

Credit Market Frictions and Trade Liberalizations*

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Abstract

Are credit frictions a barrier to gains from trade liberalization? We find that the answer to this depends on whether or not the debt limits of firms respond to profit opportunities. If so, exporters expand and non-exporters shrink efficiently allowing for the same percentage gains from reform as with perfect credit markets. If debt limits do not respond, reallocation is reduced and gains are lower. We then use data from a trade liberalization to distinguish between the two models. We find that firm-level changes in export behavior at the time of reform are consistent with model of responsive debt limits.

1 Introduction

Recent work has studied the role of credit constraints in economies undergoing reforms, and has concluded that financial market imperfections limit the gains from undergoing reform.¹ In this paper, we demonstrate that the way that credit constraints are modeled crucially determines their role in reform.² In particular, we contrast two commonly

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¹See, for example, Buera and Shin (2011 and 2013) and Song, Storesletten and Zilibotti (2011).

²This is a different question than how much credit market frictions matter for aggregate productivity in steady state, as studied in Midrigan and Xu (2013).

used types of debt limits: what we refer to as *forward-looking* debt limits, following Albuquerque and Hopenhayn (2004), and *collateral constraints* or *backward-looking* debt limits. The backward-looking constraint is an exogenous leverage ratio, modeled as a fixed parameter. The forward-looking constraint arises endogenously and may respond when non-financial reforms occur in the economy. Many of our results extend to a variety of real reforms, such as reductions in intersectoral or firm-level distortions, but we focus on trade liberalization for two reasons. First, trade liberalization is a clear example of exactly the type of reform that requires reallocation among firms, as emphasized by the recent trade literature (see Melitz (2003) and Eaton and Kortum (2004)). Second, we show that trade liberalization provides a means of distinguishing between these two types of credit frictions. Using firm-level data from Colombia, we study the effects of a trade reform on the export behavior of firms. We show that the two specifications have opposite predictions for how young firms respond to the reform, and that the response observed provides suggestive evidence in favor of the forward-looking specification.

We extend a dynamic Melitz (2003) trade model to include credit market frictions in the form of debt limits. Our formulation takes both the forward-looking and backward-looking versions as special cases. With forward-looking debt limits, the amount of debt that firms can sustain is limited by the value of continuing to operate the firm (that is, the discounted stream of future income to the firm). With backward-looking debt limits (or collateral constraints), the amount that firms can borrow is at most an exogenous fraction of their capital stock. The key difference between these specifications is how credit limits are affected by the firm's future profitability. With forward-looking constraints, higher future profits allow firms to sustain more debt. With collateral constraints, future profits do not affect debt limits.

We demonstrate that both specifications of credit frictions are consistent with the empirical relationship between credit and export decisions at the firm level analyzed in a recent literature surveyed in Manova (2010). In particular, both specifications can account for the fact that access to credit affects both export participation and the amount that firms export. Moreover, these specifications have similar predictions for the life cycle path of firms. In both models, young firms are small and grow over time until they reach their optimal scale. In each, firms generally do not find it optimal to enter export markets when their capital stocks are small.

The main contribution of this paper is to show that these models have different implications for gains from trade reform both at the aggregate and at the firm level. We show that the percentage increase in steady state consumption from a trade reform in the forward-looking specification is *the same* as in a corresponding model with perfect credit

markets. The gains are analytically the same in a special case, and are very close in magnitude in more general, calibrated examples. However, with collateral constraints, the percentage gains from trade are *lower* than with perfect credit markets. The important difference is on the extensive margin of adjustment. In the model with forward-looking debt limits, future exporters are able to sustain higher debt after the trade liberalization than before, even in periods *before* they enter the export market. This allows young, productive firms to start to export earlier. With collateral constraints, entering the export market requires asset accumulation. Non-exporters are less profitable after trade reform (due to increased wages) so they accumulate assets more slowly. Therefore, with collateral constraints productive, young (low net worth) firms are unable to enter export markets, while less productive, old (high net worth) firms are able to enter. This creates perverse selection into the export market that lowers the gains from trade reform. This demonstrates that taking into account the endogenous response of credit markets to reform is important when evaluating the potential gains from policy changes in countries with low quality credit markets.

Moreover, this difference in gains from trade is not transitory but permanent. We focus on the long run gains from trade by comparing stationary equilibria before and after the trade reform. Each firm has a probability of death each period, and a measure of new firms are born in every period. This contrasts with much of the existing literature that considers infinitely-lived firms. In that case, financial frictions slow down the transition between stationary equilibria. Instead, with our overlapping generations structure the financial frictions have permanent effects, since new firms are more financially constrained than older ones.

Given the difference in implications for the two forms of the constraint, we look for evidence to support one specification or the other. This is difficult to do in a cross-section because both models have similar implications for the lifecycle of firms. However, the model has different implications for how firms respond to trade liberalization. Trade liberalization increases the profits of exporters, but decreases the profits of non-exporters. All firms are born as non-exporters, then may in any period pay the fixed cost to become exporters. With a forward-looking constraint, a new firm that will eventually export has higher value than a new firm that does not, which allows the future exporter to sustain a higher level of debt. This allows them to borrow more and pay the fixed cost to export earlier in their life after the reform than before. However, with the backward-looking constraint, a new firm that will eventually export is not able to borrow more and, because they are now less profitable, it takes them longer to accumulate assets after the reform than before. Therefore, the two specifications are different in that the forward-looking

case implies that the increase in export participation is concentrated among younger firms, while the backward-looking case implies that it's concentrated among older firms.

Using firm-level data from Colombia from 1981-91, which includes a series of reforms affecting trade in the mid-1980s, we show that, while all firms increase export activity following the reform, the increase is concentrated among young firms. We simulate the Colombian reforms in a small open economy version of our model and show that the forward-looking model shares this prediction, while the backward-looking model predicts the opposite. We interpret this as suggestive evidence supporting the forward-looking specification of credit constraints.

While this evidence is not conclusive, it is consistent with results found in other studies of how lending decisions are made. Recent work by Li (2015) considers whether or not firms' one-year-ahead profits affect the levels of firm borrowing using data from Japanese firms. She finds that this has an important level difference in the aggregate losses due to financial market frictions. We view our work as complementary, but our approach differs in two ways. First, the forward-looking specification we consider is different in that the entire discounted stream of future profits affect borrowing. Second, our main question concerns the interaction between financial constraints a non-financial real reform, not the level effect of financial frictions. Although in a quite different context, Brunt (2006) shows that banks in England during the Industrial Revolution lent on the basis of expected future profitability, and effectively took long term equity positions in their borrowers. This is the type of lending implied by limited enforcement, rather than fixed collateral limits.

Related Literature This paper is related to several literatures in international trade and macroeconomics. We build on the seminal contribution of Melitz (2003) and subsequent work, such as Alessandria and Choi (2014), who analyze the gains from trade in a model with heterogeneous monopolistic competitive firms, which emphasize the role of reallocation and selection into the export market as a driver for the gains from trade. Chaney (2005) and Manova (2008, 2013) introduce credit market frictions into a Melitz (2003) framework. Both papers consider a static environment, and do not address how credit frictions affect the gains from trade, which is the central theme of our paper. Recent papers by Kohn et al (2015) and Gross and Verani (2013) study dynamic trade model with trade frictions but focus on firm-level dynamic and not effects of trade reform. Caggese and Cunat (2013) study the gains from trade reform with collateral constraints and show that gains are limited due to the extensive margin. We confirm their findings and contrast them with the forward-looking case.

The model presented here is consistent with the growing empirical literature on the

relationship between firm-level export behavior and access to credit (see Manova (2010) for a survey). This literature uses firm-level data from many different countries, and finds that access to credit is an important determinant of export participation (the extensive margin) and the scale of exports (the intensive margin). See Berman and Héricourt (2010), Minetti and Zhu (2011) and Gorodnichenko and Schnitzer (2013). This literature uses measures such as survey responses³ and leverage ratios to proxy for access to credit. The models of trade and credit frictions developed in the next sections are consistent with both findings from this literature. Amiti and Weinstein (2011) show that shocks to banks impact the export behavior of borrowers.

This paper is also related to the literature that studies how aggregate gains from a trade liberalization are affected by including institutional and technological details in trade models. Arkolakis, Costinot and Rodriguez-Clare (2012) show that all of a large class of trade models have the same implications for welfare gains from trade given *ex post* realizations of changes in trade flows. We are interested in evaluating *ex ante* how a given reduction in tariffs affects welfare with and without credit market frictions. This is similar in spirit to Atkeson and Burstein (2010), who show that modeling innovation decisions has no effect on aggregate gains from trade. Similarly, Kambourov (2009) shows that labor market frictions reduce gains from a trade liberalization.

We model credit market frictions following two specifications widely used in the macroeconomics literature. First, our forward-looking specification extends Albuquerque and Hopenhayn (2004) to a general equilibrium trade model with a discrete choice to export. See Cooley, Marimon and Quadrini (2004) for an application in a closed economy context. Second, we analyze collateral constraints following Evans and Jovanovic (1989), which has been used in many papers, such as Midrigan and Xu (2013). A similar constraint is used in Buera, Kaboski and Shin (2011).

Finally and most importantly, our paper contributes to the literature that analyzes how credit market frictions affect reallocation in economies undergoing reform. Buera and Shin (2013) show that collateral constraints slow down the reallocation process following a reform, because it takes time for productive but low net-worth firm to accumulate sufficient assets to start a business and operate at full scale⁴. Likewise, Song, Storesletten and Zilibotti (2011) consider a similar mechanism for the case of technological growth in China, showing that collateral constraints generate misallocation between constrained,

³For instance, in Minetti and Zhu (2011) they use a firm-level Italian data set that includes answers to the question, "In 2000, would the firm have liked to obtain more credit at the market interest rate?"

⁴Buera and Shin (2011) obtain similar results in an open economy environment (no intratemporal trade) considering debt limits that depend not only on the installed capital stock (collateral constraints) but also on period profits.

productive private firms and unconstrained, less productive state-owned firms. These results all depend on the backward-looking nature of the financial constraints. If the debt limits have a forward-looking component, as in the specification that follows Albuquerque and Hopenhayn (2004), then productive firms can start a business and operate at a larger scale sooner after the reform or technological improvement, and they do not have to accumulate a large stock of assets to do so. Jermann and Quadrini (2007) consider a similar mechanism in the context of news shocks where they show that a signal of future productivity immediately relaxes the firms' enforcement constraints. The second contribution of our paper is to suggest which micro-level evidence can help in telling these two formulations of credit market frictions apart. By looking at the 1985 Colombian trade reform, we provide evidence – albeit indirect – that the forward-looking specification of the debt limits is more in line with the data.

In Sections 2, 3, and 4 we build and characterize a model of trade and consider two types of credit frictions. In Section 5 we discuss the difference in implications between these two specifications for trade reform both at the firm level and for aggregates. In Section 6 we provide some evidence in favor of the forward-looking specification. Section 7 concludes.

2 Model

Time is discrete, denoted by $t = 0, 1, \dots$ and there is no aggregate uncertainty. There are two symmetric countries, home and foreign, with variables for the foreign country are denoted with superscript f . Each country is populated by a unit measure of identical households, competitive final good producers, and monopolistic competitive firms each producing an intermediate differentiated product.

2.1 Household Problem

The stand-in household in each country inelastically supplies 1 unit of labor each period. He chooses final good consumption c_t and bond holdings b_{t+1} to maximize his lifetime utility

$$(1) \quad \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0,1)$ is the discount factor and u is increasing, differentiable and concave, subject to the sequence of budget constraints

$$(2) \quad c_t + q_t b_{t+1} \leq w_t + b_t + \Pi_t + T_t \quad \forall t \geq 0$$

expressed in terms of the final good in each country. Here w_t is the wage, q_t is the intertemporal price, Π_t is the sum of profits from the operation of firms and T_t are lump-sum transfers from the government (revenue from tariffs). The problem for the stand-in household in the foreign country is symmetric.

2.2 Final Goods Producers

The final good in the home country is produced using the following CES aggregator:

$$(3) \quad y_t = \left[\omega \int_{I_t} y_{dt}(i)^{\frac{\sigma-1}{\sigma}} di + (1-\omega) \int_{I_{xt}^f} y_{xt}^f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where I_t is the set of active domestic firms at time t , I_{xt}^f is the set of foreign firms that export at t , $y_{dt}(i)$ is the output of firm i in I_t , $y_{xt}^f(i)$ is the output of firm i in I_{xt}^f . The final good in the foreign country is produced analogously. The parameter ω indexes home bias in the production of the final good. The elasticity of substitution among goods is $\sigma > 1$.

Final goods producers are competitive. A representative firm solves the following static problem:

$$\max_{y_t, y_{dt}, y_{xt}} y_t - \int_{I_t} p(i) y_{dt}(i) di - \int_{I_{xt}^f} (1 + \tau_t) p(i) y_{xt}^f(i) di$$

subject to (3). One can then derive the inverse demand functions faced by domestic and foreign producers for the intermediated good i :

$$(4) \quad p_{dt}(y(i)) = \omega y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}}$$

$$(5) \quad p_{xt}^f(y(i)) = \frac{1-\omega}{1+\tau_t} y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}}$$

2.3 Intermediate Goods Producers

A mass of monopolistic competitive intermediate goods producers are operated by entrepreneurs in each country. In every period a mass $\delta \in (0, 1)$ of entrepreneurs is born. Each operates a firm and is endowed with a new variety of the intermediate good. At birth the entrepreneur draws a type (z, ϕ) , where z is the firm's productivity and $\phi \in \{0, 1\}$ indicates if the firm has the ability to export or not⁵. If $\phi = 1$ the firm can pay a fixed cost f_x in any period to enter the export market the following period, while if $\phi = 0$ the firm does not have that option. We can think of this as an extreme form of heterogeneity in the export fixed costs. For simplicity, z and ϕ are independently distributed. Productivity z is drawn from a distribution Γ , and the indicator ϕ is a Bernoulli random variable with parameter ρ .⁶ The type of the firm remains constant through time⁷. The firm can produce its differentiated variety using the following constant returns to scale technology:

$$(6) \quad y = zF(k, l) = zk^\alpha l^{1-\alpha}, \quad \alpha \in (0, 1)$$

where l and k are the labor and capital employed by the firm, and y is total output produced, which the firm splits between domestic and export sales. Every period the production technology owned by the firm becomes unproductive with probability δ . To be able to export, a firm of type $\phi = 1$ must pay a sunk cost f_x in period t to be able to export in all the subsequent period conditional on surviving.

The firm has to borrow to finance its operations each period and to pay the export fixed cost f_x if it is profitable to do so. We consider a decentralization where firms have access to a rental market for capital. We denote the rental capital rate by r_t . Firms can save across periods in contingent securities that pay one unit of the final good next period conditional on the firm's survival. All firms start with a_0 units of the final good, which are transferred to them by the household. Entrepreneurs are paid of dividend d_t from the operation of the firm. We are assuming that a_0 is the maximum one-time transfer that the household can make to the firm not subject to the debt limit⁸. That is, in any period it

⁵This feature of the model is useful to match the fact that there are large, productive firms that are non-exporters, and to generate reallocation after trade reform even if there are no fixed costs.

⁶Note that even if z and ϕ are not correlated, the model generates a positive correlation between productivity and export status because only most productive firms select into the export market as in standard Melitz model.

⁷Our goal is to compare the forward-looking limited enforcement model with the backward-looking collateral constraints model. Adding idiosyncratic uncertainty would require us to specify the completeness of debt contracts.

