Capital Mobility and Optimal Fiscal Policy without Commitment: A Rationale for Capital Controls?

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ABSTRACT

Should a benevolent government impose capital controls? Should it tax at a different rate capital income of domestic and foreign agents? To answer these questions, I study the best equilibrium outcome of a Ramsey taxation model for a small open economy. If the government has commitment, there is no need to introduce capital controls. In contrast, when the government lacks commitment, imposing capital controls on inflows is necessary to support the efficient allocation. The optimal policy calls for a lower capital income tax for domestic agents. Inducing domestic households to save more and to postpone consumption not only increases continuation utility, but it also helps to relax the future commitment problem for the government. If the international interest rate is equal to the time discount factor of domestic residents then capital controls are only temporary: the optimal equilibrium outcome converges to a continuation Ramsey plan in the long-run. Further, I show through numerical examples that capital controls are more important for countries that expect to grow more in the future. Finally, I extend the model to a two country general equilibrium environment and show that the optimality of capital controls is maintained in this setting.

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1. Introduction

In the last two decades, there has been a rise in private capital flows. In particular, FDI to low and middle income economies increased substantially since the 1990s[1]. Despite this rise in international capital flows, Aizenman, Pinto and Radziwill (2007) document that approximately 90 percent of the stock of capital in developing countries is self-financed and that countries with higher self-financing ratios grew faster in the 1990s than those with lower ratios[2]. In this paper, I provide a mechanism through which policies aimed at increasing self-financing ratios - capital controls on inflows - are welfare and growth improving. In particular, I investigate whether or not it is optimal for a benevolent government to influence private foreign asset positions by taxing capital income of domestic residents and foreigners at different rates. That is, should a benevolent government impose capital controls?

To answer these questions, I study the best equilibrium outcome path of a Ramsey taxation model for a small open economy (SOE) when the government cannot commit. Under commitment (see Correia (1996) and Atkeson et al. (1999)), the optimal Ramsey policy for a SOE prescribes no capital controls; under standard assumptions, taxes on capital income are zero in every period for both the domestic and the foreign residents. In contrast, if the government cannot commit to future policies, I show that it is optimal to impose capital controls on inflows. The optimal policy prescribes that *capital income of foreigners is taxed more heavily than that of domestic households* on the transition to the steady state. Moreover, this wedge between the domestic and foreign after-tax-return on capital invested in the country is decreasing over time. In particular, if the international interest rate is equal to the inverse of the discount factor of the domestic agents, then capital controls are vanishing over time and the steady state tax rate on capital income is zero for both foreign and domestic residents as in the Ramsey policy. If the international interest rate is lower than the inverse of the discount factor, then the optimal policy dictates that the wedge between domestic and foreign tax rates, although decreasing, persists also in the steady state.

The optimality of taxing capital income of foreigners at a higher rate may look counter-intuitive, especially for a country with a low capital stock that might greatly benefit from

[1] See for instance Broner et al. (2011), Table 8.
[2] This fact is related to the "Capital Allocation Puzzle" in Gourinchas and Jeanne (2011).

receiving capital inflows from foreign investors to smooth consumption over time and to increase the capital stock in the country. One would naively suggest that it may be optimal to provide some sort of tax advantage to induce foreigners to invest. This argument is not correct if the government lacks commitment. Intuitively, if a large fraction of the capital stock is owned by foreigners, the domestic government finds it difficult to resists pressure from domestic agents who will want an increase in tax on capital income in exchange for lower taxes on labor and more government expenditure. Anticipating this, foreigners are not willing to invest today. On the other hand, the government is able to sustain a lower level of taxes on capital income - and hence a higher capital stock - if a larger share of domestic capital is owned by domestic agents.

Back-loading leisure and consumption - both private and public - is optimal, as typical in an economy with one-sided lack of commitment. In fact, back-loading consumption is not only trading off current utility with future utility, but it is also helping to relax the future commitment problem. Since the government does not directly control private consumption, it has to subsidize household savings in order to induce the domestic household to save more (or to borrow less) than what they would if they face the international interest rate. This implies that the government taxes the capital income of foreign residents at a higher rate. Equivalently, the government could tax foreign debt held by domestic residents at a higher rate. Differential tax rates are necessary because private domestic households do not internalize the fact that by postponing consumption, they are relaxing the commitment problem for the government. This allows the economy to sustain a higher level of capital, and hence, higher wages and welfare.

I then study the limiting behavior of the optimal outcome path for the SOE under two assumptions about the international interest rate. If the international interest rate is equal to the rate of time preference of households then there are no capital controls in the long-run; the domestic household and the domestic government accumulate sufficient assets such that the continuation of a solution with commitment can be sustained. Capital income taxes for domestic and foreigners converge to zero. Instead, if the international interest rate is lower than the rate of time preference, then the economy converges to a steady state with higher capital income taxes for foreign residents than for their domestic counterparts.

To assess the potential welfare gains of adopting the optimal policy, I compare the best equilibrium outcome with the one in which I impose the restriction that the tax rate on capital income must be equal for both domestic and foreign agents. This restriction eliminates the possibility that the government might manipulate the saving decision of the domestic household; there is no wedge between the international interest rate and the after tax rate of return faced by domestic households. I show, in numerical simulations, that the gains from manipulating domestic savings depend crucially on the expected TFP growth of the country. The gains are limited (on the order of a 0.2 % increase in life-time consumption) if there is no growth in TFP, but can be substantial if the country is expecting high TFP growth in the future. Thus, the normative recommendations of this paper are more relevant for fast growing economies.

Finally, the results derived for a SOE generalize to a two-country general equilibrium environment. In particular, I show that capital controls are a feature of a best equilibrium outcome also under coordination between asymmetric countries. Capital controls are not the result of a "prisoner's dilemma" argument. Thus, banning capital controls through some form of international agreement is not welfare improving.

Related Literature This paper is related to the vast literature on one-sided lack of commitment. Several contributions emphasize the optimality of *back-loading* in such economies because it relaxes future commitment problems. In particular, with respect to government lack of commitment in an open economy context, the closest works to this paper are Aguiar, Amador and Gopinath (2009) and Aguiar and Amador (2011b). Both papers, following Thomas and Worrall (1994), study the best equilibrium outcome of a game between a benevolent domestic government and competitive foreign investors. Abstracting from different incentives between the government and the households in the country, competitive private domestic agents in the country are not explicitly modeled. The main contribution of my paper is to explicitly model the behavior of the private domestic sector. This allows me address the central question of the paper: How should capital income earned by foreigners and domestic residents be taxed? In fact, as noted in Gourinchas and Jeanne (2011), these papers "*rely, implicitly or explicitly, on the assumption that the government can control the* *volume of net capital flows. This is not true in the frictionless neoclassical model, because the accumulation of reserves by the government should be offset one-for-one by higher capital inflows. This must be prevented by friction, either natural (low financial development) or policy-induced (capital controls)*"[3]. In this paper, I characterize the optimal capital controls.

This paper is also closely related to Aguiar and Amador (2011a). They consider the same policy game as I do in this paper, but they focus on a different aspect of the optimal policy. Their main result is that labor income taxes are front-loaded when the domestic stand-in household is impatient relative to the international interest rate. In particular, there are no taxes on labor and positive taxes on capital in the steady state, the opposite of what happens in a closed economy Ramsey problem with commitment. They do not characterize the path of capital income taxes for domestic and foreign investors on the transition to the steady state of the economy.

This paper is also related to the Ramsey taxation literature, see Chari and Kehoe (1999). In particular, the optimal Ramsey policy for a SOE when the government can commit has been studied by Correia (1996) and Atkeson et al. (1999). This paper also relates to the literature that studies optimal policy when the government cannot commit in a closed economy, as in Chari and Kehoe (1990), Phelan and Stacchetti (2001), Fernandez-Villaverde and Tsyvinski (2002), Reis (2008) and Benhabib and Rustichini (1997). Quadrini (2005) and Klein et al (2005) consider a two-country open economy environment, but restrict themselves to a Markov-Perfect equilibrium and assume that the government must balance its budget period by period.

Other papers examine reasons why it might be optimal to impose capital controls. The recent literature on "macro-prudential" policies emphasizes the beneficial role of capital controls in economies with credit market frictions and pecuniary externalities, see for instance Caballero and Lorenzoni (2007), Jeanne and Korineck (2010), Mendoza and Bianchi (2011) and the related recent IMF policy work by Ostry et al (2010). Closely related are Jeske (2006) and Wright (2006), who show how subsidies to capital inflows can be welfare improving in an open economy environment with decentralized credit markets and limited enforcement of debt contracts because the subsidies correct for a pecuniary externality. In this paper,

[3] See also Jeanne (2011)

I abstract from the existence of a pecuniary externality. Finally, Costinot, Lorenzoni and Werning (2011) study optimal capital controls motivated by terms of trade manipulation. In their environment, capital controls are inefficient and there are gains from coordination. In a two-country extension of my environment, I show that this result no longer holds; capital controls arise also under coordination.

The rest of the paper is organized as follows. In section 2, I describe the environment and I define a sustainable equilibrium for the policy game. In section 3, I characterize the best equilibrium outcome and prove the main results of the paper. In section 4, I compare the best equilibrium outcome with one in which the government cannot use capital controls for economies under various time profiles for TFP. In section 5, I generalize the results for a general equilibrium 2 country model. Section 6 concludes.

2. Environment

Consider a SOE version of the economy considered in Chari and Kehoe (1999) and Phelan and Stacchetti (2001). Time is discrete and denoted by $t = 0, 1, \ldots$. There are three agents in the economy: a continuum of homogeneous domestic households, a domestic government, and a risk-neutral foreign lender. The preference of each domestic household over sequences of consumption, hours worked, and public good are represented by a time-separable utility function:

$$(1) \quad \sum_{t=0}^{\infty} \beta^t \left[U(c_t, n_t) + G(g_t) \right]$$

where $\beta \in (0, 1)$ is the discount factor, c_t is consumption, n_t are hours worked, and g_t is the public good.

Assumption 1. $U(c, n) : \mathbb{R}_+^2 \to \mathbb{R}$ is strictly increasing in c and decreasing in n, strictly concave, C^1, and satisfies the following Inada conditions: $\lim_{c \to 0} U_c(c, n) = +\infty \; \forall n$, $\lim_{n \to 0} U_n(c, n) = 0 \; \forall c$, and $\lim_{n \to \bar{N}} U_n(c, n) = -\infty \; \forall c$. $G(g) : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing, strictly concave, C^1, and $\lim_{g \to 0} G'(g) = +\infty$.

In each period the unique final good y_t can be produced domestically using a constant return to scale technology:

$$(2) \quad y_t = F(K, N) = K^\alpha N^{1-\alpha} + (1 - \delta_k)K$$

where K and N are aggregate capital and hours worked respectively, $\alpha \in (0, 1)$, and $\delta_k \in (0, 1]$ is the capital depreciation rate.

Each domestic households is initially endowed with k_0 unit of capital installed in the domestic technology. Foreigners have k_0^* unit of capital installed in the domestic technology. We will denote the total capital installed in the domestic technology as $K = k + k^*$. Physical capital fully depreciates within the period.

The government is benevolent. In each period, it can make lump-sum transfers T_t to the households, but it cannot impose lump-sum taxes. To finance government expenditures, g_t, it must levy linear taxes on labor and capital income. Thus, a policy for the government is $\pi_t = (\tau_t^n, \tau_t^k, \tau_t^a, \tau_t^*, T_t, \delta_t) \in \Pi \equiv \mathbb{R}^5 \times [0, 1]$, where:

1. $\tau_t^n \in [\underline{\tau}, 1]$ is the labor income tax.
2. $\tau_t^k \in [\underline{\tau}, 1]$ is the tax on capital income earned in the country for domestic households
3. $\tau^a \in [\underline{\tau}, 1]$ is the tax on capital income earned abroad for domestic households
4. $\tau^* \in [\underline{\tau}, 1]$ is the tax on capital income earned in the country for foreign investors
5. $T_t \geq 0$ are lump-sum trasfers to the domestic households
6. $\delta \in [0, 1]$ is the fraction of debt the government is repaying (partial default).

where $\underline{\tau} < 0$ is some lower bound on taxes that will be chosen sufficiently low so that it won't bind. I fix the upper bound of the tax rate to be equal to one in order to be able to characterize the worst equilibrium of the game analytically. The results derived in section 3 can be generalized to the case where taxes are bounded by some $\bar{\tau} < 1$ as in Phelan and Stacchetti (2001).

