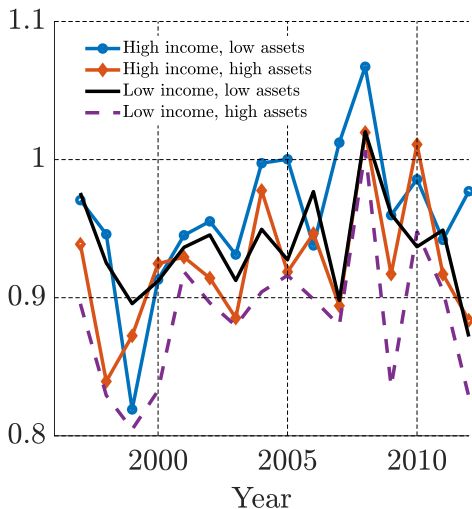
$$\begin{aligned}\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) [1 + \tau_a(s^t, s_{t+1})] \\ &= \sum_{v_{t+1}} \Pr(v_{t+1} | z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^\sigma\end{aligned}$$

In particular:

- Group individuals by income and assets
 - Logic: In Huggett economy income and assets sufficient statistic for v^t
- Within each group i , compute

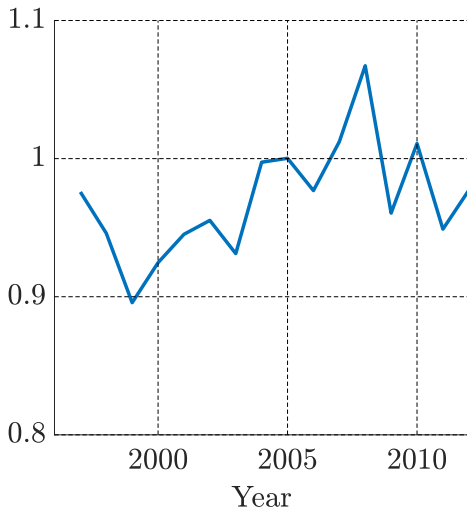
$$\bar{\beta}_{it+1} = \frac{1}{N_i} \sum_{j=1}^{N_i} (1 + \tau_{a,jt+1})$$

Path for $\bar{\beta}_{it}$ for each group



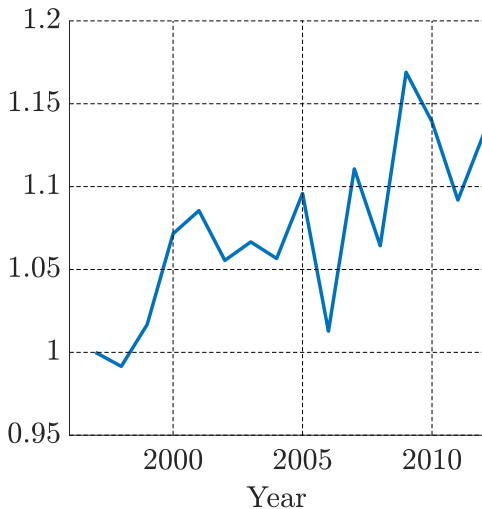
Income rich/asset poor typically have high expected risk-sharing wedges

Path for $\max_i \bar{\beta}_{it}$



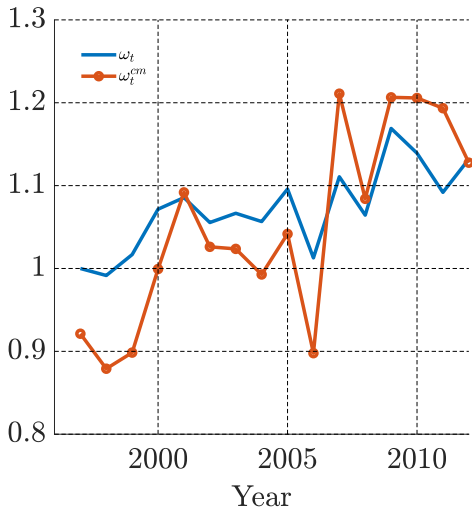
Imperfect risk-sharing \rightarrow As if RA is more patient in Great Recession

Path for $\omega(z^t)$



Disutility of labor increases in Great Recession

Path for $\omega^c(z^t)$

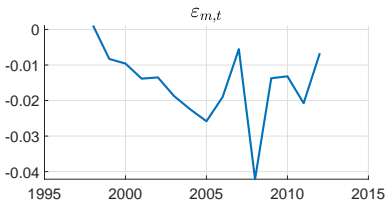
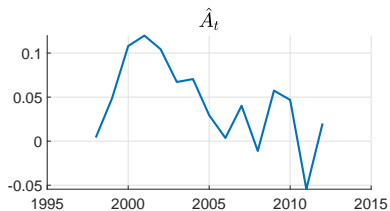
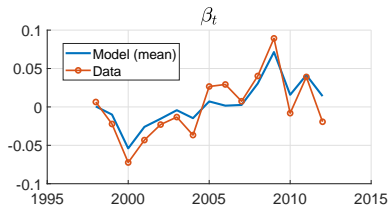