⁸Clearly if there was not a bound on such transfers this channel would eliminate the credit friction.

must be that

$$(7) \quad d_t \geq 0$$

where d_t are the dividends distributed by the firm. Firms can issue *intra-period* debt at a zero net interest rate. We first present a general formulation, then consider two cases in the next section. The amount that can be borrowed depends on their assets at the beginning of the period:

$$(8) \quad b_t \leq \bar{B}_t^i(a_t; z, \phi)$$

The firm's problem can be conveniently written recursively using assets or cash on hand, a , together with its export status and type (z, ϕ) as state variables. The problem of the firm that has already paid to enter the export market can be written as choosing dividend distribution d , new assets a' to solve:

$$(9) \quad V_t^x(a, z, \phi) = \max_{d, a'} d + q_t(1 - \delta)V_{t+1}^x(a', z, \phi)$$

subject to

$$\begin{aligned} d + (1 - \delta)q_t a' &\leq \pi_t^x(a, z) \\ d &\geq 0 \end{aligned}$$

The production plan y_d, y_x, k, l and the intra-period debt b are chosen to maximize period profits $\pi_t^x(a, z)$:

$$(10) \quad \pi_t^x(a, z) = \max_{y_d, y_x, l, b, k} p_{dt}(y_d)y_d + p_{xt}(y_x)y_x - w_t l - r_t k - b$$

subject to

$$\begin{aligned} y_d + y_x &\leq zF(k, l) \\ k &\leq a + b \\ b' &\leq \bar{B}_t^x(a; z, \phi) \end{aligned}$$

For a firm that has not yet paid the fixed cost to start exporting, denoted with the superscript nx , the recursive formulation of its problem is the same, with the addition of the

discrete decision to export or not:

$$(11) \quad V_t^{nx}(a, z, \phi) = \max_{d, a', x} d + (1 - \delta)q_t [x\phi V_{t+1}^x(a', z, \phi) + (1 - x)V_{t+1}^{nx}(a', z, \phi)]$$

subject to

$$\begin{aligned} d + (1 - \delta)q_t a' + x f_x &\leq x \pi_t^x(a - f_x, z) + (1 - x) \pi_t^{nx}(a, z) \\ d &\geq 0, x \in \{0, 1\} \end{aligned}$$

where x is an indicator variable that takes the value of 1 if the firm pays the fixed cost to export and zero otherwise. The period profits, $\pi_t^{nx}(k, z)$, are given by the following static problem:

$$(12) \quad \pi_t^{nx}(a, z) = \max_{y_d, y_x, l, b, k} p_{dt}(y_d)y_d - w_t l - r_t k - b$$

subject to

$$\begin{aligned} y_d &\leq zF(k, l) \\ k &\leq a + b \\ b' &\leq \bar{B}_t^{nx}(a; z, \phi) \end{aligned}$$

We will denote the policy functions of the firms associated with the above problems as $\{d_t^{nx}, a_t^{nx}, k_t^{nx}, b_t^{nx}, y_{dt}^{nx}, y_{xt}^{nx}, l_t^{nx}, x_t\}_{t=0}^{\infty}$ and $\{d_t^x, a_t^x, k_t^x, b_t^x, y_{dt}^x, y_{xt}^x, l_t^x\}_{t=0}^{\infty}$ for non-exporters and exporters respectively.

2.4 Equilibrium

To define an equilibrium for the economy we need to keep track of the evolution of the measure of operating firms over (a, z, ϕ) and export status. Denote by λ_t^{nx} and λ_t^x the measure of non-exporting and exporting firms at the beginning of the period over (a, z, ϕ) respectively after the entry of new firms, and let $\lambda_t = (\lambda_t^{nx}, \lambda_t^x)$. The measure of non

exporters evolves over time according to

$$(13) \quad \lambda_{t+1}^{nx}(\mathcal{A}, \mathcal{Z}, \Phi) = (1 - \delta) \int 1 \{x_t(\mathbf{a}, z, \phi) = 0, \mathbf{a}^{nx}(\mathbf{a}, z, \phi) \in \mathcal{A}, z \in \mathcal{Z}, \phi \in \Phi\} d\lambda_t^{nx} \\ + \delta \rho \int_{\mathcal{Z}} 1 \{ \mathbf{a}_0 \in \mathcal{A}, z \in \mathcal{Z}, 1 \in \Phi \} d\Gamma \\ + \delta(1 - \rho) \int_{\mathcal{Z}} 1 \{ \mathbf{a}_0 \in \mathcal{A}, z \in \mathcal{Z}, 0 \in \Phi \} d\Gamma$$

where \mathcal{A} and \mathcal{Z} are sets of asset and productivity respectively. The measure of exporters evolves over time according to

$$(14) \quad \lambda_{t+1}^x(\mathcal{A}, \mathcal{Z}) = (1 - \delta) \int 1 \{ \mathbf{a}_t^x(\mathbf{a}, z, \phi) \in \mathcal{A}, z \in \mathcal{Z}, \phi \in \Phi \} d\lambda_t^x + \\ + (1 - \delta) \int 1 \{ x_t(\mathbf{a}, z, \phi) = 1, \mathbf{a}_t^{nx}(\mathbf{a}; z, \phi) \in \mathcal{A}, z \in \mathcal{Z}, \phi \in \Phi \} d\lambda_t^{nx}$$

Market clearing in the final good market requires that

$$(15) \quad y_t = c_t + K_{t+1} - (1 - \delta_k)K_t + y_{ft}$$

where y_{ft} is the total investment in export fixed cost in period t :

$$(16) \quad y_{ft} = f_x \int x_t(\mathbf{a}, z, \phi) d\lambda_t^{nx}$$

Market clearing in the rental capital market requires that

$$(17) \quad K_t = \sum_{i \in \{nx, x\}} \int k_t^i(\mathbf{a}, z, \phi) d\lambda_t^i$$

The labor market feasibility is given by

$$(18) \quad 1 = \sum_{i \in \{nx, x\}} \int l_t^i(\mathbf{a}, z, \phi) d\lambda_t^i$$

For the bond market to clear, it must be that

$$(19) \quad b_t + b_t^f + A_t + A_t^f = K_t + K_t^f$$

where A_t is the aggregate amount of assets held by firms:

$$(20) \quad A_{t+1} = (1 - \delta) \sum_{i \in \{nx, x\}} \int a_t^i(a, z, \phi) d\lambda_t^i + \delta \int_Z a_t^{nx}(a_0, z, \phi) d\Gamma$$

We can then define a *symmetric equilibrium* for the economy. Given debt limits $\{\bar{B}_t\}_{t=0}^\infty$, an initial distribution of firms $\lambda_0 = \lambda_0^f$, capital stock $K_0 = K_0^f$, bonds holdings $b_0 = b_0^f$, and a sequence of tariff $\{\tau_t, \tau_t^f\}_{t=0}^\infty$ such that $\tau_t = \tau_t^f$, a symmetric equilibrium consists of household's allocations $\{c_t, b_{t+1}\}_{t=0}^\infty$, prices $\{p_t, w_t, r_t, q_t\}_{t=0}^\infty$, inverse demand functions $\{p_{xt}, p_{dt}\}_{t=0}^\infty$, firms decision rules $\left\{ \left\{ d_t^i, b_t^i, k_t^i, y_{dt}^i, y_{xt}^i, l_t^i \right\}_{i \in \{nx, x\}}, x_t \right\}_{t=0}^\infty$, aggregate capital $\{K_t\}_{t=0}^\infty$, and measure of firms $\{\lambda_t\}_{t=0}^\infty$ such that: 1) the households' allocations solve the problem (1) subject to (2) where the aggregate dividend distribution is given by

$$(21) \quad \Pi_t = \sum_{i \in \{nx, x\}} \int d_t^i(a, z, \phi) d\lambda_t^i,$$

and the lump-sum transfers are given by

$$(22) \quad T_t = \tau_t \left\{ \sum_{i \in \{nx, x\}} \int p_{xt} \left[y_{xt}^{if}(a, z, \phi) \right] y_{xt}^{if}(a, z, \phi) d\lambda^{if} \right\};$$

2) the firms' decision rules are optimal for (11) and (9); 3) the inverse demand functions are given by (4) and (5); 4) the rental capital rate is given by $r_t = 1/q_t - (1 - \delta_k)$; 5) the markets for final good, rental capital, labor and bonds clear, that is, (15), (17), (18), and (19) hold; 6) the measures of firms evolve according to (14) and (13).

In our analysis we will focus on a *symmetric stationary equilibrium* for the economy and in a transition from a high tariff steady state to a high tariff steady state.

3 Credit Market Frictions

We now turn to specify two polar case for the borrowing constraint that firms We will contrast two popularly used specifications that are widely used in the literature. We refer to the first as the *forward-looking* specification, which follows Albuquerque and Hopenhayn (2004), and to the second as the *backward-looking* specification, following Evans and Jovanovic (1989) among others. Intermediate cases have been analyzed in Buera, Kaboski, and Shin (2011) and Li (2015). We choose to consider the two extreme to make our point in the starkest possible way.

3.1 Forward-Looking Specification

In our first specification for the debt limits (8), we derive debt limits faced by the firm that arise from the inability of firms to commit to repay their debt obligations. Credit contracts are not enforceable in the sense that every period the entrepreneur can choose to default on their outstanding debt.⁹ After default, the entrepreneur can divert a proportion θ of the funds advanced for the next period's capital stock for personal benefits that are consumed immediately. Also with probability $1 - \xi$, the entrepreneur loses its production technology. If the technology survives the default, the entrepreneur is able to continue to operate the firm without the assets or debt previously accumulated.¹⁰ The corresponding debt limit \bar{B}^i for $i \in \{x, nx\}$ is implicitly defined by:

$$(23) \quad V_t^i(a) = \theta \left[\bar{B}_t^i(a; z, \phi) + a \right] + \xi v_0(z, \phi)$$

where $v_0(z, \phi) = V_{t+1}^{nx}(0, z, \phi)$. This corresponds to the debt limit being “*not too tight*” in the terminology of Alvarez and Jermann (2000). The parameters θ and ξ index to the quality of financial markets¹¹. If $\theta = 0$, then entrepreneurs have nothing to gain from default and credit constraints never bind. In this formulation, firms are able to borrow even if they have zero assets. For simplicity we set $a_0 = 0$.

The key feature of this specification is that debt limits depend on the future profitability of the firm. That is, the higher the present value of the firm, $V^i(a)$, the more debt it can sustain.

3.2 Backward-Looking Specification

The backward-looking specification is a collateral constraint, with a debt limit (8) for $i = x, nx$ given by:¹²

$$(24) \quad \bar{B}_t^i(a; z, \phi) = \frac{1 - \theta}{\theta} a$$

⁹This may seem strange because households both own the firms and the debt lent to them, so the entrepreneurs would be effectively defaulting on themselves. We follow the formulation of Jermann and Quadrini (2012). There are a continuum of households each composed of workers and entrepreneurs. Savings across households are pooled and loans made from the pool, so that default by any one (measure zero) entrepreneur would have no effect on payments out of the pool, but would increase that firm's present value of income paid to their household.

¹⁰Within a stationary equilibrium, this is equivalent to a period of exclusion from financial markets.

¹¹As in Jermann-Quadrini (2012), we do not restrict $\theta \in [0, 1]$. This can be interpreted as there being some probability that, following default, the entrepreneur cannot be punished.

¹²This is equivalent to require that $b \leq \theta k$.

for some $\theta \in [0, 1]$. That is, a firm can borrow only up to a multiple $(1 - \theta)/\theta$ of its assets. A common interpretation for this formulation is that entrepreneur cannot commit to repay his intra-period debt but the only punishment for doing so is the loss of a fraction $1 - \theta$ of the capital stock. In particular, default does not result in the destruction of the firm's technology nor in exclusion from credit markets. In this case, new entrepreneurs must be endowed with some assets in order to begin operation, $a_0 > 0$. Again, θ parameterizes the quality of financial markets, where higher values of θ imply lower financial market quality.

The backward-looking debt limits depend only on the amount of profits that the firm has reinvested in the past, a , and not on future profitability. This aspect contrasts with the forward looking case. This difference is crucial for the two specifications to differ in their implications for the response of the economy to a trade reform.

As demonstrated in Rampini and Viswanathan (2010), the key difference between the forward-looking and backward-looking specifications is the existence of a dynamic punishment in the forward-looking environment. Because the backward-looking specification has no dynamic punishment, the firm's cash on hand in the current period is sufficient to determine their maximum borrowing.

4 Characterization of Equilibrium

Before analyzing the effect of a trade reform, we characterize the symmetric stationary equilibrium for the economy. We show that both specifications of credit market frictions are able to account for the relationship between export behavior and access to credit documented in the empirical literature: (i) the probability that a firm is an exporter is decreasing with measures of firm-level financial constraints, and (ii) firms' sales and exports grow over time and are decreasing in the credit constraints it faces.

In a stationary equilibrium, all prices and aggregate quantities are constant over time. Therefore, we will drop the dependence on time in this section. First we consider a relaxed problem where the borrowing constraint is dropped. It is easy to see that the production decisions are independent of the firm's debt level, and solve the following static problem:

(25)

$$\pi^*(z, \phi) = \max_{l, k, y_d, y_x, x} \omega y^{1/\sigma} y_d^{1-1/\sigma} + x \phi \frac{1 - \omega}{1 + \tau} y^{1/\sigma} y_x^{1-1/\sigma} - wl - rk - x\phi(1 - q(1 - \delta))f_x$$

subject to

(26)
$$y_d + xy_x \leq zF(k, l)$$

Given prices w, q , tariff τ and aggregate final output y , denote the solutions to this problem $\{l^*(z, \phi), k^*(z, \phi), y_d^*(z, \phi), y_x^*(z, \phi), x^*(z, \phi)\}$. These would be the firms' decision rules in a standard Melitz (2003) model. We say that a firm reaches its optimal scale whenever $k = k^*(z, \phi)$.

The following proposition fully characterizes the evolution of a firm over time. The proof is relegated to the online appendix.¹³

Proposition 1 *When debt limits are given by (23) or (24) then:*

- (i) *Firms issue no dividends until they reach their optimal scale¹⁴;*
- (ii) *\exists cut-off productivity level z_x s.t. the firm will eventually export iff $\phi = 1$ and $z \geq z_x$;*
- (iii) *$\forall z \geq z_x \exists \hat{a}(z, 1)$ s.t. firms export iff $\phi = 1$ and $a \geq \hat{a}(z, 1)$;*
- (iv) *If $z' > z \geq z_x$ and $T(z)$ is the age when a firm starts exporting, then $T(z') \leq T(z)$.*

Part (i) states that dividend distributions are back-loaded. Because the firm discounts at the equilibrium interest rate, the value of increasing their assets is always greater than the value of distributing dividends whenever the debt limit is binding. Therefore, the firm distributes no dividends so that it can increase its assets as quickly as possible. This allows the firm's capital stock to grow over time until it reaches its unconstrained scale $k^*(z, \phi)$. After reaching optimal scale the dividend policy of the firm is arbitrary, so long as they maintain enough assets to sustain their optimal scale of capital¹⁵.

Part (ii) states that only more productive firms will export, as in Melitz (2003), but now with the qualification that they will *eventually* export. When a firm is constrained to operate at an inefficiently low scale they may find it profitable to wait several periods to enter the export market. Part (iii) states that the firm's export status depends on both productivity and assets. For each productivity type z , there is an asset cut-off $\hat{a}(z, 1)$ such that it is profitable to start to export only if a firm has assets above that threshold. Firms with low assets are borrowing constrained and their capital stock is too low to make it profitable to pay the fixed cost to export.

¹³This characterization extends Albuquerque and Hopenhayn (2004) to an environment with a discrete choice of increasing the number of markets in which the firm operates.

¹⁴This statement is minorly qualified. The period before the firm reaches its optimal scale it is only required to distribute a low enough level of dividends that it will still be able to operate at full scale in the next period. In that period, zero dividends is optimal, but not uniquely optimal.

¹⁵Two extreme cases would be the following: 1) after reaching optimal scale, all firms maintain just enough cash on hand to sustain their optimal capital stock and always distribute the rest of its profit in dividends, or 2) all firms retain all of their earnings by saving in risk-free debt. Either of these dividend policies, or any intermediate case, is consistent with the same allocation within a stationary equilibrium. In the first case, the household receives the dividends directly from the firms, while in the second they receive them indirectly through increased borrowing.

Finally, part (iv) states that more productive firms enter export markets younger. This is true for two reasons. First, the value of being an exporter is increasing in the productivity of the firm. The minimal amount of assets necessary to justify the fixed cost to be an exporter, $\hat{a}(z, 1)$, is decreasing in z . Second, more productive firms accumulate assets more quickly because they earn higher profits. Moreover, in the forward looking specification (23), more productive firms are able to borrow more because the value of the firm (left hand side of (23)) is increasing in z : For a given value of assets, default is less attractive the higher is the productivity of the firm.

The typical life-cycle path predicted by the model is as follows. After the initial productivity draw there is no uncertainty (except for exogenous exit) and firms are fully characterized by their productivity and their age. The amount of capital that a firm can sustain is initially low, then it increases over time as firms use period profits to accumulate assets (no dividend distributions). Likewise, labor usage and domestic sales (which are the static solutions to (12) above) are also initially low and grow over time with the capital stock. More productive firms eventually find it optimal to pay the fixed cost to enter the export market because they are able to sustain a larger capital stock, which increases the value of being an exporter. Then labor, domestic sales and export sales for a given capital stock are the solution to (10). Again, export sales remain at suboptimal levels as long as the firm's capital stock is constrained below its optimal scale. In finite time, the firm is able to sustain its optimal capital stock, and labor, domestic sales and export sales are constant forever after that.