The government budget constraint in period t is:

$$(3) \quad \delta_t b_t + g_t + T_t = \tau_t^n w_t n_t + \tau_t^k R_t k_t + \tau_t^a R^* a_t + \tau_t^* R_t k_t^* + q_{t+1} b_{t+1}$$

The government does not posses a *commitment technology.* Taxes and the default decision are chosen sequentially, *after* capital has been installed and capital therefore is inelastically supplied for the period. This is the source of the time inconsistency problem.

The timing protocol of the game is as follows:

1. Enter the period with (k_t, k_t^*, a_t, b_t)
2. The government sets $\pi_t = (\tau_t^n, \tau_t^k, \tau_t^a, \tau_t^*, T_t, \delta_t) \in \Pi$
3. Each households chooses $(c_t, n_t, k_{t+1}, s_{t+1})$ that satisfy the budget constraint

$$(4) \quad c_{t+1} + k_{t+1} + a_{t+1} \leq (1 - \tau_t^k) R_t k_t + (1 - \tau_t^a) R^* a_t + (1 - \tau_t^n) w_t n_t + T_t$$

   and foreigners choose $k_{t+1}^* \geq 0$.
4. Finally the government collects tax revenues and finances g_t with tax collection according to (3).

Before formally defining a sustainable equilibrium for the policy game, it is useful to define a competitive equilibrium for the economy

A. Competitive Equilibria

DEFINITION 1. *Given* (k_0, k_0^*, a_0, b_0) *and a sequence of international interest rates* $\{R_t^*\}_{t=0}^{\infty}$*, a competitive equilibrium for the SOE is a sequence of policies* $\{g_t, \pi_t\}_{t=0}^{\infty}$*, an allocation for the stand-in domestic household* $\{c_t, n_t, k_{t+1}, a_{t+1}\}_{t=0}^{\infty}$*, an allocation for the foreign investors* $\{k_t^*\}_{t=0}^{\infty}$*, and prices* $\{w_t, R_t, q_{t+1}\}_{t=0}^{\infty}$ *such that:*

1. *Given prices and policies, the stand-in household maximizes his utility (1) subject to the sequence of budget constraints (4)*
2. *Optimality conditions for the foreign investors:*

$$\begin{aligned} R_{t+1}^* &\geq (1 - \tau_{t+1}^*) R_t, \quad \text{"} = \text{"} if k_{t+1}^* > 0 \quad \forall t \geq 0 \\ q_{t+1} &= \frac{\delta_{t+1}}{R_{t+1}^*} \quad \forall t \geq 0 \end{aligned}$$

   are satisfied
3. $w_t = F_n(K_t, n_t), \quad R_t = F_k(K_t, n_t) \quad \forall t \geq 0$

4. The government budget constraint (3) holds $\forall t \geq 0$.

If an allocation x is implementable with the "FDI" equilibrium defined above, then $\tilde{x} = \{c_t, n_t, g_t, K_t = k_t + k_t^*, B_t = k_t^* - a_t\}_{t=0}^{\infty}$ can be decentralized with an alternative equilibrium notion in which all the investment in the country is done by domestic households that are allowed to *borrow*, and the government tax instruments are: tax on labor, $\tilde{\tau}_t^n$, and capital income (all earned by domestic residents), $\tilde{\tau}_t^k$, and tax on foreign borrowing and lending, $\tilde{\tau}_t^B$. The household budget constraint in this case can then be written as

$$c_t + K_{t+1} - B_{t+1} \leq (1 - \tilde{\tau}_t^k) R_t K_t + (1 - \tilde{\tau}_t^k) w_t n_t - (1 - \tilde{\tau}_{t+1}^B) R_t^* B_t$$

There are capital controls if $\tilde{\tau}_{t+1}^B \neq 0$[4]. This decentralization may offer a clearer interpretation of capital controls. However, if one assumes, as it is natural, that private agents also cannot commit to repay their debt obligations and the only punishment they face after default is the exclusion from credit markets in the future, then I can show that no private debt can be supported in equilibrium. That is, the debt limit would be $B_{t+1} \leq 0$, as in my "FDI" decentralization, where I impose that $a_t \geq 0$. So, if private agents cannot commit, the only form of *private* foreign liability that can be supported in a deterministic equilibrium is FDI. I will later show that the domestic government can instead support a positive amount of foreign debt because a government default can trigger a self-fulfilling expectation of high capital income taxes. Therefore, defaulting is more costly for the government than for an individual atomistic agent. This observation can rationalize the fact that most of the international capital flows to emerging economies are either FDI or are intermediated through the government.

B. Sustainable Equilibria

I now formally define a symmetric[5] subgame perfect equilibrium for the policy game. I will use the concept of a sustainable equilibrium (SE) developed in Chari and Kehoe (1990) in order to accomodate the fact that in the game I consider there is one strategic player

[4]It is easy to check that $\tilde{\tau}_{t+1}^B \neq 0 \iff \tau_{t+1}^* \neq \tau_{t+1}^k$. In particular there are capital controls on inflows $\tilde{\tau}_{t+1}^B < 0 \iff \tau_{t+1}^* > \tau_{t+1}^k$.

[5]All domestic households follows the same strategy. The same is true for the foreign investors.

(the government), and a continuum of non strategic players (the domestic households and the foreign investors). Let $h^t = (\pi_0, \pi_1, ..., \pi_t) \in H^t$ denote a public history of events up to time t, and let $h^{-1} = \emptyset$. The government strategy, $\sigma = \{\sigma_t\}_{t=0}^{\infty}$, the domestic household's strategy, $f = \{f_t\}_{t=0}^{\infty}$, and the foreigners' strategy, $f^* = \{f_t^*\}$, are measurable functions of public histories[6]. That is:

$$\sigma_t : H^{t-1} \to \Pi, \qquad f_t : H^t \to \mathbb{R}^4, \qquad f_t^* : H^t \to \mathbb{R}$$

with $\sigma_t = \pi_t$, $f_t = (c_t, n_t, k_{t+1}, a_{t+1})$, and $f_t^* = k_{t+1}^*$. With some abuse of notation, I will use the same letter to refer to strategies and outcomes. For any strategy, define the on path utility for the domestic household after history h^{t-1} as follows:

$$W(\sigma, f, f^*|h^{t-1}) = \sum_{r=0}^{\infty} \beta^r [U(c_{t+r}(h^{t+r}), n_{t+r}(h^{t+r})) + G(g_{t+r}(h^{t+r}))]$$

where $h^{t+r} \succeq h^{t-1}$ is induced by σ. Finally, define $p = (w, R, q)$ as follows:

$$\begin{aligned} w_t(h^t; \sigma, f, f^*) &= F_n(K_t(h^{t-1}), n_t(h^t)), \quad R_t(h^t; \sigma, f, f^*) = F_k(K_t(h^{t-1}), n_t(h^t)) \\ q_t(h^t; \sigma, f, f^*) &= \frac{\delta_{t+1}(h^t)}{R_{t+1}^*} \end{aligned}$$

Definition 2. *A sustainable equilibrium (SE) is a triple (σ, f, f^*) such that $\forall$ (t, h^{t-1}) and associated $(k_t(h^{t-1}), k_t^*(h^{t-1}), a_t(h^{t-1}), b_t(h^{t-1}))$:*

1. $\forall \tilde{\sigma} \quad W(\sigma, f, f^*|h^{t-1}) \geq W(\tilde{\sigma}, f, f^*|h^{t-1})$
2. $\{f_{t+r}(h^{t+r})\}_{r=0}^{\infty}, \{f_{t+r}^*(h^{t+r})\}_{r=0}^{\infty}, \{\pi_{t+r}(h^{t+r-1})\}_{r=0}^{\infty}$, $\{p_{t+r}(h^{t+r})\}_{r=0}^{\infty}$, *where $h^{t+r} \succeq h^{t-1}$ is induced by σ, constitutes a competitive equilibrium.*

In a SE, the requirement of optimality for the strategic player, the government, is the standard game theoretic one. Domestic households and foreign investors are non-strategic: they take current policies, prices and the evolution of future histories as unaffected by their actions. Optimality for these players is captured by the requirement that equilibrium strate-

[6] Restricting households and foreign investors to condition their actions on the public history is without loss of generality, see Phelan and Stacchetti (2001).

gies induce allocations and prices that constitute a competitive equilibrium. Moreover, this notion of equilibrium requires that the government, the domestic households and the foreign investors behave optimally after every history, not just the ones induced by the equilibrium strategy σ. This perfection requirement captures the lack of a commitment technology on the side of the government.

In the rest of the paper, I will characterize the equilibrium outcome of the *best* SE of the policy game, that is, the one that attains the highest utility for the stand-in domestic household. I then contrast it with the solution of a Ramsey problem.

3. Characterization of the Best Equilibrium Outcome

Following the seminal work of Abreau (1988) and Chari and Kehoe (1990), I can characterize the *best* SE outcome by solving a simple programming problem, using the fact that any equilibrium outcome path can be supported with a trigger strategy that calls for the worst SE (in terms of utility) after any deviation.

A. Set of Equilibrium Outcomes

First, I can characterize the set of allocations that can be implemented as a competitive equilibrium as follows:

LEMMA 1. Given τ_0^k, τ_0^a, τ_0^*, δ_0, and (k_0, k_0^*, a_0, b_0), an *interior* allocation $x \equiv \{c_t, n_t, k_{t+1}, k_{t+1}^*, a_{t+1}, g_t\}_{t=0}^{\infty}$ is part of a competitive equilibrium for the SOE iff x satisfies

$$(5) \quad \sum_{t=0}^{\infty} \beta^t \left[U_{ct}c_t + U_{nt}n_t\right] = A_0$$

where $A_0 = U_{c0}\left[(1-\tau_0^k)F_{k0}k_0 + (1-\tau_o^a)R^*a_0\right]$, $\forall\ t \geq 1$

$$(6) \quad \sum_{s=0}^{\infty} \beta^s \left[U_{ct+s}c_{t+s} + U_{nt+s}n_{t+s}\right] \geq 0$$

$$(7) \quad \sum_{t=0}^{\infty} Q_t \left[g_t + c_t + k_{t+1} + R^*k_t^* - F(K_t, n_t)\right] - R^*a_0 + \tau_0^* F_k(K_0, n_0)k_0^* = -b_0\delta_0$$

where $Q_0 = 1$ and $\forall t \geq 1$ $Q_t \equiv \prod_{s=1}^{t}(1/R_s^*)$.

Proof. Appendix. □

Denoting $\underline{V}(k, k^*, a, b)$ as the value of the *worst* SE starting from (k, k^*, a, b), I can characterize the set of equilibrium outcome paths as follows.

LEMMA 2. Given (k_0, k_0^*, a_0, b_0), an *interior* allocation $x \equiv \{c_t, n_t, k_{t+1}, k_{t+1}^*, a_{t+1}, g_t\}_{t=0}^{\infty}$ is part of the outcome path of a SE iff it satisfies (5), (6), (7) with $\delta_0 = 0$ if $b_0 > 0$ and $\delta_0 = 1$ if $b_0 \leq 0$, and $\tau_0^k = \tau_0^* = \tau_0^a = 1$, and $\forall t \geq 1$, it satisfies the "sustainability" constraint:

$$(8) \quad \sum_{s=0}^{\infty} \beta^s [U(c_{t+s}, n_{t+s}) + G(g_{t+s})] \geq \underline{V}(k_t, k_t^*, a_t, b_t)$$

Proof. Standard. □

Given this characterization of the set of allocations that can be supported as a SE, it follows that the best equilibrium outcome path solves the following programming problem:

$$(\mathrm{P}) \quad \overline{V}(k_0, k_0^*, a_0, b_0) = \max_{\{c_t, n_t, k_{t+1}, a_{t+1}, k_{t+1}^*, g_t\}} \sum_{t=0}^{\infty} \beta^t [U(c_t, n_t) + G(g_t)]$$

subject to (5), (6), (7) and (8) where $\delta_0 = 0$ when $b_0 > 0$ and $\delta_0 = 1$ when $b_0 \leq 0$, and $\tau_0^k = \tau_0^* = \tau_0^a = 1$. To characterize (P), one has to characterize the value of the *worst* SE, $\underline{V}$. One way to do this is to use the technique developed in Abreu et al (1990) and Phelan and Stacchetti (2001) to numerically characterize the set of SE of the policy game. Given the unitary upper bound on capital income taxes, τ_t^k, τ_t^a and τ_t^*, I can characterize the worst SE of the policy game analytically. The predictions of the model do not change if I exogenously specify a function $\underline{V}(k, k^*, a, b) : \mathbb{R}^4 \rightarrow \mathbb{R}$ to characterize the punishment for a government that deviates from the plan, as in Benhabib and Rustichini (1997).