Imperfect risk sharing \rightarrow As if RA wants to work more in Great Recession

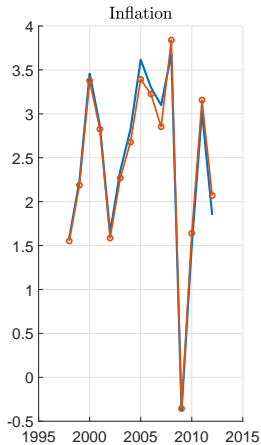
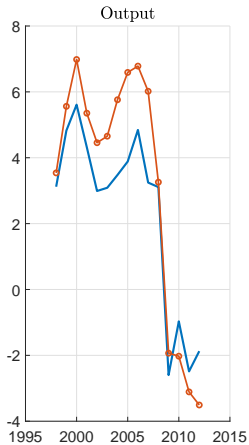
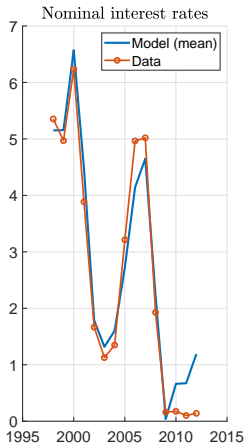
Estimation and filtering

- A_t follows AR(1), $\varepsilon_{m,t}$ iid, $\{\max_i \beta_{it}, \omega_t\}$ follow VAR(1) process
- We set $\sigma = 1$, $\nu = 1$, $\mu = 1.2$, $\Pi^* = 1.02$
- Remaining parameters: $[\kappa, \rho_i, \gamma_\pi, \gamma_{\Delta y}]$ and those of stochastic process $\{A_t, \varepsilon_{m,t}, \max_i \bar{\beta}_{it}, \omega_t\}$
- Use equivalent RA economy to evaluate likelihood function and estimate parameters using $\mathbf{Y}_t = \{\hat{Y}_t, \pi_t, i_t, \beta_{jt}, \omega_t\}$ as observables
- After estimation, back-out structural shocks using particle filter

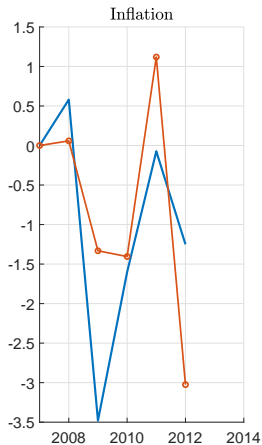
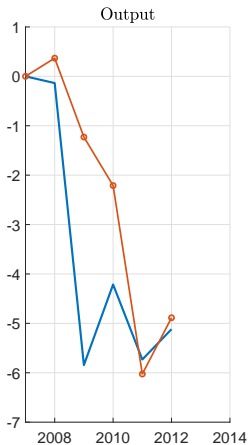
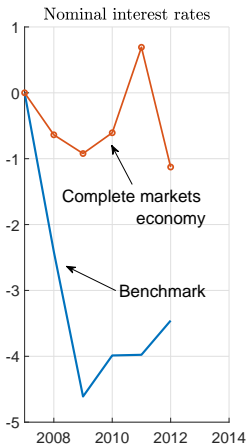
Filtered shocks