Thus credit market frictions in the form of debt limits (23) or (24) affect firm level export decisions along the extensive and intensive margin: Firms whose debt limits are binding are both less likely to be exporters and export at smaller scale. This is consistent with the findings of the empirical literature on the relationship between export behavior and access to credit discussed before.¹⁶ Despite having similar implications for firm-level dynamics in a stationary equilibrium, the two specifications of credit market frictions have different implications for the aggregate effects of a trade reform, as we will show next.

5 Effects of Trade Liberalization

In this section, we evaluate if credit market imperfections reduce the gains from a bilateral tariff reduction. We show that with forward-looking debt limits the gains from trade

¹⁶Notice that firms with binding debt limits have higher leverage ratios and would identify themselves as constrained in survey responses.

are *not* affected by the quality of financial markets, while with backward-looking debt limits the gains from trade are lower than in an economy with perfect credit markets. The key mechanism that we will highlight throughout is how the debt limits that firms face respond to trade reform. With the backward-looking constraint, the amount firms are able to borrow depends only on their history of capital accumulation, and does not directly respond to the reform. However, with the forward-looking constraint, the fact that exporting firms are more profitable makes default less attractive and increases their debt limits.

5.1 Forward-Looking Case: Analytical Result

We first show that the percentage gains from trade are the same in a model with perfect credit markets as they are in a model with forward-looking constraints. We first consider a special case by setting $f_x = 0$. In that case, all firms with $\phi = 1$ are exporters both before and after the trade liberalization, while all firms with $\phi = 0$ are not. Then because the set of exporting firms is not affected by trade reform, the only margin of adjustment is the intensive margin. With perfect credit markets, trade liberalization causes factors of production to be reallocated from non-exporters to exporters. In principle, financial frictions could be a barrier to that reallocation. The following proposition shows *analytically* that this is not true with the forward-looking specification of borrowing constraints.

Proposition 2 *Under the forward-looking specification with $f_x = 0$, for any change in tariffs the steady state percentage changes in aggregate output and wages are independent of θ and ξ . Furthermore, firm-by-firm the percentage change in capital usage is independent of θ and ξ .*

A formal proof of Proposition 2 is provided in the appendix, but here we sketch our approach. When tariffs are reduced exporting firms are more profitable, so for any debt level and capital stock, the value of not defaulting has increased. Therefore, exporting firms can sustain higher debt levels than before the liberalization allowing the firm to operate at a greater scale. The opposite is true for non-exporters who, because wages have increased, are less profitable after the tariff reduction than before.

In particular, the relaxation of the borrowing constraints for exporters is such that all exporting firms increase their scale by exactly the same proportion for every age and every productivity level as in the model with perfect credit markets. This is because the optimal scale of production and the present value of firms' dividends increase by exactly the same proportion. Consider a reform in which tariffs have been reduced from τ to τ' . Let $\mathbf{s} = (y, w, \tau)$ be the aggregate state before the reform and conjecture that the

aggregate post reform is given by $\mathbf{s}' = (\Delta_y y, \Delta_w w, \tau')$ where Δ_y and Δ_w are the changes in output and wages in the economy with perfect credit markets. Letting Δ_k be the change in the capital stock of exporting firms in the economy with perfect credit markets after the reform, we have that:

$$\Delta_k \pi^j(k; \mathbf{s}) = \pi^j(\Delta_k k; \mathbf{s}') \text{ and } \Delta_k V^x(\mathbf{a}; z, \mathbf{s}) = V^x(\Delta_k \mathbf{a}; z, \mathbf{s}')$$

This implies that if the path $\{a_t, k_t, b_t, d_t\}_{t=0}^{\infty}$ is feasible and optimal for a firm of type (z, ϕ) with aggregate state \mathbf{s} , then $\{\Delta_k a_t, \Delta_k k_t, \Delta_k b_t, \Delta_k d_t\}_{t=0}^{\infty}$ is feasible and optimal for the firm given the aggregate state $\mathbf{s}' = (\Delta_y y, \Delta_w w, \tau')$. Debt limits (23) are satisfied with this new path because, noting that $v_0(z, \mathbf{s}) = V^x(0; z, \mathbf{s})$:

$$\begin{aligned} V^x(\mathbf{a}; z, \mathbf{s}) &= \theta [\bar{B}^x(\mathbf{a}; z, \mathbf{s}) + \mathbf{a}] + \xi v_0(z, \mathbf{s}) \\ \implies V^x(\Delta_k \mathbf{a}; z, \mathbf{s}') &= \theta [\bar{B}^x(\Delta_k \mathbf{a}; z, \mathbf{s}') + \Delta_k \mathbf{a}] + \xi v_0(z, \mathbf{s}') \end{aligned}$$

We can then verify that markets clear at the new allocation and therefore $\mathbf{s}' = (\Delta_y y, \Delta_w w, \tau')$ is the aggregate state after the tariff reduction. Hence, though credit frictions do create an inefficient allocation of inputs across firms, they do not limit the reallocation of inputs and the percentage change in output following a trade reform.

Because they typically abstract from fixed cost, this result extends directly to closed economy models as demonstrated in the appendix. Other types of reform that affect the indirect profit function of the firm multiplicatively, such as taxes on revenues or inputs, as well as other types of distortions across firms. Moreover, it also holds for other specifications of the right hand side of (23). For instance, the same result goes through if, instead of the capital stock, the entrepreneur was able to abscond with working capital, period revenues, period profits, or any linear combination thereof.

If $f_x > 0$ we are not able to prove the analogue of Proposition 2. The presence of the fixed cost breaks down the value function's homogeneity property that is used in the proof. Despite not holding exactly, the numerical results below clearly indicate that the difference in the percentage change in consumption, output, and exports that follows a bilateral tariff reduction between an economy with perfect credit markets and one with debt limits of the form (23) is very small.

5.2 Quantitative Exercise

To evaluate the effects of a trade liberalization in general equilibrium, we calibrate both specifications of the model and analyze the response to a bilateral, unforeseen reduction

of tariffs.

5.2.1 Calibration

To calibrate our model we make use of the Colombian *Annual Survey of Manufactures* (ASM), which is described in detail in Roberts and Tybout (1997). This dataset covers all manufacturing plants with ten or more employees and provides data on items including sales, exports, input usage (employees, capital and energy), age, and subsidies at the plant level. Plants are classified by 3 digit SIC industry. There are 66,921 plant-year observations after removing observations that are inconsistently coded. A trade liberalization occurred in 1985-86 in Colombia, so we calibrate our model to the 1981-84 period. A detailed description of the reform can be found in the next section.

Table 1 lists the parameter values used. The parameters α (the Cobb-Douglas parameter), β (the discount factor), σ (the elasticity of substitution) and δ_k (capital depreciation) are set to standard values. The survival probability is set to match the average age of operating firms in the data set during the pre-liberalization period. We assume that the ex-ante productivity distribution Γ is distributed log-normal(0,s).

We calibrate the model with forward-looking and backward-looking constraints separately, with parameter values given in columns (a) and (b) in Table 1. We have six parameters to calibrate in each model: f_x , ω , s , ρ , ξ , and θ under the forward-looking specification and f_x , ω , ρ , a_0 , and θ in the backward-looking specification. They are set jointly to match six moments from Colombia in the years 1981-1984. These moments are: 1) the fraction of firms that export, 2) exports as a fraction of GDP, 3) the average difference in labor usage between exporters and non-exporters, 4) the average annual growth rate in labor usage before age 10, and 5) the fraction of firms that export before age 10. Age 10 was chosen because that is the first age for which the average growth rate of firms is 0%. The values of these moments in the model and data are given in Table 2. For comparison, we do a third calibration for the model without credit constraints shown in column (c) in Table 1. Here we only have four parameters to calibrate (f_x , ω , s , and ρ) and we match the first four listed moments.

5.2.2 Results

We consider the effects on the model economy of an unforeseen, bilateral reduction in tariffs from 50% to 13% (these are the Colombian manufacturing tariff rates before and after the reform from Attanasio et al (2004)). The results are reported in Table 3. We compare the steady state effects of a bilateral tariff reduction in the calibrated version of the model

under the forward-looking and backward-looking specifications, each compared to an economy with perfect credit markets. As reported in Table 3, we can see that the gains from trade in the forward-looking specification are very similar to those in the model with perfect credit markets with the same parameters (changes in consumption of 7.1% versus 7.2%). However, the gains are lower under the backward-looking specification compared to the perfect credit markets benchmark by more than a full percentage point (6.4% versus 7.5%).

We also compare the effects predicted by our calibrated model with credit market frictions to a calibrated model with perfect credit market (calibration (c)). Again, Table 3 shows us that the gains from trade with the forward-looking specification are similar to those with perfect credit markets, while the gains under the backward-looking specification are lower.

5.2.3 Sensitivity

We test the sensitivity of these findings to variation in parameter values. We vary the fixed cost f_x , the elasticity of substitution σ and the quality of financial markets θ .

The first panel of Figure 1 shows the results of varying the fixed cost to export from one third to three times its calibrated value in both specifications of the model. In this figure, we plot the percentage gains from the trade reform in the perfect credit markets model (horizontal axis) and the model with each specification of debt limits (vertical axis). We see that high values of the fixed cost does increase the gap between percentage gains under perfect credit market and financial frictions under each specification. However, the difference with the forward-looking constraint is at most 0.7 percentage points (7.7% versus 8.4%) when the fixed cost is three times its calibrated value. This effect is more dramatic for the backward-looking specification, where the difference is as large as 2 percentage points (6.8% versus 8.8%). The example in the next subsection will demonstrate the importance of the fixed cost in determining this difference.

The second panel of Figure 1 shows how the gains from trade change as the elasticity of substitution σ is varied. Here it ranges from 1.5 to 10. As σ is varied, the gains from trade for the same change in tariffs also varies widely. However, we can see that the same basic story holds: under the forward-looking specification the gains from trade are similar to those with perfect credit markets, while with the backward-looking specification they are lower. It is noteworthy that when σ is low (between 1.5 and 3) gains from trade are actually somewhat higher with the forward-looking constraint than with perfect credit markets (by at most 1 percentage point when $\sigma = 2.5$), so that the forward-looking and backward-looking specifications have opposite predictions about how the

gains from trade relate to the perfect credit markets case. This amplification disappears when the $\sigma = 2.5$ economy is calibrated to the moments described above.

Lastly, Figure 2 compares the gains predicted by the model as θ is varied. In both panels, the solid lines are pre-reform and post-reform levels of final output as θ is varied. The dotted line is the percentage increase in output from the perfect credit markets model applied to the pre-reform level in each specification. Therefore, if the model with debt constraints and the perfect credit markets model have exactly the same percentage increase in output, the dotted line and upper solid line would coincide. We see that this is the case for the forward-looking specification, but not for the backward-looking specification. In the backward-looking case, we can see that the gap between the two widens as θ is increased (worse credit markets).

5.2.4 Discussion and the role of the extensive margin

From these results we can then conclude that under the forward-looking specification, the difference in gains from trade is not significant confirming the results in our special case with no fixed cost. Furthermore, as the sensitivity analysis shows, the larger are fixed costs, the further from this result we get. However, we would need a counterfactually high fixed cost to make the difference significant (that is, one that implies a very small number of exporting firms).

Under the backward-looking specification gains are limited by the presence of financial frictions. Next we illustrate the mechanism behind this result. In particular, we show that the inability of young firms to borrow sufficiently to enter the export market is the key factor that lowers gains from trade, which is consistent with the findings in Caggese and Cunat (2013). We illustrate this directly with an example in which the fixed cost to export is zero and the model is calibrated under both specifications of the borrowing constraint¹⁷. The results are reported in Table 4. As we can see, with no extensive margin of trade the difference between the perfect credit markets model and both specifications of the borrowing constraint are small. That the percentage gains with the forward-looking specification and perfect credit markets are exactly the same is analytically true by Proposition 2.

The reason that the extensive margin is so important in the backward-looking specification is as follows. All firms start as non-exporters and must accumulate sufficient

¹⁷The calibration strategy is the same as before, except that we have one fewer parameter to calibrate (f_x), and two fewer moments (the size difference between exporters and non-exporters, and the percentage of young firms that export). Since the model is overidentified, we arbitrarily set ξ in the forward-looking specification and α_0 in the backward-looking specification to the values from the baseline calibration.

assets to be able to become exporters. Since trade reform makes non-exporters less profitable (because wages have increased), they accumulate assets more slowly after the reform than before. Under the backward-looking specification, this directly implies that they cannot borrow as much after the reform compared to before. Notice that this is not the true in the forward-looking case. There, the fact that the firm will be an exporter in the future allows it to borrow more from the beginning of its life. Therefore, whether or not young firms are able to become exporters is the key factor that determines how financial frictions affect gains from trade. This is highlighted in Figure 8, which shows the age at which firms enter export markets as a function of their productivity before and after reform. In the forward-looking environment, all exporters enter at younger ages, while in the backward-looking environment, firms take longer to become exporters.

We can then conclude that we have two possible answers to our original question: do financial frictions limit gains from trade reform? In the backward-looking model gains are less than in a perfect credit markets benchmark. However, in the limited enforcement model, the difference in gains is negligible. It is then important to distinguish between these two forms of debt limits. As discussed in Section 4, these models are difficult to distinguish using firm level data from a stationary environment because they have very similar implications for firms dynamics. In the next section we will argue that a trade reform provides a means of distinguishing them.

6 Distinguishing Between Credit Constraints

In this section, we provide a means of distinguishing between the forward-looking and backward-looking specifications of the debt limits using data from a trade reform. We will show that the experience of Colombia in the 1980s provides evidence in favor of the forward-looking specification.

6.1 Colombian Reform

We use data from the Colombian *Annual Survey of Manufacturers*, as detailed in the previous section. The major benefit of using this data is that there was a major period of reform in the middle of sample. Through the early 1980s, Colombia had increasingly high tariff rates and quotas (see Roberts (1996)). This trend reversed in 1985, when Colombia agreed to a Trade Policy and Export Diversification Loan from the World Bank. Tariffs were substantially reduced and trade subsequently increased (see Fernandes (2007)). Figure 3 shows large increases in exports at both the aggregate and firm level. Also, changes in

exchange rate policy led to a major real exchange rate depreciation (see Figure 4). Though not equivalent to a trade liberalization, a large real exchange rate devaluation has, for our purposes, the same effect of a reduction in tariff from the foreign country: an increase in the value of being an exporter compared to being a non-exporter.

6.2 Difference in Implications

As the previous section demonstrates, the important difference between the two specifications of debt limits is whether or not credit constraints restrict the ability of firms to become exporters following trade reform. In the backward-looking case, firms are only able to export once they have accumulated sufficient assets. Since the profitability of young, non-exporting firms is decreased after the reform, it takes longer to accumulate assets and, therefore, credit constraints diminish the extensive margin of exporting. This predicts that the incidence of export activity across firms will be shifted away from young firms (who are more credit constrained) and toward older firms (who are less credit constrained). Alternatively, under the forward-looking specification firms that will eventually export are able to borrow more from the beginning of their lifetimes, which allows them to become exporters. Furthermore, since the profitability of exporting has increased, firms may choose to become exporters earlier in their lives.

In the case of Colombia, we illustrate the export activity of firms by age in Figures 5 and 6. Figure 5 shows the fraction of firms that export conditional on age before and after the reform. As we can see, there is a very large increase in export activity among the youngest firms. In Figure 6, we create a cumulative distribution function of exporters aged 1 to 20 before and after the reform. Panel a shows this in the raw data. Panel b shows the results controlling for industry effects, year effects, and industry-specific age trends, where the industry-specific age trend is to control for the possibility that firms in some industries may enter export markets at earlier ages than in others.¹⁸ This shows that export activity increased by the most among young firms, independent of overall changes in export activity. Given the argument above and in the previous section, this provides support for the forward-looking case relative to the backward-looking case. We formalize this with simulations in the next subsection.

¹⁸The graph looks similar with and without these industry-specific age trends.