B. Worst SE of the Policy Game

Allowing the government to tax 100 percent of capital income[7], $\tau_t^k, \tau_t^a, \tau_t^* \leq 1$, the worst SE of the policy game is quite simple. On path (i) in any period, the government is taxing capital income of any sources to the maximum extent; (ii) no new investment is

[7]For the characterization of the worst SE it does not matter if the government imposes taxes on capital income or wealth, as long as I allow the maximal rate to be 100%.

undertaken by domestic households and foreign investors, so the capital stock in the country is depreciating at a rate δ_k, $K_t = (1-\delta_k)^t K_0$; (iii) the government is defaulting on its foreign debt whenever $b \geq 0$ and it is not allowed to borrow abroad, but it can save abroad to smooth public and private consumption over time; and (iv) labor income taxes and lump-sum transfers solve a simple static Ramsey problem.

Formally, let $(\underline{\sigma}, \underline{f}, \underline{f^*})$ be the strategy that attains the worst SE. The strategy for the domestic household is defined as follows: $\forall\ t \geq 0$, $\forall\ h^t \in H^t$

$$\begin{aligned} \underline{k}_{t+1}(h^t) &= (1-\delta)\underline{k}_t(h^{t-1}) \\ \underline{a}_{t+1}(h^t) &= 0 \end{aligned}$$

and $\underline{c}(h^{t-1}, \pi_t)$ and $\underline{n}(h^{t-1}, \pi_t)$ are such that they solve the following two equations:

$$\begin{aligned} v'\left(\underline{n}(h^{t-1}, \pi_t)\right) &= u'\left(\underline{c}(h^{t-1}, \pi_t)\right)(1-\tau^n(\pi_t))\, F_n\left(\underline{n}(h^{t-1}, \pi_t), K(h^{t-1})\right) \\ \underline{c}(h^{t-1}, \pi_t) &= (1-\tau^n(\pi_t))\, F_n\left(\underline{n}(h^{t-1}, \pi_t), K(h^{t-1})\right)\underline{n}(h^{t-1}, \pi_t) + T(\pi_t) \end{aligned}$$

The investment strategy for the foreign investor is simply given by $\underline{k}^*_{t+1}(h^t) = (1-\delta)\underline{k}^*_t(h^{t-1})$ $\forall\ t \geq 0$, $\forall\ h^t \in^t H^t$.

The strategy for the government is given by:

$$\begin{aligned} \underline{\tau}^k(h^{t-1}) &= \underline{\tau}^*(h^{t-1}) = \underline{\tau}^a(h^{t-1}) = 1 \quad \forall\ t \geq 0, \forall\ h^t \in H^t \\ \underline{\delta}(h^{t-1}) &= \begin{cases} 1 & \text{if } h^{t-1} \text{ is associated with } b(h^{t-1}) \geq 0 \\ 0 & \text{otherwise} \end{cases} \qquad \forall\ t \geq 0, \forall\ h^t \in H^t \end{aligned}$$

Finally, $\forall t \geq 0$, $\forall h^{t-1}$ and associated $(k_t(h^{t-1}), k^*_t(h^{t-1}), a_t(h^{t-1}), b_t(h^{t-1}))$, let $\underline{\tau^n}(h^{t-1})$ and $\underline{T}(h^{t-1})$ be implied by the solution to the following *static* Ramsey problem:

$$(9) \quad \omega_t(K, \mathrm{fa}) = \max_{c,n,g} U(c,n) + G(g)$$

subject to

$$\begin{aligned} u'(c)c - v'(n)n &\geq 0 \\ c + g &\leq A_t F(K, n) + R_t^* \text{fa} + \underline{b}'(h^{t-1}) \end{aligned}$$

given that $\underline{b}(h^{t-1})$ solves

$$(10) \quad \Omega(\underline{K}(h^{t-1}), \text{fa}(h^{t-1})) = \max_{b' \leq 0} \omega_t(\underline{K}(h^{t-1}), R^* \text{fa}(h^{t-1}) + b') + \beta\Omega((1-\delta)\underline{K}(h^{t-1}), -b')$$

where $fa \equiv a + \max\{0, -b\}$.

LEMMA 3. $\forall$ (k, k^*, a, b), $(\underline{\sigma}, \underline{f}, \underline{f^*})$ is the worst SE of the policy game, and its value is given by

$$\underline{V}(k, k^*, a, b) = \Omega(k + k^*, a + \max\{-b, 0\})$$

Proof. Appendix. $\square$

Then, given $\underline{V}(k, k^*, a, b) = \Omega(k + k, a + \max\{0, -b\})$, I can characterize the best equilibrium outcome by solving (P).

C. Solution with Commitment

Before I move to characterizing the best equilibrium outcome, I first consider the solution to the problem when the government has a commitment technology. The optimal plan under commitment, the Ramsey plan, is the solution to a relaxed version of the probelm (P), where we can drop the sustainability constraints (6) and (8). The next proposition characterizes the solution to the Ramsey problem for the SOE.

PROPOSITION 1. *[Ramsey Solution] The Ramsey plan and associated tax rates are such that*

1. $\tau_t^* = 0 \ \forall t \geq 2$
2. *If* $\beta R_t^* = 1$ *or* $U(c, n) = \frac{c^{1-\eta}}{1-\eta} - v(n)$ *for some* $\eta \geq 0$ *then* $\tau_t^k = 0 \ \forall t \geq 2$
3. *If* $\beta R_t^* = 1 \ \forall t$ *then* $c_t = c$, $n_t = n$, $g_t = g$ *and* $K_t = K$ *for all* $t \geq 1$.
4. *If* $\lim_{t \to \infty} \beta R_t^* < 1$ *then* $\{c_t^R\} \downarrow 0$, $\{g_t^R\} \downarrow 0$, $\{n_t^R\} \to \bar{N}$, $\{K_t^R\} \to K^R > 0$

Proof. See Atkeson et al (1999) and Correia (1996). □

The first part of the proposition states that if the government has commitment, then it is not optimal to tax capital of foreign investors. That is, the marginal product of capital installed in the country is equal to the international interest rate

(11) $F_{kt} = R_t^* \quad \forall t \geq 2$

The second part of the proposition says that if $\beta R^* = 1$ or $u(c)$ is CRRA then it is also optimal to not tax capital income of domestic residents for all $t \geq 2$. Hence, in this case, it is not optimal to impose capital controls. The case with $u(c)$ being CRRA is analogous to the closed economy set-up in Chari and Kehoe (1999). The case with $\beta R^* = 1$ strengthen the seminal Chamley (1986) - Judd (1987) result about the optimality of zero capital income taxes in steady state for a closed economy under general preferences. In fact, if $\beta R_t = 1$ for $t \geq 1$, then the economy reaches its steady state in period $t = 1$ and consuption, leisure, government consumption and capital stock are all constant for all $t \geq 1$. See Atkeson et al (1999) for a complete discussion of these results. Finally, the last part of the proposition states that if $\lim_{t \to \infty} \beta R_t^* < 1$ then consumption - both private and public - and leisure are converging to zero. Because the domestic household is relatively impatient with respect to the international interest rate, it is optimal to front-load consumption and leisure. The economy then is running a current account deficit that is initially large and later it repays its debt.

Before constrasting the solution with commitment to the one without, it is usefull to understand under which conditions (if any) the Ramsey plan can be sustained as an equilibrium outcome when the government lacks commitment, that is, when $\forall t \geq 1$

$$V_t^R \equiv \sum_{s=0}^{\infty} \beta^s \left[U(c_{t+s}^R, n_{t+s}^R) + G(g_{t+s}^R) \right] \geq \underline{V}(k_t^R, k_t^{*R}, a_t^R, b_t^R)$$

where a superscript R denotes the Ramsey plan.

Proposition 2. *[Sustainability of the Ramsey Plan] (i) Suppose first that $\beta R_t^* = 1$ $\forall t \geq 1$. Then, $\forall \beta \in (0,1)$ $\exists$ $\bar{A} > 0$ sufficiently large such that $\forall \, (a_0 - b_0) \geq \bar{A}$ the Ramsey* *plan is sustainable. (ii) If, instead, $\beta R_t^* < 1$, $\forall(k, k^*, a, b)$ the solution of the problem with commitment is not sustainable.*

Proof. Appendix. □

Consider first the case with $\beta R_t^* = 1\ \forall t \geq 1$. From part 3 of Proposition 1, it follows that one only has to check that

$$V_1^R \equiv \frac{U(c^R, n^R) + G(g^R)}{1-\beta} \geq \underline{V}(k^R, k^{*R}, a^R, b^R)$$

If the government and/or the domestic household has a sufficiently large assets position then the solution to the problem with commitment is sustainable. If the government asset position is sufficiently large (i.e. if $-b_0$ is sufficiently large), then it can finance the desidered amount of public expenditure using almost entirely public savings, without having to rely on distortionary labor income taxes. In this case, the incentive to renege on the promise to not tax capital income is weak because the benefits from substituting distortionary labor tax with a non-distortionary capital levy are small. The exact same argument works if household wealth at $t = 0$ is sufficiently large. In fact, at time zero the government can impose a full capital levy and save its proceeds. Thus, part (i) of Proposition 1 emphasizes the importance of government savings as a determinant of the ability of the government to maintain its promises: the higher are the assets of the government (the lower is its debt), the lower is its incentive to renege on its promises not to tax capital income.

Instead, if $\beta R_t^* < 1$, the continuation of a Ramsey plan is *not* sustainable, no matter what the initial asset positions of the domestic households and the governments are. As shown in part 4 of Proposition 1, if $\beta R_t^* < 1$ leisure and consumption (public and private) are converging to zero as time goes to infinity, while the capital stock is converging to a positive constant. Thus, the continuation utility for the stand-in households in the country is converging to its lower bound

$$\lim_{t\to\infty} V_t^R = \frac{u(0) - v(\bar{N}) + G(0)}{1-\beta} = 0$$

Because the domestic capital stock is owned entirely by foreigners, the goverment will then find it optimal to impose a capital levy and default on its debt, achieving a higher continuation utility, $\Omega(K, FA < 0) = \Omega(K, 0) > 0$.

D. Optimality of Capital Controls

I now characterize the best sustainable outcome path under the assumption that preferences are separable with respect to consumption and leisure and they are characterized by constant elasticity of substitution:

ASSUMPTION 2. $U(c,n) : \mathbb{R}_+^2 \to \mathbb{R}$ is given by $U(c,n) = \frac{c^{1-\eta}}{1-\eta} - \chi\frac{n^{1+\gamma}}{1+\gamma}$ with $\gamma > 0$ and $\eta \in (0,1)$.

The main result of the paper is that it is optimal to more heavily tax the capital income of foreigners whenever the sustainability constraint is binding.

PROPOSITION 3. *[Capital Controls] Under A2, the best sustainable outcome path is such that* $\forall t \geq 2$

1. $\tau_t^* \geq 0$, *strictly if the sustainability constraint binds at t*
2. $\tau_t^k \leq \tau_t^*$, *strictly if the sustainability constraint binds at t*

Proof. Appendix. □

A formal proof of the statement that deals with the non-convexity of the constraint set is relagated to the appendix. To get some intuition, let λ, $Q_t\phi_t$,$\beta^t\mu_t$ be the Lagrangian multipliers associated with the implementability constraint (5), the consolidated budget constraint (7), and the sustainability constraint (8), respectively. A necessary condition for an optimal allocation is:

$$(12) \quad Q_t\phi_t\left[F_{kt+1} - R_{t+1}^*\right] = \beta^{t+1}\mu_{t+1}\Omega_{k,t+1} \qquad \forall t \geq 1$$

Marginally increasing the stock of capital owned by foreigners in the country increases the resources available to the country as a whole by $F_{kt+1} - R_{t+1}^*$, i.e. the output produced minus the compensation (net of taxes) to foreign investors. This increases the objective function in (P) by $Q_t\phi_t\left[F_{kt+1} - R^*_{t+1}\right]$. When the government cannot commit, there is an extra cost for increasing the stock of capital. This is captured by the term on the right hand side of (12). Increasing the stock of capital tightens the sustainability constraint. This introduces a wedge between the international interest rate and the marginal product of capital in the SOE that is not present in the case with commitment. Combining this optimality condition with the necessary first order condition for foreign investors which must hold in any interior competitive equilibrium,

$$(13) \quad R^*_{t+1} = (1 - \tau^*_{t+1})F_{kt+1}$$

I can obtain the following expression for τ^k_{t+1}:

$$(14) \quad \tau^*_{t+1} = \frac{\beta^{t+1}\mu_{t+1}}{Q_t\phi_t}\frac{\Omega_{k,t+1}}{F_{k,t+1}} \in [0,1)$$

Thus, the foreign investors capital income is taxed at a strictly positive rate whenever the sustainability constraint is binding, i.e. when $\mu_{t+1} > 0$.