Equilibrium outcomes: model and data

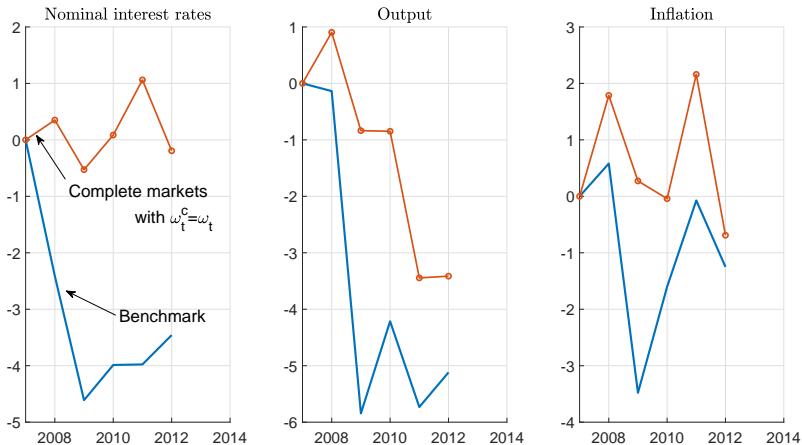


Contribution of imperfect risk-sharing



Milder recession without heterogeneity

Contribution of imperfect risk-sharing



Negative output effects due to increase in propensity to save

Discussion

- In simple NK model, heterogeneity affects aggregates through $[\beta_{it}, \omega_t]$
 - True also in more sophisticated versions (capital, price indexation, etc.)
- Advantages of our procedure
 - Agnostic about nature of idiosyncratic risk and market incompleteness
 - By construction we account for macro and micro data
 - Can perform calculation in benchmark business cycle models
- Disadvantages of our procedure
 - Wedges are not fundamental “shocks”, we cannot say what moves them
 - Cannot study optimal policy
 - No feedback between micro wedges

Conclusion

- Novel framework to evaluate macro models with heterogeneous agents
- Measure micro wedges using CEX and PSID
- Used micro wedges to evaluate business cycle implications of NK models with heterogeneous agents
 - Imperfect risk-sharing during crisis can induce sizable output losses
 - Effects due to increase in propensity to save of income rich/asset poor
- We are working on
 - Disentangling driving forces: precautionary savings vs. debt limits
 - Sensitivity of counterfactuals (adding capital)
 - Other counterfactuals (effects of monetary policy shocks)

Huggett (1993) with tight borrowing limits

- Model details
 - No capital accumulation
 - Aggregate and idiosyncratic risk. Households trade one-period bond
 - Elastic labor supply
 - Competitive labor, goods and financial markets
- Implications for wedges
 - Efficiency wedge due to idiosyncratic income risk
 - Risk sharing wedge due to incomplete markets
 - No labor wedge

Preferences, technology and shocks

- Households have preferences

$$U(c, l) = \log(c) - \chi \frac{l^{1+\nu}}{1+\nu}$$

subject to

$$\begin{aligned} c(s^t) + b(s^t) &\leq W(s^t)l(s^t) + b(s^{t-1})R(z^{t-1}) \\ b(s^t) &\geq 0 \end{aligned}$$

(Financial autarky in equilibrium: no borrowing \rightarrow no savings)

- Technology for producing final good

$$Y(z^t) = A(z^t) \sum_{v^t} p(v^t|z^t) e(v^t) l(s^t) \quad \mathbb{E}_{z^t}[e(v^t)] = 1$$

Equilibrium

Optimality and budget constraint

$$\begin{aligned}W(s^t) &= A(z^t)e(v^t) \\ \chi l(s^t)^\nu &= \frac{A(z^t)e(v^t)}{c(s^t)} \\ c(s^t) &= A(z^t)e(v^t)l(s^t)\end{aligned}$$

So, the allocation is given by

$$\begin{aligned}l(s^t) &= \chi^{-\frac{1}{1+\nu}} \\ c(s^t) &= e(v^t)A(z^t)\chi^{-\frac{1}{1+\nu}}\end{aligned}$$

Wedges

- Efficiency wedge:

$$\theta(v^t) = W(s^t)/W(z^t) = \frac{A(z^t)e(v^t)}{A(z^t)} = e(v^t)$$

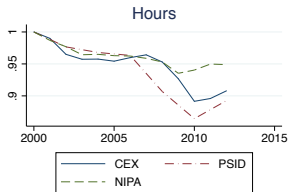
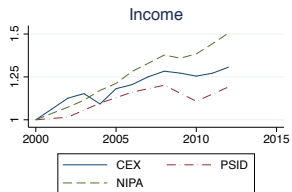
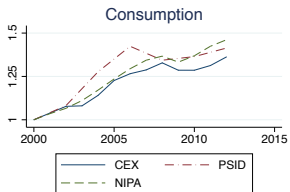
- Risk-sharing wedge:

$$\tau_a(s^t, s_{t+1}) = \left[\frac{C(z^{t+1})/C(z^t)}{c(s^{t+1})/c(s^t)} \right] - 1 = \frac{e(v^t)}{e(v^{t+1})} - 1$$

- Labor wedge:

$$\tau_l(s^t) = 0$$

Comparison with NIPA Aggregates



A simple example

- Assume $\sigma = 1$
- Law of motion for idiosyncratic efficiency

$$\Delta \log[\theta(v_t)] = -\frac{\sigma(z_t)}{2} + \sigma(z_t)\varepsilon_{v,t}$$