6.3 Small Open Economy Simulation

To precisely compare the predictions of both models with the outcome of the reform in Colombia, we now simulate the effects of a trade reform and real exchange rate depreciation in line with those experienced in Colombia. To do this, we reformulate the model as a small open economy for two reasons. First, during this period Colombia was very different from its trading partners and accounted for a small share of world trade, so that we think of the small open economy case as being more realistic than an economy with two identical countries. Second, the reforms that Colombia underwent were highly asymmetric in nature: reductions in import tariffs and real exchange rate devaluation. Considering a small open economy allows us to impose a real exchange rate decline by exogenously changing the price level in the rest of the world.

In the small open economy model¹⁹ intermediate goods are exported abroad, and the domestic country buys a single foreign intermediate good that has price P . The country is small in the sense that P is exogenous. We analyze the effect of the following reform: a simultaneous reduction of import tariffs from 50% to 13% (the average manufacturing tariff rates before and after reform from Attanasio et al. (2004)), and an increase in P . Since the domestic price level is numeraire, an increase in P is equivalent to a real exchange rate depreciation. In our baseline exercise, we match the observed 44% depreciation.

We calibrate the small open economy model to match the same moments as in the two country case. Additionally, we have the initial price of the imported good P , which we choose to match imports as a fraction of GDP. The calibrated values and targets are in Tables 5 and 6. Then we can compare the incidence of exporting across firms of different ages to what was observed in the data. The first panel of Figure 7 shows the results for the forward-looking case, and the second panel for the backward-looking case. This shows the pattern described above. In the forward-looking case, the increase in export activity is concentrated among the young, while in the backward-looking case it is concentrated among older firms. The two panels of Figure 8 show why this is true. Here, the horizontal axis is firm productivity, and the vertical axis is the age when a firm first becomes an exporter. In the forward-looking model, firms of every productivity level become exporters earlier in their lives after the reform than before. In the backward-looking case, just the opposite is true. Though more firms are exporters after the reform than before, young firms actually take longer to become exporters after the reform than before. This is completely at odds with the change in the pattern of export status observed among young firms in the Colombian reform.

¹⁹A formal description of the small open economy framework is in the appendix.

As is a common shortcoming of this class of trade models, the large real exchange rate depreciation generates a counterfactually large increase in exports as a fraction of GDP. As an alternative exercise, we consider the case where the increase in P is calibrated to match the change in exports as a fraction of GDP observed in Colombia before and after the reform. The results are given in both panels of Figure 9. As in the baseline case, the forward-looking case agrees with the predictions from the data. Furthermore, in this case, the shift in the composition of export activity is quantitatively similar to that observed in the data. However, again the backward-looking case has the opposite prediction. We take this as suggestive evidence in favor of the forward-looking specification of the credit constraint.

7 Conclusion

In this paper we show that the gains from a trade reform under credit market frictions are sensitive to the way credit market frictions are modeled. We consider two polar specifications for credit constraints, what we refer to as the forward-looking specification and the backward-looking or collateral constraint specification. We first show that these two models have importantly different predictions for the gains from undergoing a reform. We argue that the extensive margin is critical in generating such differences. We further show indirect evidence from a trade reform in Colombia provides evidence in favor of the forward-looking specification. Although the model and data are related to a trade reform, we believe these results are more widely applicable.

We interpret our results as demonstrating that models with fixed collateral constraints may be misleading when analyzing economies undergoing reform or structural change. While a collateral constraint may be a good approximation to an underlying financial market imperfection in a stationary economy, it fails to address the endogenous response of financial markets in economies undergoing change. This may be important in contexts other than trade reform.

In future work, we would like to consider the case where borrowers are privately informed about their types. We have assumed here that lenders set debt limits according to the publicly known type of the borrower. If the profitability of exporting was known only to the borrower, they may be able to misrepresent it to lenders, be able to borrow more than they otherwise would, then default on their debts. Debt contracts that take this possibility into account may respond very differently to reform than those considered here.

In this paper we abstracted from idiosyncratic productivity shocks to isolate the dif-

ference between these two financial environments. This abstraction is potentially important²⁰, but introducing them would require one to specify whether or not debt is contingent on realizations of individual productivity. Typically, models with collateral constraints have only non-contingent debt²¹, while models with limited enforcement usually have contingent debt. In future work we plan to explore this distinction.

Finally, we assumed that the nature of the credit market constraint is a fixed feature of the environment. The nature of the credit market frictions depends of external enforcement by government and institutions that determine the persistence of the punishment and the nature of the intermediaries. Both features are endogenous. In particular, a reform or a technological innovation may drive innovation in the credit sector. This endogenous feedback link may strengthen our conclusion that credit market frictions do not necessarily affect negatively the outcome of a reform.

8 References

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²⁰Midrigan and Xu (2013) show that volatility in firm-level productivity can have large effects on the ability of firms to overcome their collateral constraints through self-financing, though they provide evidence that productivity is very persistent in the micro data.

²¹Rampini and Viswanathan (2010) is a notable exception.

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9 Tables

Table 1. Parameters Value:

- (a) Calibration for the economy with forward-looking debt limits
 (b) Calibration for the economy with backward-looking debt limits
 (c) Calibration for the economy with perfect credit markets

Parameter	Symbol	(a)	(b)	(c)
Discount Factor	β	0.96	0.96	0.96
Cobb-Douglass Parameter	α	0.3	0.3	0.3
Capital Depreciation	δ_k	0.05	0.05	0.05
Elasticity of Substitution	σ	5	5	5
High Tariffs	τ_H	0.50	0.50	0.50
Low Tariffs	τ_L	0.13	0.13	0.13
Survival Probability	δ	0.07	0.07	0.07
Std of Productivity	s	0.39	0.41	0.40
Export fix cost	f_x	0.14	0.15	0.19
Home Bias	ω	0.50	0.50	0.50
% Firms that can Export	ρ	0.54	0.57	0.52
Enforcement Parameter	θ	1.71	0.37	-
Probability of Starting New Firm	ξ	0.96	-	-
Initial Assets	a_0	-	0.005	-

Table 2. Target Statistics: Data and Model

Target	Data	Model		
		(a)	(b)	(c)
Exports/GDP	11%	11%	11%	11%
% Firms Exporters	12%	12%	12%	12%
Average Exporter Size Difference	3.9	3.9	3.9	3.9
St. Deviation of Log(Employees)	1.1	1.1	1.1	1.1
Annual Firm Growth, 1 to 10 years	5%	5%	5%	-
% Firms Exporting, 1 to 10 years	8%	8%	8%	-

Data: IMF IFS and Colombian Annual Survey of Manufacturers using years 1981-1984.

Table 3. Steady State Comparison

	% Change in			
	c	y	w	$\Delta(x/y)$
Forward-Looking (a)	7.1	7.5	5.8	7.4
Perfect Credit Markets (a)	7.3	7.7	6.0	7.1
<i>Difference</i>	-2%	-2%	-3%	
Backward-Looking (b)	6.4	6.8	5.6	6.8
Perfect Credit Markets (b)	7.5	8.0	6.2	7.1
<i>Difference</i>	-16%	-15%	-10%	
Perfect Credit Markets (c)	7.3	7.9	5.9	7.5

$\Delta(x/y)$ is the change in exports as a fraction of output.

Table 4 Steady State Comparison, $f_x = 0$ case

	% Change in			
	c	y	w	$\Delta(x/y)$
Forward-Looking (a)	5.4	5.7	4.4	4.4
Perfect Credit Markets (a)	5.4	5.7	4.4	4.4
<i>Difference</i>	0%	0%	0%	
Backward-Looking (b)	5.3	5.5	4.2	4.1
Perfect Credit Markets (b)	5.5	5.8	4.4	4.3
<i>Difference</i>	-4%	-4%	-3%	
Perfect Credit Markets (c)	5.4	5.7	4.3	4.3

$\Delta(x/y)$ is the change in exports as a fraction of output.

Table 5 Small Open Economy, Target Statistics: Moments and Data

Target	Data			
	(a)	(b)	(c)	
Exports/GDP	11%	11%	11%	11%
% Firms Exporters	12%	12%	12%	12%
Imports/GDP	14%	14%	14%	14%
Average Exporter Size Difference	3.9	3.9	3.9	3.9
St. Deviation of Log(Employees)	1.1	1.1	1.1	1.1
Annual Firm Growth, 1 to 10 years	5%	5%	5%	-
% Firms Exporting, 1 to 10 years	8%	8%	8%	-

Data: IMF IFS and Colombian Annual Survey of Manufacturers using years 1981-1984.

Table 6 Small Open Economy Parameter Values

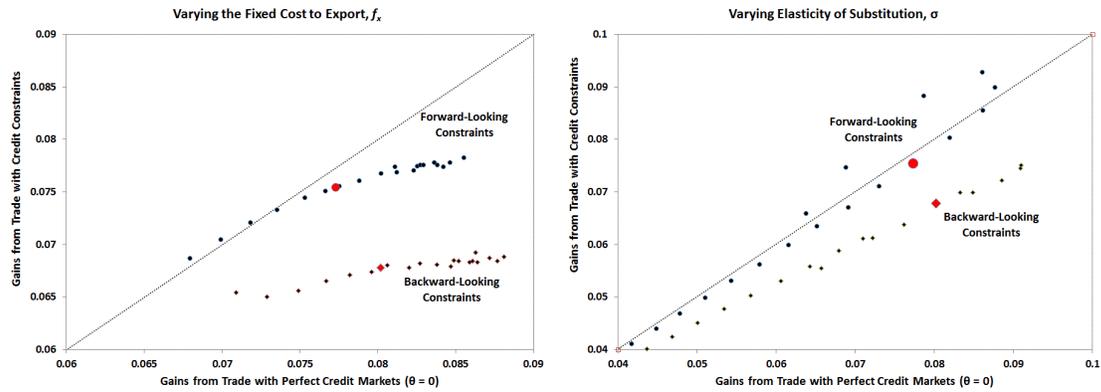
(a) Calibration for the economy with forward-looking debt limits

(b) Calibration for the economy with backward-looking debt limits

Parameter	Symbol	(a)	(b)
Discount Factor	β	0.96	0.96
Cobb-Douglas Parameter	α	0.3	0.3
Capital Depreciation	δ_k	0.05	0.05
Elasticity of Substitution	σ	5	5
Import Tariffs, Pre-Reform	τ_I	0.50	0.50
Import Tariffs, Post-Reform	τ'_I	0.13	0.13
Export Tariffs	τ_X	0.05	0.05
Survival Probability	δ	0.07	0.07
Foreign Price, Pre-Reform	P	0.90	0.91
Foreign Price, Post-Reform (i)	P'	1.58	1.60
Foreign Price, Post-Reform (ii)	P'	1.03	1.04
Std of Productivity	s	0.35	0.39
Export fix cost	f_x	1.84	1.42
Home Bias	ω	0.39	0.36
% Firms that can Export	ρ	0.53	0.49
Enforcement Parameter	θ	0.73	0.20
Probability of Starting New Firm	ξ	0.96	-
Initial Assets	a_0	-	0.03

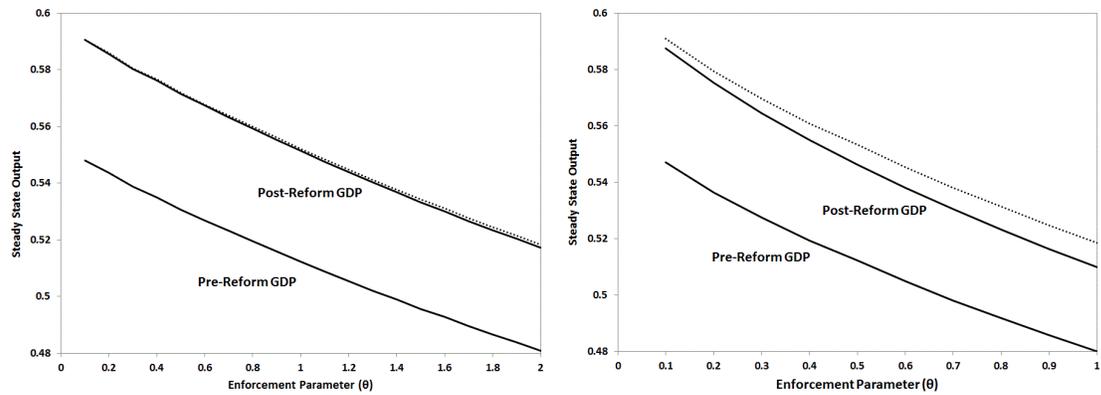
10 Figures

Figure 1. Gains from trade varying fixed costs and elasticity of substitution



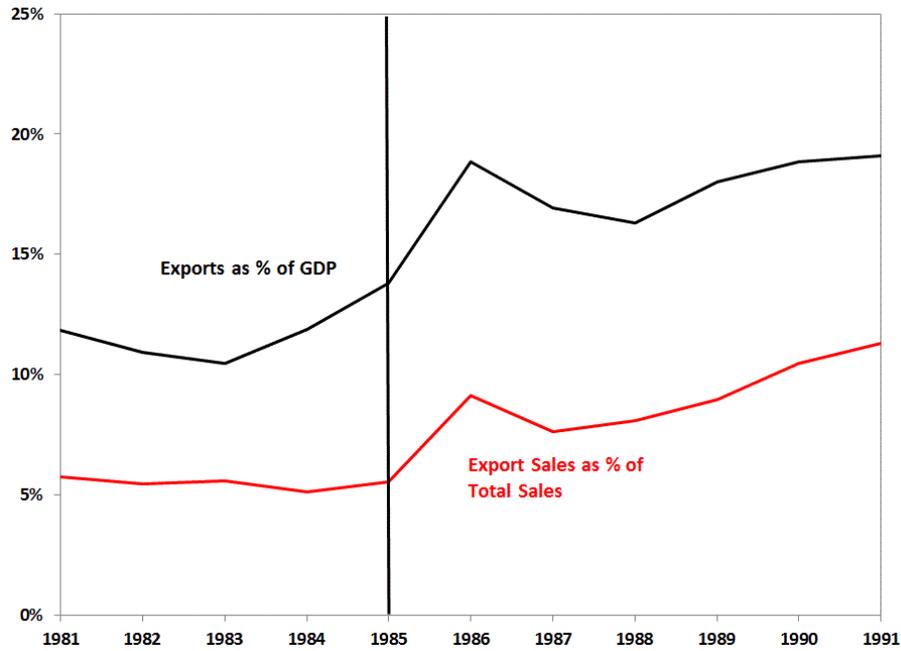
Percentage increase in steady state consumption from the decrease in tariffs in both specifications of borrowing constraints compared to the model with no credit constraints. Gains in the calibrated economies are emphasized.

Figure 2. Changes in Steady State Output, varying credit market quality



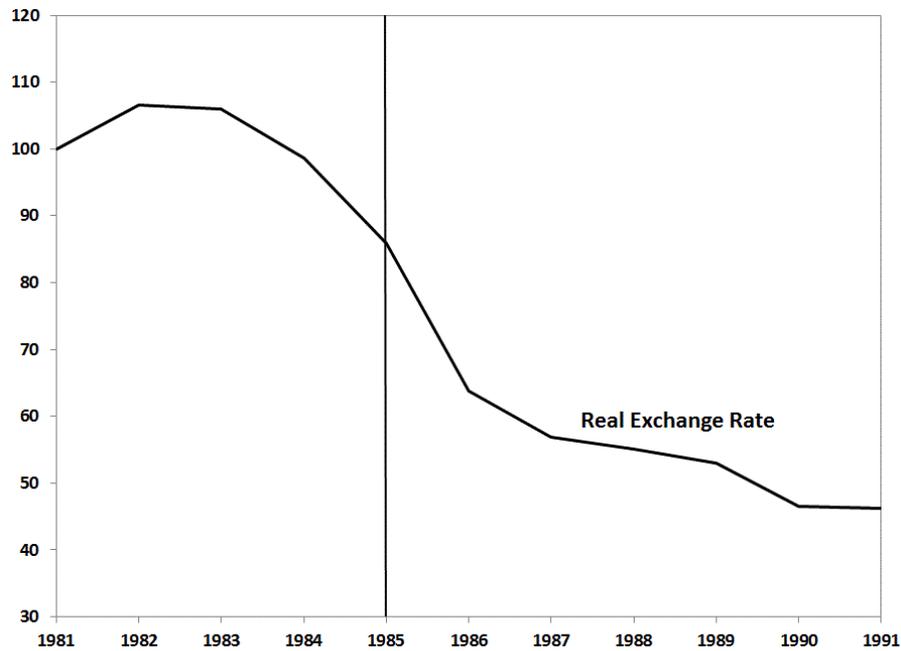
The left panel shows output before and after reform in the economy with forward-looking constraints, and the right panel shows the same with backward-looking constraints. The dotted line is the pre-reform level of output increased by the same percentage as in the model with perfect credit markets.