To understand why it is optimal to tax domestic residents at a lower rate than foreign investors, consider the following necessary condition for an interior optimal allocation:

$$(15) \quad u'(c_t) = \beta R^*_{t+1}u'(c_{t+1}) + \frac{\mu_{t+1}}{1 + (1-\eta)\lambda + M_t}R^*_{t+1}u'(c_{t+1})$$

where $M_t \equiv \sum_{s=0}^{t}\mu_s$ is the cumulative Lagrangian multiplier on the sustainability constraints (8). Condition (15) equates the marginal cost of postponing consumption from t to $t+1$, $u'(c_t)$, with its marginal benefits. Postponing consumption not only increases tomorrow's utility by $\beta R^*_{t+1}u'(c_{t+1})$, but it also helps to relax the sustainability constraint at $t+1$. The latter effect is captured by the second term on the right hand side of (15). Combining (15) with the Euler equation for the domestic household which must hold in any interior competive equilibrium

$$(16) \quad u'(c_t) = \beta u'(c_{t+1})(1 - \tau^k_{t+1})F_{kt+1}$$

and (13), I obtain the following expression for the optimal τ^k_{t+1} $\forall t \geq 1$:

$$(17)\quad \tau^k_{t+1} = \tau^*_{t+1} - (1-\tau^*_{t+1})\frac{\mu_{t+1}}{1+(1-\eta)\lambda + M_t} \leq \tau^*_{t+1}$$

Thus, because $\tau^*_{t+1} \leq 1$, it follows from (17) that whenever the sustainability constraint is binding, $\tau^k_{t+1} < \tau^*_{t+1}$. That is, the capital income of domestic residents is taxed at a lower rate than that of foreigners or, equivalently, there are capital controls on inflows.

The intuition for the optimality of capital controls is that when the fraction of installed capital owned by foreigners, k^*/K, is high, the government finds it more difficult to resist the temptation to impose a big capital levy. In fact, if the ratio k^*/K is high, imposing a capital levy allows the government to raise funds via a non distortionary tax, and to transfer resources from foreign investors to domestic households. As k^*/K decreases, the second effet is no longer present. Hence, it is easier for the government to credibly commit to low capital income taxes[8].

The optimality of capital controls is a robust feature of the environment. Above, I characterized the best SE of the policy game, using the worst SE to punish a deviation by the government. If instead I followed Benhabib and Rustichini (1997) and exogenously specified a continuation value $\Omega(K, fa) : \mathbb{R}^2_+ \to \mathbb{R}$ for a government that chooses to deviate from the equilibrium path of plays, I can show the result still holds; the conclusions of Proposition 3 hold true for all Ω that are strictly increasing in K, (weakly) increasing in fa, and differentiable in K[9]. Therefore, my results are not specific to using the value of the worst SE as a punishment for a deviation. In particular, optimality of capital controls holds if Ω is such that $\forall K, fa, fa'$ $\Omega(K, fa) = \Omega(K, fa')$,as in Aguiar and Amador (2011a, 2011b).

An interesting feature of the model is that, despite the fact that the government can save after a deviation (default), a strictly positive level of debt can be sustained in equilibrium, contrary to the finding in Bulow and Rogoff (1989). In fact, in my formulation, the prospect of a default is worse than in Bulow-Rogoff (1989) in that a deviation by the government not only triggers exclusion from borrowing, but also a drop in investment and therefore in output.

[8] See Gourinchas and Jeanne (2005) for a static model with a similar mechanism.

[9] The differentiability of Ω is just a technical requirement and it is not necessary, as shown in the proof of Proposition 3 in the appendix. In fact, the value of the worst SE is not differentiable at $(K, 0)$ $\forall K \geq 0$.

However, despite the possibility of sustaining positive government debt, it is efficient to run down public debt. In closely related papers, Aguiar and Amador (2011a, 2011b) emphasize the role of public foreign asset accumulation in relaxing the sustainability constraint for the government. The same conclusion carries over to my environment with one caveat. As argued in Proposition 2 part 1, a higher government assets position - or lower public debt - leads to loosening of the sustainability constraints. In fact, the government has little to gain from raising funds through a non-distortionary taxsince it can finance g_t mainly using its positive assets position and relying very little on distortionary labor income taxation. However, I can show that the government is accumulating assets only if the domestic capital stock is owned entirely by the domestic households ($k/K = 1$). This is the content of Proposition 4.

PROPOSITION 4. *The best sustainable outcome path is such that* $\forall\ t \geq 1$, *whenever the sustainability constraint is binding, either (i)* $k_t = K_t$ *and* $b_t \in \mathbb{R}$, $a_t \geq 0$, *or (ii)* $k_t < K_t$ *and* $b_t \geq 0$, $a_t = 0$.

E. Convergence to a Steady State

I now characterize the long-run behavior of the best equilibrium outcome. Are capital controls temporary or permanent? I show that if the international interest rate, R_t^*, converges to the inverse of the discount factor β, that is $\lim_{t\to\infty} \beta R_t^* = 1$, then the economy is converging to a steady state with no taxes on capital income and no capital controls. Instead, if the international interest rate is lower than the discount factor, $\lim_{t\to\infty} \beta R_t^* < 1$, then the economy is converging to a steady state with positive taxes on foreign investment and capital controls on inflows. Consider first the case with $\lim_{t\to\infty} \beta R_t^* = 1$.

PROPOSITION 5. *[Steady State with* $\beta R_\infty^* = 1$*] Under A2, if* $\lim_{t\to\infty} \beta R_t^* = 1$ *then the economy converges to an interior steady state such that: (i) the sustainability constraint is not binding and (ii):* $\tau^k, \tau^{k*}, \tau^a \to 0$ *and* $\tau^n \to \tau_\infty^n > 0$.

Proof. Appendix. □

From (15) and the fact that $\lim_{t\to\infty} \beta R_t^* = 1$, for t sufficiently large, it follows that the consumption profile for the domestic household is increasing whenever the sustainability constraint is binding ($\mu_{t+1} > 0$) and is constant otherwise. A similar argument can be made for public consumption and leisure. Hence, $\{c_{t+s}, g_{t+s}, \bar{N} - n_{t+s}\}_{t=0}^{\infty}$ and, therefore, the continuation utility is weakly increasing. It will be strictly increasing if the sustainability constraint is binding. The gist of the proof is to show that, in the limit, the sustainability constraint cannot bind ($\mu_t \to 0$). The logic of the proof is by contradiction. Suppose that $\lim_{t=0} \mu_t > 0$, then it must be that $c_t, g_t \to \infty$ and $n_t, K_t \to 0$. I can then show that for t large enough there is a continuation plan (with constant consumption, hours worked, and capital) which attains higher utility, is resource feasible, and is sustainable. Then the original plan cannot be optimal. Thus, when $\beta R^* = 1$, the optimal outcome path is such that consumption (public and private) and leisure are back-loaded in order to relax the sustainability constraint until it is no longer binding. When the sustainability constraint no longer binds, the optimal policy with commitment can be implemented. Therefore, hours worked and private and public consumption are constant and there are no capital income taxes for both domestic households and and foreign investors. Thus, capital controls are not a feature of the optimal policy in the long-run.

A typical best sustainable outcome path for the case with $\beta R_t^* = 1 \; \forall t \geq 1$ is displayed in Figure 1, together with the best outcome under commitment. First, notice that initially capital income of the domestic agents is taxed at a lower rate than the capital income of foreign investors. Both taxes are strictly positive and approach zero as time elapses, as stated in Proposition 5. Second, because it is optimal to back-load utility in order to relax future sustainability constraints, private and public consumption are increasing over time and converge to their steady state level from below. Hours worked are also increasing over time. This is not guaranteed, as there are two opposing forces: (i) on one hand, to relax future sustainability constraints, the government wants to backload leisure $(1 - n_t)$, and (ii) on the other hand, it is less efficient to work at early dates since capital is increasing over time.

Next, I consider the case with $\lim_{t\to\infty} \beta R_t^* < 1$. I show that in this case capital controls are a feature of the optimal policy also in the long-run.

Proposition 6. *[Steady State with $\beta R_\infty^* < 1$] Under A2, if $\lim_{t\to\infty} \beta R_t^* < 1$ then the economy converges to an interior steady state $(c_\infty, n_\infty, g_\infty, K_\infty) \in \mathbb{R}_{++}^4$ such that (i) the sustainability constraint is binding, and (ii) $\tau_t^* \to \tau_\infty^* > 0$, $\tau_t^k \to \tau_\infty^k < \tau_\infty^*$, and $\tau^n \to \tau_\infty^n = 0$.*

Proof. Appendix. □

To understand the logic behind Proposition 6, notice that because $\lim_{t\to\infty} \beta R_t^* < 1$, for T sufficiently large, $\beta R_T^* < 1$. Thus, the domestic residents are impatient relative to the international interest rate. From equation (15) one can see that there are two opposing forces: (i) relative impatience induces the domestic household to *front-load* consumption, and (ii) binding sustainability constraints calls for *back-loading* consumption. The same is true for government consumption and for leisure. As t goes to infinity, two things can happen: either the economy is converging to an interior steady state $(c_\infty, n_\infty, g_\infty, K_\infty) \in \mathbb{R}^4_{++}$ or it is converging to an "immiseration" steady state $(c_\infty, n_\infty, g_\infty, K_\infty) = (0, 0, 0, 0)$. The idea of the proof is to show that no optimal path can converge to the immiseration steady state. I can find a sustainable plan starting from the "immiseration" steady state that attains higher utility by having positive foreign capital invested in the country and using the increased output to increase government consumption. The government can commit not to tax this capital at expropriatory rate because the marginal return on capital in unbounded for K greater than but sufficiently close to zero. The economy then must converge to an interior steady state. For consumption to converge to some positive constant (instead of converging to zero), from (15), it must be that $\mu_t \to \mu_\infty > 0$. This and (14) imply that $\tau_t^* \to \tau_\infty^* > 0$. Moreover, combining the necessary household Euler equation, (16), with the one for the foreign investors, (13), it follows that

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta R^* \frac{1-\tau_{t+1}^k}{1-\tau_{t+1}^*} \to 1 \Rightarrow \lim_{t\to\infty} \tau_t^k = \tau_\infty^k < \tau_\infty^*$$

Thus it is optimal to have capital controls also in the long run. Finally, as shown in parallel work by Aguiar and Amador (2011b), labor income taxes converge to zero if $\beta R^* < 1$. See their paper for an exhaustive discussion of the result about labor income taxes.

In summary, the optimal policy with commitment calls for no capital controls; there are no taxes on capital income for both domestic and foreign investors. Instead, when the government cannot commit, capital controls on inflows are optimal on the transition to the steady state. In the long run, capital controls are a feature of the best equilibrium outcome only if the domestic agents are impatient relative to the international interest rate ($\beta R^* < 1$).

When $\beta R^* = 1$, the government and the private domestic agents accumulate sufficient assets that the government can credibly commit not to tax capital income[10]. In that case, both the marginal product of capital and the intertemporal marginal rate of substitution of the domestic households are equated to the international interest rate.

4. Comparison to an Environment without Capital Controls

To assess the potential welfare gains of adopting the optimal policy, I compare the best equilibrium outcome with the one in which the government cannot impose capital controls, i.e. where $\tau_t^k = \tau_t^*$. This restriction eliminates the possibility for the government to manipulate the saving decision of the domestic households. In any equilibrium outcome, it must be that

$$(18) \quad u'(c_t) = \beta R_{t+1}^* u'(c_{t+1})$$

That is, there is no wedge between the international interest rate and the intertemporal marginal rate of substitution for the domestic household.