- Asset market structure
 - Households can only trade a risk-free bond
 - Face a tight borrowing limit: $b(s^t) \geq 0$

In equilibrium financial autarky: every agent is hand-to-mouth

- Labor supply is the same for all households ($\sigma = 1$)
- Individual consumption: $c(s^t) = \theta(v_t)C(z^t)$

Idiosyncratic risk and aggregate demand

The risk sharing wedge in this model is

$$1 + \tau_a(s^t, s_{t+1}) = \frac{\theta(v_t)}{\theta(v_{t+1})} = \exp\{-\Delta \log[\theta(v_{t+1})]\}$$

We can compute the “micro block”

$$\begin{aligned}\beta(v^t, z^{t+1}) &= \beta \sum_{v^{t+1}} \Pr(v^{t+1} | v^t, z^{t+1}) \exp\{-\Delta \log[\theta(v_{t+1})]\} \\ &= \beta \exp\{\sigma(z^{t+1})\} \\ \omega(z^t) &= 1\end{aligned}$$

Key mechanism: high expected $\sigma(z_{t+1})$ increases precautionary motives. Higher desired savings manifests itself in the aggregate as increase in β

In benchmark NK models, these shocks lead to a fall in aggregate demand

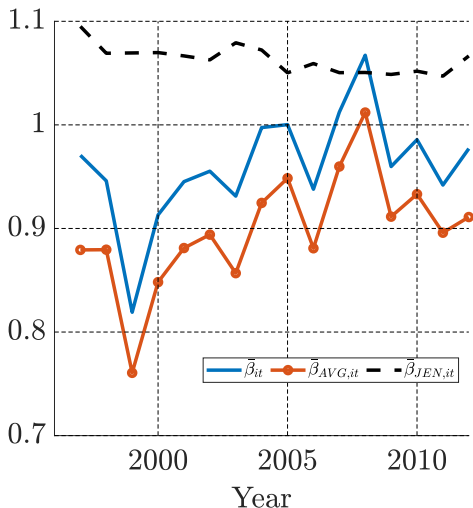
What drives variation in $\max_i \bar{\beta}_{it+1}$?

Focus on income rich/asset poor group

$$\bar{\beta}_{it} = \beta \underbrace{\left[\frac{C_t/C_{t-1}}{\frac{1}{N_i} \sum_{j=1}^{N_i} c_{jt}/c_{jt-1}} \right]}_{\bar{\beta}_{AVG,it}} \underbrace{\sum_{j=1}^{N_i} \left[\frac{\sum_{j=1}^{N_i} c_{jt}/c_{jt-1}}{c_{jt}/c_{jt-1}} \right]}_{\bar{\beta}_{JEN,it}}$$

- $\bar{\beta}_{it}$ can increase if, on average, consumption share of that group between $t - 1$ and t falls relative to average
- $\bar{\beta}_{it}$ can increase if Jensen component increases (e.g. higher cross-sectional dispersion in consumption growth)

What drives variation in $\max_i \bar{\beta}_{it+1}$?



Increase in $\max_i \bar{\beta}_{it+1}$ during Great Recession due to a decline, on average, in the consumption share of this group

Bayesian estimation

We set $\sigma = 1$, $\nu = 1$, $\mu = 1.2$ (Gust et al.), $\Pi^* = 1.02$, and β so that annual nominal rate is 4.5% in deterministic steady state

Parameter	Distribution	Prior		Posterior	
		Mean	Standard deviation	Mean	90% Interval
$4 \times \kappa$	Gamma	85.00	15.00	90.79	[67.41, 115.75]
ρ_i	Beta	0.50	0.28	0.57	[0.29, 0.85]
γ_π	Normal	0.00	1.00	1.51	[0.85, 2.14]
$\gamma_{\Delta y}$	Normal	0.00	1.00	0.58	[0.21, 0.92]
ρ_a	Beta	0.50	0.28	0.52	[0.25, 0.79]
$\Phi_{\beta, \beta}$	Beta	0.50	0.28	0.76	[0.61, 0.92]
$\Phi_{\omega, \omega}$	Beta	0.50	0.28	0.73	[0.33, 0.99]
$100 \times \sigma_a$	InvGamma	1.00	1.00	7.23	[3.36, 10.39]
$100 \times \sigma_m$	InvGamma	1.00	1.00	1.91	[0.98, 2.81]
$100 \times \sigma_\beta$	InvGamma	1.00	1.00	1.88	[1.13, 2.60]
$100 \times \sigma_\omega$	InvGamma	1.00	1.00	3.80	[2.54, 5.05]