Figure 3. Evidence of Liberalization: Colombia 1981-1991 (Data)



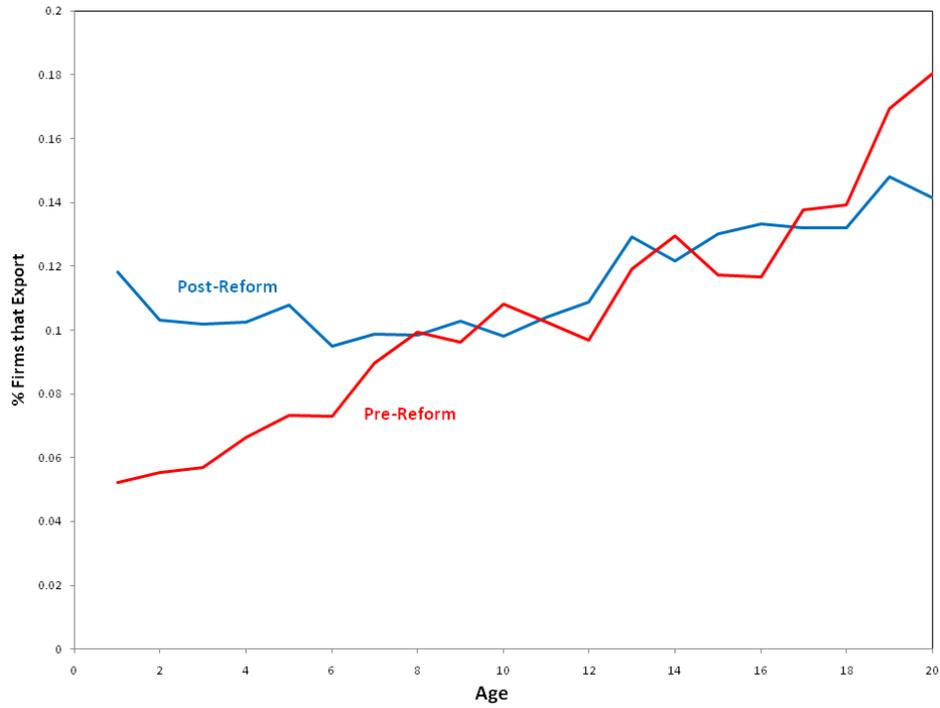
Sources: IMF IFS, and Colombian Annual Survey of Manufacturers

Figure 4. Real Exchange Rates (Data)



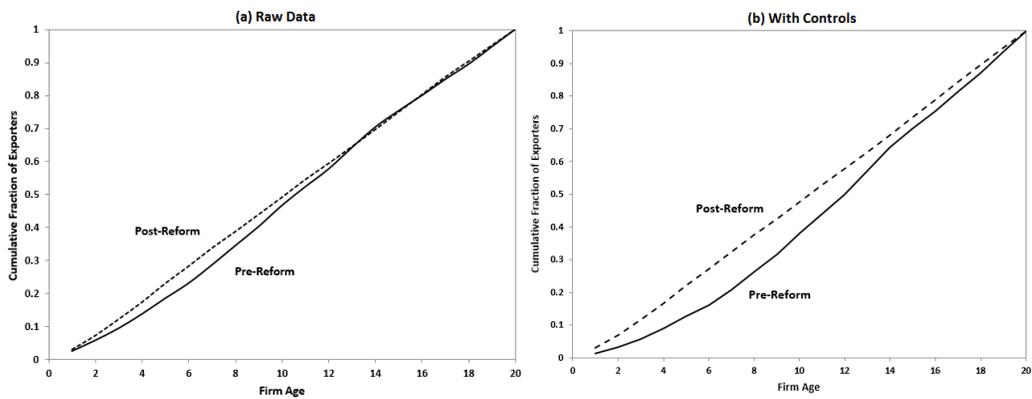
Source: IMF IFS. We normalize 1981 to 100.

Figure 5. Trade Reform: Extensive Margin for Young Firms (Data)



Source: Colombian Annual Survey of Manufacturers, 1981-1991

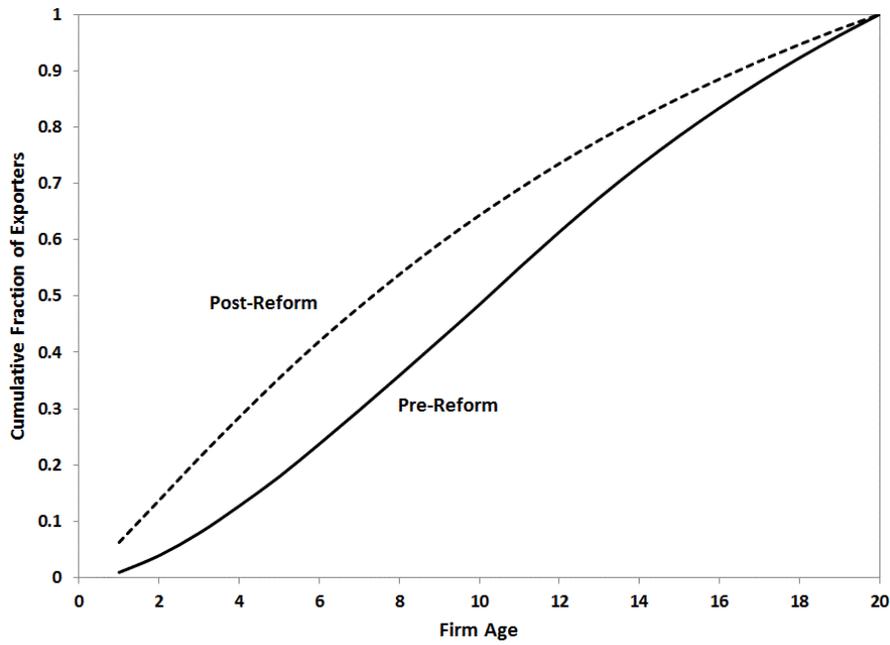
Figure 6 Cumulative Distribution Function of Age for Exporters (Data)



Source: Colombian Annual Survey of Manufacturers, 1981-1991

Figure 7 Cumulative Distribution Function of Age for Exporters (SOE Model, case (i))

Forward-looking debt limits:



Backward-looking debt limits:

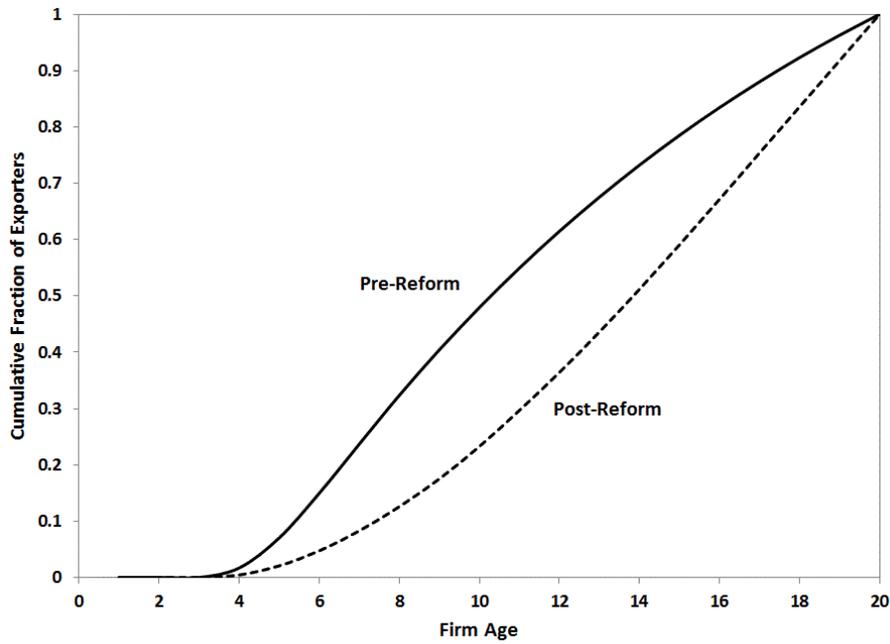
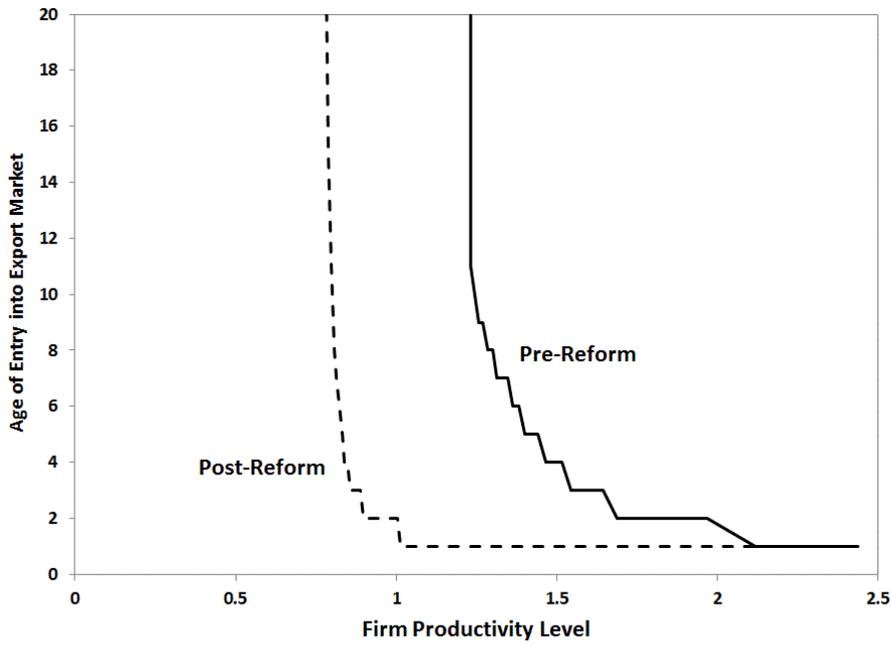


Figure 8 Age of Entering the Export Market by Productivity
(SOE model case (i))

Forward-looking debt limits:



Backward-looking debt limits:

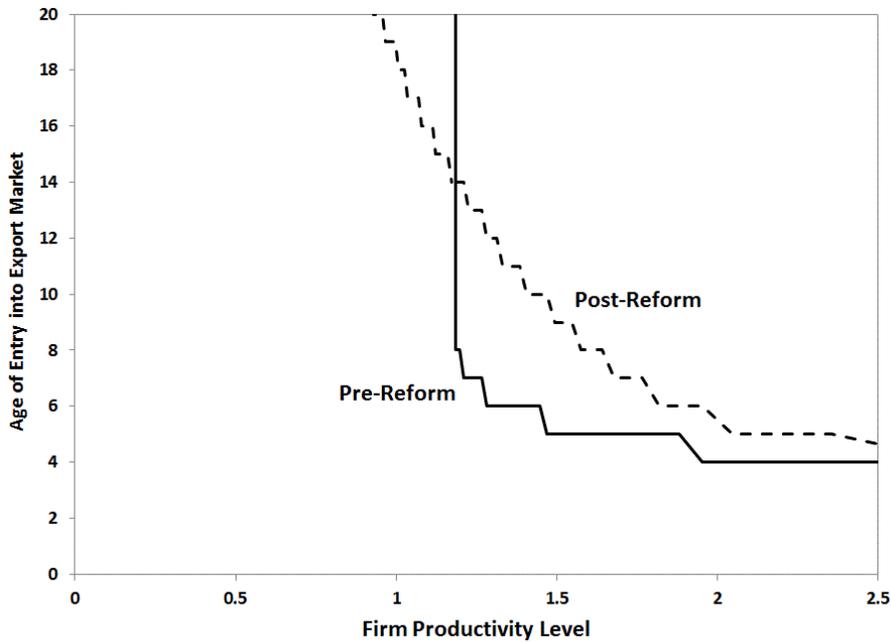
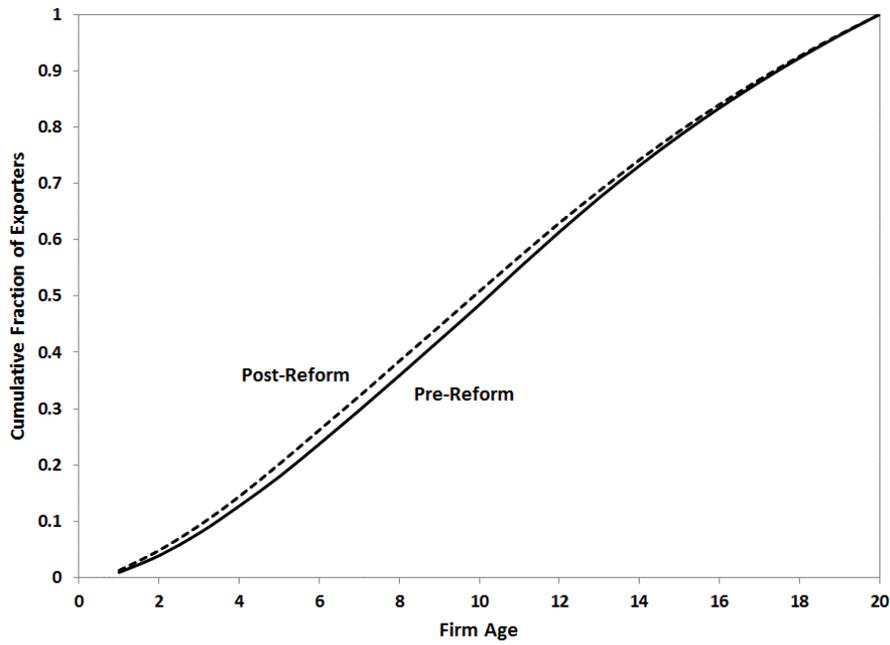
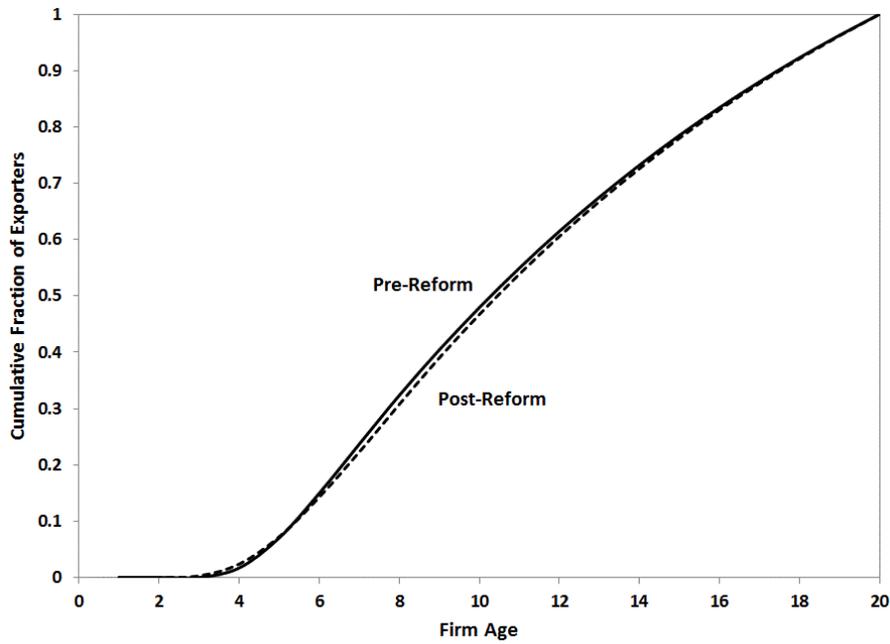


Figure 9 Cumulative Distribution Function of Age for Exporters (SOE model, case (ii))

Forward-looking debt limits:



Backward-looking debt limits:



A Appendix

A.1 Proof of Proposition 2

With $f_x = 0$, all firms with $\phi = 1$ always export. Let the aggregate state of the economy be $\mathbf{s} = (y, w, \tau)$ and let $D_0(\tau) = \omega$ and $D_1(\tau) = (\omega^\sigma + (\frac{1-\omega}{1+\tau})^\sigma)^{1/\sigma}$. Let $\Delta y, \Delta w, \Delta D$ and $\Delta(1+\tau)$ be defined by $\Delta x = x'/x$, where primes denote post-reform variables.