Assuming that $\beta R_t^* = 1 \ \forall t \geq 1$, the optimal outcome path for the economy without capital controls is shown in Figure 2, compared to the best outcome path for the economy analyzed in the prevous sections. To allow for a clean comparison, I assume that the government can impose a very high capital levy on the domestic household ($\tau_0^k = 1$), so that the right hand side of (5) is always equal to zero, making the first order conditions with respect to time zero variables symmetric to the ones for $t \geq 1$. When $\tau_t^k = \tau_t^* \ \forall t \geq 1$, consumption is constant over time because of (18). Leisure is back-loaded; this implies that domestic households are working more in periods with low levels of capital than in periods when the capital stock is high. Moreover, it also implies that if the economy is experiencing positive TFP growth, households work less when productivity is high. This is because, with a constant consumption profile, the government has a stronger incentive to back-load labor and public consuption to increase future continuation utility and to relax future sustainability constraints. To do so, the labor income tax will be increasing over time, while it is decreasing over time if the government can impose capital controls. As shown in Figure 2, the economy

[10]As show in Proposition 2, if $\beta R^* = 1$ and the economy has a sufficiently large foreign asset position, the transition to the steady state can last one period only. It is slower for lower initial foreign asset positions.

is converging to a steady state with no capital income taxes for both domestic households and foreigners. So, the sustainability constraint is not binding in the limit. The difference with the case where the government is allowed to impose capital controls is that the government must rely too heavily on back-loading government consumption and leisure when it cannot manipulate the consumption profile of the domestic households. This generates welfare losses.

An interesting finding is that the gains from being able to tax capital income of domestic and foreign residents at different rates crucially depend on the profile of TFP over time. Modify the production function (2) to be such that $y_t = Z_t F(K_t, n_t)$ for some deterministic sequence for TFP, $\{Z_t\}_{t=0}^{\infty}$, such that $Z_t \to Z_\infty$ as $t \to \infty$. With this specification, all the results derived in the previous sections go through. In my numerical examples, the gains - measured in terms of CEV in consumption - are modest when $\{Z_t\}_{t=0}^{\infty}$ is constant over time, on the order of 0.2% increase in life-time consumption. However, they can be substantial if $\{Z_t\}_{t=0}^{\infty}$ is growing over time. Moreover, the gains are increasing in the growth rate of TFP. This suggests that the normative recommendations of this paper are more relevant for fast growing economies.

5. General Equilibrium: 2-Country Case

The results derived in the previous sections generalize to a general equilibrium environment where the international interest rate is endogenous and there is strategic interaction amongst governments. In particular, capital controls arise under coordination among heterogeneous countries and they are not the result of a "prisoner's dilemma" sort of argument. Any outcome in the set of best sustainable equilibrium outcome of the game between the two governments and the households in the two countries cannot be improved in a Pareto sense by a planner subject to the constraint that the outcome (i) has to be a competitive equilibrium for the economy, and (ii) has to satisfy the sustainability constraints for each country. Thus, "coordination" between governments cannot improve upon a best sustainable outcome of the game between the two.

Moreover, a ban on capital controls will generate welfare losses. It amounts to restricting the set of instruments available to the government without generating any benefits from "coordination". These results are interesting in relation to the work in Costinot, Lorenzoni and Werning (2011) which studies the optimality of capital controls motivated by manipulation of the terms of trade for a two-country pure exchange economy. The policy implications of their model for a growing economy closely resembles those from my model, despite the fact that the mechanisms are completely different. However, in their economy, capital controls are inefficient so it would be optimal for an international organization like the IMF to ban capital controls since imposing capital controls is individually optimal but there are gains from coordination. In the case considered here, with lack of commitment and a production economy, it is no longer efficient to ban capital controls.

6. Conclusion

In this paper, I provide a mechanism through which policies aimed at increasing the self-financing ratio - controls on capital inflows - are welfare improving. It can account for the fact that countries with higher self-financing ratios are growing faster during the 1990s, as is documented in Aizenman et al. (2007). When the government doesn't have commitment, it is optimal to tax capital income of domestic residents at a lower rate to stimulate domestic capital accumulation, which helps to relax future commitment problems. Capital controls persist in the long-run only if the domestic households are impatient relative to the international interest rate. I further show that the policy recomendations are more relevant for countries that face higher TFP growth.

One important question that is left for future research is: Can the mechanism in this paper account for the capital allocation puzzle? Gourinchas and Jeanne (2011) documented that the direction of capital flows to developing countries is at odds with the predictions of the standard neoclassical growth model. Capital does not flow to countries that grow and invest more; the opposite is true. Moreover, the capital allocation puzzle is driven mainly by public flows. There is a strong positive correlation between TFP growth and accumulation of foreign assets by the government, as documented in Aguiar and Amador (2011b) and Gourinchas and Jeanne (2011). As Gourinchas and Jeanne (2011) suggest: "*the solution to the allocation puzzle should be looked for at the nexus between growth, saving and reserve accumulation. Why do countries that grow more also accumulate more reserves, and why is this reserve accumulation not offset by capital inflows to the private sectors*?". The basic insights of my paper can be generalized to a model where the government cannot commit to costly reforms that are optimal ex-ante but not ex-post.[11] Modifying the environment in such a way will generate an endogenous link between TFP growth, capital controls, and accumulation of foreign assets by the government.

Furthermore, in this paper I abstracted from uncertainty. This model, augmented with shocks to productivity and the international interest rate, offers a framework for studying how the government should respond to shock to the international interest rate, and how the differential between domestic and foreign capital income taxes moves over the cycle.

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8. Appendix: Proofs.

A. Proof of Lemma 1

($\Rightarrow$) Let $x \equiv \{c_t, n_t, k_{t+1}, k^*_{t+1}, a_{t+1}, g_t\}_{t=0}^{\infty}$ be part of a competitive equilibrium. Then it must satisfy the necessary focs for the stand-in household's problem:

$$\begin{aligned}
U_{ct} &= \beta U_{ct+1}(1-\tau^k_{t+1})F_{kt+1} \\
U_{ct} &= \beta U_{ct+1}(1-\tau^a_{t+1})F_{kt+1} \\
-U_{nt} &= U_{ct}(1-\tau^n_t)F_{nt}
\end{aligned}$$

and the TVCs

$$\begin{aligned}
\lim_{T\to\infty} \beta^T U_{cT}(1-\tau^k_T)F_{kT}k_T &= 0 \\
\lim_{T\to\infty} \beta^T U_{cT}(1-\tau^a_T)R^* a_T &= 0
\end{aligned}$$

and from the foreign investors:

$$\begin{aligned}
R^*_t &= (1-\tau^*_t)F_{kt} \\
q_{t+1} &= \delta_{t+1}/R^*_{t+1}
\end{aligned}$$

Therefore, substituting out for prices and taxes in the household's budget constraint at $t=0$ and iterating forward (invoking the TVCs), I obtain the implementability condition (5). In the same way $\forall t \geq 1$ I obtain that

$$\sum_{s=0}^{\infty} \beta^s \left[U_{ct+s}c_{t+s} + U_{nt+s}n_{t+s}\right] = U_{ct}\left[(1-\tau^k_t)F_{kt}k_t + (1-\tau^a_t)R^*_t a_t\right] \geq 0$$

where the fact that the rhs is greather than zero comes from the fact that in any equilibrium $k_t, a_t, U_{ct}, (1-\tau^k_t)F_{kt}, (1-\tau^a_t)R^*_t \geq 0$. Hence $\forall\ t \geq 1$ (6) must hold in any competitive equilibrium. Finally, since the budget constraint for the government

$$\delta_t b_t + g_t = \tau^n_t w_t n_t + \tau^k_t R_t k_t + \tau^a_t R^* a_t + \tau^*_t R_t k^*_t + q_{t+1}b_{t+1}$$

and the domestic household

$$
\begin{aligned}
c_{t+1}+k_{t+1}+a_{t+1} &= (1-\tau_t^k)R_tk_t+(1-\tau_t^a)R^*a_t+(1-\tau_t^n)w_tn_t \\
&= -\left[\tau_t^n w_tn_t+\tau_t^kR_tk_t+\tau_t^aR^*a_t\right]+R_tk_t+w_tn_t+R_t^*a_t
\end{aligned}
$$

must hold with equality in any equilibrium, I can combine them substituting out for prices to obtain:

$$
\begin{aligned}
\delta_tb_t+g_t &= \left[\tau_t^n w_tn_t+\tau_t^kR_tk_t+\tau_t^aR^*a_t\right]+\tau_t^*R_tk_t^*+q_{t+1}b_{t+1} \\
&= \left[R_tk_t+w_tn_t+R_t^*a_t-(c_{t+1}+k_{t+1}+a_{t+1})\right]+\tau_t^*R_tk_t^*+\frac{\delta_{t+1}}{R_{t+1}^*}b_{t+1} \\
&= \left[F_{kt}k_t+F_{nt}n_t+R_t^*a_t-(c_{t+1}+k_{t+1}+a_{t+1})\right]+\tau_t^*F_{kt}k_t^*+\frac{\delta_{t+1}}{R_{t+1}^*}b_{t+1} \\
&= \left[F_{kt}k_t+F_{nt}n_t+R_t^*a_t-(c_{t+1}+k_{t+1}+a_{t+1})\right]+\left(F_{kt}-R_t^*\right)k_t^*+\frac{\delta_{t+1}}{R_{t+1}^*}b_{t+1} \\
&= F(k_t+k_t^*,n_t)+R_t^*a_t-(c_{t+1}+k_{t+1}+a_{t+1})-R_t^*k_t^*+\frac{\delta_{t+1}}{R_{t+1}^*}b_{t+1} \\
\iff\quad & F(k_t+k_t^*,n_t)+R_t^*a_t-\delta_tb_t=g_t+c_t+k_{t+1}+R_t^*k_t^*+a_{t+1}-\frac{\delta_{t+1}}{R_{t+1}^*}b_{t+1}
\end{aligned}
$$

Then, iterating forward I obtain:

$$
\sum_{t=0}^{\infty}Q_t\left[g_t+c_t+k_{t+1}+R^*k_t^*-F(K_t,n_t)\right]-R^*a_0+(1-\tau_0^*)F_k(K_0,n_0)k_0^*=-b_0\delta_0
$$

Hence the consolidated budget constraint for the country (7) must also hold, as desired.

($\Leftarrow$) Suppose $x\equiv\{c_t,n_t,k_{t+1},k_{t+1}^*,a_{t+1},g_t\}_{t=0}^{\infty}$ satisfies (5), (6) and (7). Let taxes be such that $\forall\ t\geq 0$

$$
\begin{aligned}
\tau_{t+1}^p &: \; R_t^*=(1-\tau_t^*)F_{kt} \\
\tau_{t+1}^k &: \; U_{ct}=\beta U_{ct+1}(1-\tau_{t+1}^k)F_{kt+1} \\
\tau_{t+1}^a &: \; U_{ct}=\beta U_{ct+1}(1-\tau_{t+1}^a)R_{t+1}^* \\
\tau_t^n &: \; -U_{nt}=U_{ct}(1-\tau_t^n)F_{nt}
\end{aligned}
$$

and $\delta_t=1\ \forall t\geq 1$. Let factor prices be $w_t=F_{nt}$ and $R_t=F_{kt}$ and $q_{t+1}=1/R_{t+1}^*\ \forall t\geq 0$.

Then the allocation satisfies the (sufficient) focs for the household optimization problem at the constructed prices and taxes and it is affordable due to (5). It also satisfies the optimality conditions for the foreign investors.

I am then left to show that the allocation, constructed taxes and prices satisfy the government budget constraint. To this end, I construct government debt $\{b_{t+1}\}_{t=0}^{\infty}$ recursively as follows:

$$R_t^* b_t = \tau_t^n w_t n_t + \tau_t^k R_t k_t + \tau_t^a R^* a_t + \tau_t^* R_t k_t^* + b_{t+1} - g_t$$

to make the government consumption $\{g_t\}_{t=0}^{\infty}$ affordable. Notice that $\lim_{t\to\infty} b_t$ is going to be bounded because of (7). □

B. Sketch of Proof of Lemma 3

Claim 1. Can construct strategy to support as an equilibrium outcome the allocation $\underline{x}$ that solves the problem above. Call this strategy $(\underline{\sigma}, \underline{f}, \underline{f^*}) \in SE(k, k^*, a, b)$.Then $\underline{V}(k, k^*, a, b) = \underline{V} = W(\underline{\sigma}, \underline{f}, \underline{f^*})$.

Claim 2. If x is an equilibrium outcome path for the game thenn it is an equilibrium outcome path for a similar game when the government can also tax investment.

Claim 3. $\underline{V} = \min v$ such that $\exists(\sigma^i, f^i, f^{*i}) \in SE^i(k, k^*, a, b) : v = W^i(\sigma^i, f^i, f^{*i})$ where a superscript i refers to the game where the government can tax investment also

Claim 4. $\underline{V} = \min v$ such that $\exists(\sigma, f, f^*) \in SE(k, k^*, a, b) : v = W(\sigma, f, f^*)$.

This final claim is implied by the previous two claims. □

C. Proof of Proposition 2

First consider the case with $\beta = 1/R_t^*\ \forall t \geq 1$. The Ramsey problem can be written as:

$$V_0^R(K_0, a_0 - b_0) = \max_{\{c_t, n_t, g_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[U(c_t, n_t) - G(g_t)\right]$$

subject to

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t \left[U_{ct} c_t + U_{nt} n_t \right] &\geq 0 \\
\sum_{t=0}^{\infty} \beta^t \left[c_t + g_t + K_{t+1} - F(K_t, n_t) \right] &= a_0 - b_0
\end{aligned}
$$

Notice that for $a_0 - b_0$ sufficiently large, the implementability constraint doesn't bind. In fact, for $a_0 - b_0 \geq \bar{A}'$ sufficiently high, it is possible to finance the "first best" level of public expenditure - the solution to the above problem dropping the implementability constraint, denoted by a superscript FB - without levying any taxes:

$$
a_0 - b_0 \geq \sum_{t=0}^{\infty} \beta^t g_t^{FB}(K_0, a_0 - b_0) = \frac{g^{FB}(K_0, a_0 - b_0)}{1 - \beta}
$$

Hence, for $a_0 - b_0 \geq \bar{A}'$ $V_0^R(K_0, a_0 - b_0) = V_0^{FB}(K_0, a_0 - b_0)$ and $V_1^R(K_0, a_0 - b_0) = V_1^{FB}(K_0, a_0 - b_0)$, where

$$
V_1^R(K_0, a_0 - b_0) \equiv \sum_{t=1}^{\infty} \beta^{t-1} \left[U(c_t, n_t) - G(g_t) \right]
$$

Then, since $\underline{V}_1 < V_1^{FB}$, since V_1^R is continuous in $a_0 - b_0$, $\underline{V}_1 \leq V_1^R$ for $a_0 - b_0 \geq \bar{A} < \bar{A}'$, proving part (i).

Consider now the case with $\beta R^* < 1$. The solution with commitment is such that

$$
\{c_t^R\} \downarrow 0, \;\; \{g_t^R\} \downarrow 0, \; \{n_t^R\} \rightarrow \bar{N}, \; \{K_t^R\} \rightarrow K^R > 0
$$

That is, it is optimal to front-load consumption and leisure. It is easy to see that for t sufficiently high and for $\varepsilon > 0$ sufficiently small

$$
V_t^R = \sum_{s=0}^{\infty} \beta^s \left[u(c_{t+s}^R) - v(n_{t+s}^R) + G(g_{t+s}^R) \right] \leq \frac{u(0) - v(\bar{N}) + G(0)}{1 - \beta} + \varepsilon < \Omega(K_t)
$$

Then the Ramsey plan cannot be supported as a SE if $\beta R^* < 1$, proving the second part of the proposition. $\square$

D. Proof of Proposition 3:

Under assumption A2 the implementability condition (5) in (P) can be written as

$$\sum_{t=0}^{\infty} \beta^t \left(c_t^{1-\eta} - \chi n_t^{1+\gamma}\right) = c_0^{-\eta} A_0$$

The constraint set in (P) is not convex. Moreover, the plan $x = \{c_t, n_t, k_{t+1}, k_{t+1}^*, a_{t+1}, b_{t+1}, g_t\}_{t=0}^{\infty}$ that solves (P) is not necessarily interior. In fact, it is likely a_{t+1} and b_{t+1} are at a corner as shown in Proposition 4. Thus, using Lagrangian methods to characterize the full problem (P) is not feasible. However, if x is the solution to (P), then it must be that $\{c_t, n_t, k_{t+1}, k_{t+1}^*, g_t\}_{t=0}^{\infty}$ solves

$$\text{(P')} \quad \max_{\{c_t, n_t, k_{t+1}, k_{t+1}^*, g_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\eta}}{1-\eta} - \chi \frac{n_t^{1+\gamma}}{1+\gamma} + G(g_t)\right]$$

subject to

$$(19) \quad (\lambda) \quad \sum_{t=0}^{\infty} \beta^t \left(c_t^{1-\eta} - \chi n_t^{1+\gamma}\right) = c_0^{-\eta} A_0$$

$$(20) \quad \left(\beta^t \psi_t\right) \quad \sum_{s=0}^{\infty} \beta^s \left(c_{t+s}^{1-\eta} - \chi n_{t+s}^{1+\gamma}\right) \geq 0 \quad \forall t \geq 1$$

$$(21) \quad (Q_t \phi_t) \quad F(k_t + k_t^*, n_t) + R_t^*(a_t - b_t) \geq g_t + c_t + k_{t+1} + R_t^* k_t^* + (a_{t+1} - b_{t+1}) \quad \forall t \geq 0$$

$$(22) \quad \left(\beta^t \mu_t\right) \quad \sum_{s=0}^{\infty} \beta^s \left[U(c_{t+s}, n_{t+s}) + G(g_{t+s})\right] \geq \Omega(k_t + k_t^*, a_t + \max\{-b_t, 0\}) \quad \forall t \geq 1$$

$$k_t, k_t^* \geq 0 \quad \forall t \geq 1$$

given the optimal path for $\{a_{t+1}, b_{t+1}\}_{t=0}^{\infty}$. Thus, if the optimal $\{c_t, n_t, k_{t+1}, k_{t+1}^*\}_{t=0}^{\infty}$ is interior, then it must satisfy the following necessary foncs wrt $c_t, n_t, g_t, k_{t+1}, k_{t+1}^*$ $\forall t \geq 1$:

$$(23) \quad 0 = \beta^t \left[1 + (1-\eta)\lambda + M_t + (1-\eta) P_t\right] c_t^{-\eta} - Q_t \phi_t$$

$$(24) \quad 0 = \beta^t \left[1 + (1+\gamma)\lambda + M_t + (1+\gamma) P_t\right] \chi n_t^{\gamma} - Q_t \phi_t F_{nt}$$

$$(25) \quad 0 = \beta^t \left(1 + M_t\right) G'(g_t) - Q_t \phi$$

$$
\begin{aligned}
(26) \quad 0 &= -Q_t\phi_t - \beta^{t+1}\mu_{t+1}\Omega_{k,t+1} + Q_{t+1}\phi_{t+1}F_{kt+1} \\
(27) \quad 0 &= Q_{t+1}\phi_{t+1}\left[F_{kt+1} - R^*_{t+1}\right] - \beta^{t+1}\mu_{t+1}\Omega_{k,t+1}
\end{aligned}
$$

where λ, $\beta^t\psi_t$, $Q_t\phi_t$, and $\beta^t\mu_t$ are the Lagrange multipliers associated with (19), (20), (21), (22) respectively, and $M_t \equiv \sum_{s=1}^{t}\mu_s$ $\forall t \geq 1$ and $P_t \equiv \sum_{s=1}^{t}\psi_s$. For a proof for the fact that multiplier for this class of problems are in ℓ_1 see Rustichini (1998).

The fonc wrt k^*_{t+1}, (27), can be rearranged as follows:

$$
R^*_{t+1} = F_{kt+1} - \frac{\beta^{t+1}\mu_{t+1}}{Q_t\phi_t}\Omega_{k,t+1}
$$

Therefore, since in any equilibrium it must be that $R^*_{t+1} = (1-\tau^*_{t+1})F_{kt+1}$, we have that $\forall t \geq 1$

$$
\tau^*_{t+1} = \frac{\beta^{t+1}\mu_{t+1}}{Q_t\phi_t}\frac{\Omega_{k,t+1}}{F_{k,t+1}} \geq 0
$$

strictly if $\mu_{t+1} > 0$, establishing part 1 of the proposition.

To establish part 2, notice that I can combine (26) and (27) to obtain:

$$
\begin{aligned}
\beta^{t+1}\mu_{t+1}\Omega_{k,t+1} &= Q_{t+1}\phi_{t+1}\left[F_{kt+1} - R^*_{t+1}\right] = -Q_t\phi_t + Q_{t+1}\phi_{t+1}F_{kt+1} \\
&\Rightarrow Q_t\phi_t = Q_{t+1}\phi_{t+1}R^*_{t+1} \Rightarrow \phi_t = \phi_{t+1}
\end{aligned}
$$

Then, using (23) to substitute for $Q_t\phi_t$ and $Q_{t+1}\phi_{t+1}$ I get:

$$
\begin{aligned}
&\beta^t\left[1+(1-\eta)\lambda + M_t + (1-\eta)P_t\right]c_t^{-\eta} = \beta^t\left[1+(1-\eta)\lambda + M_{t+1} + (1-\eta)P_{t+1}\right]c_{t+1}^{-\eta}R^*_{t+1} \\
&\Rightarrow \frac{c_t^{-\eta}}{\beta R^*_{t+1}c_{t+1}^{-\eta}} = \left(\frac{1+(1-\eta)\lambda + M_{t+1} + (1-\eta)P_{t+1}}{1+(1-\eta)\lambda + M_t + (1-\eta)P_t}\right) = 1 + \frac{\mu_{t+1} + (1-\eta)\psi_{t+1}}{1+(1-\eta)\lambda + M_t + (1-\eta)P_t}
\end{aligned}
$$

Therefore, since in any competitive equilibrium it must be that $c_t^{-\eta} = \beta c_{t+1}^{-\eta}(1-\tau^k_{t+1})F_{kt+1}$, it follows that $\forall t \geq 1$

$$
\begin{aligned}
(1-\tau^k_{t+1}) &= (1-\tau^*_{t+1})\left(1 + \frac{\mu_{t+1} + (1-\eta)\psi_{t+1}}{1+(1-\eta)\lambda + M_t + (1-\eta)P_t}\right) \\
&\Rightarrow \tau^k_{t+1} = \tau^*_{t+1} - (1-\tau^*_{t+1})\frac{\mu_{t+1} + (1-\eta)\psi_{t+1}}{1+(1-\eta)(\lambda + P_t) + M_t} \leq \tau^*_{t+1}
\end{aligned}
$$

strictly if $\mu_{t+1} > 0$, since $(1-\tau^*_{t+1}) \in (0,1]$ as $\tau^* \leq 1$. This establish part 2 of the Proposition. $\square$

E. Proof of Proposition 4

Let $\hat{x}$ be the optimal solution to (P). I will concentrate with public NFA position, the argument for the private position is identical. Suppose for contradiction that there exists a $t \geq 1$ such that the sustainability constraint is binding, $\hat{k}_t < \hat{K}_t$, and that is the government is saving abroad ($b_t < 0$). Consider the alternative plan x constructed from $\hat{x}$ and some $\varepsilon > 0$ sufficiently small as follows:

$$
\begin{aligned}
b_t &= \hat{b}_t + \varepsilon \leq 0, \quad k_t = \hat{k}_t + \varepsilon > 0, \quad k^*_t = \hat{k}^*_t - \varepsilon + \xi(\varepsilon) \geq 0, \quad g_t = \hat{g}_t + \zeta(\varepsilon) > \hat{g}_t \\
b_s &= \hat{b}_s, \quad k_s = \hat{k}_s, \quad k^*_s = \hat{k}^*_s, \quad g_s = \hat{g}_s \quad \forall s \neq t \\
c_s &= \hat{c}_s, \quad n_s = \hat{n}_s, \quad a_s = \hat{a}_s \quad \forall s \geq 0
\end{aligned}
$$

where $\xi(\varepsilon), \zeta(\varepsilon) > 0$ are defined as follows. $\xi(\varepsilon) > 0$ is chosen such that

$$
\text{(28)} \quad \Omega(\hat{K}_t, \hat{b}_t + \hat{a}_t) = \Omega(\hat{K}_t + \xi, \hat{b}_t - \varepsilon + \hat{a}_t)
$$

Notice that $\forall \varepsilon > 0$ such a $\xi > 0$ exists by continutity of Ω and the fact that it is strictly increasing in both arguments. Finally, $\zeta(\varepsilon) > 0$ is defined by

$$
c_t + \hat{g}_t + \zeta(\varepsilon) \; + k_{t+1} + a_{t+1} - b_{t+1} + R^*_t k^*_t = F(K_t, n_t) + R^*_t \left(a_t - b_t\right)
$$