First we prove some properties for the economy with perfect credit markets. Recall that $k^*(z, \phi; \mathbf{s})$ and $l^*(z, \phi; \mathbf{s})$ are the solution to (25).

Lemma 3 $k^*(z, \phi; \mathbf{s})$ is homogeneous of degree 1 in y , degree σ in D , degree $\sigma - 1$ in z , and degree $(1 - \alpha)(1 - \sigma)$ in w ; $l^*(z, \phi; \mathbf{s})$ is homogeneous of degree 1 in y , degree σ in D , degree $\sigma - 1$ in z , and degree $(\alpha - 1)\sigma - \alpha$ in w .

Proof. Letting λ be the Lagrangian multiplier on the constraint, the first order conditions of the unconstrained firm imply:

$$k/l = \frac{\alpha}{1 - \alpha} \frac{w}{r}$$

and

$$\lambda = w \frac{1}{1 - \alpha} \frac{1}{z} \left(\frac{k}{l} \right)^{-\alpha} = w \frac{1}{1 - \alpha} \frac{1}{z} \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^{-\alpha} = \text{const} \times \frac{w^{1-\alpha} r^\alpha}{z}$$

Then, notice that

$$\begin{aligned} y_d &= (1 - 1/\sigma)^\sigma \omega^\sigma \lambda^{-\sigma} y \propto \omega^\sigma \left(\frac{w^{1-\alpha} r^\alpha}{z} \right)^{-\sigma} y \\ y_x &= \phi (1 - 1/\sigma)^\sigma \left(\frac{1 - \omega}{1 + \tau} \right)^\sigma \lambda^{-\sigma} y \propto \left(\frac{1 - \omega}{1 + \tau} \right)^\sigma \left(\frac{w^{1-\alpha} r^\alpha}{z} \right)^{-\sigma} y \end{aligned}$$

Therefore using the production function:

$$y = z \left(\frac{k}{l} \right)^\alpha \quad l = z \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^\alpha l$$

it follows that

$$(27) \quad l^*(z, \phi; \mathbf{s}) = y(z, \phi; \mathbf{s}) \left[z \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^\alpha \right]^{-1} \\ \propto z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha}$$

$$(28) \quad k^*(z, \phi; \mathbf{s}) \propto z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha+1} = z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)(\sigma-1)}$$

as wanted.

Consider now an economy with limited enforcement. Define

$$v_t(z, \phi; \mathbf{s}) = \sum_{s=0}^{\infty} (q(1-\delta))^s d_{t+s}(z, \phi; \varphi) = \frac{v_0(z, \phi; \varphi)}{(q(1-\delta))^t}$$

be the present value of future dividends for a firm of age t . The second equality comes from Proposition 1: whenever the borrowing constraint is binding there are no dividends paid. Hence, v_t grows at rate $1/(q(1-\delta))$. Let $k(v, z, \phi; \mathbf{s})$ and $l(v, z, \phi; \mathbf{s})$ be the solution to:

$$(29) \quad \pi(v, z, \phi; \mathbf{s}) = \max_{y_x, y_d, l, k} \omega y^{1/\sigma} y_d^{1-1/\sigma} + \phi \frac{(1-\omega)}{1+\tau} y^{1/\sigma} y_x^{1-1/\sigma} - \omega l - r k$$

subject to (26) and

$$(30) \quad q(1-\delta)v \geq \theta k + \xi v_0(z, \phi; \mathbf{s})$$

For v sufficiently high, the enforcement constraint is not binding and the firm operates at its optimal scale $k^*(z, \phi; \mathbf{s})$ and makes profits $\pi^*(z, \phi; \mathbf{s})$. Define $v^*(z, \phi; \mathbf{s}) = \theta k^* + \xi v_0$ as the smallest value of v needed to sustain optimal scale, and $T^*(z, \phi; \mathbf{s}) = \lceil \log(v_0/v^*) / \log(q(1-\delta)) \rceil$ as the number of periods it takes the firm to reach optimal scale.

Given that the financial sector makes zero expected profits, in equilibrium the initial value of the firm v_0 is the solution to:

$$v_0(z, \phi; \mathbf{s}) = \sum_{t=0}^{\infty} (q(1-\delta))^t \pi \left(\frac{v_0(z, \phi; \mathbf{s})}{(q(1-\delta))^t}, z, \phi; \mathbf{s} \right)$$

We now prove a series of Lemmas that we will use in the proof of the Proposition.

Lemma 4 $k(v, z, \phi; \mathbf{s})$ is given by

$$(31) \quad k(v, z, \phi; \mathbf{s}) = \min \left\{ k^*(z, \phi; \mathbf{s}), \frac{q(1-\delta)v - \xi v_0(z, \phi; \mathbf{s})}{\theta} \right\}$$

and the indirect profits function is given by

$$(32) \quad \pi(v, z, \phi; \mathbf{s}) = C \omega^{\frac{(\alpha-1)(\sigma-1)}{1+\alpha(\sigma-1)}} D_{\phi}^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} k(v, z, \phi; \mathbf{s})^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} - r k(v, z, \phi; \mathbf{s})$$

where C is a constant.

Proof. The solution for k is given by

$$(33) \quad k(v, z, \phi; \mathbf{s}) = \min \left\{ k^*(z, \phi; \mathbf{s}), \frac{q(1-\delta)v - \xi v_0(z, \phi; \mathbf{s})}{\theta} \right\}$$

Combining first order conditions, the solution to the problem is given by the solution to the following equations:

$$(34) \quad y_d = \omega^\sigma (1 - 1/\sigma)^\sigma y \lambda^{-\sigma} = \omega^\sigma (1 - 1/\sigma)^\sigma y \left[\frac{(1-\alpha)z}{w} \left(\frac{k}{l} \right)^\alpha \right]^\sigma$$

$$(35) \quad y_x = \left(\frac{1-\omega}{1+\tau} \right)^\sigma (1 - 1/\sigma)^\sigma y \lambda^{-\sigma} = \left(\frac{1-\omega}{1+\tau} \right)^\sigma (1 - 1/\sigma)^\sigma y \left[\frac{(1-\alpha)z}{w} \left(\frac{k}{l} \right)^\alpha \right]^\sigma$$

Then I can use the production function to solve for l :

$$(36) \quad l = \left(\frac{(1 - 1/\sigma)^\sigma \left[\omega^\sigma + \left(\frac{1-\omega}{1+\tau} \right)^\sigma \right] y \left[\frac{(1-\alpha)z}{w} \left(\frac{k}{l} \right)^\alpha \right]^\sigma}{zk^\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$= \text{const} \times D_\phi^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} w^{\frac{-\sigma}{1+\alpha(\sigma-1)}} k^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}}$$

Plugging these solutions back in the objective function, it follows that when the enforcement constraint is binding we have that:

$$(37) \quad \pi_\phi(v, z; \mathbf{s}) = D_\phi(\tau) y_d^{1/\sigma} \left[zk^\alpha l^{1-\alpha} \right]^{1-1/\sigma} - wl - rk$$

Plugging in the above yields

$$\pi_\phi(v, z; \mathbf{s}) = C w^{\frac{(\alpha-1)(\sigma-1)}{1+\alpha(\sigma-1)}} D_\phi^{\frac{\sigma}{1+\alpha(\sigma-1)}} y^{\frac{1}{1+\alpha(\sigma-1)}} z^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} k(v, z, \phi; \mathbf{s})^{\frac{\alpha(\sigma-1)}{1+\alpha(\sigma-1)}} - rk(v, z, \phi; \mathbf{s})$$

where C is a constant.

Lemma 5 *With perfect credit market we have that*

$$(38) \quad \Delta_{k,\phi} = \Delta_{D,\phi}^\sigma \Delta_y \Delta_w^{(\alpha-1)(\sigma-1)}$$

Proof. It follows directly from (28).

Lemma 6 $v_0(z, \phi; \mathbf{s})$ is homogeneous of degree $\sigma - 1$ in z , σ in D_ϕ , 1 in y , and $(\alpha - 1)(\sigma - 1)$

in w . That is, \exists a scalar \tilde{v}_0 such that $\forall(z, \phi, \mathbf{s})$

$$(39) \quad v_0(z, \phi; \mathbf{s}) = \tilde{v}_0 z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)(\sigma-1)}$$

Proof. We now proceed by guess and verify. Suppose that $v_0(z, \phi; \mathbf{s})$ takes the form in (39). Then, given the guess $v^*(w) = \theta \tilde{k} w^{(\alpha-1)(\sigma-1)} + \xi \tilde{v}_0 w^{(\alpha-1)(\sigma-1)} = \tilde{v}^* w^{(\alpha-1)(\sigma-1)}$. Hence it follows that

$$(40) \quad k(v, z, \phi; \mathbf{s}) = \min \left\{ k^*(z, \phi; \mathbf{s}), \frac{q(1-\delta)v - \xi v_0(z, \phi; \mathbf{s})}{\theta} \right\} \propto z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)(\sigma-1)}$$

Lastly, it can be shown that $\forall t \geq 0$

$$(41) \quad \pi(v_t(w); w) = \pi \left(\frac{\tilde{v}_0}{(q(1-\delta))^t}; 1 \right) w^{(\alpha-1)(\sigma-1)}$$

by combining (32) and (40). Thus, using (41) in the definition of v_0 it follows that $v_0(z, \phi; \mathbf{s})$ is homogeneous of degree $\sigma - 1$ in z , σ in D_ϕ , 1 in y , and $(\alpha - 1)(\sigma - 1)$ in w as wanted.

Thus, the above lemmas imply that if the path $\{k_t(z, \phi), b_t(z, \phi), d_t(z, \phi)\}_{t=0}^\infty$ for a firm of type (z, ϕ) with aggregate state \mathbf{s} , then $\{\Delta_k k_t(z, \phi), \Delta_k b_t(z, \phi), \Delta_k d_t(z, \phi)\}_{t=0}^\infty$ is optimal for the aggregate state $s' = (\Delta_y y, \Delta_w w, \tau')$. We are now left to show that labor and good market clear. We denote variables from the perfect credit markets environment with superscript PC and from the forward-looking environment FL. First we prove a lemma that shows that the financial friction induces a distortion across age, but not across productivity levels.

Lemma 7 *Suppose s^{PC} is an equilibrium in the perfect credit markets environment and s^{FL} is an equilibrium in the forward looking environmet. Then $\forall z, \phi$,*

$$l^{\text{PC}}(z, \phi; s^{\text{PC}}) = \delta \sum_t (1-\delta)^t l^{\text{FL}} \left(\frac{v_0(z, \phi)}{((1-\delta)q)^t}, z, \phi; s^{\text{FL}} \right)$$

$$y_d^{\text{PC}}(z, \phi; s^{\text{PC}})/y^{\text{PC}} = \delta \sum_t (1-\delta)^t y_d^{\text{FL}} \left(\frac{v_0(z, \phi)}{((1-\delta)q)^t}, z, \phi; s^{\text{FL}} \right) / y^{\text{FL}}$$

and

$$y_x^{\text{PC}}(z, \phi; s^{\text{PC}})/y^{\text{PC}} = \delta \sum_t (1-\delta)^t y_x^{\text{FL}} \left(\frac{v_0(z, \phi)}{((1-\delta)q)^t}, z, \phi; s^{\text{FL}} \right) / y^{\text{FL}}$$

Proof. We only show the case with l as the other cases are analogous. Combining (39)

and (36),

$$\forall t, z, \phi, l^{\text{FL}} \left(\frac{v_0(z, \phi)}{((1-\delta)q)^t}, z, \phi; \mathbf{s} \right) \propto z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha}$$

As above, we already know that

$$\forall z, \phi, l^{\text{PC}}(z, \phi; \mathbf{s}) \propto z^{\sigma-1} D_\phi^\sigma y w^{(\alpha-1)\sigma-\alpha}$$

Then the fact that labor markets clear in both cases implies

$$\begin{aligned} 1 &= \rho \int_Z l^{\text{PC}}(z, 0; s^{\text{PC}}) d\Gamma(z) + (1-\rho) \int_Z l^{\text{PC}}(z, 1; s^{\text{PC}}) d\Gamma(z) \\ 1 &= \delta \int_Z \sum_t (1-\delta)^t \left[\rho l^{\text{FL}} \left(\frac{v_0(z, 0)}{((1-\delta)q)^t}, z, 0; s^{\text{FL}} \right) + (1-\rho) l^{\text{FL}} \left(\frac{v_0(z, 1)}{((1-\delta)q)^t}, z, 1; s^{\text{FL}} \right) \right] d\Gamma(z) \end{aligned}$$

Using the above facts yields,

$$\begin{aligned} 1 &= l^{\text{PC}}(1, 0; s^{\text{PC}}) \left(\rho \int_Z z^{\sigma-1} d\Gamma(z) + \left(\frac{D_1}{D_0} \right)^\sigma (1-\rho) \int_Z z^{\sigma-1} d\Gamma(z) \right) \\ 1 &= \delta \sum_t (1-\delta)^t l^{\text{FL}} \left(\frac{v_0(1, 0)}{((1-\delta)q)^t}, 1, 0; s^{\text{FL}} \right) \left(\rho \int_Z z^{\sigma-1} d\Gamma(z) + \left(\frac{D_1}{D_0} \right)^\sigma (1-\rho) \int_Z z^{\sigma-1} d\Gamma(z) \right) \end{aligned}$$

Combining these equations and multiplying through by any value of $z^{\sigma-1}$ or D_ϕ yields the result.

Now we check the labor market clearing condition. Again, let $\mathbf{s} = (y, w, \tau)$. The post-reform labor market clearing condition is:

$$\begin{aligned} 1 &= \rho \int_Z l^{\text{PC}}(z, 0; s^{\text{PC}'}) d\Gamma(z) + (1-\rho) \int_Z l^{\text{PC}}(z, 1; s^{\text{PC}'}) d\Gamma(z) = \\ &= \frac{\Delta_y}{\Delta_w^{1+(1-\alpha)(\sigma-1)}} \left[\rho \int_Z l^{\text{PC}}(z, 0; s^{\text{PC}}) d\Gamma(z) + \Delta_D^\sigma (1-\rho) \int_Z l^{\text{PC}}(z, 1; s^{\text{PC}}) d\Gamma(z) \right] \end{aligned}$$

Applying the above lemma we get:

$$\begin{aligned} 1 &= \frac{\Delta_y \delta}{\Delta_w^{1+(1-\alpha)(\sigma-1)}} \times \\ &\times \int_Z \sum_t (1-\delta)^t \left[\rho l^{\text{FL}} \left(\frac{v_0(z, 0)}{((1-\delta)q)^t}, z, 0; s^{\text{FL}} \right) + \Delta_D^\sigma (1-\rho) l^{\text{FL}} \left(\frac{v_0(z, 1)}{((1-\delta)q)^t}, z, 1; s^{\text{FL}} \right) \right] d\Gamma(z) \end{aligned}$$

which implies

$$1 = \delta \int_Z \sum_t (1 - \delta)^t \left[\rho l^{\text{FL}} \left(\frac{v_0(z, 0)}{((1 - \delta)q)^t}, z, 0; s^{\text{FL}'} \right) + (1 - \rho) l^{\text{FL}} \left(\frac{v_0(z, 1)}{((1 - \delta)q)^t}, z, 1; s^{\text{FL}'} \right) \right] d\Gamma(z)$$

Hence, labor market clearing is satisfied for the forward-looking case. The perfect credit markets goods market clearing condition is equivalent to:

$$1 = \int_Z \left[\omega \left(\frac{y_d^{\text{PC}}(z, 0; s^{\text{PC}'})}{y^{\text{PC}}} \right)^\gamma + (1 - \omega) \rho \left(\frac{y_x^{\text{PC}}(z, 1; s^{\text{PC}'})}{y^{\text{PC}}} \right)^\gamma \right] d\Gamma(z)$$

which implies

$$1 = \omega \Delta_w^{(1-\sigma)(1-\alpha)} \int_Z \left(\frac{y_d^{\text{PC}}(z, 0; s^{\text{PC}'})}{y^{\text{PC}}} \right)^\gamma d\Gamma(z) + \\ + (1 - \omega) \Delta_w^{(1-\sigma)(1-\alpha)} \Delta_D^{\sigma-1} \rho \int_Z \left(\frac{y_x^{\text{PC}}(z, 1; s^{\text{PC}'})}{y^{\text{PC}}} \right)^\gamma d\Gamma(z)$$

Applying the above lemma yields:

$$1 = \omega \delta \Delta_w^{(1-\sigma)(1-\alpha)} \left[\int_Z \sum_t (1 - \delta)^t \left(\frac{y_d^{\text{FL}}(v_0(z, 0)/(q(1 - \delta))^t, z, \phi; s^{\text{FL}'})}{y^{\text{FL}}} \right)^\gamma d\Gamma(z) + \right. \\ \left. + \frac{1 - \omega}{\omega} \Delta_D^{\sigma-1} \rho \int_Z \sum_t (1 - \delta)^t \left(\frac{y_x^{\text{FL}}(v_0(z, 1)/(q(1 - \delta))^t, z, 1; s^{\text{FL}'})}{y^{\text{FL}}} \right)^\gamma d\Gamma(z) \right]$$

which implies

$$1 = \omega \delta \left[\int_Z \sum_t \delta^t \left(\frac{y_d^{\text{FL}}(v_0(z, 0)/(q(1 - \delta))^t, z, 0; s^{\text{FL}'})}{y^{\text{FL}'}} \right)^\gamma d\Gamma(z) + \right. \\ \left. + \frac{1 - \omega}{\omega} \rho \int_Z \sum_t (1 - \delta)^t \left(\frac{y_x^{\text{FL}}(v_0(z, 1)/(q(1 - \delta))^t, z, 1; s^{\text{FL}'})}{y^{\text{FL}'}} \right)^\gamma d\Gamma(z) \right]$$

which is goods market clearing in the forward-looking environment. Hence, the changes in prices from the perfect credit markets environment are also the equilibrium changes in prices in the forward-looking environment. This is equivalent to the statement of Proposition 2.