Because $\xi(\varepsilon) > 0$, it is evident that x attains higher utility than $\hat{x}$. Then, to prove that $\hat{x}$ cannot be an optimal plan, it is sufficient to show that x is in the constraint set of (P). First, notice that if x satisfies the implementability constraint then x does too since c and n are unchanged. Consider now the sequence of sustainability constraints. $\forall s \geq t$ they are unchanged. At t instead

$$\begin{aligned}\sum_{s=0}^{\infty}\beta^s\left[U(c_{t+s},n_{t+s})+G(g_{t+s})\right] &> \sum_{s=0}^{\infty}\beta^s\left[U(\hat{c}_{t+s},\hat{n}_{t+s})+G(\hat{g}_{t+s})\right]=\Omega(\hat{K}_t,\hat{b}_t+\hat{a}_t)\\ &= \Omega(\hat{K}_t+\xi,\hat{b}_t-\varepsilon+\hat{a}_t)=\Omega(K_t,b_t+a_t)\end{aligned}$$

by definition of $\xi(\varepsilon)$. Then the sustainability constraint holds at t, and so it does $\forall s<t$ because the RHS is unchanged and the LHS is increased. So, finally, I must check the consolidated budget constraint at $t-1$, since $\forall s\neq t,t-1$ the consolidated budget constraint is the same under $\hat{x}$ and x, and at t it holds by definition of $\zeta(\varepsilon)$. To this end, notice that at $t-1$

$$\begin{aligned}&\hat{c}_{t-1}+\hat{g}_{t-1}+\hat{k}_t+\hat{a}_t-\hat{b}_t+R^*\hat{k}_{t-1}=F(\hat{k}_{t-1}+\hat{k}^*_{t-1},\hat{n}_{t-1})+R^*\left(\hat{a}_{t-1}-\hat{b}_{t-1}\right)\\ \Rightarrow\ &\hat{c}_{t-1}+\hat{g}_{t-1}+(k_t-\varepsilon)+a_t-(b_t-\varepsilon)+R^*_{t-1}\hat{k}_{t-1}=F(\hat{k}_{t-1}+\hat{k}^*_{t-1},\hat{n}_{t-1})+R^*_{t-1}\left(\hat{a}_{t-1}-\hat{b}_{t-1}\right)\\ \Rightarrow\ &c_{t-1}+g_{t-1}+k_t+a_t-b_t+R^*_{t-1}k^*_{t-1}=F(k_{t-1}+k^*_{t-1},n_{t-1})+R^*_{t-1}\left(a_{t-1}-b_{t-1}\right)\end{aligned}$$

Then x is feasible and attains higher utility. Hence $\hat{x}$ cannot be optimal. □

F. Proof of Proposition 5:

Using the fact that $\beta R^*=1$, I can then rewrite the foncs wrt c_t, g_t and n_t as follows:

$$c_t^{\eta}=\frac{\beta^t}{Q_t}\frac{1+\lambda(1-\eta)+M_t}{\phi}=\frac{1+\lambda(1-\eta)+M_t}{\phi}\tag{29}$$

$$1/G'(g_t)=\frac{\beta^t}{Q_t}\frac{1+M_t}{\phi}=\frac{1+M_t}{\phi}\tag{30}$$

$$\begin{aligned}\frac{F_{n,t}}{\chi n_t^{\gamma}}&=\frac{(1-\alpha)K_t^{\alpha}n_t^{-\alpha}}{\chi n_t^{\gamma}}=\frac{\beta^t}{Q_t}\frac{1+\lambda(1+\gamma)+M_t}{\phi}=\frac{1+\lambda(1+\gamma)+M_t}{\phi}\\ \iff n_t^{-(\gamma+\alpha)}&=\frac{\chi}{(1-\alpha)K_t^{\alpha}}\frac{1+\lambda(1+\gamma)+M_t}{\phi}\geq\frac{\chi}{(1-\alpha)\bar{K}^{\alpha}}\frac{1+\lambda(1+\gamma)+M_t}{\phi}\end{aligned}\tag{31}$$

Recall that $M_t\equiv\sum_{s=1}^{t}\mu_s$. Suppose for contradiction that $\mu_t\nrightarrow 0$, implying that $M_t\to+\infty$. Then, since it must be that if $k_0+k_0^*\in[0,\bar{K}]$ then $K_t\in[0,\bar{K}]$ $\forall t\geq 1$, the RHS of the three equations above is converging to $+\infty$. Thus, it must be that also the LHS must converge to $+\infty$. Therefore we have that $c_t\to+\infty$, $n_t\to 0$, and $g_t\to+\infty$. Moreover, notice that the necessary foc wrt k_t^* is

$$(32)\quad F_{Kt} - 1/\beta = \frac{\mu_t}{\phi}\Omega_{Kt} \iff \beta\alpha\left(\frac{n_t}{K_t}\right)^{1-\alpha} = 1 + \beta\frac{\mu_t}{\phi}\Omega_{Kt} \geq 1 \iff \beta\alpha n_t^{1-\alpha} \geq K_t^{1-\alpha}$$

Since $n_t \to 0$ then also $K_t \to 0$. Now, because $n_t \to 0$ and $K_t \to 0$, $\forall\varepsilon > 0\ \exists\ T(\varepsilon) \in \mathbb{N}$ sufficiently large such that $\forall t \geq T(\varepsilon)\ K_t \leq \varepsilon$ and $n_t \leq \varepsilon$ and then $y_t \leq \varepsilon$. I will now show that the plan $x = \{c_t, n_t, g_t, k_t, k_t^*, b_t\}_{t=0}^{\infty}$ cannot be optimal. To this end, consider an alternative plan $\hat{x}(\varepsilon) = \{\hat{c}_t, \hat{n}_t, \hat{g}_t, \hat{k}_t, \hat{k}_t^*, \hat{b}_t\}_{t=0}^{\infty}$ defined as follows:

$$\begin{aligned}
\hat{x}_t &= x_t \quad \forall t = 0, 1, ..., T(\varepsilon) \\
\hat{c}_t &= \hat{c}, \quad \hat{n}_t = \hat{n}, \quad \hat{g}_t = \hat{g}, \quad \hat{k}_t = k_{T(\varepsilon)+1}, \quad \hat{k}_t^* = k_{T(\varepsilon)+1} \quad \forall t \geq T(\varepsilon) + 1
\end{aligned}$$

where $\hat{c}$, $\hat{n}$, $\hat{g}$ are defined as

$$\begin{aligned}
(33)\quad \frac{\hat{c}^{1-\eta}}{1-\beta} &= \sum_{t=1}^{\infty} \beta^{t-1} c_t^{1-\eta} \\
(34)\quad \frac{\hat{n}^{1+\gamma}}{1-\beta} &= \sum_{t=1}^{\infty} \beta^{t-1} n_t^{1+\gamma} \\
(35)\quad \frac{\hat{g}}{1-\beta} &= \frac{1}{\beta} FA_{T+1} + \frac{F(\hat{K}, \hat{n}) - R^*\hat{K}}{1-\beta} - \frac{\hat{c}}{1-\beta}
\end{aligned}$$

where

$$\begin{aligned}
FA_{T+1} &\equiv Y_T - (c_t + g_T) \\
Y_T &\equiv F(K_T, n_T) + R^* a_0 + (1 - \tau_0^*) F_k(K_0, n_0) k_0^* - R^* b_0 \\
\hat{K} &= \hat{k}_t + \hat{k}_t^* \forall t \geq T(\varepsilon) + 1
\end{aligned}$$

Finally, $\{\hat{a}_t, \hat{b}_t\}_{t \geq T(\varepsilon)+1}^{\infty}$ can be obtained residually. Notice that by construction $\hat{x}(\varepsilon)$ satisfies the implementability constraint (5), and the consolidated budget constraint of the country (7). To prove that x cannot be optimal for (P), it suffices to show that $\hat{v}_{T(\varepsilon)+1} \equiv \frac{u(\hat{c}) - v(0) + G(\hat{g})}{1-\beta} >$ $v_{T(\varepsilon)+1}$. In fact, this implies that $\hat{x}(\varepsilon)$ satisfies the sustainability constraint $\forall t \geq 0$ as $\forall t =$

$0, 1, ..., T(\varepsilon)$

$$\hat{v}_t = \sum_{s=0}^{T(\varepsilon)-t} \beta^s \left(u(c_{t+s}) - v(n_{t+s}) + G(g_{t+s})\right) + \beta^{T(\varepsilon)+1} \hat{v}_{T(\varepsilon)+1} > v_t \geq \Omega(K_t, FA_t) = \Omega(\hat{K}_t, F\hat{A}_t)$$

and $\forall t \geq T(\varepsilon) + 1$

$$\hat{v}_t = \frac{u(\hat{c}) - v(\hat{n}) + G(\hat{g})}{1-\beta} > v_{T(\varepsilon)+1} = \Omega(K_{T(\varepsilon)+1}, FA_{T(\varepsilon)+1}) = \Omega(\hat{K}_t, F\hat{A}_t) = \Omega(\hat{K}, F\hat{A})$$

Thus if x is feasible in (P) so is $\hat{x}(\varepsilon)$. Moreover, if $\hat{v}_{T(\varepsilon)+1} \equiv \frac{u(\hat{c}) - v(\hat{n}) + G(\hat{g})}{1-\beta} > v_{T(\varepsilon)+1}$ then x cannot be an optimal plan since $x(\varepsilon)$ is feasible for (P) and it attains a higher utility: $\hat{v}_0 > v_0$. First, notice that from (33) and (34) we have by strict concavity of u that

$$\frac{\hat{c}^{1-\eta}}{1-\beta} = \sum_{t=1}^{\infty} \beta^{t-1} c_t^{1-\eta} \Rightarrow \frac{\hat{c}}{1-\beta} + \Delta(\varepsilon) = \sum_{t=1}^{\infty} \beta^{t-1} c_t$$

for some $\Delta(\varepsilon) > 0$. Second, notice that (35) implies that

$$\begin{aligned}
(36) \quad \frac{\hat{g}}{1-\beta} &= \frac{1}{\beta} FA_{T+1} + \frac{F(\hat{K}, \hat{n}) - R^* \hat{K}}{1-\beta} - \frac{\hat{c}}{1-\beta} \\
&= \left[\sum_{t=1}^{\infty} \beta^{t-1} \left(g_{T(\varepsilon)+t} + c_{T(\varepsilon)+t}\right) - \sum_{t=1}^{\infty} \beta^{t-1} \left(F(K_{T(\varepsilon)+t}, n_{T(\varepsilon)+t}) - R^* K_{T(\varepsilon)+t}\right) \right] \\
&\quad + \frac{F(\hat{K}, \hat{n}) - R^* \hat{K}}{1-\beta} - \frac{\hat{c}}{1-\beta} \\
&= \sum_{t=1}^{\infty} \beta^{t-1} g_{T(\varepsilon)+t} + \Delta(\varepsilon) - \sum_{t=1}^{\infty} \beta^{t-1} \left(F(K_{T(\varepsilon)+t}, n_{T(\varepsilon)+t}) - R^* K_{T(\varepsilon)+t}\right) \\
&\quad + \frac{F(\hat{K}, \hat{n}) - R^* \hat{K}}{1-\beta} \\
&\geq \sum_{t=1}^{\infty} \beta^{t-1} g_{T(\varepsilon)+t} - \sum_{t=1}^{\infty} \beta^{t-1} F(K_{T(\varepsilon)+t}, n_{T(\varepsilon)+t}) - \frac{R^* \hat{K}}{1-\beta} \\
&\geq \sum_{t=1}^{\infty} \beta^{t-1} g_{T(\varepsilon)+t} - \frac{(1+R^*)\varepsilon}{1-\beta}
\end{aligned}$$

Then I can pick $\varepsilon > 0$ sufficiently close to zero such that (36) and strict concavity of $G(.)$ imply that

$$(37) \quad \frac{G(\hat{g})}{1-\beta} > \sum_{t=1}^{\infty} \beta^{t-1} G(g_{T(\varepsilon)+t})$$

which implies that $\hat{v}_{T(\varepsilon)+1} \equiv \frac{u(\hat{c})-v(0)+G(\hat{g})}{1-\beta} > v_{T(\varepsilon)+1}$. Hence $\hat{x}(\varepsilon)$ is feasible for (P) and it attains a higher utility than x. Hence x cannot be an optimal plan.Therefore it must be that $M_t \to M_\infty < +\infty$ and (a necessary condition for this) $\mu_t \to \mu_\infty = 0$. Therefore, from (29)-(31), it follows that $c_t \to c_\infty$, $n_t \to n_\infty$, $g_t \to g_\infty$ and from (32) we have that

$$(38) \quad F_{kt+1} \to R^*$$

All these results imply that $\tau_t^k, \tau_t^{k*}, \tau_t^a \to 0$ and finally

$$(39) \quad \tau_t^n = \frac{\lambda(\eta+\gamma)}{1+\lambda(1+\gamma)+M_t} \to \frac{\lambda(\eta+\gamma)}{1+\lambda(1+\gamma)+M_\infty} > 0$$

as wanted. $\square$

G. Proof of Proposition 6:

The foncs wrt c_t, g_t and n_t can be rewritten as follows:

$$\begin{aligned}
(40) \qquad c_t^\eta &= \frac{\beta^t}{Q_t}\frac{1+\lambda(1-\eta)+M_t}{\phi} = \frac{\beta^t}{Q_t}\frac{1+\lambda(1-\eta)}{\phi} + \frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to \frac{\beta^t}{Q_t}\frac{M_t}{\phi} \\
(41) \qquad n_t^{-(\gamma+\alpha)} &= \frac{\chi}{(1-\alpha)K_t^\alpha}\frac{\beta^t}{Q_t}\frac{1+\lambda(1+\gamma)+M_t}{\phi} \\
(42) \qquad 1/G'(g_t) &= \frac{\beta^t}{Q_t}\frac{1+M_t}{\phi} \to \frac{\beta^t}{Q_t}\frac{M_t}{\phi}
\end{aligned}$$

Since $M_t \equiv \sum_{s=0}^{\infty} \mu_s$ with $\mu_t \geq 0\ \forall t \geq 0$, then $\{M_t\}_{t=0}^{\infty}$ is an increasing sequence. Thus, either $M_t \to M < \infty$ or $M_t \to \infty$. Consider now the behavior of $\left\{\frac{\beta^t}{Q_t}\frac{M_t}{\phi}\right\}_{t=0}^{\infty}$ as $t \to \infty$. There are three possibilities: (i) $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to 0$, (ii) $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to \tilde{M}_\infty \in (0,\infty)$, or (iii) $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to +\infty$. Case (iii) can be ruled out using an argument similar to the one used in Proposition 3. I can then consider only case (i) and (ii).