A.2 Small Open Economy Framework

The small open economy model differs from the two country framework in three ways.

First, there is an imported good M with price P that enters the production function of the domestic final good as follows:

$$y = \left[\omega \int_I y_d(i)^{\frac{\sigma-1}{\sigma}} di + (1 - \omega)M^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Second, we derive the inverse demand functions for domestic firms that sell abroad as the solution to the following problem of the final goods producer in the rest of the world:

$$\max_{Y, M, y_x} P^*Y - PM - (1 + \tau_x) \int_{I_x} p_x^f(i) y_x(i) di$$

subject to

$$Y = \left((1 - \mu) \int_{I_x} y_x(i)^{\frac{\sigma-1}{\sigma}} di + \mu M \right)^{\frac{\sigma}{\sigma-1}}$$

The solution to this implies an inverse demand function:

$$p_x^f(i) = \frac{1 - \mu}{1 + \tau_x} P^* Y^{1/\sigma} y_x(i)^{-1/\sigma}$$

The rest of the world is large, so domestic exports have no impact on the foreign price level. Formally, we consider parameter values for μ that satisfy $(1 - \mu) = Y^{-1/\sigma}$, then consider the limit as Y goes to infinity. Then the foreign price level converges to $P^* = P$ and the inverse demand function limits to

$$p_x^f(i) = \frac{P}{1 + \tau_x} y_x(i)^{-1/\sigma}$$

Lastly, as in the data, trade is not balanced. Therefore, we are assuming that bonds are internationally traded with a gross interest rate of $1/\beta$, and that the economy is initially endowed with a net foreign asset position such that their net payments on debt are exactly equal to domestic net exports.

Besides these three features, the model is exactly the same as the two country case.

B Online Appendix (Not for Publication)

B.1 Proof of Proposition 1

(i) Consider first a firm that already paid the fixed cost f_x . We can write the dynamic problem of the firm using cash-on-hand as the unique state variable as follows:

$$V^x(a, z) = \max_{d, k', b'} d + q(1 - \delta)V^x(a', z)$$

subject to

$$\begin{aligned} d + (1 - \delta)qa' &\leq \pi^x(a) \\ d &\geq 0 \end{aligned}$$

It can be shown that V^x is differentiable and concave in a and $V^x(a) \geq 1$ with $V^x(a) = 1$ for all $a \geq a^*$ and $V^x(a) > 1$ for $a < a^*$. Letting λ and η be the multiplier associated with the budget constraint and the non-negativity on dividends respectively, the focs for the problem are:

$$(42) \quad d : 0 = 1 - \lambda + \eta$$

$$(43) \quad a' : 0 = q(1 - \delta)\lambda - q(1 - \delta)V^x(a')$$

and the envelope condition:

$$V^{nx'}(a) = \lambda\pi^{x'}(a)$$

We want to show that if $a' < a^*$ then $\eta > 0$. Suppose for contradiction that $\eta = 0$. Then (42) implies that $\lambda = 1$ and in turn (43) implies that

$$1 - V^x(a') = 0$$

but $V^x(a) > 1$ if $a < a^*$ thus $1 - V^x(a') < 0$ yielding a contradiction. Then it must be that $\eta > 0$ and $d = 0$.

Consider a non-exporter now. If it is never optimal to export, the same logic we used for an exporter goes through (notice that in this case V^{nx} is concave and differentiable). Instead, if it will be optimal to export at some date, V^{nx} is not necessarily concave and differentiable everywhere. Letting T be the period in which a firm with initial cash on

hand a will start to export, we can write the problem as follows:

$$V^{nx}(a) = \max_{\{d_t, a_{t+1}\}_{t=0}^T} \sum_{t=0}^T \beta^t (1-\delta)^t d_t + \beta^{T+1} (1-\delta)^{T+1} V^x(a_{T+1})$$

subject to

$$\begin{aligned} d_t + q(1-\delta)a_{t+1} &\leq \pi^{nx}(a_t) \quad \text{for } t = 0, \dots, T-1 \\ d_T + q(1-\delta)a_{T+1} &\leq \pi^{nx}(a_T - f_x) - f_x \\ d_t &\geq 0 \end{aligned}$$

Let $\beta^t(1-\delta)^t \lambda_t$ and $\beta^t(1-\delta)^t \eta_t$ be the multiplier associated with the budget constraint and the non-negativity on dividends respectively, the focs for the problem are:

$$(44) \quad 0 = 1 - \lambda_t + \eta_t \quad \text{for } t = 0, \dots, T$$

$$(45) \quad 0 = \lambda_t - \lambda_{t+1} \pi'(a_{t+1}) \quad \text{for } t = 0, \dots, T-1$$

$$(46) \quad 0 = \lambda_T - V^{x'}(a_{T+1})$$

Starting at $T+1$, suppose that $a_{T+1} < a^*$ and for contradiction that $\eta_T = 0$. Then it must be that $\lambda_T = 1$. This and (46) imply that

$$1 - V^{x'}(a') = 0$$

but $V^{x'}(a_{T+1}) > 1$ thus $1 - V^{x'}(a') < 0$ yielding a contradiction. Hence $\eta_T > 0$ and $d_T = 0$. Now combine (44) at t and $t+1$ with (45) at t we obtain:

$$\eta_t = \lambda_t - 1 = \lambda_{t+1} \pi'(a_{t+1}) - 1 \geq \lambda_{t+1} - 1 = \eta_{t+1}$$

Thus, if $\eta_{t+1} > 0$ then $\eta_t > 0$. This in turn implies that as long as any borrowing constraint is binding in the future then there is no dividend distributions as wanted. When no borrowing constraint in the future are binding then the firm's optimal dividend policy is indeterminate. Thus, without loss of generality we can set $d = 0$ to characterize the firm's value and policy functions.

To prove the remaining parts of Proposition 1 we will consider the forward and backward looking case separately.

Forward-Looking Constraint In this case it is convenient to write the problem in (9) and (11) using their dual formulation. This can be thought of as an optimal contracting problem between the entrepreneur and competitive, risk-neutral financial intermediaries. Financial intermediaries offer the entrepreneur long-term contracts that specify production plans and the value of the dividends paid to the entrepreneur. It is then straightforward to write this problem recursively using the discounted sum of promised dividend payments v as well as the export status of the firm as state variables. Denote the value functions of the financial intermediaries as $W^{nx}(v, z, \phi)$ and $W^x(v, z, \phi)$. The problem of the financial intermediary can be written as:

$$W^x(v, z, \phi) = \max -rk + p_d(y_d)y_d + p_x(y_x)y_x - wl - d + q(1 - \delta)W^x(v', z, \phi)$$

subject to

$$\begin{aligned} y_d + y_x &\leq zk^\alpha l^{1-\alpha} \\ d + q(1 - \delta)v &= v \\ v &\geq \frac{\theta}{q(1 - \delta)}k + \frac{\xi}{q(1 - \delta)}v_0 \end{aligned}$$

and for a firm that has not paid the fixed cost already:

$$W^{ns}(v, z, \phi) = -rk + p_d(y_d)y_d - wl - d - xf_x + q(1 - \delta) [xW^x(v', z, \phi) + (1 - x)W^{nx}(v', z, \phi)]$$

subject to

$$\begin{aligned} y_d + y_x &\leq zk^\alpha l^{1-\alpha} \\ d + q(1 - \delta)v &= v \\ v &\geq \frac{\theta}{q(1 - \delta)}k + \frac{\xi}{q(1 - \delta)}v_0 \end{aligned}$$

For notational convenience define

$$\Pi^x(v, z) = \max_{y_d, y_x, l, k} [p_d(y_d)y_d + p_x(y_x)y_x - wl] - rk$$

subject to

$$y_d + y_x \leq zF(k, l)$$

$$v \geq \frac{\theta}{q(1-\delta)}k + \frac{\xi}{q(1-\delta)}v_0$$

and

$$\Pi^{nx}(v, z) = \max_{y_d, l, k} [p_d(y_d)y_d + p_x(y_x)y_x - wl] - rk$$

subject to

$$y_d \leq zF(k, l)$$

$$v \geq \frac{\theta}{q(1-\delta)}k + \frac{\xi}{q(1-\delta)}v_0$$

By part (i) we can set $d_t = 0$ without loss for all t and rewrite the intermediary's problem as follows:

$$W^{nx}(v, z, \phi) = \max\{\Pi^{nx}(v, z) + q(1-\delta)W^d\left(\frac{v}{q(1-\delta)}, z, \phi\right);$$

$$\Pi^x(v, z) - f_x + q(1-\delta)W^x\left(\frac{v}{q(1-\delta)}, z, \phi\right)\}$$

$$W^x(v, z, \phi) = \Pi^x(v, z) + q(1-\delta)W^x\left(\frac{v}{q(1-\delta)}, z\right)$$

Finally, the minimum equity value for the firm to operate at its efficient scale is given by:

$$v^*(z, \phi) \equiv \min\{\arg \max_v \{\max\{\Pi^{nx}(v, z), \phi\Pi^x(v, z)\}\}\}$$

A firm will eventually reach v^* , because $v_t = \frac{v_0}{((1-\delta)q)^t}$. Then, for $v' \geq v^*$ a domestic firm with inside equity value v' will start exporting iff

$$\frac{\Pi^{x*}(z)}{1 - (1-\delta)q} - \frac{\Pi^{nx*}(z)}{1 - (1-\delta)q} \geq f_x$$

as in a standard Melitz model. Since the LHS is strictly increasing in z , there exists a cut-off z_x s.t. the above condition holds for all $z \geq z_x$.

We now prove part (iii) and (iv). To this end consider

$$\begin{aligned}
W^x(v, z) - W^{nx}(v, z) &= \Pi^x(v, z) + q(1 - \delta)W^x\left(\frac{v}{q(1 - \delta)}, z\right) - \\
&\quad - \max\left\{\Pi^{nx}(v, z) + q(1 - \delta)W^{nx}\left(\frac{v}{q(1 - \delta)}, z\right); W^x(v, z) - f_x\right\} \\
&= \min\{\Pi^x(v, z) - \Pi^{nx}(v, z) + \\
&\quad + q(1 - \delta)\left(W^x\left(\frac{v}{q(1 - \delta)}, z\right) - W^{nx}\left(\frac{v}{q(1 - \delta)}, z\right)\right); f_x\} \\
&= \min\left\{\Delta\Pi(v, z) + q(1 - \delta)\left(W^x\left(\frac{v}{q(1 - \delta)}, z\right) - W^{nx}\left(\frac{v}{q(1 - \delta)}, z\right)\right); f_x\right\}
\end{aligned}$$

The following lemma shows that the value of becoming an exporter weakly increases with v .

Lemma 8 (a) $\forall z W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in v , and (b) $\forall v W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in z .

Proof. Define $T : C(\mathbb{R}_+ \times \mathbb{R}_+) \rightarrow C(\mathbb{R}_+ \times \mathbb{R}_+)$ as

$$Tf(v, z) = \min\left\{\Delta\Pi(v, z) + q(1 - \delta)f\left(\frac{v}{q(1 - \delta)}, z\right); f_x\right\}$$

where $C(\mathbb{R}_+ \times \mathbb{R}_+)$ is the space of continuous and bounded functions. T satisfies the Blackwell's sufficient conditions for a contraction mapping. Then T is a contraction, and $W^x - W^{nx}$ is its unique fixed point.

To prove (a), let $C'(\mathbb{R}_+ \times \mathbb{R}_+)$ be the set of continuous, bounded and weakly increasing function in their first argument. $C'(\mathbb{R}_+ \times \mathbb{R}_+)$ is a closed set, hence by Corollary 3.1 in Stokey, Lucas and Prescott (1989) it suffices to show that $\forall f \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ $Tf \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ to prove that $W^x - W^{nx}$ is increasing in its first argument. Fix z , let $f \in C'(\mathbb{R}_+ \times \mathbb{R}_+)$ and $v' > v$:

$$\begin{aligned}
Tf(v', z) &= \min\left\{\Delta\Pi(v', z) + q(1 - \delta)f\left(\frac{v'}{q(1 - \delta)}, z\right); f_x\right\} \\
&\geq \min\left\{\Delta\Pi(v, z) + q(1 - \delta)f\left(\frac{v}{q(1 - \delta)}, z\right); f_x\right\} = Tf(v, z)
\end{aligned}$$

as wanted, because $\Delta\Pi(v, z)$ is increasing in v , and f is weakly increasing by assumption. Then we established (a). The exact same argument can be used to prove (b) noticing that $\Delta\Pi(v, z)$ is increasing in z also.

Thus, if $z \leq z_x$ a firm will never export since for all v $W^x(v, z) - W^{nx}(v, z) \leq W^x(v^*, z) - W^{nx}(v^*, z) < f_x$. Vice versa, if $z \geq z_x$, then the firm will eventually export, proving (ii).

To prove (iii), notice that if $z \geq z_x$ the firm will eventually export, and the fact that $W^x(v, z) - W^{nx}(v, z)$ is increasing in v implies that there exists a unique threshold $\tilde{v}(z)$ such that a firm will export iff $v \geq \tilde{v}(z)$.

Lastly, we prove (iv) by showing that if $z' > z$ then $\tilde{v}(z')/v_0(z') \leq \tilde{v}(z)/v_0(z)$, implying $\tilde{T}(z') \leq \tilde{T}(z)$. Let $z' > z \geq z_x$. The fact that $W^{nx}(v, z)$ is strictly increasing in z for all v implies that $v_0(z') > v_0(z)$, since v_0 is such that $W^{nx}(v_0(z), z) = 0$. To prove the proposition it is sufficient to show that $\tilde{v}(z') < \tilde{v}(z)$. By the previous lemma $\forall v$ $W^x(v, z) - W^{nx}(v, z)$ is weakly increasing in z . Thus, if $W^x(\tilde{v}(z), z) - W^{nx}(\tilde{v}(z), z) = f_x$ then $W^x(\tilde{v}(z), z') - W^{nx}(\tilde{v}(z), z') \geq f_x$ since $z' > z$, therefore $\tilde{v}(z') \leq \tilde{v}(z)$ as wanted.

To relate this to the "cash on hand" formulation, notice that the cash on hand for a firm with value v is given by $W^i(v)$ for $i = x, nx$, which is a monotone relation in v . Hence, all statements about v are also true for a .