Consider first case (ii). Suppose that $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to \tilde{M}_\infty < \infty$. Then (40) and (42) directly imply that $c_t \to c_\infty = \tilde{M}_\infty^{1/\eta} \in (0, M^{1/\eta}]$ and $g_t \to g_\infty = \ell(\tilde{M}_\infty) \in (0, \ell(M)]$, where ℓ is the inverse of $1/G'(g)$. In order for $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to \tilde{M}_\infty$, it must be that $\mu_t \to \mu_\infty = \left(\frac{1-\beta R^*}{\beta R^*}\right)\tilde{M}_\infty > 0$. Hence, from the fonc wrt k_t^*

$$(43) \quad F_{Kt} - R^* = \frac{\mu_t}{\phi}\Omega_{Kt} \to \frac{\mu_\infty}{\phi}\Omega_{K\infty} > 0$$

and (41) it follows that $K_t \to K_\infty > 0$ and $n_t \to n_\infty \in (0, \bar{N}]$. Moreover, combining (43) with the Euler equation for the foreingn investors, $R^* = (1-\tau_t)F_{kt}$, I obtain that as $t \to \infty$

$$(44) \quad \tau_t^* \to \frac{F_{K\infty} - R^*}{F_{K\infty}} = \frac{\mu_t}{\phi}\frac{\Omega_{Kt}}{F_{K\infty}} > 0$$

Combining the household Euler equation with the one for the foreign investors I can write $\forall t$

$$(45) \quad \frac{u'(c_t)}{u'(c_{t+1})} = \beta R^* \frac{1-\tau_{t+1}^k}{1-\tau_{t+1}^*} \to 1 \iff \lim_{t\to\infty} \tau_t^k = \tau_\infty^k < \tau_\infty^*$$

Finally, from (40) and (41) I have that $\forall t$

$$\begin{aligned} v'(n_t) &= u'(c_t)F_{nt}\frac{1+\lambda(1+\gamma)+M_t}{1+\lambda(1-\eta)+M_t} \\ &= u'(c_t)F_{nt}\left(1 - \frac{\lambda(\gamma+\eta)}{1+\lambda(1-\eta)+M_t}\right) \end{aligned}$$

then labor income taxes are converging to zero as $t \to \infty$:

$$\tau_t^n = \frac{\lambda(\eta+\gamma)}{1+\lambda(1-\eta)+M_t} \to \frac{\lambda(\eta+\gamma)}{1+\lambda(1-\eta)+M_\infty} = 0$$

Consider now case (i). If $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to 0$ then from (40) and (42) one gets that $c_t \to 0$ and $g_t \to 0$. Hence the LHS of the sustainability constraint it is converging to its lower bound with $n_t \to 0$ and $K_t \to 0$. In fact, suppose that $n_t \to n > 0$, then $\forall K \geq 0$

$$\frac{u(0)}{1-\beta} - \frac{v(n)}{1-\beta} + \frac{G(0)}{1-\beta} < \Omega(K, 0) \leq \Omega(K, A) \quad \forall A \geq 0$$

then the sustainability constraint is violated. Then it must be that $n_t \to 0$, and this implies that also $K_t \to 0$, otherwise the sustainability constraint won't hold. Therefore, if $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to 0$

then the economy converges to a steady state such that $(c_\infty, n_\infty, g_\infty, K_\infty) = (0,0,0,0)$. Call this the "immiseration" steady state.

Finally, I want to argue that it cannot be that $\frac{\beta^t}{Q_t}\frac{M_t}{\phi} \to 0$. To this end, I will show that I can find an alternative plan, $\hat{x}$, that starting from the "immiseration" steady state - i.e. zero assets - attains strictly higher utility ($\varpi > \Omega(0,0)$, the value of the immiseration steady state), and it is sustainable, i.e. it attains higher utility than the worst SE $\forall t$, $\varpi \geq \Omega(k,0)$, where k is the aggregate capital prescribed by the alternative strategy. To define $\hat{x}$, let $\{\underline{c}_t, \underline{n}_t, \underline{g}_t\}_{t=0}^{\infty}$ be the optimal plan associated with $\Omega(\kappa, 0)$, the worst SE starting from some $\kappa > 0$ and zero foreign assets. Then define $\hat{x}$ as follows: $\forall t = 0,1,...,T$ $\hat{x}_t = x_t$, and for $t \geq T$ let

$$\hat{c}_t = \underline{c}_t, \quad \hat{n}_t = \underline{n}_t, \quad \hat{a}_t = \hat{k}_t = 0, \quad \hat{k}_t^* = \kappa > 0, \quad \hat{b}_t = \underline{b}_t \tag{46}$$

$$\hat{g}_t = \underline{g}_t + (F(\kappa, \hat{n}_t) - R^*\kappa) - F\left((1-\delta)^t\kappa, \underline{n}_t\right) \tag{47}$$

Notice that since

$$\sum_{t=0}^{\infty} \beta^t \left(\hat{c}_t^{1-\eta} - \chi \hat{n}_t^{1+\gamma}\right) = \sum_{t=0}^{\infty} \beta^t \left(\underline{c}_t^{1-\eta} - \chi \underline{n}_t^{1+\gamma}\right) = 0$$

then $\hat{x}$ satisfies (5) for T sufficiently high. This is due to the fact that $c_t, n_t \to 0$ implies that $\lim_{T\to\infty} \sum_{t=0}^{\infty} \beta^t \left(c_{T+t}^{1-\eta} - \chi n_{T+t}^{1+\gamma}\right) \to 0$[12]. By construction (7) must also hold. It is then left to show that (i) $\hat{x}$ is sustainable and (ii) it is an improvement over x. Since $\forall n > 0$ $\lim_{K\to 0} F_K(K,n) = \infty$, from (47) it follows that for κ small enough, $\hat{g}_t > \underline{g}_t$. This is because

$$\begin{aligned}
(F(\kappa, \hat{n}_t) - R^*\kappa) &\approx F\left((1-\delta)^t\kappa, \underline{n}_t\right) + F_K\left((1-\delta)^t\kappa, \underline{n}_t\right)\left[1-(1-\delta)^t\right]\kappa - R^*\kappa \\
&= F\left((1-\delta)^t\kappa, \underline{n}_t\right) + \left\{F_K\left((1-\delta)^t\kappa, \underline{n}_t\right)\left[1-(1-\delta)^t\right] - R^*\right\}\kappa \\
&> F\left((1-\delta)^t\kappa, \underline{n}_t\right)
\end{aligned}$$

[12] To be precise, I should add some $\varepsilon > 0$ sufficiently small to $\hat{c}_T = \underline{c} + \varepsilon$.

for $\kappa > 0$ sufficiently close to zero. Then it follows that

$$\hat{v}_T = \sum_{t=0}^{\infty} \beta^t \left[u(\hat{c}_{T+t}) - v(\hat{n}_{T+t}) + G(\hat{g}_{T+t}) \right] > \Omega(\kappa, 0) > \Omega(0,0) = v_\infty = v_T - \varepsilon$$

Thus $\hat{x}$ is sustainable and it attains higher utility than x, which, therefore, cannot be optimal. Therefore, it must be true that the economy is in case (ii). $\square$

9. Appendix: Figures.

Figure 1: Example of outcome path with and without commitment on the government side

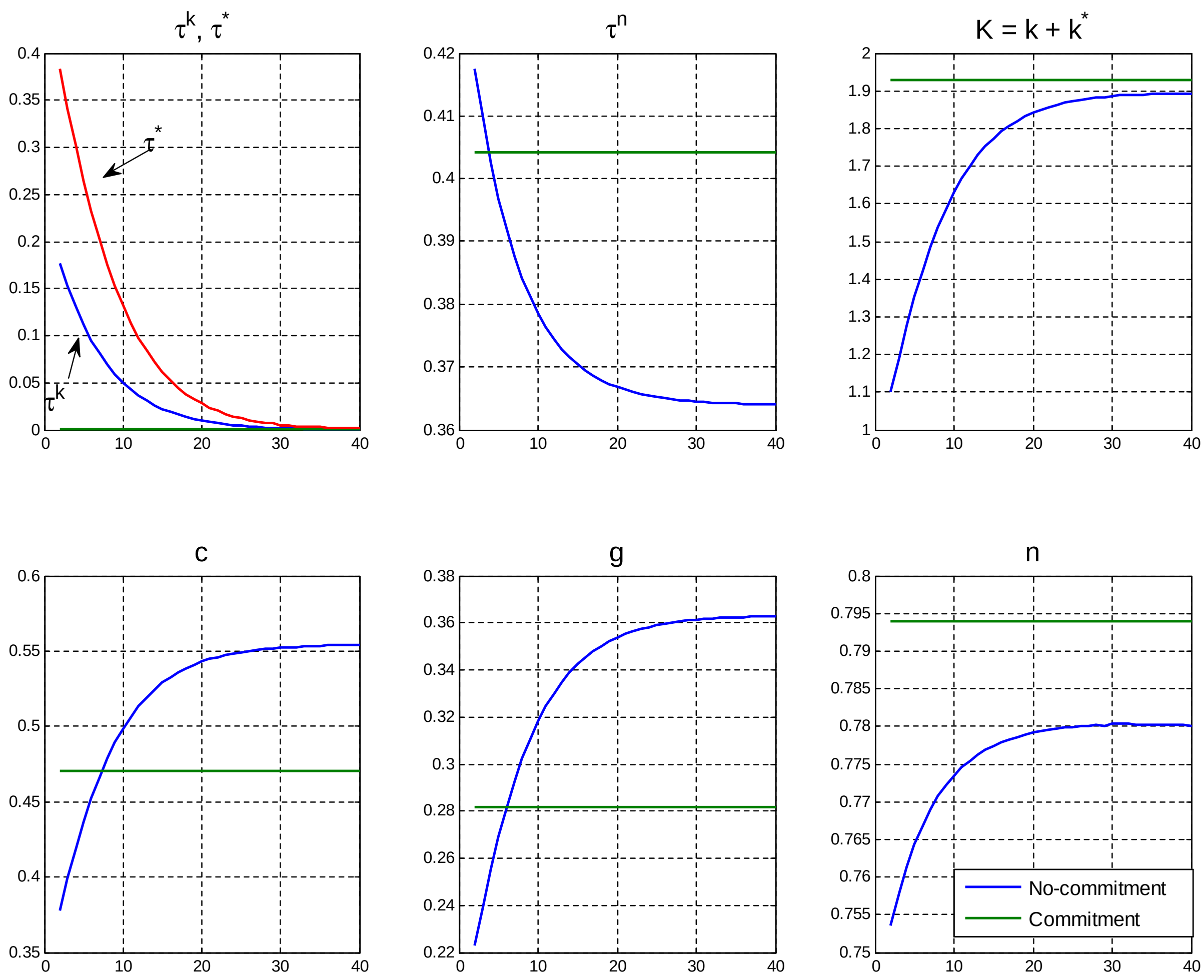

Figure 2: Example of outcome path without commitment with and without capital controls