Backward-Looking Constraint Proof of part (iii):

Lemma 9 *Consider a restricted problem in which firms can only choose to either pay the fixed cost in the first or second Let $x(a, z) = 0$ be the decision to not export in the first period, and $x(a, z) = 1$ be the decision to export in the first period. Then $\exists \hat{a} : \forall a < \hat{a}, x(a, z) = 0$, and $\forall a \geq \hat{a}, x(a, z) = 1$.*

Proof. In this restricted problem, the fact that all firms must be exporters after the second period (and the fact that z does not change) implies that the objective of the firm is equivalent to maximizing third period assets. Then the decision to export today or tomorrow yields the following payouts:

If the firm exports today (here assuming all constraints are binding to simplify notation):

$$x(a, z) = 1 \implies q(1 - \delta)a_x = \pi^x \left(\frac{\pi^x(a - f_x, z)}{1 - (1 - \delta)q}, z \right)$$

and if they export the next period:

$$x(a, z) = 0 \implies q(1 - \delta)a_{nx} = \pi^x \left(\frac{\pi^{nx}(a, z)}{1 - (1 - \delta)q} - f_x, z \right)$$

The firm then chooses whichever is greater. Define $\Delta(a, z) \equiv q(1 - \delta)[a_x - a_{nx}]$. Let $F(a, z) \equiv \pi^x(a - f_x, z) - \pi^{nx}(a, z) + (1 - q(1 - \delta))f_x$. Note that $\text{sign}(F(a, z)) = \text{sign}(\Delta(a, z))$. Then any zero of the function F is also a zero of the function Δ . We can show that F is a

strictly increasing function of a :

$$F_1(a, z) \equiv \pi_1^x(a - f_x, z) - \pi_1^{nx}(a, z) > 0$$

which is true because π^x and π^{nx} are concave, and $\forall a, z, \pi_1^x(a, z) > \pi_1^{nx}(a, z)$.

Then notice that $F(f_x, z) < 0$ and (assuming that $z \geq z_x$) $F(a^*, z) > 0$. Therefore, $\exists \hat{a} \in [f_x, a^*]$ that has the cutoff properties described in the statement of the lemma.

To complete the proof, we demonstrate that the cutoff found in the restricted problem corresponds to the cutoff in the general problem.

First, consider firms with asset values $a < \hat{a}(z)$. Our claim is that the firm does not export with that level of assets. For contradiction, suppose that they did. Then, by the definition of \hat{a} given in the lemma, we know that the firm could generate strictly greater profits by, instead, delaying their decision to export by one period. Hence, exporting this period is not optimal.

Second, consider firms with asset values $a \geq \hat{a}(z)$. The next lemma shows that for these firms the restriction on the periods when they can export is not binding.

Lemma 10 *Suppose a firm prefers to export this period instead of one period in the future. Then the firm prefers to export this period rather than any period in the future.*

Proof. We prove this by induction. The base step is true by hypothesis. Let $a^k(t)$ be the asset level of a firm k periods in the future who chooses to enter the export market in period t .

Using the fact that the firm's objective is equivalent to maximizing their assets whenever they are constrained, to complete the proof we need only show that $a^{k+1}(k) < a^{k+1}(1) \implies a^{k+2}(k+1) < a^{k+2}(1)$. Notice that the fact that $a'(a, z)$ is increasing in a means that $a^{k+1}(k) < a^{k+1}(1) \implies a^{k+2}(k) < a^{k+2}(1)$, so it is sufficient to show that $a^{k+2}(k+1) < a^{k+2}(k)$. But this follows immediately from the previous lemma, the fact that $a \geq \hat{a}(z)$, and the fact that $a'(a, z)$ is increasing in a . This completes the proof.

Therefore, $\forall a \geq \hat{a}(z)$, the fact that they prefer to export this period rather than the following period implies that they prefer to export this period rather than wait until any other period. Therefore, $\hat{a}(z)$ is the threshold level of assets that determines export status.

Proof of part (iv):

Here we use the fact that $a'(a, z)$ is increasing in z and that $\hat{a}(z)$ is decreasing in z . The fact that $a'(a, z)$ is increasing in z follows immediately from the fact that $\pi^{nx}(a, z)$ is increasing in z . To prove that $\hat{a}(z)$ is decreasing in z we make use of the characterization in the proof to part (iii).

Recall that $\hat{a}(z)$ solves $F(\hat{a}(z), z) = 0$. Then the implicit function theorem implies:

$$\frac{d\hat{a}}{dz} = -\frac{[\pi_2^{nx}(a, z) - \pi_2^x(a - f_x, z)]}{[\pi_1^{nx}(a, z) - \pi_1^x(a - f_x, z)]} < 0$$

The sign follows from the fact that for $j \in \{nx, x\}$, π^j is concave in the first argument, $\pi_{21}^j > 0$, $\forall a, z$, $\pi_1^x(a, z) > \pi_1^{nx}(a, z)$ and $\pi_2^x(a, z) > \pi_2^{nx}(a, z)$.

Therefore, starting from assets a_0 , firms with higher productivity both have faster asset growth and a lower asset threshold to enter the export market. Hence, $T(z)$ is decreasing in z .

C Closed Economy

In this section we show that the implications of Proposition 2 are not peculiar to a trade model and they extend to a closed economy framework where there are firm-specific distortions along the lines of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

Environment. We consider a closed economy version of the monopolistic competitive setting considered above. The problem for the stan-in household is the same as in our trade economy. In particular, recall that they inelastically supply one unit of labor. The final consumption good is produced by competitive firms using a CES aggregator. The final good in the home country is produced using the following CES aggregator:

$$(47) \quad y_t = \left[\int_{I_t} y_t(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\sigma/(\sigma-1)}$$

where I_t is the set of active firms. One can then derive the inverse demand functions faced by producers for the intermediated good i :

$$(48) \quad p_t(y(i)) = \omega y_t^{\frac{1}{\sigma}} y(i)^{-\frac{1}{\sigma}}$$

A mass of monopolistic competitive intermediate goods producers are operated by entrepreneurs. In every period a mass $\delta \in (0, 1)$ of entrepreneurs is born. Each operates a firm and is endowed with a new variety of the intermediate good. Productivity z is drawn from a distribution Γ and it remains constant through time. The firm can produce its differentiated variety using the following constant returns to scale technology:

$$(49) \quad y = zF(k, l) = zk^{\alpha}l^{1-\alpha}, \quad \alpha \in (0, 1)$$

where l and k are the labor and capital employed by the firm, and y is total output produced. Every period the production technology owned by the firm becomes unproductive with probability δ .

The firms are subject to idiosyncratic policy distortions. As in Restuccia and Rogerson (2008), we let τ denote a firm-level tax rate. Its value is revealed once the firm draws its productivity z . We also assume that the value of this tax rate remains fixed for the duration of the time for which the establishment is in operation. The type of a firm is then (z, τ) . We assume that τ is draw from some probability distribution $P(\cdot|z)$. We allow for z and τ to be correlated. (In our trade model, a tariff may be thought of a negative tax on high productivity firms and a subsidy to low productivity firms). We further assume that eventual revenues of the subsidy are lump-sum rebated to the households (or viceversa they are taxed).

As before, the firm has to borrow to finance its operations each period. We consider a decentralization where firms have access to a rental market for capital. We denote the rental capital rate by r_t . Firms can save across periods in contingent securities that pay one unit of the final good next period conditional on the firm's survival. All firms start with a_0 units of the final good, which are transferred to them by the household. Firms are subject to debt limits and a non-negativity on dividend payouts:

$$(50) \quad b_t \leq \bar{B}_t(a_t, z, \tau)$$

$$(51) \quad d_t \geq 0$$

The firm's problem can be conveniently written recursively using assets or cash on hand, a , together with its productivity type (z, τ) as state variable:

$$(52) \quad V_t(a, z, \tau) = \max_{d, a'} d + (1 - \delta)q_t V_{t+1}(a', z, \tau)$$

subject to

$$d + (1 - \delta)q_t a' \leq \pi_t(a, z, \tau)$$

$$d \geq 0$$

where profits $\pi_t(a, z, \tau)$ are given by the following static problem:

$$(53) \quad \pi_t(a, z, \tau) = \max_{y, l, b, k} (1 - \tau)y_t^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} - w_t l - r_t k - b$$

subject to

$$\begin{aligned} y &\leq zF(k, l), \quad k \leq a + b \\ b' &\leq \bar{B}_t(a, z, \tau) \end{aligned}$$

We will denote the policy functions of the firms associated with the above problems as $\{d_t, a'_t, k_t, b_t, y_t, l_t\}_{t=0}^{\infty}$.

Equilibrium. To define an equilibrium for the economy we need to keep track of the evolution of the measure of operating firms over (a, z, τ) . Denote such measure by λ_t . The measure of non exporters evolves over time according to

$$(54) \quad \begin{aligned} \lambda_{t+1}(A, Z, T) &= (1 - \delta) \int 1 \{a'(a, z, \tau) \in A, z \in Z, \tau \in T\} d\lambda_t \\ &+ \delta \rho \int_Z 1 \{a_0 \in A, z \in Z, \tau \in T\} dP d\Gamma. \end{aligned}$$

Market clearing in the final good market requires that

$$(55) \quad y_t = c_t + K_{t+1} - (1 - \delta_k)K_t$$

Market clearing in the rental capital market requires that

$$(56) \quad K_t = \int k_t(a, z, \tau) d\lambda_t$$

The labor market feasibility is given by

$$(57) \quad 1 = \int l_t(a, z, \tau) d\lambda_t$$

For the bond market to clear, it must be that

$$(58) \quad b_t + A_t = K_t$$

where A_t is the aggregate amount of assets held by firms, $A_{t+1} = (1 - \delta) \int a'_t(a, z, \tau) d\lambda_t + \delta \int_Z a'_t(a_0, z, \tau) dP d\Gamma$.

We can then define a *symmetric equilibrium* for the economy in a way analogous to the one in text for the trade model.

Consider a stationary equilibrium for this economy so we can drop the dependence

on time. Under the forward looking specification of debt limits,

$$V(a, z, \tau) = \theta (\bar{B}(a, z, \tau) + a) + \xi V(0, z, \tau),$$

let aggregates be denoted by

$$Y = Y(\theta, \xi, P), \quad C = C(\theta, \xi, P), \quad K = K(\theta, \xi, P), \quad w = w(\theta, \xi, P)$$

where we let them depend on (θ, ξ) and P to emphasize the dependence of aggregates from idiosyncratic distortion and level of credit market frictions. A version of Proposition 2 holds in this environment:

Proposition 2'. *Under the forward-looking specification, for any change in distortions P , the steady state percentage changes in aggregate output and wages are independent of θ and ξ . Furthermore, firm-by-firm the percentage change in capital usage is independent of θ and ξ .*

The proof of this proposition mimicks is similar to the one of Proposition 2. Consider an economy with perfect credit markets that undergoes a reform that changes the idiosyncratic distortions from P to P' . Let $\Delta w^* = w'/w$ and $\Delta y = y'/y$ be the steady state change in wages and final output and $\Delta k^*(z, \tau, \tau')$ and $\Delta l^*(z, \tau, \tau')$ be the change in capital and labor inputs used by a firm of productivity z that faces taxes τ pre-reform and τ' post-reform. Note that since we are considering a steady state, the rental rate of capital must equal $r = 1/\beta + \delta$ both pre and post reform. Clearly, since

$$(59) \quad \frac{k}{l} = \frac{\alpha}{1-\alpha} \frac{w}{r}$$

it must be that

$$(60) \quad \frac{\Delta k^*(z, \tau, \tau')}{\Delta l^*(z, \tau, \tau')} = \Delta w^* \Rightarrow \Delta k^*(z, \tau, \tau') = \Delta w^* \Delta l^*(z, \tau, \tau').$$

Moreover, the optimal k^* must satisfy

$$(61) \quad \begin{aligned} k^* &= \left[\frac{\sigma-1}{\sigma} (1-\tau) \left(z \left(\frac{\alpha}{1-\alpha} \frac{w}{r} \right)^{\alpha-1} \right)^{\frac{\sigma-1}{\sigma}} \frac{\alpha}{r} \right]^{\sigma} y \\ &\Rightarrow \Delta k^*(z, \tau, \tau') = \left(\frac{1-\tau'}{1-\tau} \right)^{\sigma} \Delta w^{*(\alpha-1)(\sigma-1)} \Delta y^* \\ &\Rightarrow \Delta w^* = \left[\frac{\Delta k^*(z, \tau, \tau')}{\Delta y^*} \left(\frac{1-\tau}{1-\tau'} \right)^{\sigma} \right]^{\frac{1}{(\alpha-1)(\sigma-1)}} \end{aligned}$$

Consider now an economy with imperfect credit markets indexed by (θ, ξ) . We now argue that if $\{k(a, z, \tau), l(a, z, \tau)\}$ and w where part of the equilibrium for the economy post reform then $w' = \Delta w^* \times w$ and inputs usage for a firm (a, z, τ') is given by

$$\{k(a, z, \tau'), l(a, z, \tau')\} = \{\Delta k^*(z, \tau, \tau') \times k(a, z, \tau), \Delta l^*(z, \tau, \tau') \times l(a, z, \tau)\}$$

To see that this is the case, it is again convenient to work with the dual formulation of the firm's problem. Let $k(v, z, \phi; \mathbf{s})$ and $l(v, z, \phi; \mathbf{s})$ be the solution to:

$$(62) \quad \Pi(v, z, \tau; \mathbf{s}) = \max_{l, k} (1 - \tau) y_t^{\frac{1}{\sigma}} [z(1 - \tau)F(k, l)]^{\frac{\sigma-1}{\sigma}} - w(\mathbf{s})l - rk$$

subject to

$$(63) \quad q(1 - \delta)v \geq \theta k + \xi v_0(z, \tau; \mathbf{s})$$

Given that the financial sector makes zero expected profits, in equilibrium the initial value of the firm v_0 is the solution to:

$$v_0(z, \tau; \mathbf{s}) = \sum_{t=0}^{\infty} (q(1 - \delta))^t \Pi\left(\frac{v_0(z, \tau; \mathbf{s})}{(q(1 - \delta))^t}, z, \tau; \mathbf{s}\right)$$

since on path the evolution of v_t conditional on no exit is given by

$$v_t = \frac{v_0(z, \tau; \mathbf{s})}{(q(1 - \delta))^t}$$

for our conjecture to be true we just have to verify that v_0 goes up by a factor of $\Delta k^*(z, \tau, \tau')$, i.e.

$$\frac{v_0(z, \tau'; \mathbf{s}')}{v_0(z, \tau; \mathbf{s})} = \Delta k^*(z, \tau, \tau'; \mathbf{s}, \mathbf{s}')$$

To see that this is the case, consider Π evaluated at the conjectured new policies:

$$\begin{aligned}
\Pi' &= (1 - \tau') y'^{\frac{1}{\sigma}} [zF(k', l')]^{\frac{\sigma-1}{\sigma}} - w(s') l' - rk' \\
&= (1 - \tau') y'^{\frac{1}{\sigma}} \left[zk' \left(\frac{\alpha}{1 - \alpha} \frac{w'}{r} \right)^{\alpha-1} \right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha} k' \\
&= (1 - \tau') y'^{\frac{1}{\sigma}} \left[z \Delta k^* k \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \left[\frac{\Delta k^*}{\Delta y^*} \left(\frac{1 - \tau}{1 - \tau'} \right)^\sigma \right]^{\frac{1}{(\alpha-1)(\sigma-1)}} \right)^{\alpha-1} \right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha} k' \\
&= (1 - \tau') y'^{\frac{1}{\sigma}} \left[z \Delta k^* k \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^{\alpha-1} \left(\frac{\Delta k^*}{\Delta y^*} \right)^{\frac{1}{\sigma-1}} \left(\frac{1 - \tau}{1 - \tau'} \right)^{\frac{\sigma}{\sigma-1}} \right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha} \Delta k^* k \\
&= (1 - \tau') \left(\frac{y'}{\Delta y^*} \right)^{\frac{1}{\sigma}} \left[zk \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^{\alpha-1} \left(\frac{1 - \tau}{1 - \tau'} \right)^{\frac{\sigma}{\sigma-1}} \Delta k^* \frac{\sigma}{\sigma-1} \right]^{\frac{\sigma-1}{\sigma}} - \frac{r}{\alpha} \Delta k^* k \\
&= \left[(1 - \tau) y^{\frac{1}{\sigma}} z (1 - \tau') k \left(\frac{\alpha}{1 - \alpha} \frac{w}{r} \right)^{\alpha-1} - \frac{r}{\alpha} k \right] \Delta k^* = \Pi \Delta k^*
\end{aligned}$$

as wanted. Note that the first equality is the definition of Π' , in the second we used (59), in the third (61), the last steps are simple algebraic manipulations.

To verify this is indeed an equilibrium one is left to show that the market clearing conditions are satisfied. This follows from the fact that initial allocation clears market and so the allocations in the economy with perfect credit markets. The proof is identical to the one provided for Proposition 2.